8.4.3 Energy Equation

In the downstream flow region the energy equation for a streamline at a depth \( y \) above the bed is:

\[
E(y) = \rho(y) * g * (x + y \cos \alpha) + \int_0^{\infty} \rho(h) * g * \cos \alpha * \, dh + \rho(y) * \left(\frac{\alpha(y)}{2}\right)^2
\]

With the same method as that used by WOOD (1985) the parameter \( E \) is defined as:

\[
E = (1 - C_{\text{mean}})^2 * \left[ \int_0^1 (1 - C) * u^2 * \, dy \right]^{1/3} \left[ \int_0^1 (1 - C) * u^2 * \, dy \right]^{1/3}
\]

The velocity distribution (fig. 8-12) may be estimated from equation [8.9] and the parameter \( E \) becomes:

\[
E = (1 - C_{\text{mean}})^2 * \left[ \int_0^1 2 \right]^{1/3} (1 - C) * y^6 * \, dy \left[ \int_0^1 \frac{2}{3} \right]^{1/3} (1 - C) * y^6 * \, dy
\]

where \( C \) is computed from equation [8.5]. \( E \) is only function of the mean air concentration. Numerical computations of \( E \) are plotted in fig. 8-14 with data on spillway model (appendix P).

The energy equation may be rewritten in term of \( X = \frac{x}{d} \), \( d' = \frac{d}{d^*} \), \( W = \frac{W}{d^*} \) and \( Fr = \frac{q_w}{\sqrt{g^* \cdot d^3}} \)

where \( d = d^* \) at \( x = 0 \) and then becomes:

\[
\frac{d}{d^*} \frac{d'}{dx} = \frac{\sin \alpha \cdot \left( 1 + d^* \frac{d \alpha}{dx} \right) - S_f + E \cdot \frac{d'}{W} + \frac{Fr^2}{\rho_c} \cdot \frac{d'W}{dx}}{\cos \alpha \cdot \frac{E \cdot Fr^2}{\rho_c} \cdot \frac{d^3}{d^*}}
\]

The continuity equation for air and the energy equation provide two simultaneous differential equations which can be solved using numerical method with the value of \( C_c \) for each \( \alpha \).

8.4.4 Application

8.4.4.1 Introduction

In the particular case a spillway with constant width the flow parameters \( d \) and \( C_{\text{mean}} \) are obtained from a simple system of equations:

\[
\frac{d}{dx} C_{\text{mean}} = \frac{u_x \cdot \cos \alpha}{q_w} \cdot (C_c \cdot C_{\text{mean}}) \cdot (1 - C_{\text{mean}}) \cdot d^*
\]

[8.34]
\[
\frac{d}{dx'} = \frac{\sin \alpha \left( 1 + \frac{d}{dx'} \right) - S_f}{\cos \alpha - \frac{E \cdot F_r^2}{d^3}}
\]

which can be solved numerically. From the knowledge of \(d\) and \(C_{mean}\) the air concentration distribution (eq. [8.5]) and the velocity distribution (eq. [8.9] & [8.10]) can then be calculated at any point in the downstream flow region.

The above equations were used to reproduce the air entrainment on spillway model and details of the computer program are presented in appendix P.

On spillway model (\(\alpha = 52.33^\circ\)) the initial conditions of calculations are taken at the start of the downstream flow region (fig. 8-15). The complete set of data is presented in appendix C and the main results are summarized in table 8-3. It is remarkable that the flow conditions (\(d_s, C_s\)) at the start of the downstream flow region are almost independent of the flow discharge, the subpressure \(\Delta P\) in the cavity and the air flow provided by the air supply system \(Q_{air}\).

These initial flow conditions (\(C_s, d_s\)) are only function of the depth of water \(d_0\) in the approach flow region of the aerator (table 8-3) and the position of the start of the downstream flow region is function of the position of the impact point of the jet. The latter is calculated from the jet trajectory equations developed in chapter 5, where the subpressure \(\Delta P\) in the cavity is obtained from the air demand through the air inlets (chapter 6).

The results indicate that, for high Froude numbers (\(Fr > 7\)), the flow conditions in the flow regions downstream of an aerator are almost independent of the quantity of air supplied by the air supply system of the aerator.

The air entrainment on spillway model is reproduced downstream of the aerator for a particular discharge (fig. 8-16) where the origin of the graph is the end of the deflector and the initial conditions are obtained from table 8-3.

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{d_0/t_s} & \text{d_s/t_s} & \text{Y90\%t_s} & \text{C_s} \\
\hline
0.77 & 0.87 & 1.27 & 0.32 \\
1.15 & 1.10 & 1.53 & 0.26 \\
2.70 & 2.10 & 2.40 & 0.12 \\
\hline
\end{array}
\]

Table 8-3 - Initial air concentration parameters of the downstream flow region

\[\text{q}_w = 0.40 \text{ m}^3 \text{/m} \cdot \text{d}_0/t_s = 1.15 \]
\[v_r = 16 \text{ cm/s} \cdot d_s = 0.039 \text{ m} \cdot C_s = 26\%\]
8.4.4.2 The application of the proposed method to the Clyde Dam Spillway

The Clyde dam spillway (New Zealand) has a 70 m long steep spillway ($\alpha = 51.34^\circ$ & 50.19$^\circ$) followed by a stilling basin (50 m long) with a 8:1 reverse slope which ends by a flip bucket (fig. 8-17). The aerator is located 39 m below the reservoir flood level and 28 m above the stilling basin invert. Complete details are presented in appendix D.

The equations [8.34] and [8.35] determine the air entrainment on the spillway and on the stilling basin and hence it is possible to compare the air entrainment on the spillway with and without aerator. It must be remarked that the equation [8.35] was obtained for a hydrostatic pressure distribution. The transition F-G between the spillway and the stilling basin occurs with a circular turn ($R = 20$ m) and HENDERSON (1966)\(^1\) shows that the increase of pressure due to the turn is small for $d/R > 10$ (i.e. for a depth of water $d$ lower than 2 m). Hence the assumption of the hydrostatic pressure distribution is reasonable for $d < 2$ m.

For a water discharge $q_w = 21$ m$^3$/s/m the surface roughness $k_s = 3$ mm gives a friction factor $f = 0.025$.

The only value of the rise bubble velocity on prototype is the result from CAIN's (1978) data on Aviemore dam. This result ($v_B = 40$ cm/s) may be used for the flow downstream of the point of inception (spillway without aerator). The extrapolation of the results obtained on spillway model may be affected by scale effects and hence for the flow downstream of the aerator the rise bubble velocity on prototype is taken as that on Aviemore dam ($v_B = 40$ cm/s).

For this water discharge and surface roughness the point of inception of air entrainment for the spillway without aerator was computed from KELLER and RASTOGI (1977)\(^2\). These calculations provide $S_1 = 73$ m and $d_1 = 0.675$ m where $S_1$ is the curviline co-ordinate along the spillway surface from the origin B (fig. 8-15) and $d_1$ the flow depth. This position corresponds to the start of the turn at the end of the spillway (point F).

For a water discharge $q_w = 21$ m$^3$/s/m the water depth in the approach flow region of the aerator (point D) is $d_0 = 0.75$ m and the computed impact point of the jet (chapter 5) is located 13.5 m downstream of the end of the deflector. Assuming that the downstream flow region starts 19.5 m downstream of the end of the deflector, the initial air concentration $C_*$ and flow depth $d_*$ are estimated from the results on spillway model (table 8-3) : $C_* = 0.26$ and $d_* = 0.72$ m.

The results are summarized in table 8-4 for this particular discharge and the location of the points (fig. 8-18) is detailed in appendix F.

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\(^1\)Open Channel Flow
F.M. HENDERSON

\(^2\)Design Chart for Predicting Critical Point on Spillways
R.J. KELLER and A.K. RASTOGI
At the position H in the middle of the stilling basin the air concentration profiles are plotted in fig. 8-19 for both cases: 1- without aerator and 2- with aerator. The results suggest that without aerator cavitation would occur on the spillway and on the stilling basin because the air concentration near the spillway bottom is below the required minimum to prevent cavitation damage (chapter 2).

For this particular discharge the presence of the aerator should prevent cavitation erosion on the spillway and for most of the stilling basin but damage may occur at the end of the stilling basin (between H and J) as the air concentration in the layer close to the spillway surface (C = 2-3 %) is below the required minimum 5-7 % (chapter 2).

It must be emphasized that these results depend critically on the assumed rise bubble velocity \( u_r \), the friction factor \( f \), the initial air concentration \( C_s \), the initial flow depth \( d_s \). It worth noting that the greatest uncertainty applies on the assumed rise bubble velocity \( u_r \).

<table>
<thead>
<tr>
<th>Point</th>
<th>S (m)</th>
<th>( \alpha ) (°)</th>
<th>d (m)</th>
<th>C\text{mean} (%)</th>
<th>d (m)</th>
<th>C\text{mean} (%)</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>56.92</td>
<td>50.19</td>
<td>0.675</td>
<td>0.720</td>
<td>0.26</td>
<td>Start of the downstream flow region.</td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>64.91</td>
<td>50.19</td>
<td>0.687</td>
<td>0.701</td>
<td>0.28</td>
<td>End of spillway. Start of circular tur.</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>85.65</td>
<td>-7.125</td>
<td>0.720</td>
<td>0.710</td>
<td>0.28</td>
<td>End of tur. Start of stilling basin.</td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>105.00</td>
<td>-7.125</td>
<td>0.826</td>
<td>0.774</td>
<td>0.23</td>
<td>End of stilling basin.</td>
<td></td>
</tr>
<tr>
<td>J</td>
<td>126.73</td>
<td>-68.20</td>
<td>0.903</td>
<td>0.853</td>
<td>0.17</td>
<td>End of flip bucket.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.938</td>
<td>0.16</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 8-4 - Results for \( q_w = 21 \text{ m}^3/\text{s} \) and \( u_r = 40 \text{ cm/s} \)

8.4.5 Aerator Spacing

Air concentration downstream of an aerator and in the layer close to the spillway surface is of primary importance for cavitation erosion protection. In the impact point region the turbulence in the flow redistributes the air bubbles. The air concentration distribution in the downstream flow region tends to the equilibrium distribution of the particular slope and this can be estimated from the equations [8.32] & [8.33].

The spillway surface will not be protected from cavitation erosion when the air concentration near the spillway floor falls below a minimum amount of 5 to 10 % (chapter 2) and this means an average air concentration \( C_{\text{mean}} \) below 30 %.

On a steep spillway (\( \alpha > 20^\circ \)) the air concentration distribution downstream of an aerator will tend to the equilibrium air concentration distribution (\( C_e > 30 \% \)). If the average air concentration at the start of the downstream flow region is high enough (\( C_{\text{mean}} > 25-30 \% \)) all the length downstream of the first aerator will be protected and no additional aerator will be required as long as \( \alpha > 20^\circ \). If the spillway slope becomes lower than 20° or for a flat spillway (\( \alpha < 20^\circ \)) the flow may be de-aerated and an additional aerator will be required when the average air concentration \( C_{\text{mean}} \) becomes lower than 30 %.

8.5 Conclusion

A complete analogy between the flow downstream of an aerator and the flow on a free-surface aerated spillway has been developed. In the gradually varied flow region downstream of the point of inception a simple analysis based on the continuity equation
and the energy equation provides two simultaneous differential equations which can be solved by numerical method and reproduce air entrainment on a spillway.

The same equations may be derived in the flow downstream of an aerator. Predictions of air entrainment can be computed by this method and this provides a satisfactory method to calculate the aerator spacing. However the estimation of the rise bubble velocity in the downstream flow region requires additional prototype measurements.

The distribution of air concentration in the gradually varied flow region downstream of an aerator seems to be almost independent from the quantity of air supplied by the aerator. For the flow in such a region the aerator acts like inducing the commencement of air entrainment far upstream of the real position of the device.
CHAPTER NINE

CONCLUSION

1. The Mechanisms of Air Entrainment

The air entrainment above an aerator is characterized by: 1- air entrainment through the upper and lower air-water interfaces of the water jet called nappe entrainment, 2- plunging jet entrainment at the intersection of the jet and the rollers and 3- air recirculation within the cavity below the jet.

When the jet leaves the deflector the pressure distribution within the flow changes from an almost hydrostatic pressure gradient to a negative pressure gradient and the air bubbles become subjected to a fall velocity engendered by the pressure gradient. Hence air entrainment through the upper free surface of the jet almost always occurs.

At the lower air-water interface of the jet, the air entrainment process is affected by the sudden change of shear stress and the component of turbulence perpendicular to the interface. The interaction between the air recirculation in the cavity beneath the nappe (of finite volume) and the effect of the pressure gradient on the bubble velocity is an additional complicating parameter.

At the rear of the cavity below the jet air bubbles and air pockets are entrapped by the jet impinging on the free-surface of the pool formed. Experimental results [ERVINE et AHMED (1982)1] indicate that plunging jet entrainment occurs above a critical velocity \( V_c \) and for highly turbulent jet [ERVINE et al. (1980)2] obtained \( V_c \approx 0.8 \text{ m/s} \).

The air recirculation may occur through the roller surface and the experimental results suggest that this process may be affected by the pressure gradient in the cavity.

The experiments indicate the dominant effect of the nappe entrainment through both the upper and lower free-surfaces in the aeration processes for high velocity turbulent water jets.

2. The Aeration Region

The study of the air demand of the aerator provides the relationship between the air discharge supplied by the air inlets and the subpressure in the cavity. Combining with the pressure loss equation of the air supply system this gives the operating point of the aerator characterized by the subpressure beneath the nappe \( \Delta P \) and the air discharge \( Q_{air\_inlet} \).

From the flow conditions and the subpressure in the cavity the jet trajectory calculations provide: 1- the volume and shape of the cavity below the jet, 2- the position of the impact point, and 3- the angle of the jet with the spillway bottom.

In the free-surface aeration region the inner core of the jet is clear water and no air exchange occurs between the upper and lower regions of the flow. Simple considerations on the continuity equation show that the air concentration distribution, the velocity

1. A Scaling Relationship for a Two-Dimensional Vertical Drop Shaft
   D.A. ERVINE and A.A. AHMED

2. Effect of Turbulence Intensity on the Rate of Air Entrainment by Plunging Water Jets
   D.A. ERVINE, E. McKEOGH and E.M. ELSAWY
distribution and hence the air entrainment through the upper and lower interfaces are deduced from five parameters \( Y_{90}^{\text{lower}}, Y_{10}^{\text{lower}}, Y_{10}^{\text{upper}}, Y_{90}^{\text{upper}}, Q_{\text{air inlet}} \).

When the inner core of the flow becomes aerated (i.e. in the fully aerated flow region) air exchange may occur between the upper and lower regions of the flow but the mechanisms remain obscure. At the present time no instrumentation is available to measure the air exchange. The nappe entrainment is then directly function of the plunging jet entrainment, the air recirculation and the air discharge from the air supply system, and its quantification requires greater knowledge of the plunging jet entrainment and the air recirculation.

3. The Impact Point Region

The study of air entrainment in the impact point region is very complex. A strong de-aeration occurs in the impact region and the flow is subject to a rapid redistribution of air. This may be caused by the high pressure gradient at the impact point.

For high Froude numbers the quantity of air entrained within the flow at the end of the impact point region appears to be almost independent of the air supply system and to be little affected by the Froude number. Further investigations in this regions are required but it must be emphasized that this will require new instrumentation. The high turbulence level and the rapid change of direction of the flow in this region implies that two-dimensional measurements must be obtained.

4. The Downstream Flow Region

An analogy between the flow downstream of an aerator and free-surface aeration indicates that the air concentration distribution tends to the equilibrium distribution. Hence for steep spillway the flow will continue to be aerated downstream of the aerator and on flat spillway deaeration will occur downstream of the aerator.

A simple analysis on the continuity equations and the energy equation provides two simultaneous differential equations which can be solved and this provides an estimation the air concentration distribution, the velocity distribution and the rate of aeration or deaeration of the flow. The extensions of these calculations downstream of an aerator bring a new method for the determination of aerator spacing.
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