DSM in Practice: Performance Results of Iterative Water-filling Implemented on ADSL Modems

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Abstract—New techniques emerge in the DSL world to increase the bit rate and deployment range of particular services. These techniques are better known as Dynamic Spectrum Management (DSM) applied at different levels. The increasing DSM levels can be seen as an evolution towards increasing coordination between multiple DSL lines: from level 1 and 2 (multi-user power allocation resulting in crosstalk avoidance) to level 3 (multi-user detection resulting in crosstalk mitigation). In this paper we focus on DSM at level 1 and in particular on a specific algorithm called iterative water-filling which has been implemented on ADSL modems. Measured performance results are given showing a big performance increase.

Index Terms—Digital Subscriber Line (DSL), Dynamic Spectrum Management (DSM), vectoring, multi-user power allocation, DSL Access Multiplexer (DSLAM), ADSL.

I. INTRODUCTION

Due to increasing line attenuation with increasing line length, the deployed services on long loops are low-bit-rate services. As each operator has a minimum-bit-rate service, customers on very long loops are not granted ADSL connections. Increasing the bit rate on long loops has thus a double effect, on the one hand it provides the customers on long loops with a higher bit rate, eventually enabling Video-over-DSL, and on the other hand it increases the operator’s DSL customer base. Indeed, increasing the bit rate on long loops is comparable to increasing the deployment range for a certain service.

When cable plant modifications are allowed, then the well known evolutionary scenario of remote deployment [1] comes into the picture, which means extending optical fibers up to a flexibility point such as a street cabinet (Fiber-To-The-Cabinet = FTTCab), where a subtending DSLAM, also called remote terminal (RT), is installed. This way the length of the copper twisted pair is substantially reduced and this allows extending the frequency band used for transmission by means of new DSL flavors such as ADSL2+ (up to 2.2 MHz) and VDSL (up to 12 MHz).

On the other hand, when no cable plant modifications are taken into account, a technology enabling an increase in bit rate on long loops is Dynamic Spectrum Management (DSM) [2]. DSM is an adaptive form of spectrum management [3] and is based on automatic detection and/or prevention of service faults caused by crosstalk, hence it increases the reliability of the DSL services. On top of that, DSM enables higher data rates. DSM gets its gains from more or less coordinating transmission on multiple DSL lines. Coordination between DSL lines provides gains, because the lines in the same cable binder are coupled by crosstalk. We remark that the naming “dynamic spectrum management” originates from adaptive multi-user power allocation techniques, but the meaning of the term DSM has widened to include also multi-user detection techniques.

In [2] a distinction is made between DSM at level 0, 1, 2, and 3 according to the degree of coordination. Level-0 DSM means no coordination between the lines. DSM at level 1 means that the bit rates and possibly the transmit powers and noise margins are reported to and controlled by a Spectrum Management Center (SMC). At level 2 the received signal and noise Power Spectral Densities (PSD) are reported to the SMC and the transmit PSDs are controlled by the SMC. At level 1 and 2 gains in rate/reach are originating from adaptive multi-user power allocation techniques, resulting in crosstalk avoidance. At level 1 the actual transmit PSDs are computed in each transceiver, hence the multi-user power control is distributed [4], whilst at level 2 the actual transmit PSDs are computed centrally in the SMC. Level 3 is the highest DSM level at which all co-located transceivers jointly process the received symbols for upstream transmission and the transmit symbols for downstream transmission. At level 3 gains are originating from multi-user detection techniques, resulting in either crosstalk cancellation for upstream transmission or crosstalk precompensation for downstream transmission. An overview of the future of DSL implementing DSM with increasing coordination levels and migrating to remotely deployed DSL is given in [6].
In this paper we concentrate on DSM at level 1, and in particular on the specific algorithm called iterative water-filling. First we recapitulate in Section II the DSL channel characteristics, followed by the multi-user power allocation technique (iterative water-filling) in Section III. In Section IV we give an overview of iterative water-filling practically implemented on ADSL modems together with measurement results. Finally we draw a conclusion in Section V.

II. THE DSL CHANNEL

A. Crosstalk noise

Individual wires carrying electric signals radiate electromagnetically. The telephone wires are therefore twisted together to reduce the electromagnetic coupling between pairs. Still, a small amount of signal leakage and, reversing, electromagnetic pick-up can be observed between pairs in the same binder. The Far-End crosstalk (FEXT), the most important crosstalk component in Frequency Division Duplex (FDD) systems, results from signals traveling in the same direction on the twisted pairs. This FEXT is seen as noise deteriorating the signal at the receiver.

The Discrete MultiTone (DMT) modulation is adopted in the ADSL standard [5] and is a frequency-domain modulation technique. The main benefits of DMT modulation can be found in, on the one hand, bit loading and, on the other hand, power loading.

B. DMT modulation

The Discrete MultiTone (DMT) modulation is adopted in the ADSL standard [5] and is a frequency-domain modulation technique. The main benefits of DMT modulation can be found in, on the one hand, bit loading and, on the other hand, power loading.

Bit loading is calculated on a per tone basis, as given by equation (1), and depends on the SNR at the receiver.

\[
b_i^j = \log_2 \left( 1 + \frac{\text{SNR}_i(k)}{\Gamma_i} \right) \\
= \log_2 \left( 1 + \frac{|H_{1i}(k)|^2 S_i(k)}{\Gamma_i (|H_{12}(k)|^2 S_i(k) + N_i(k))} \right) \tag{1}
\]

In equation (1) \( k \) represents the tone index, \( N_i(k) \) denotes all the noises different from self-crosstalk, and \( \Gamma_i \) is equal to the Shannon gap including noise margin and coding gain.

This bit loading allows the modem to adapt to the changing line conditions by dynamically varying the constellation used on each tone.

Power loading allows the modem to vary the power transmitted at each tone. This makes it possible for the modems to tune the SNR at the receiver and is thus closely related to the bit loading.

III. MULTI-USER POWER ALLOCATION

Equation (1) tells us that the bit loading for user 1 depends on the crosstalk coming from user 2. If the crosstalk increases on a particular carrier, fewer bits can be put on the carrier. The same is true for user 2, the crosstalk coming from user 1 interferes with the signal of user 2. The optimisation of the overall bit rate of both users, under a total power constraint on each user, can be performed by means of a cost function given by equation (2).

\[
\max_{S_1(k), S_2(k)} J(S_1(k), S_2(k)) \\
= \sum_k \log_2 \left( 1 + \frac{S_1(k) h_{11}^2(k)}{\Gamma_1 (S_1(k) + S_2(k) h_{12}^2(k))} \right) \\
+ \sum_k \log_2 \left( 1 + \frac{S_2(k) h_{22}^2(k)}{\Gamma_2 (S_1(k) + S_2(k) h_{12}^2(k))} \right) \\
+ \gamma_1 \cdot \left( P_1 - \sum_k S_1(k) \right) + \gamma_2 \cdot \left( P_2 - \sum_k S_2(k) \right) \tag{2}
\]

Equation (2) is the sum of the bit rates of both users together with the Lagrange multipliers taking into account the total power restriction of both users. Taking the derivatives of equation (2) to \( S_1(k) \) and \( S_2(k) \) and putting them equal to zero gives a quadratic solution for both \( S_1(k) \) and \( S_2(k) \). In case more users are involved, the solution becomes very complex and coordination between the lines is needed.

The objective is to obtain a simple equation where no coordination between the lines is needed. This can be achieved by introducing an approximation of much higher background noise than self-crosstalk noise. It results in
The water-filling equation can be rewritten as

\[
\max_{S_1(k), S_2(k)} J(S_1(k), S_2(k)) = \sum \log_2 \left( 1 + \frac{S_1(k) \cdot h_{11}^2(k)}{\Gamma_1 N_1(k)} \right) + \sum \log_2 \left( 1 + \frac{S_2(k) \cdot h_{22}^2(k)}{\Gamma_2 N_2(k)} \right) + \gamma_1 \left( P_1 - \sum S_1(k) \right) + \gamma_2 \left( P_2 - \sum S_2(k) \right)
\]

Taking the derivative of the cost function with respect to \( S_1(k) \) for user 1 and making it equal to zero gives us the optimum given by (4).

\[
\frac{\partial J}{\partial S_1(k)} = 0 = \frac{1}{S_1(k) + \frac{\Gamma_1 N_1(k)}{h_{11}^2(k)}} - \lambda_1
\]

Rewriting equation (4) results in the water-filling equation for user 1 as indicated by (5).

\[
S_1(k) = \frac{1}{\lambda_1} - \frac{\Gamma_1 N_1(k)}{h_{11}^2(k)}
\]

Therefore, after replacing the background noise with the total noise in equation (5), the optimal response of a modem to another modem interfering with it, under total power constraint, is known as the water-filling power allocation. Indeed, no coordination is applied between the lines and the noise measured by one modem is the total noise: background noise plus interference. Iterative water-filling converges to the Nash equilibrium where each modem’s power allocation is the optimal response to the other modem’s power allocation [4]. The water-filling equation can be rewritten as indicated by (6).

\[
S_1(k) = K_1 - \frac{\Gamma_1 \tilde{N}_1(k)}{h_{11}^2(k)}
\]

\( K_1 \) denotes the water-filling level and \( \tilde{N}_1(k) = N_1(k) + |H_{12}(k)|^2 S_2(k) \), i.e. the background noise plus interference, which equals the total noise.

Looking to Equation (1), to have one bit on a carrier, the SNR must be at least as big as \( \Gamma_1 \), which can be approximated by \( \Gamma_1 = 12 \text{ dB} \). Indeed, the Shannon gap to achieve a Bit Error Rate (BER) of \( 10^{-7} \) is approximately equal to 9.75 dB. Adding to this a noise margin of 6 dB minus a coding gain of 3.75 dB, one gets an overall value of 12 dB for \( \Gamma_1 \), which results in (7).

\[
S_1(k) \geq \Gamma_1 \frac{\tilde{N}_1(k)}{h_{11}^2(k)} = 16 \frac{\tilde{N}_1(k)}{h_{11}^2(k)}
\]

The signal is thus, on the usable tones, at least 16 times bigger than the Noise-to-Channel ratio. Taking this into account together with equation (6), the transmit PSD can be approximated by (8).

\[
S_1(k) = K_1
\]
IV. ITERATIVE WATER-FILLING IN PRACTICE

Fig. 2 shows the DSM (level 1) demonstrator at Alcatel Research & Innovation labs, which has provided the results shown in Fig. 3, Fig. 4 and Fig. 5. The demonstrator allows switching from normal mode to DSM mode. In DSM mode some modem parameters are switched to ensure PA (=FM) operation and in addition the DSL transceivers switch from a normal modem software build to a DSM modem software build. Some changes have been made to the modem software to allow iterative water-filling.

Fig. 2. DSM demonstrator at Alcatel Research & Innovation labs consisting of an Alcatel ASAM 7300 (high density) with a 12-lines ADSL LT board of which 4 lines are connected to 4 Thomson ADSL NT’s (SpeedTouch Home) by means of a 4-quad France Telecom 0.4 mm cable. There are 2 long lines of 5000 m and 2 short lines of 2000 m.

Fig. 3. Downstream ADSL transmit power spectral density (PSD) (solid) of the ATU-C transmitting over the 2000 m (short) loop, which results from water-filling the noise-to-channel ratio (dotted) and has a nominal (average) PSD (dashed): without DSM and with DSM illustrating power back-off.

The changes in the software consist of, in the first place, to allow boosting above –40dBm/Hz and enhanced transmit power reduction to as low as –60 dBm/Hz. A second topic of software changes concerns the sync symbols in showtime. Once in showtime, the modems react to upcoming and disappearing noises coming from neighbouring lines. In case a modem starts up with a high noise due to many disturbers, the transmit PSD will be calculated to achieve the needed SNR to attain the target bit rate. If the noise decreases due to neighbouring lines stopping transmitting power, the modem will automatically decrease its transmit PSD as the SNR is higher than needed. The transmit PSD of the sync symbols has to be low enough compared to the transmit PSD of the data symbols to avoid Inter-Symbol Interference (ISI) from the sync symbols into the data symbols. This can be either achieved by ensuring a low transmit PSD of the sync symbols, either by adapting the transmit PSD of the sync symbols equal to the data symbol transmit PSD variation.

Fig. 3 shows how iterative water-filling boils down to flat power back-off on a short loop, while maintaining a bit rate of 4 Mbit/s and a noise margin of 6 dB.

Fig. 4 shows how IWF boils down to boosting on a long loop. Without DSM only an average of 208 kbit/s is achieved. With DSM not less than 1280 kbit/s is reached on average over the two lines, which is an increase of over 500%!

Fig. 4. Downstream ADSL transmit power spectral density (PSD) (solid) of the ATU-C transmitting over the 5000 m (long) loop, which results from water-filling the noise-to-channel ratio (dotted) and has a nominal (average) PSD (dashed): without DSM and with DSM illustrating boosting.

Fig. 5 depicts the rate region for the short and long line with and without DSM. It is clear that DSM allows extending the rate region substantially.

Fig. 5. Rate region for the short and long loop scenario: without DSM (dotted, circles) and with DSM (solid, plusses).
Alternatively DSM allows to extend the reach of the 4 Mbit/s service on the long loop from 3100 m to 3650 m, while maintaining the length of the short loop at 2000 m. This is an increase of more than 17%. Remark that these results are given here only to give an idea of the potential of DSM. The results achievable in the field will depend on the noise environment and the loop length distribution.

Although these results look very promising, IWF also has a number of drawbacks. Firstly as shown in Fig. 5 iterative water-filling means boosting on the long loops. Boosting implies breaking the spectral mask constraints, hence spectral compatibility with other services is not assured, not according to method A of the spectrum management standard [3] anyway. Fortunately also an alternative analytical method B has been standardised [3]. Method B ensures spectral compatibility of a new technology not by imposing a spectral mask, but by ensuring that the new technology does not harm the specified basis systems by computing its impact on, for example, the bit rate of these basis systems. Hence spectral compatibility of the transmit PSDs proposed by IWF will have to be checked by means of method B.

A second important drawback of IWF is illustrated in Fig. 6. This figure shows that DSM reduces the noise margin on the short line significantly, which means that, if, for example, a new DSL line is activated, the short line could go out of sync. Without special precautions DSM could introduce stability problems. So before DSM at level 1 can be introduced in the field, a more in-depth study on spectral compatibility and stability is required.

![SNR Graph](image)

**Fig. 6.** SNR (Signal-to-Noise Ratio) of the ATU-R1 transmitting over the 2000 m (short) loop: SNR required for bit loading (solid) (with and without DSM), SNR including fixed noise margin of 6 dB (with DSM), SNR including maximal noise margin (without DSM).

**V. CONCLUSION**

In this paper the practical implementation of iterative water-filling is investigated together with its performance on the cable farm at Alcatel’s Research & Innovation lab. Taking into account the properties of the DSL channel together with the DMT modulation characteristics, the overall binder bit rate can be increased by applying multi-user power allocation techniques. These techniques can be categorised as Dynamic Spectrum Management (DSM) at level 0, 1, or 2 according to the level of coordination between the lines. (Level-3 DSM is based on multi-user detection)

We focused on DSM at level 1 and we recapitulated how water-filling results from a binder bit rate optimisation by introducing an approximation on the noise. Applying water-filling to DMT systems makes it possible to approximate it by flat transmit spectra decreasing consequently the complexity of the power allocation algorithm. Applying water-filling iteratively on the different modems results in the Nash equilibrium. Iterative water-filling is then considered as DSM at level 1 as no coordination between the lines is needed.

The performance results of iterative water-filling implemented on ADSL modems show a significant performance increase compared to the actual deployment mode used by the operators (Margin Adaptive mode). Both bit rate and reach are increased and shown with measurement results. The rate-region is plotted for a particular deployment case showing the huge advantages of iterative water-filling.

Finally some drawbacks of iterative water-filling are pinpointed as to be studied carefully to come up with spectral compatible and stable implementations.

**REFERENCES**


