Abstract—This Paper presents a novel voltage stability analysis and assessment method based on an observation of the sensitivity of generator reactive power outputs to changes in loading at buses in a power system. This information can be used to assess the relative stability of a power system and to determine the sets of generators that cause voltage instability when they lose voltage control and the associated buses that provide the mechanisms for collapse. This method has been tested on a modified Cigré ‘Nordic’ test system and on the large scale Queensland Transmission system and the results of these tests are provided in this paper. This method is compared to another widely excepted voltage stability assessment method.

Index Terms—Coherent Bus Groups, Voltage Stability, Sensitivity analysis.

I. INTRODUCTION

With the advent of deregulated energy markets and the growing desire to fully utilise existing transmission equipment and infrastructure voltage stability issues are becoming increasingly critical. The need for an easy and reliable voltage stability assessment method is of great importance. The sensitivity analysis technique [1] at the heart of the assessment method presented in this paper was originally created as an improvement to a widely accepted voltage stability method presented by Robert Schlueter [2, 3] but it has been found to be a useful voltage stability analysis tool in its own right. The proposed voltage stability assessment method determines the set combinations of generators that cause voltage instability when they lose voltage control and the associated buses that provide the mechanisms for collapse when additional loading beyond the current state leads to this collapse. In using this information system operators and planners can ensure that either the particular generator combinations do not reach their limits or that the critical loads are decreased so that they do not contribute to the collapse.

II. ASSESSMENT METHODS

The voltage stability security assessment method proposed by Schlueter [2] finds coherent bus groups in a system that have similar VQ curve minima and share a similar set of exhausted generators at this VQ minima. Schlueter calls these groups of coherent buses voltage control areas and their associated reactive reserve basins. According to Schlueter the voltage control areas and their associated reactive reserve basins are the agents for both clogging and loss of voltage control instability [6]. The main problem with Schlueter’s method is that it involves a fairly high degree of trial and error and involves the computation of VQ curves at a number of buses before the individual coherent bus groups can be found [1].

The voltage stability assessment method presented in this paper is based on a technique originally provided by Alvarado [7], which determines the sensitivity of the reactive power flow on a transmission line to an injection of reactive power at a bus in the system. In order to understand how this sensitivity is utilised in the proposed method it is crucial to realise that the power produced by a generator is equivalent to the flow through the transformer branch, or generator branch, as it shall be called, connecting this generator to the system. In this way the sensitivity of a generator branch, and therefore generator, to an injection of reactive power, or alternatively a change in load, can be determined. Details of the algorithm used in this method have been reported in a previously submitted paper [1]. The summary of this algorithm is now presented.
Sensitivity Method:

1. Obtain line flow Jacobian ($J_f$) which relates the flows at either end of a line to changes in voltage magnitudes and angles

$$J_f = \begin{bmatrix} \frac{\partial f_p}{\partial V} & \frac{\partial f_p}{\partial V} \\ \frac{\partial f_q}{\partial V} & \frac{\partial f_q}{\partial V} \end{bmatrix}$$

*(The subscripts $p$ and $q$ denote real and reactive power flows)*

2. Obtain power flow Jacobian ($J$) which relates injected powers to voltage magnitudes and angles

$$J = \begin{bmatrix} \frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial V} \end{bmatrix}$$

3. Determine the sensitivities of reactive power flows to an injection of reactive power at a bus using the formula given in Equation 1 (an in-depth description of this formula is given in the appendix). The sensitivity elements of interest in this method are the reactive flow- reactive injection sensitivities.

$$\begin{bmatrix} \frac{\partial F_p}{\partial Q} \\ \frac{\partial Q_p}{\partial Q} \\ \frac{\partial Q_q}{\partial Q} \end{bmatrix} = Jf^* \left( J \setminus ER \right)$$

Equation 1

In this method only the generator branch sensitivities to reactive injections at PQ buses and no generator buses (slack and PV) are obtainable. This is considered to be acceptable as we are more interested in changes in load, not generation, and it is therefore not as necessary to observe the sensitivity of generator flow to injections by other generators.

III. RESULTS

Voltage instability and collapse is generally considered to be a reactive power problem and it is therefore useful to examine the production, transmission and consumption of reactive power. There are several ways in which a power system’s operating status can be changed in a manner known to contribute to collapse.

1. Load increase
2. Action of tap changing transformers
3. Generators, Synchronous Condensers or Static Var Compensators (SVC) reaching reactive power limits
4. Contingencies such as line tripping and generator outages

Observing the sensitivity of generator reactive power outputs to changes in loading at buses in a power system can help to explain the impact of these changes. In this results section these relationships between system changes and sensitivities will be illustrated with the help of two power systems, a modified Cigré Nordic test system that can be seen in Figure 1, and the Queensland Transmission system, which can be seen in two figures, Figure 2 and Figure 3. This modified Nordic system is based on the CIGRE Nordic test system[4] and differs from this standard test system in one area only. The step-up transformers in this modified system have been modelled externally and the reactive limits of the generators increased to allow for the additional losses in the transformers. This has been done to ensure an accurate indication of the loading limit of the buses in the system is obtained. It has been found in previous investigations that accurate loading limit results cannot be obtained if the transformers are modelled internally[8].

This section will also illustrate and discuss how the change in sensitivities resulting from generators, synchronous condensers or SVC limiting can be used to determine sets of generators that cause voltage instability when they lose voltage control and the associated buses that provide the mechanisms for collapse.

A selection of sensitivity values found for the Modified Nordic Test System is shown in Table 1. These values illustrate the impact that the first two system change categories, change in bus loading and transformer tap operation, have on the system sensitivities found. Note the negative sign of the sensitivity values found, as we are actually looking at an injection of reactive power, or in other words, a decrease in loading at the bus. The buses of interest were chosen as they lay in different parts of the system and the generators of interest were chosen for the same reason. Sensitivity values were obtained for the base case load flow solution, the load flow solution when the load at the bus 43 was increased by 1 and 5 percent respectively and load flow solution when the tap setting on the transformer between buses 4044 and 1044 was increased by one tap setting.

As can be seen in Table 1 a change in loading at bus 43 not only increases the magnitude of the sensitivity of generators to a change in load at this bus but also increases the sensitivity of generators in the system to changes at other buses. To understand why this is the case it is useful to consider that as the loading is increased the flows in the system are also increased and as a result the reactive losses in the system are increased. These losses must be accounted for by either reactive supply from capacitor banks being connected into the system or by the generators in the system. In the case that we are looking at new capacitor banks are being added so the generators must supply the additional reactive power and therefore their sensitivity to increases in loading also increases.

To further illustrate the fact that an increase in loading has the effect of increasing the sensitivities in the system the loads...
in the southern region of the Queensland system, that surrounding and including the capital city Brisbane and which can be seen in Figure 3, were increased by a relatively small amount of 15MW. It was observed that the sensitivity of the southern region generators to the southern region buses increased in comparison to the base case. For example, the sensitivity of the Wivenhoe unit number 1 to a changing in load at the Victoria Park load bus changed from -0.0152 to -0.0155 and the sensitivity of the Blackwall SVC to the change in load at the Victoria Park load bus changed from -2.3664 to -2.4034.

Table 1 Selection of sensitivity values found for Modified Nordic System

<table>
<thead>
<tr>
<th>Generator</th>
<th>412</th>
<th>431</th>
<th>441</th>
<th>451</th>
<th>462</th>
<th>472</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>BUS 43</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Base</td>
<td>-0.0455</td>
<td>-0.1228</td>
<td>-0.1655</td>
<td>-0.2375</td>
<td>-0.0851</td>
<td>-0.0176</td>
</tr>
<tr>
<td>Bus 43 load+1%</td>
<td>-0.0466</td>
<td>-0.1249</td>
<td>-0.1680</td>
<td>-0.2402</td>
<td>-0.0863</td>
<td>-0.0182</td>
</tr>
<tr>
<td>Bus 43 load+5%</td>
<td>-0.0521</td>
<td>-0.1361</td>
<td>-0.1807</td>
<td>-0.2543</td>
<td>-0.0922</td>
<td>-0.0204</td>
</tr>
<tr>
<td>4044-1044 incr. tap</td>
<td>-0.0454</td>
<td>-0.1225</td>
<td>-0.1652</td>
<td>-0.2370</td>
<td>-0.0849</td>
<td>-0.0178</td>
</tr>
</tbody>
</table>

| **BUS 1012** |     |     |     |     |     |     |
| Base      | -0.1825 | -0.0275 | -0.0214 | -0.0150 | -0.0079 | -0.0491 |
| Bus 43 load+1% | -0.1828 | -0.0278 | -0.0218 | -0.0153 | -0.0081 | -0.0492 |
| Bus 43 load+5% | -0.1845 | -0.0298 | -0.0236 | -0.0168 | -0.0088 | -0.0498 |
| 4044-1044 incr. tap | -0.1825 | -0.0275 | -0.0214 | -0.0150 | -0.0079 | -0.0491 |

| **BUS 61** |     |     |     |     |     |     |
| Base      | -0.0341 | -0.1039 | -0.2255 | -0.1340 | -0.4092 | -0.0131 |
| Bus 43 load+1% | -0.0348 | -0.1052 | -0.2270 | -0.1354 | -0.4102 | -0.0134 |
| Bus 43 load+5% | -0.0362 | -0.1119 | -0.2348 | -0.1425 | -0.4155 | -0.0147 |
| 4044-1044 incr. tap | -0.0341 | -0.1038 | -0.2253 | -0.1337 | -0.4091 | -0.0131 |

| **BUS 1041** |     |     |     |     |     |     |
| Base      | -0.0526 | -0.1421 | -0.2037 | -0.4334 | -0.1313 | -0.0205 |
| 4044-1044 incr. tap | -0.0523 | -0.1414 | -0.2026 | -0.4310 | -0.1306 | -0.0204 |

Looking again now at Table 1 the impact of tap changer action on the sensitivities found can be observed for the modified Nordic system. When the tap ratio of the transformer between the 400kV bus 4044 and 130kV bus 1044 was forced up by one tap setting to simulate the control action that would be performed if the 130 kV sub system containing bus 1044 required its voltage to be increased it was noticed that the sensitivities of the system decreased in magnitude. As can be seen in Table 1. Many of the sensitivities though were decreased by almost insignificant amounts to the point at which they appear in this table to have not changed in value. This is especially true of the sensitivity values associated with bus 1012. This bus is located in a different section of the system some considerable distance from the transformer of interest such that a change in the tap setting would appear to have had little impact on the flows, and therefore losses and sensitivities to this bus.

The impact of transformer taps on system sensitivities can also be illustrated by looking at the taps between the 275kV and 132kV Belmont buses in the southern region of the Queensland system. These taps were increased in the base load flow data and the sensitivity values were recalculated for the system. In this case it was observed that the sensitivity values increased rather than decreased at many of the southern buses, especially to southern generators. For example the sensitivity element representing the output of the Blackwall SVC to an injection of reactive power at the Victoria Park load bus was changed from -2.3664 to -2.4653. Unlike the case already shown for the Modified Nordic System the action of this tap changers would appear to have had an unbeneficial impact on the flows, losses and sensitivities in the system.

In summary it can therefore be pointed out that when the loading of a bus or indeed of a whole section of a system is increased the sensitivities associated with buses and generators in that section of the system also increase. On the other hand, when transformer taps operate to improve a falling voltage (i.e. the tap ratio increases) the sensitivities of buses and generators in the section of the system containing the transformer will either increase or decrease depending on the impact such a change has on the system.

The third category of system change that has been known to contribute to collapse is the situation when Generators, Synchronous Condensers or Static Var Compensators (SVC) reach their reactive power limits. When these generators, synchronous condensers or SVC limit the sensitivities also increase and in some cases, as will be shown, can cause bifurcation, instability and collapse as indicated by a qualitative change in system behaviour as a result of slow and continuous variations in system parameters, in this case reactive power generation. This can be verified by a sudden change in sign or noticeably large increase in magnitude [9].

The Nordic test system can now be used to illustrate the impact of generator limiting. In the previously submitted paper [1] coherent bus groups were found for the modified Nordic test system as shown in Figure 1. The coherent groups found for this system are shown in Table 2.

Table 2 Modified Nordic System Coherent Bus Groups

<table>
<thead>
<tr>
<th>Bus(s)</th>
<th>4062</th>
<th>4061</th>
<th>4051</th>
<th>4047</th>
<th>4043</th>
<th>4042</th>
<th>4041</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 1</td>
<td>51</td>
<td>47</td>
<td>43</td>
<td>42</td>
<td>41</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Group 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Group 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Group 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Group 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Group 6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Group 7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*(All other buses are either generator buses or belong to their own individual one bus group)*
The set of reactive reserve generators determined for coherent bus group number 1, namely buses 4062 and 62, were the generators located at buses 122, 431, 442, 462, 4631 and 4632. Sensitivity calculations have been carried out using the current base load flow data for this system but the bus types for these particular reactive reserve generator buses have been set to PQ rather than PV so as to simulate these generators as being limited. A new set of sensitivity values was subsequently obtained. When these values were obtained it was noticed that the sign of the generator sensitivities to load changes had changed sign for a number of buses in the system compared with the base case. Such a change in sign would indicate that the system might have undergone bifurcation, as bifurcation theory assumes that power system parameters vary slowly which is clearly contradicted by this distinct change in system parameters. In fact it was found that if this particular set of generators had their limits set to their current base case values the buses at which the sensitivities changed sign could not have their load increased without a solution failure occurring. This means that if this particular set of generators is limited and the bus loadings are increased at any of the buses where a change in sign was observed than the system will suffer from instability and collapse.
It was noticed that the generator sensitivities to a change in load at bus 51 also changed sign when the group 1 generators where simulated as limited. This was particularly interesting as the reactive reserve basin for this bus was found to be different from that of group number 1. In the case of bus 51, and bus 4051 which make up coherent bus group 3, the reactive reserve generators where determined to be 122, 143, 431, 442 and 451. In a subsequent load flow analysis it was found that if not only this particular set of generators had their limits set to their current values but the set of generators as set out for group 1 also had their limits set to current values then the load at bus 51 could not be increased without a solution failure occurring. When the loading is on bus 51 alone the flows in the system will cause a certain groups of generators to lose control and lead the system to collapse. That does not mean that this is the only group of generators that once limited will not allow the load on bus 51 to be increased without occurring instability and collapse. By running through a number of different combinations of generator sets it should be possible to determine the sets of generators that when limited will lead to instability and the buses load increase will lead to this collapse.

The Queensland System, shown in the two figures Figure 2 and Figure 3 was analysed to determine if the relationship between the changes in the system sensitivities could be equated to system instability and collapse in this system as well. The Blackwall SVC is located in the southern region of Queensland and for the purposes of this study it has been modelled as synchronous condenser. When Blackwall had its limit set to its current output it was found that loads in the northern region could be increased without causing the system solution to fail but loads in the southern and central regions could not be increased without the system load flow solution failure. The sensitivity values were noticeably larger than the base case when the Blackwall unit bus was set to PQ type. For example the sensitivity of the Swanbank-A number 1 unit, located in the south region of Queensland, to a change in load at the Victoria Park load bus increased from 0.0237 to 4.1430. The Wivenhoe 1 and Swanbank-B 1 units, also located in the south region, had their sensitivities to a change in load at Victoria Park increased from 0.0152 to 11.2 and 0.0177 to 6.56 respectively. The extreme jump in sensitivities means any increase in load will increase flows in the system drastically and will lead other generators to limit quickly. In fact it can be seen that when Blackwall unit and the generators at the Wivenhoe station or when the Blackwall unit and the generators at the Swanbank-B station are modelled as PQ buses (i.e. modelled as if limited) the sensitivities for the southern region of the system change sign indicating a bifurcation and collapse. A large change in state itself could be seen to be an indicator that a bifurcation has occurred.

When the Blackwall unit was modelled as limited the northern buses were not affected and the sensitivities for this region reflected this situation. It was ascertained that when the combination of generators at the Barron Gorge, Karreya, stations and Ross SVC were set at their limits the system solution failed if any load in the northern region was increased. When these units were modelled as PQ buses the sensitivities in the northern region predictably changed sign while the southern region sensitivities remained unchanged.

**IV. Conclusions**

This paper has provided a voltage stability assessment method that can allow generator combinations to be determined that must not be allowed to reach their limits and has also determined which buses can have the most impact on these generators. Reducing loads at these buses may be useful in reducing the reactive output of these generators away from their limits. If a generator is close to limiting the sensitivities could be calculated with this generator limited and it can be determined if its limiting will lead to collapse. In Schlueter’s voltage stability assessment method he finds the reactive reserve basin for a bus, being the set of generator limited at the bottom of the VQ curve. But as this paper has shown this is not the only combination of generators, which if limited the load at the bus of interest cannot be increased without the system failing. The analysis of the Nordic and Queensland systems have highlighted the usefulness of the proposed method and indicated its suitability as an alternative voltage stability analysis tool.

**V. References**

VI. APPENDIX

\[
\begin{bmatrix}
\frac{df_p}{dQ} & \frac{df_p}{df_q} & \frac{df_p}{df_q} \\
\frac{dQ}{dV} & \frac{dQ}{dV} & \frac{dQ}{dV} \\
\end{bmatrix} = Jf^*(J \setminus ER)
\]

The symbol “\(\div\)” denotes the left matrix divide function (i.e. If \(Ax = B\) then \(x = A\div B\)). This is effectively the same as the equation \(J^*\cdot ER\). The ER matrix is an error matrix set up to simulate the injected power \(\Delta Q\). ER is set up similar to the power flow Jacobian, \(J\) in that the top rows of \(ER\) correspond to the non-slack buses in the system and to \(\Delta P\) real power injections and the bottom rows correspond to the PQ buses in the system and to \(\Delta Q\) imaginary power injections. The columns of \(ER\) correspond to all system buses. The value of 1 is placed at the relative positions of the system’s PQ buses in the bottom section of the matrix to represent \(\Delta Q\) injections at these buses.

The 5-bus test system shown in Figure 4 will be used to illustrate this formula in more depth. (The slack bus is bus 4)

![Figure 4 Simple 5 bus system](image)

\[
\begin{bmatrix}
\frac{dF_p}{dQ} & \frac{dF_p}{dQ} & \frac{dF_p}{dQ} & \frac{dF_p}{dQ} & \frac{dF_p}{dQ} \\
\frac{dQ}{dV} & \frac{dQ}{dV} & \frac{dQ}{dV} & \frac{dQ}{dV} & \frac{dQ}{dV} \\
\end{bmatrix} = Where X = J^*\cdot ER;
\]

\[
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & C_{14} & C_{15} \\
C_{21} & C_{22} & C_{23} & C_{24} & C_{25} \\
C_{31} & C_{32} & C_{33} & C_{34} & C_{35} \\
C_{41} & C_{42} & C_{43} & C_{44} & C_{45} \\
C_{51} & C_{52} & C_{53} & C_{54} & C_{55} \\
\end{bmatrix} \cdot \begin{bmatrix}
C_{11} & C_{12} & C_{13} & C_{14} & C_{15} \\
C_{21} & C_{22} & C_{23} & C_{24} & C_{25} \\
C_{31} & C_{32} & C_{33} & C_{34} & C_{35} \\
C_{41} & C_{42} & C_{43} & C_{44} & C_{45} \\
C_{51} & C_{52} & C_{53} & C_{54} & C_{55} \\
\end{bmatrix} = \begin{bmatrix}
C_{11} & C_{12} & C_{13} & C_{14} & C_{15} \\
C_{21} & C_{22} & C_{23} & C_{24} & C_{25} \\
C_{31} & C_{32} & C_{33} & C_{34} & C_{35} \\
C_{41} & C_{42} & C_{43} & C_{44} & C_{45} \\
C_{51} & C_{52} & C_{53} & C_{54} & C_{55} \\
\end{bmatrix}
\]

VII. ACKNOWLEDGEMENTS

The Authors would like to take this opportunity to thank Dr Zhao Yang Dong (University of Queensland, Australia) for his suggestions and helpful advice during the preparation of this manuscript.

VIII. BIOGRAPHIES

Craig Anthony Aumuller was born in Cairns, Australia in 1974. He graduated from James Cook University, Australia in 1996 with a Bachelor of Engineering (Honours). Since graduation he has worked at the Callide B Power Station and at Connell Wagner, an Australian based international consulting engineering firm. He is currently undertaking full time PhD research at the University of Queensland, Brisbane – Australia. His interests include power systems planning, analysis and control.

![Craig Anthony Aumuller](image)

Tapan Kumar Saha was born in Bangladesh and came to Australia in 1989. Dr Saha is a Senior Lecturer in the School of Information Technology and Electrical Engineering, University of Queensland, Australia. Before joining the University of Queensland he taught at the Bangladesh University of Engineering and Technology, Dhaka, Bangladesh for three and a half years and at James Cook University, Townsville, Australia for two and a half years. He is a senior member of the IEEE and a Chartered Professional Engineer of the Institute of Engineers, Australia. His research interests include power systems, power quality, high voltage and insulation Engineering.

![Tapan Kumar Saha](image)