

ESTABLISHMENT OF CURRENT IN CONDUCTORS

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Abstract

Currents are established on the surface of conductors by the propagation of electromagnetic waves in the insulating material between them. If the load is less than the characteristic impedance of the line, multiple reflections and retransmissions eventually build up the line current to that required by the load. The currents are initially established on the surface of the conductors before diffusing relatively slowly into the interior. These transmission and diffusion processes are fundamental to basic electrical engineering science, and their effects can be observed by applying a step input to a loaded line, and observing the load and mid-line voltages. The mid-line voltage clearly shows how the diffusion effects the propagation and reveals that the initial build up of surface current causes cables to enter a diffusion limited mode where currents have to diffuse into the interior before further propagation can take place to replenish surface currents.

1. INTRODUCTION

The purpose of this work is to show that Ohms law ($I=V/Z$) relies on electromagnetic propagation [1,2] and diffusion to establish current changes in electrical systems. The well-known transmission line process is considered from an electrical science approach, dealing with magnetic and electrostatic energies, and giving displacement current a more prominent role. The same propagation and diffusion processes are also necessary to establish flux in magnetic circuits, and this work on establishing currents is seen as a desirable precursor to that.

In cables the 'stray capacitance' of a line is sometimes seen as an undesirable effect that limits the frequency of operation. However, it is displacement currents in this capacitance that sets up the line current on the surface of the conductor, by transmission line action, at velocities approaching the speed of light, and it is essential for current flow. Once set up on the surface of the conductors by propagation in the insulating medium, the current then diffuses relatively slowly into the interior, driven by the longitudinal voltage drop along the surface of the conductors (ie. current concentration gradient). The diffusion velocity depends the conductivity, permeability, thickness of the conductor, and the frequency of the excitation.

Direct currents eventually get uniformly distributed throughout the cross section of the conductor, while alternating currents only have time to penetrate to the skin depth before flowing out again and reversing direction.

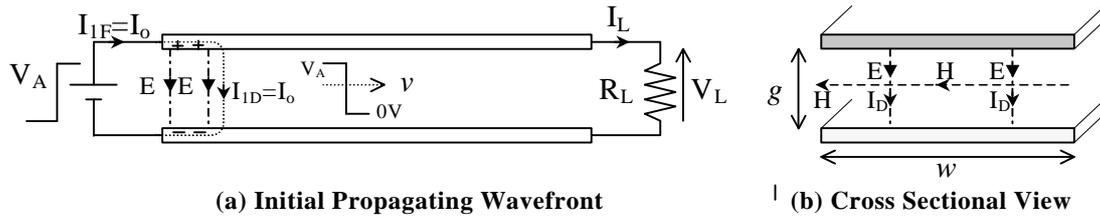
A similar situation exists in the case of magnetic circuits, where 'leakage flux' is often considered undesirable. However, this is not the case since the magnetic flux is initially set up on the surface of the magnetic core/laminations by multiple reflections of the leakage flux, before diffusing into the interior.

2. ESTABLISHING SURFACE CURRENTS

In electrical engineering transmission-line propagation is traditionally considered to be a distributed process in which an applied voltage progressively charges up the line capacitance, though the series inductance of the line, as it moves towards the load. Very little is normally stated about displacement current, surface currents, or diffusion. However, an alternative approach, based on the physical processes involved, is taken, that puts a more direct emphasis on the electrostatic and magnetic energies of the line, and gives the displacement current a more prominent role. It is the transverse displacement current that establishes the magnetic field at the wavefront, not the longitudinal conduction current, as implied in the more traditional approach. The longitudinal conduction current simply maintains the magnetic field behind the propagating wavefront.

2.1 Initial Propagation Towards Load

Consider a 2-core rectangular copper cable in a loss-less insulating medium with capacitance C F/m, and inductance L H/m, terminated with a load resistance R_L as indicated in Fig 1.



Transverse Electric Field, $E \approx \frac{V_A}{g}$ V/m, neglecting fringing.

Displacement Current Density, $J_D = \frac{\partial D}{\partial t} = \epsilon \frac{\partial E}{\partial t}$ A/m²

Displacement Current at Wavefront, $I_{1D} = \int_s J_D \cdot ds = I_o = \frac{V_A}{Z_o}$ A

Magnetic Field at Wavefront, $Curl H = J_D, \therefore H \approx \frac{I_{1D}}{w}$ A/m,

Flux Density at Wavefront, $B = \mu H$ Wb/m²

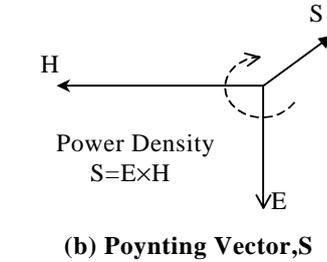
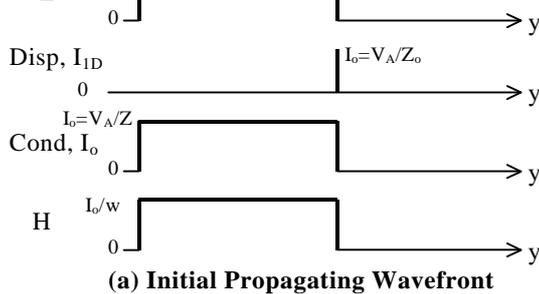
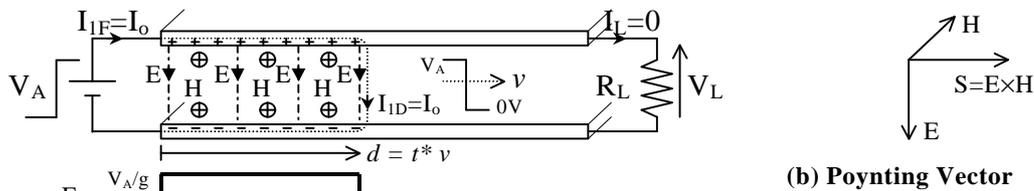


Fig. 1 Initial Wave Propagation Down Transmission Line

The characteristic field impedance of the cable, $Z_f = Z_o = \sqrt{L/C}$, and the velocity of propagation, $v = 1/\sqrt{LC}$. When a potential V_A is suddenly applied across the input of the cable, transmission line propagation and conductor diffusion processes are initiated, that finally result in a steady-state current, that is uniformly distributed throughout the whole cross section of the conductors. The initial voltage step sets up a transverse electric field E , with a displacement current $I_{1D} = CV_A v = V_A/Z_o$, and its \rightarrow

associated H field, in the medium separating the two conductors. This electromagnetic (E & H) field propagates down the line at a velocity v , 'pulling' along the initial line conduction current $I_o = I_{1D}$. This longitudinal line conduction current takes the path of least energy and initially flows along the surface of the conductor, before beginning to diffuse relatively slowly into the interior of the conductor, by means of the small longitudinal ohmic voltage drop along its length. The initial wavefront has voltage V_A , and displacement current $I_{1D} = I_o = V_A/Z_o$, as shown in Fig. 2.



Displacement Current, $I_{1D} = I_o = V_A/Z_o$
 Line Conduction Current, $I_{1F} = I_o$
 Line Voltage, $V_{1F} = V_A$
 Power Supplied to Line = $V_A I_o$ Watts
 Electrostatic Energy of Line = $\frac{1}{2} C V_A^2$ J/m.
 Magnetic Energy of Line = $\frac{1}{2} L I_o^2$ J/m.

Fig. 2 Initial Wave Propagation Down Transmission Line

The transverse displacement current I_{1D} establishes the magnetic field H at the wavefront, while the longitudinal conduction current, continuously flowing from the supply, maintains H behind the wavefront. The energy for the electrostatic and magnetic fields that are being established in the line medium flows

from the supply via the Poynting vector. Behind the wavefront is a line medium that has been energised to carry a current, $I_{1F} = I_o = V_A/Z_o$, at a voltage $V_{1F} = V_A$. The front leaves behind a line charged with electrostatic energy $\frac{1}{2} C V_A^2$ and magnetic energy $\frac{1}{2} L I_o^2$, J/m length of line. These energies are

maintained behind the front by the continuous power being supplied from the source via the Poynting Vector. When this initial wavefront reaches the load, the line is charged to the potential of approximately V_A , and carries a current $I_o = V_A/Z_o$.

2.2 Establishing Line Current to Match Load

When the wavefront reaches the load end of the line, there are three sources of energy available:

- Stored electrostatic energy in the medium due to the electric field E .
- Stored magnetic energy in the medium due to the current and resultant magnetic field H . →

- The continuous power coming from the source via the energised line, supplying power $V_A I_o$, to the load by the Poynting vector, $S = E \times H$.

What happens now depends upon the load resistance. If this initial line current $I_o < V_A/R_L$, then it is insufficient for the load. There is a deficiency $I_d = V_A/R_L - I_o$, in the line conduction current, and the magnetic energy in the line medium is insufficient for the load. The load voltage falls below V_A by $V_d = I_d (Z_o/R_L)$. This reduction in voltage, $DV = -V_d$, now propagates back up the line with a corresponding displacement current, $I_{D2} = -V_d/Z_o$, leaving behind it a line charged to a voltage $V_A - V_d$, carrying a conduction current $I_o + V_d/Z_o$ as indicated below in Fig 3.

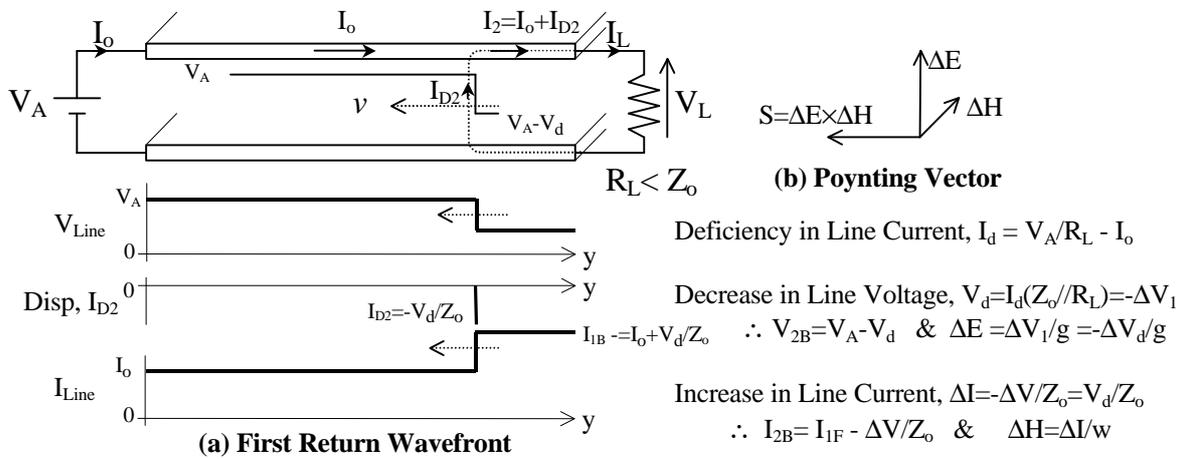


Fig. 3 First Reflected Wave Propagation when $R_L < Z_o$

On this backward leg there is no change in current supplied from the source, and the reduction in the potential energy of the line plus the reflected source power (V_d^2/Z_o), is converted to an increase in magnetic energy, with a resultant increase in conduction current, in the line behind the returning wave front.

2.3 Multiple transitions To Establish Current

When this return wavefront reaches the source the voltage is returned to V_A , and the resultant rise of $+V_d$ is propagated down the line towards the load, with a displacement current ($+V_d/Z_o$). As in the first run, this second run again adds electrostatic and magnetic energies to the medium in the line, leaving behind the wavefront a line that is energised to a voltage V_A , carrying a conduction current, $I_{2F} = I_{1F} + 2V_d/Z_o$. After numerous back and fro transitions of the line its medium is correctly energized, to carry the load current $I_L = V_A/R_L$ at the voltage V_A , and supplies the correct power to the load via the Poynting vector.

1st Forward Transition,

$$V_{1F} = V_A, \quad I_{1F} = V_A / Z_o$$

1st Backward Transition,

$$\text{Load Current Deficiency, } I_{1d} = \left(\frac{V_{1F}}{R_L} - I_{1F} \right)$$

$$\text{Now, } \Delta V_1 = -I_{1d} (Z_o // R_L)$$

$$= \left(I_{1F} - \frac{V_{1F}}{R_L} \right) (Z_o // R_L), \text{ and is -ve for } R_L < Z_o.$$

$$\text{so, } V_{1B} = V_{1F} - \Delta V_1, \quad \& \quad I_{1B} = I_{1F} + \Delta V_1 / Z_o$$

nth Transition,

$$\text{Forward, } V_{nF} = V_A,$$

$$I_{nF} = I_{(n-1)B} + \Delta V_{(n-1)} / Z_o \quad \dots (1a)$$

$$\text{Backward, } \Delta V_n = -I_{nd} (Z_o // R_L)$$

$$\Delta V_n = (V_{nF} - I_{(n-1)F} R_L) Z_o / (Z_o + R_L) \quad \dots (1b)$$

$$V_{nB} = V_A + \Delta V_n, \quad I_{nB} = I_{nF} - \Delta V_n / Z_o \quad \dots (1c)$$

Matlab plots of these functions are shown in Fig 4 for an input step voltage of 10 Volts when $R_L = Z_o/5$. The build up of load voltage, and hence load current, is shown in the familiar stepped output of Fig 4.

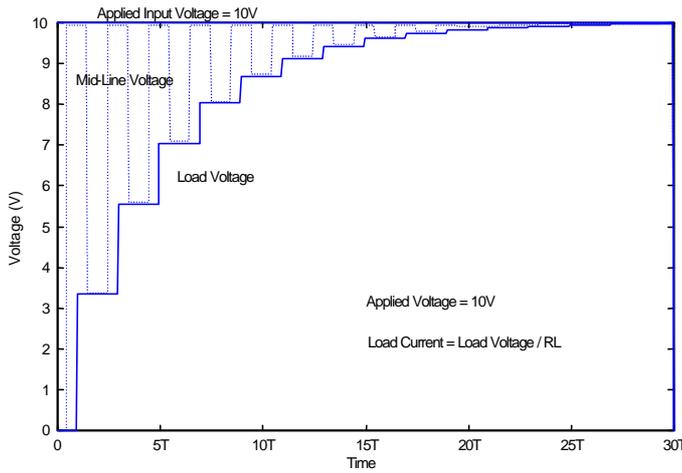


Fig 4. Transitions for Increasing Load Current

The back and fro transitions can be seen by observing the mid-line voltage, where a small gap has been introduced at the top and bottom of the transitions to distinguish them from the input and load voltages. →

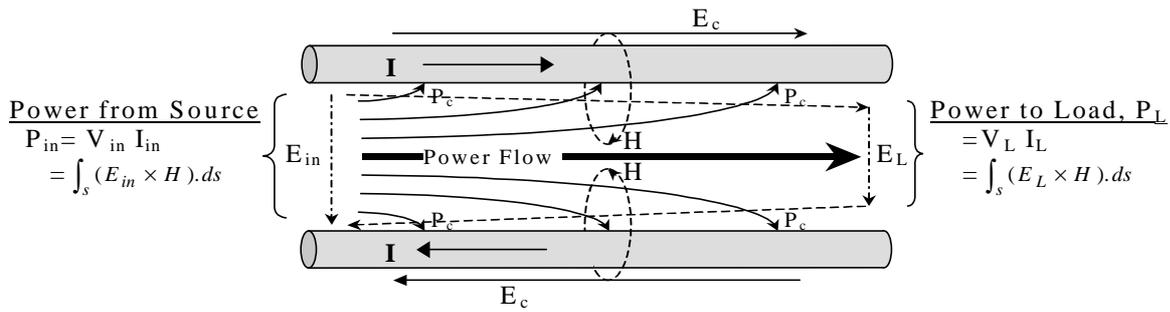


Fig 5 Power Flow into Conductors and Load via Poynting Vector.

This power flows into the conductors to support ohmic and eddy current losses, and establish internal magnetic fields within the conductors. It causes the transition wavefronts to become more and more gradual, so that the original square wave transitions eventually become triangular, as shown in Fig6.

$$V_{input} = \text{Propagation Voltage} + \text{Volt Drop Along Conductors} + V_{Load} .$$

Propagation ceases when the input voltage is completely used up by the load plus the longitudinal voltage drop along the conductors.

2.5 Velocity of Propagation

Assuming that the front is moving at velocity, v , then

Displacement Current,

$$I_D = \partial q / \partial t = CV_A v = (ew/g) Egv = eEwv$$

This displacement current produces a magnetic field H , in the medium, directed in the $-x$ direction.

$$\text{Now, } \text{Curl} H = J_D \text{ so } \oint H \cdot dl = \int_s J_D \cdot ds = I_D$$

It is important to note that these electromagnetic wave transitions occur until the difference between the input and load voltages is completely taken up by the longitudinal ohmic voltage drop along the conductors.

2.4 Power Flow

Immediately after the input voltage is applied, there is no conduction current or load voltage, so all of the input voltage is used to propagate changes, via electromagnetic transitions of the line. With each transition of the line, the every increasing surface currents cause higher and higher voltage drops along the supply and return conductors, and also raise the output voltage. Thus less and less voltage is available for propagation, and the amplitude of the transitions gradually decrease, as indicated by the midline voltage of Fig 3. The increasing longitudinal voltage drops along the conductors, cause an increasing amount of source power to flow in through the surface of the conductors, to support the diffusion process and ohmic losses as indicated below in Fig 5.

$$\therefore \text{Magnetic Field, } H = I_D / w = eE v$$

$$\& \text{ Flux Density, } B = \mu H = \mu e E v$$

Thus the magnetic field is proportional to the velocity of the wavefront, and is itself moving at velocity v .

This moving magnetic field produces a ‘back’ electric field, E_b that opposes the applied field E , that caused it, and limits the velocity of propagation.

$$\text{Curl} E_b = -\partial B / \partial t ,$$

$$\text{so } V_b = \oint E_b \cdot dl = -\int_s \partial B / \partial t$$

$$\text{Back emf, } V_b = E_b g = -Bgv ,$$

$$\text{so } E_b = -Bv = -\mu e E v^2$$

For a loss-less medium such as air, this induced electric field E_b , is equal and opposite to the applied E .

$$\therefore \mu e v^2 = \frac{E_b}{E} = 1 \quad \& \quad \text{Velocity, } v = \frac{1}{\sqrt{\mu e}}$$

It should be noted that the velocity of propagation depends on the magnetic field H , and hence the

wavefront displacement current, produced by the driving electric field E . Comparing the field impedance of 377 ohms for free space, with the resistivity of $1.8 \times 10^{-8} \Omega\text{-m}$ for copper, gives some insight as to why the velocities of electromagnetic wavefronts in good conductors are so slow, even without consideration of the very high attenuations due to eddy current losses.

3. DIFFUSION OF SURFACE CURRENT INTO THE CONDUCTORS

All changes in conductor currents are established on the surface of conductors by propagation of electromagnetic wave displacement currents in the insulating material, and is easily demonstratable by a simple experiment on a common 3-core flat cable [3].

If the load is less than the characteristic impedance of the line, multiple reflections and retransmissions eventually build up the line current to that required by the load. These currents are initially layered onto the near surfaces of the conductors between the supply and return lines at velocities approaching the speed of light, and quickly surround the whole conductor, to take the path of least energy, before diffusing relatively slowly into the interior.

$$\text{Diffusivity, } a = \frac{1}{sm} \quad \text{m}^2/\text{sec},$$

$$\text{Skin Depth, } d = \sqrt{\frac{2a}{w}} \quad \text{m}$$

$$\text{Diffusion Phase Velocity, } u = wd \quad \text{m/sec}$$

The diffusion velocity depends on the thickness of the conductor, and the frequency of the excitation, and such effects are difficult to appreciate. Fortunately, the more easily appreciated diffusion process for conduction of heat into solids is similar [4]. Heat diffusion is opposed by the heat capacitance (cp) of a material, and aided by its thermal conductivity (k). However in the electrical case both the electrical conductivity (σ) and the permeability (μ) slow down the diffusion process.

Material	Diffusivity, a (m^2/sec)	
	Thermal= k/cp	Magnetic= $1/\sigma\mu$
Copper	1.14×10^{-4}	137×10^{-4}
Trans Steel	0.18×10^{-4}	0.4×10^{-4}

It can be seen that the thermal and electrical diffusivities of transformer steels are similar to each other, and give an appreciation of the slow diffusion rates of surface flux into magnetic core laminations. The electrical diffusivity of copper is approx 100 times its thermal. Since diffusion velocities are proportional to the $\sqrt{\alpha}$, electric fields (and hence

surface currents) will diffuse into copper only 10 times faster than heat, as indicated in the table below for a frequency of 50 Hz .

Material	Phase Velocity, $u = wd$ (m/sec)	
	Thermal	Magnetic
Copper	0.27	2.93
Trans Steel	0.11	0.16

These comparisons with heat give some appreciation of the impossible situation that would exist if currents had to diffuse longitudinally through the whole length of copper conductors instead of transversely across half their thickness. It is very fortunate that surface currents are initially established along the length of conductors at velocities around c , by means of the displacement currents in the insulating medium.

Thus all current changes actually move into the conductors from their outside surfaces and only have to diffuse through the thickness of the conductors (usually half the diameter), rather than along the length of the cable from the power source. If it were not for the displacement currents setting up the surface currents in the first instance, energy transmission, other than via relatively steady dc currents, by copper conductors would be virtually impossible because of the long diffusion times and attenuations.

The familiar skin effect results from the fact that the surface currents only have time to penetrate the conductor to the skin depth in the $\frac{1}{4}$ period of the operating frequency. Surface currents move into the conductor on the rising part of the positive surface current waveform. Once the surface currents reach their sinusoidal peak and begin to fall, the interior currents move back out towards the surface of the conductor. At 50Hz the skin depth in copper is approx 10mm and is about as far as the wave can propagate into the copper in 5ms quarter period.

This diffusion process is the means whereby surface current penetrates into the interior of conductors, and surface magnetic flux into the interior of magnetic cores. It reduces the ohmic voltage drop along the conductor as the current redistributes itself throughout the whole cross section of the conductor, so $I = V/R$.

4. EXPERIMENTAL RESULTS

The result of applying a 10V step is to a 50m length of loaded line is shown below in Fig 6, where the electromagnetic wave transitions, that build up the surface current, can be clearly seen in the midpoint voltage. Diffusion causes the rise and fall times of the midpoint voltage transitions to continually increase, and the levels themselves to fall short of the limiting values of applied voltage (V_A), and load voltage (V_L).

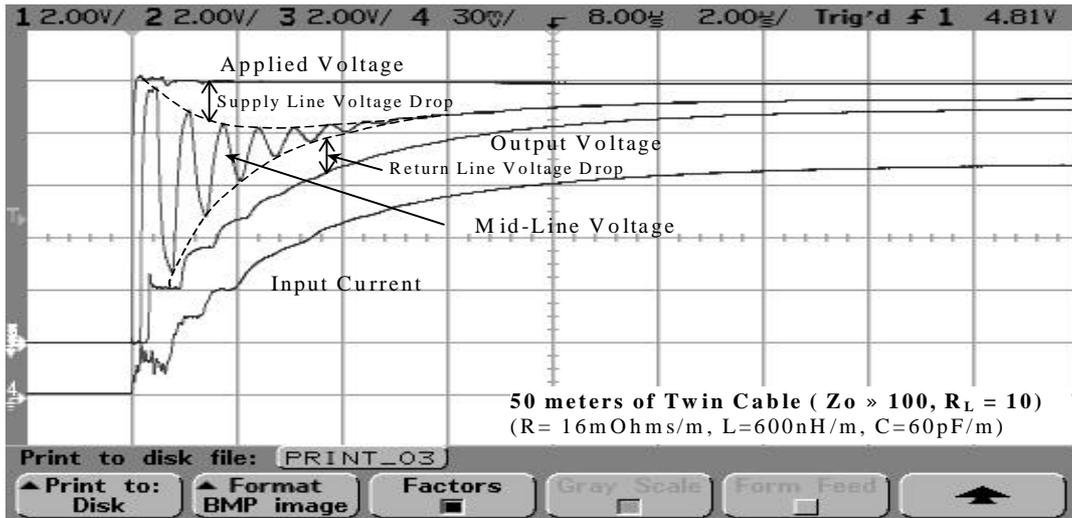


Fig 6. Measured Voltages and Current in 50m Cable when $R_L = Z_0/10$ (2ms/div)

This is because the current is initially established on the surface of the conductors, and flows through a fairly high surface resistance. As the current builds up on the surface, it also diffuses into the interior of the conductors, with a corresponding decrease in the effective resistance. Eventually the longitudinal line

voltage drops falls towards their normal dc values as indicated in Fig 6..

The effect of current diffusion can also be clearly seen in Fig 7, which shows the result of applying a step current of 1Amp to a 20m length of 3mm diameter copper conductors terminated in a short circuit.

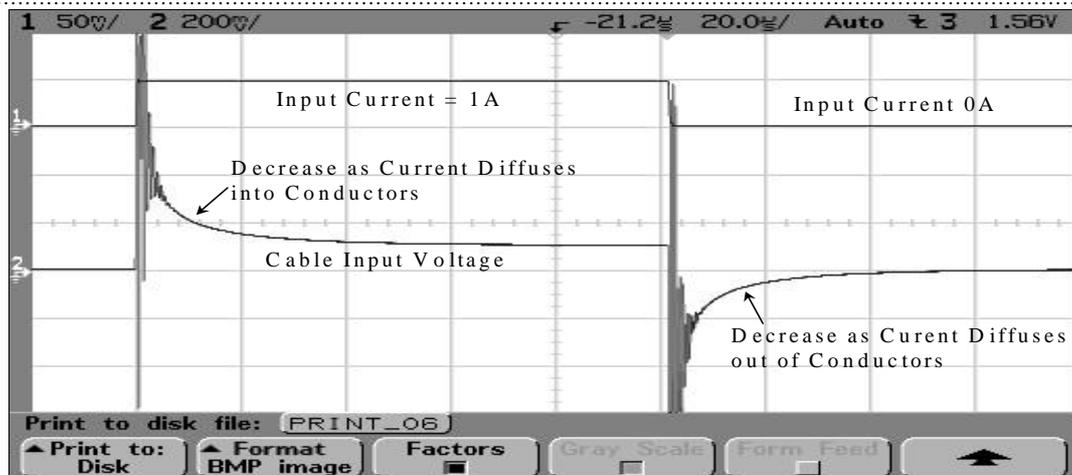


Fig 7 Slow Decrease in Voltage Drop to Current Diffusion (20ms/div)

The ever-decreasing voltage 'ringing' due to the back and fro transitions, that establish the initial line voltage changes, can be seen during the first 5 μ secs. Afterwards the further gradual reduction in line voltage drop is due to diffusion, which drives the surface currents into the interior, and gradually reduces the longitudinal voltage drop along the conductors. It can be seen that the diffusion time is approx 50 μ secs for these 3mm diameter conductors.

5. CONCLUSIONS

It has been shown that displacement currents, electromagnetic propagation, and diffusion are essential fundamentals for establishing current

changes in conductors, even at dc. The experimental results are seen to support the concepts.

6. REFERENCES

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