PART 1
CHAPTER ONE

INTRODUCTION

When water flows over a spillway there is a region of clear water with a growing boundary layer and when this reaches the free surface the turbulence in the boundary layer can initiate natural air entrainment. Over recent years some progress has been made in understanding this process and calculations can now be made in the air entraining region. When this air-water region reaches the spillway surface the fluid next to the surface is protected from cavitation damage.

With the increase in the height of dams and the greater spillway discharges per unit width, the air from the free surface does not reach the spillway surface. The irregularities on the spillway surface will in a high speed flow cause small areas of flow separation and in these regions the pressure will be lowered. If the velocities are high enough the pressure may fall to below the local vapor pressure of the water and vapor bubbles will form. When these are carried downstream into high pressure region the bubble collapses giving rise to high pressures and possible cavitation damage.

Experimental investigations show that the damage can start at clear water velocities of between 12 to 15 m/s and up to velocities of 20 m/s it may be possible to protect the surface by streamlining the boundaries, improving the surface finishes or using resistant materials (VOLKART and RUTSCHMANN [1984]1).

When air is present in the water the resulting mixture is compressible and this damps the high pressure caused by the bubble collapses (PETERKA [1953]2). If the velocities near the spillway bottom are sufficiently high, aerators must be introduced to prevent cavitation. Although these have been installed for some years, the mechanisms of air entrainment at the aerators and the slow movement of the air away from the spillway surface are not clear.

The object of this project is to study the mechanisms of air entrainment above an aerator and air distribution downstream of the aerator. A scale model of a slice of the spillway of the Clyde dam (New Zealand) was modified to carry out general studies on spillway aerators. New instrumentation and data acquisition systems were used to perform air concentration and velocity measurements in the air-water mixture.

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1 Air Entrainment Devices
P. VOLKART and P. RUTSCHMANN
Mitteilungen der Versuchsanstalt fur Wasserbau, Hydrologie und Glaziologie, N°72, Zurich, Switzerland, 1984.

2 The Effect of Entrained Air on Cavitation Pitting
A.J. PETERKA
Joint meeting paper. IAHR/ASCE, Minneapolis, USA. Aug. 1953.
CHAPTER TWO

CAVITATION AND AERATION

2.1 Cavitation

2.1.1 Mechanism of Cavitation

The fundamental process of cavitation (KNAPP et al. [1970]) is characterized by explosive growth of vapour bubbles. Bubble growth may occur either by a pressure reduction, a temperature rise or a combination of these processes.

The three main types of cavitation are: 1- the bubble appearance and growth caused by dynamic pressure reduction at essentially constant temperature (vaporous cavitation); 2- the growth of gas bubbles due to diffusion of dissolved gas, or diffusion of undissolved gas contents coming out of solution (gaseous cavitation) and 3- the usual boiling process with vapour bubbles growing continuously.

Another related case (KNAPP et al. [1970]) is the ventilation of cavities where gas is removed by entrainment without condensing.

Vaporous cavitation is the sudden expansion of a vapour bubble due to vaporization of the liquid at the bubble wall (fig. 2-1a). Gaseous cavitation is the relatively slow expansion of a gas bubble due to diffusion of dissolved or undissolved gases (fig. 2-1b and 2-1c). The growth and coalescence of vapour bubbles resulting from boiling yields vapour masses that condense slowly.

If a growing bubble (of vapour or gas) is subjected to a pressure increase (above the vapour pressure $P_v$, or above the gas pressure in the bubble), its growth will be arrested and reversed. The bubble will disappear by solution of gas and condensation of vapour (i.e. disappearance of bubble resulting from boiling process), by collapse (i.e. vaporous cavitation) or by a combination of the two (i.e. gaseous cavitation). Thus cavitation involves the entire sequence of events : bubble formation extending to bubble disappearance.

The requirements for inception of cavitation and the formation of a cavity is the presence of a nucleus or weak spot in the liquid. Nuclei-free liquids have very high tensile strengths. KNAPP et al. (1970) indicate that nuclei are probably associated with impurities including undissolved gas.

2.1.2 Effects of Cavitation

In hydraulic structures the liquid is water and the cavities are filled with vapour and air. When liquids flow at high velocities, the possibility exists that dynamic effects may cause
the local pressure to fall to vapour pressure. If this happens vapour bubbles or cavities form and subsequently collapse as they are carried out into higher local pressure regions.

Collapses occur implosively for a vapour-filled cavity with negligible gas content and the pressures resulting from bubble collapses can reach very high values in the adjacent liquid. HICKLING and PLESSSET (1964)\(^1\) showed that the maximum pressures are function of the initial nucleus size (i.e. the maximum pressure decreases when the initial nucleus size increases). This nucleus size in some case is a function of initial gas content and hence the presence of gas content may reduce the collapse pressure. The collapse of bubbles near a wall may induce cavitation erosion and this will be detailed later.

Cavitation (MILLER (1978)\(^2\)) is important because 1- it may restrict the flow capacity, 2- it may generate unacceptable noise (i.e. submarines), and 3- it may cause erosion and failure. Subsequent to cavitation damage, instabilities (i.e. erosion) may be induced by initial cavitation erosion and this may damage or destroy a system (i.e. Glenn Canyon dam).

### 2.1.3 Effect of Gas Content on Cavitation

If the gas is dissolved in the water, KNAPP et al. (1970) indicates that it will have a negligible influence on the effective tensile strength of the liquid but may affect the vapour pressure of the solution. At atmospheric pressure and ambient temperature the solubility of air in water is just under 2\% (KNAPP et al. [1970]).

Permanent pockets of undissolved gas or uncondensed vapour are a type of impurity that reduces the effective tensile strength of a liquid sample. Free-gas bubbles are weak points in the fluid where cavitation may develop and therefore the presence of gas is crucial in cavitation process for without it liquids could sustain large tension forces without rupture.

For low velocities HOLL (1960)\(^3\) showed that with high gas content, gaseous cavitation may occur at pressure higher than the critical pressure \(P_v\) for vaporous cavitation. But HOLL (1960) and RIPKEN and KILLEN (1962)\(^4\) indicated that the critical pressure for gaseous cavitation decreases asymptotically to \(P_v\) when the velocity increases and KNAPP et al. (1970) suggests that at high velocity gaseous cavitation may not exist even if the gas content is high. The magnitude of the velocity at which gaseous cavitation may not exist, has not yet been determined.

Fluids with very high gas content may cavitate at higher static pressure than clear water but this cavitation will not occur as much erosion in liquids with low gas content and this will be detailed later in this chapter.

### 2.1.4 Cavitation Number

The cavitation number is a local dimensionless measure used to characterize the susceptibility of local cavitation. When a body (fig. 2-2) is fixed in an established water flow with a local average flow velocity \(V_0\) and if the local pressure on its surface is reduced to or near \(P_v\) (water : \(P_v = 2.49 \times 10^5\) Pa at \(T = 21.1^\circ\)C and sea level, KNAPP et al. [1970]), cavitation may be expected to occur. The condition for the onset of cavitation is given by the relation:

\[\text{Cavitation Number} = \frac{P_v}{\rho g V_0^2} \]

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\(^{1}\)Collapse and Rebound of a Spherical Bubble in Water
\ R. HICKLING and M.S. PLESSSET

\(^{2}\)Internal Flow Systems
\ D.S. MILLER

\(^{3}\)An Effect of Air Content on the Occurrence of Cavitation
\ J.W. HOLL

\(^{4}\)Gas Bubbles : their Occurrence, Measurement and Influence in Cavitation Testing
\ J.F. RIPKEN and J.M. KILLEN
where $P_m$ is the minimum local pressure at any point $M$ on the surface of the body. Using Bernoulli's principle the above condition to avoid cavitation becomes:

$$\frac{P_o + \rho_w g h - P_v}{\frac{V_0^2}{2}} > \left(\frac{V^2}{V_0^2}\right) - 1$$

where $P_o$ is the pressure above the flow, $h$ is the water depth upstream of the body, $V_o$ is the average flow velocity upstream of the body and $V$ the local velocity taken outside of the boundary layer at a point $M$ where cavitation could occur. The term on the left of the inequality is called the cavitation number $\sigma$ and is equal:

$$\sigma = \frac{P_o + \rho_w g h - P_v}{\frac{V_0^2}{2}} > \left(\frac{V^2}{V_0^2}\right) - 1$$

[2.1]

The magnitude of the term on the right can be calculated for each position $M$ on a simple bodies (i.e. sphere) but for more complex configurations it must be obtained by experiments. Denoting this value $\sigma_c$ the condition for avoidance cavitation has form of the relation [2.1] :

$$\sigma > \sigma_c \quad \quad \quad \quad \quad [2.2]$$

If the system operates at $\sigma$ below $\sigma_c$, the lower the value of $\sigma$, the more severe is the cavitation action in a given system.

### 2.2 Cavitation Erosion

#### 2.2.1 Cavitation Pitting

When a cavitation bubble collapses or implodes close to or against a solid surface, an extremely high pressure is generated. This pressure acts on an infinitesimal area of the surface for a very short period of time and causes cavitation pitting.

![Fig. 2-3 - Model of jet collapse near wall - KNAPP et al. (1970)](image-url)

MAY (1987)\(^1\) indicates that damage to solid surfaces may be caused by 1- the impact of micro-jet created by the collapse of bubbles (fig. 2-3) which produce impact velocities up to 130 m/s, corresponding to an impact pressure of 150 MPa, and 2- shock waves resulting from the collapse of cavities which induce maximum impulsive pressures up to 20 MPa.

TOMITA and SHIMA (1988)\(^2\) introduce a third possibility is the interaction between a bubble and a shock wave which may also cause damage by inducing virtual jet velocities of over 200 m/s corresponding to a water hammer pressure exceeding 300 MPa.

\(^1\)Cavitation in Hydraulic Structures: Occurrence and Prevention
R.W.P. MAY

\(^2\)Mechanisms of Impulsive Pressure Generation and Damage Pit Formation by Bubble Collapse
Y. TOMITA and A. SHIMA
A succession of these high-energy impacts will damage almost any solid material. After an initial period of erosion with tiny craters or pits on surface, the erosion progress rapidly. This may be due to 1- the fatigue of the structure of the material after repeated loadings, 2- a relative vulnerability of the material beneath the surface and 3- a focus of the cavitation impacts caused by the geometry of the pits.

2.2.2 Cavitation Damage

In the past severe cavitation damage occurred on spillway and outlets. Table 2-1 presents typical cases of large cavitation erosion and damage. The information was obtained from the references: ASCE/USCOLD (1975), LOWE et al (1979), PALVEY (1982), BURGII et al. (1984), HOPPING and MASS (1987), and MEFFORD and MULLER (1987).

The solutions proposed for these problems are presented in the last column. Not all the remedies were successful (i.e. Karun Dam, Iran) and in the next sections, methods used to prevent cavitation erosion will be presented.

<table>
<thead>
<tr>
<th>Date</th>
<th>Place</th>
<th>Description</th>
<th>Remedy</th>
</tr>
</thead>
<tbody>
<tr>
<td>1983</td>
<td>Glenn Canyon Dam Colorado, USA</td>
<td>Tunnel spillway. Large erosion induced by initial cavitation.</td>
<td>Aeration slots.</td>
</tr>
<tr>
<td>1982</td>
<td>Stampede Dam Nevada, USA</td>
<td>Concrete damage in the outlets works.</td>
<td>Air slot.</td>
</tr>
<tr>
<td>1977</td>
<td>Karran Dam Iran</td>
<td>Cavitation erosion on open channel spillway induced by surface irregularities.</td>
<td>Fibre concrete and epoxy. Repair not satisfactory.</td>
</tr>
<tr>
<td>1972</td>
<td>Libby Dam Montana, USA</td>
<td>Cavitation erosion of the outlets.</td>
<td>Steel fibre concrete and aeration slits.</td>
</tr>
<tr>
<td>1970</td>
<td>Clear Creek Dam Colorado, USA</td>
<td>Cavitation erosion of the concrete in outlet conduit.</td>
<td>Installation of air vents.</td>
</tr>
<tr>
<td>1967</td>
<td>Turtle Creek Dam Kansas, USA</td>
<td>Cavitation in the concrete floor downstream of the seal place.</td>
<td>Steel plate.</td>
</tr>
<tr>
<td>1967</td>
<td>Yellowstone Dam Montana, USA</td>
<td>Tunnel spillway. Cavitation damage through the lining into the foundation rock.</td>
<td>Air slots and epoxy bounded concrete.</td>
</tr>
<tr>
<td>1966</td>
<td>Aldes-Davilla Dam Portugal</td>
<td>Cavitation erosion in the auxiliary tunnel spillway.</td>
<td>Aeration.</td>
</tr>
<tr>
<td>1964</td>
<td>Palissades Dam Idaho, USA</td>
<td>Cavitation damage downstream of the gates.</td>
<td>Air slot.</td>
</tr>
</tbody>
</table>

1Erosion of Concrete in Hydraulic Structures
ACI Committee 210

2Lessons from Dam Incidents. USA
Committee on Failures and Accidents on Large Dams

3Some Experiences with High Velocity Flow at Tarbela Dam Project
J. LOWE, H.D. BANGASH and P.C. CHAO
13th Congress ICOLD, New Delhi, India 1979, Q. 50 R. 13.

4Predicting Cavitation in Tunnel Spillways
H.T. PALVEY

5Operation of Glenn Canyon Dam Spillways
P.H. BURGII, B.M. MOYES and T.W. GAMBLE

6Cavitation Damage on the Karun Dam
P.N. HOPPING and G.R. MASS

7Cavitation Damage and Repair of Stampede Dam
B.W. MEFFORD and B.C. MULLER Jr
### 2.3 Prevention and Repair of Cavitation Damage

#### 2.3.1 Introduction

The methods of preventing cavitation damage are: 1- the decrease of the critical cavitation number (i.e. removal of the surface irregularities), 2- the increase of the resistance of the material surface to cavitation erosion (i.e. the use of steel fibre concrete), 3- a combination of two first methods (i.e. the use of steel lining) and 4- the aeration of the flow. This last method will be detailed in the next section.

In the first case (decrease of the critical cavitation number) no cavitation occurs as \( \sigma > \sigma_0 \) (eq. [2.23]). In the other cases (2, 3 & 4) cavitation occurs but the erosion resistance of the surface is increased by reinforcement of the material or by reduction of the cavitation pitting pressures (i.e. aeration). Cavitation damage is a function of the time of operation, the type of protection and the cavitation number (eq. [2.1]). FALVEY (1982) presents the results obtained from spillway tunnels.

As most of the hydraulic structures are made of concrete, the problems of cavitation erosion of concrete will dominate the next paragraphs.

#### 2.3.2 Construction Finish and Design

Flow curvature and surface protuberances cause local pressure reduction which could reach the vapour pressure to form vapour cavities or bubbles that move downstream. At higher pressures the vapour cavities can no longer exist and thus condense or collapse. Cavitation damage can result from collapses near a boundary (i.e. wall, bottom). Small surface irregularities: offsets (fig. 2-4), abrupt slope, holes, transverse grooves, protruding joints could trigger cavitation damage. Extensive laboratory research and prototype observations (BALL [1976]1, ARNDT et al. [1979]2) provide specific criteria for construction finish limits for particular cavitation numbers.

If some surface irregularities do not satisfy these criteria, cavitation and cavitation erosion occur. Cavitation damage can be prevented by reducing the size of these irregularities or by changing their shape according to the above references. The specified finishes to avoid cavitation are very restrictive: for high velocity flow \( V = 25 \text{ m/s} \) an offset height of 3 mm will produce cavitation (FALVEY [1982]). It is also possible that defects can occur on the surface concrete during the hydraulic structure's life and induce cavitation.

ARNDT et al. (1979) showed that an isolated roughness may cause cavitation at a lower velocity than with continuous roughness of the same height and suggested that this is due to higher velocities close to a smooth wall, compared to a rough wall.

In some cases when some protuberances cannot be avoided (i.e. gate slots, joints) it may be advantageous to roughen surfaces systematically.

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1 Cavitation from Surface Irregularities in High Velocity J.W. BALL

2.3.3 High resistance concretes

Recently some cavitating resistant materials have been used, but being expensive, they are reserved for small areas where cavitation cannot be avoided. The first of these materials is the high resistance and special concretes (i.e. fibre reinforced concrete and polymerized concrete).

On Aswan High dam (height : 111 m) and Nureks hydroelectric plant (height : 310 m), high resistance concrete (GALPERIN et al. [1971]) was used and the high velocity flows did not cause cavitation damage (fig. 2-5).

HOUGHTON et al. (1978) tested the performances of conventional, polymerized, fibre reinforced and polymerized steel fibre concretes subjected to high velocity water flow (V = 37 m/s). The results presented in fig. 2-6 show the depth of cavitation erosion of the test slabs as a function of the operation time. They indicate that steel fibre concrete and polymerized concrete present three times as much resistance to cavitation erosion as conventional concrete.

2.3.4 Steel plates - Epoxy Resin - Coating

Protection of the material surfaces is also possible with steel lining, epoxy resins, coatings or any combination of these materials. Steel linings are often used because of their higher resistance to cavitation pitting and their smooth surface. ABELEV et al. (1971) found that a layer of Nyrac coating applied to carbon steel reduces by almost one tenth the amount of erosion by cavitation.

MAY (1987) indicates that epoxy materials have a good cavitation resistance but great care must be taken of the connection between the resin and the concrete. Plastic and polymer coatings provided good cavitation erosion resistance and details of experimental results are presented by MAY (1987) and ABELEV (1971).

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1Cavitation in Elements of Hydraulic Structures
R.S. GALPERIN, K.K. KUZMIN, I.S. NOVIKOVA, A.G. OSKOLKOV, V.M. SEMENKOV and G.N.
TSEDRKOV

2Cavitation Resistance of Some Special Concretes
D.L. HOUGHTON, D.E. BORGE and J.A. Paxton

3Investigation of Relative Cavitation Resistance of Materials and Protective Coatings and Development of Measures against Cavitation Erosion of Hydraulic Structures Elements
A.S. ABELEV, B.G. KARELEV and I.V. PLOKHOTNIKOV
2.4 Aeration

2.4.1 Effect of Air on Cavitation

In presence of a gas content (paragraph 2.1.3) flows may cavitate at higher static pressures and MOUSSON (1937)\(^1\) and RASMUSSEN (1956)\(^2\) showed that substantial quantities of air produce a large reduction in damage rate.

RUSSELL and SHEEHAN (1974)\(^3\) suggest that entrained air is effective because: 1- if there is any air present in the vapour cavities it will cushion the cavity collapse and reduce the resulting water hammer pressure (HICKLING and FLESSET [1964]) and 2- the presence of air bubbles in the water will reduce the celerity of the shock wave, and hence the magnitude of the shock waves on the material surface.

PETERKA (1953)\(^4\) and RUSSELL and SHEEHAN (1974) performed experiments on concrete specimens and showed that air concentrations of 1-2 % reduce substantially the cavitation erosion and above 5-7% no erosion was observed. The results are plotted in figures 2-7 and 2-8 where C is the average air concentration of the flow at atmospheric pressure and ambient temperature. The tests were performed over two hours in each case and the flow velocities at the points where cavitation occurred are indicated on the figures. The results of RUSSELL and SHEEHAN were obtained with concretes of different compressive strengths.

![Graphs showing relation between weight loss and air concentration](image)

On the boundary surface the local free air concentration is zero. Indeed the shear stress applied to an air bubble at the surface would split it in micro bubbles of negligible volume (fig. 2-9). Air concentration distributions on prototype (CAIN [1978])\(^5\) and model (author's data) are plotted in fig. 2-10 in comparison with the computed equilibrium profiles (chapter

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\(^1\)Pitting Resistance of Metals of Metals Under Cavitation Conditions
J.M. MOUSSON
Trans. ASME, 59, pp. 399-408, 1937.

\(^2\)Some Experiments on Cavitation Erosion in Water Mixed with Air
R.E.H. RASMUSSEN

\(^3\)Effect of Entrained Air on Cavitation Damage
S.G. RUSSELL and G.J. SHEEHAN

\(^4\)The Effect of Entrained Air on Cavitation Pitting
A.J. PETERKA
Joint meeting paper, IAHR/ASCE, Minneapolis, USA, Aug. 1953.

\(^5\)Measurements within Self-Aerated Flow on a Large Spillway
P. CAIN
8). The results may be interpreted to indicate the presence of an air concentration boundary layer of about 10-15 mm near the spillway surface.

For particular conditions, figures 2-7 and 2-8 indicate that cavitation erosion will be reduced if the air concentration in the layers close to the bottom is above 1-2% and no damage will occur for air concentrations greater than 5-7%. In presence of a thick air concentration boundary layer, higher air concentrations may be required outside of that boundary layer.

![Fig. 2-9 - Shear stress on an air bubble at the surface](image)

2.4.2 Application - Aerator Devices

2.4.2.1 Introduction

The entrained air through the free surface of the flow may protect the spillway floor from cavitation damage if the free-surface aeration process provides a sufficient air concentration near the bottom (i.e. C > 7%). If there is not enough surface aeration (i.e. downstream of a gate) or if the tolerances of surface finish required to avoid cavitation are too severe (i.e. V > 30 m/s), air can be artificially introduced by devices called aerators and located on the spillway floor and sometimes on the side walls.

2.4.2.2 Design

A small deflection in a spillway structure (i.e. ramp, offset) tends to deflect the high velocity flow away from the spillway surface (fig. 2-11). In the cavity formed below the nappe, a local subpressure beneath the nappe (ΔP) is produced by which air is sucked into the flow (Gault, 1966).

On a spillway of slope α the three basic types of aerator are: 1- the groove characterized by its transverse area \( A_d \) (fig. 2-12c), 2- the offset of height \( h_s \) (fig. 2-12b) and 3- the ramp defined by its slope \( \phi \), the length \( L_{ramp} \) and the ramp height \( r \) (fig. 2-12a).

![Fig. 2-11 - Aerator device](image)

![Fig. 2-12 - Main aeration devices](image)
VISCHER et al. (1982) detailed the properties of the different types of aeration devices. Usually a combination of the three types (fig. 2-12d) provides the best design: the ramp dominates operation at small discharges while the groove provides space for the air supply and the offset enlarges the trajectory of the jet at higher discharges.

VOLKART and CHERVET (1983) have studied on model the behaviour of a large range of aerators and VOLKART and RUTSCHMANN (1984) present several examples of air supply system.

2.4.2.3 Applications
To illustrate different designs, details of spillways with aerators devices are now listed (table 2-2).

<table>
<thead>
<tr>
<th>Place</th>
<th>Flow conditions</th>
<th>Slope α°</th>
<th>Aerator device</th>
<th>Groove</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>V = 45 m/s</td>
<td>19.3°</td>
<td>tf = 0.2, 0.17, 0.15 m</td>
<td>No</td>
<td>Ad = 2.06 m²</td>
</tr>
<tr>
<td>Aliacra Dam</td>
<td>qw = 77 m³/s</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Argentina</td>
<td>V = 44.5 m/s</td>
<td>0.65°</td>
<td>θ = 4.1°, L = 4 m</td>
<td>No</td>
<td>Ad = 0.56 m²</td>
</tr>
<tr>
<td>Argentina</td>
<td>Q = 800 m³/s</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ullum Dam Argentina</td>
<td>Q = 2560 m³/s</td>
<td>14.0°</td>
<td>No</td>
<td></td>
<td>Aeration by lateral deflector (θ = 7.1°, h = 0.5 m).</td>
</tr>
<tr>
<td>Foz do Arela Brazil</td>
<td>V = 43 m/s</td>
<td>14.5°</td>
<td>θ = 7.125°, L = 10 m</td>
<td>No</td>
<td>Ad = 3.8 m²</td>
</tr>
<tr>
<td></td>
<td>qw = 156 m³/s</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fenglihan Dam China</td>
<td>Q = 1140 m³/s</td>
<td>26.56°</td>
<td>θ = 3.81°, L = 9 m</td>
<td>No</td>
<td>Ad = 0.9 m²</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.85°</td>
<td>θ = 7.1°</td>
<td>No</td>
<td></td>
</tr>
<tr>
<td>Calascuccia France</td>
<td>V = 31 m/s</td>
<td>4.17°</td>
<td>No</td>
<td></td>
<td>Ad = 1.07 m²</td>
</tr>
<tr>
<td></td>
<td>Q = 100 m³/s</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>King Tali Dam Jordania</td>
<td>qw = 62 m³/s</td>
<td>19.27°</td>
<td>θ = 9.9°, L = 8 m</td>
<td>No</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>19.6°</td>
<td>θ = 7.8°</td>
<td></td>
<td>Ad = 2.03 m²</td>
</tr>
<tr>
<td>M'Jara Dam Morocco</td>
<td>V = 35 m/s</td>
<td>50.2°</td>
<td>θ = 1.7°, L = 3 m</td>
<td>0.45 m</td>
<td>Ad = 3.4 m²</td>
</tr>
<tr>
<td></td>
<td>Q = 1400 m³/s</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clyde Dam New Zealand</td>
<td>V = 28 m/s</td>
<td>1.43°</td>
<td>θ = 7.3°, L = 1 m</td>
<td>0.18 m</td>
<td></td>
</tr>
<tr>
<td></td>
<td>qw = 80 m³/s</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tarbela Dam Pakistan</td>
<td>V = 49 m/s</td>
<td>5.02°</td>
<td>θ = 2.0 m, L = 3 m</td>
<td>0.025 m</td>
<td>Ad = 0.135 m²</td>
</tr>
<tr>
<td></td>
<td>Q = 2690 m³/s</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Manaro Dam Peru</td>
<td>V = 59 m/s</td>
<td>0.80°</td>
<td>θ = 0.06 m, L = 3 m</td>
<td>No</td>
<td>Ad = 0.77 m²</td>
</tr>
<tr>
<td></td>
<td>qw = 24 m³/s</td>
<td></td>
<td></td>
<td></td>
<td>Ad = 0.77 m²</td>
</tr>
<tr>
<td></td>
<td></td>
<td>24.9°</td>
<td>θ = 0.06 m, L = 3 m</td>
<td>No</td>
<td>Ad = 0.77 m²</td>
</tr>
<tr>
<td>Restitution</td>
<td></td>
<td>41.54°</td>
<td>No</td>
<td></td>
<td>Ad = 0.77 m²</td>
</tr>
<tr>
<td>Manaro I3 Peru</td>
<td></td>
<td>34.24°</td>
<td>No</td>
<td></td>
<td>Ad = 0.77 m²</td>
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<tr>
<td></td>
<td></td>
<td>36.94°</td>
<td>No</td>
<td></td>
<td>Ad = 0.77 m²</td>
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</table>

1Hydraulic Modelling of Air Slots on Open Chute Spillways
D. VISCHER, P. VOLKART and A. SIGENTHALER

2Air Slots for Flow Aeration
P. VOLKART and A. CHERVET

3Air Entrainment Devices
P. VOLKART and P. RUTSCHMANN
<table>
<thead>
<tr>
<th>Dam Name</th>
<th>Location</th>
<th>$V$ (m/s)</th>
<th>$q_w$ (m$^2$/s)</th>
<th>$\phi$ (°)</th>
<th>$\theta$ (°)</th>
<th>$L$ (m)</th>
<th>$D$ (m)</th>
<th>No</th>
<th>$A_d$ (m$^2$)</th>
<th>Aerator</th>
<th>Notes</th>
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<tr>
<td>Laibon Dam</td>
<td>Philippines</td>
<td>37</td>
<td>120</td>
<td>20.3°</td>
<td>5.7, 7.1°</td>
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<td></td>
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<td>0.425</td>
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<td>122</td>
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Table 2-2 - Examples of aerator devices on prototypes
CHAPTER THREE

MECHANISMS OF AIR ENTRAINMENT
DIMENSIONLESS PARAMETERS

3.1 The Mechanisms of Air Entrainment

3.1.1 Presentation

The regions of flow above a bottom aerator are on a long spillway (fig. 3-1): 1- the approach flow region, 2- the transition zone, 3- the aeration zone, 4- the impact point region, 5- the downstream flow region and 6- the equilibrium region.

The approach zone may be in a region where some of the surface is aerated. The transition zone coincides with the length of the ramp. The deflector changes the mean perpendicular pressure field and increases the shear stress on the spillway floor. This change alters the turbulent field and these have a strong influence on the lower air-water interface in the aeration zone.

Without a ramp there is still a pressure change at the lip of the aerator from a hydrostatic pressure distribution to a negative pressure gradient. Indeed both with and without the ramp the pressure on the upper and lower free surfaces at the lip are respectively atmospheric and the cavity pressure.

In the aeration region air is entrained by high intensity turbulent eddies close to the air-water interfaces and this type of aeration called nappe entrainment occurs on both the lower and upper free surfaces of the jet (ERVINE and FALVEY [1987]1). At the end of the deflector we observe a solid inner jet core of water (C = 0 %) which is reduced along the channel while the flow is aerated through the air-water interfaces and this region is called the free-surface aeration region. If the water jet is long enough, a fully-aerated jet region starts developing downstream of the point where the central part of the jet becomes aerated.

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1 Behaviour of Turbulent Water Jets in the Atmosphere and in Plunge Pools
D.A. ERVINE and H.T. FALVEY

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Fig. 3-1 - Flow regions above an aerator

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Fig. 3-2 - Impact point region
The bottom pressure attains its maximum at the impact point of the jet (fig. 3-2). The rollers at the rear of the cavity entrain an additional quantity of air by plunging jet entrainment. The flow is highly turbulent and a high energy loss occurs in the impact region. Variations of the position of the impact point occur and it is suggested that these are caused by subpressure oscillations which occur in the cavity below the jet.

The impact region is characterized by a rapid air concentration redistribution with a de-aeration occurring immediately downstream of the point of impact (fig. 3-2). At the end of the jet the flow is subject to a rapid change of pressure distribution from a negative pressure gradient to a maximum pressure gradient at the impact point (higher than the hydrostatic pressure gradient) and finally to the hydrostatic pressure distribution far downstream. The high pressure gradient in the impact region may cause the strong de-aeration observed at the free surface.

The downstream flow region includes a rapidly varied flow region and a gradually varied flow region. This ends when the air concentration and velocity distributions reach the equilibrium flow region far downstream of the aerator.

3.3.2 Discussion

The study of air entrainment on spillway aerator is complex because of the interaction between the different air entrainment processes and it will be shown later that the air entrainment (fig. 3-3) above an aerator is characterized by: 1- air entrainment through the upper and lower free surfaces of the water jet called nappe entrainment. 2- plunging jet entrainment at the intersection of the water jet and the rollers, and 3- air recirculation in the cavity below the jet.

Let consider the control volume ABCDEF in the flow region above the aerator (fig. 3-4). We define: $q_{air}^{upper}$ the net air entrainment through the upper free surface BC, $q_{air}^{lower}$ the net air entrainment through the lower-air-water interface AF, $q_{air}^{plung}$ the plunging jet entrainment, $q_{air}^{recirc}$ the air recirculation and $Q_{air}^{inlet}$ the air discharge supplied by the air inlets.

The continuity equation for the air phase in the control volume ABCDEF of the flow is:

$$q_{air}^{CD} - q_{air}^{AB} = q_{air}^{upper} + q_{air}^{lower} + q_{air}^{plung} - q_{air}^{recirc}$$  \[3.1\]

where $q_{air}^{AB}$ is the initial quantity of air entrained within the flow through AB and $q_{air}^{CD}$ is the quantity of air entrained within the flow through CD.

The continuity equation applied to the cavity below the jet is:

$$\frac{q_{inlet}}{W} + q_{air}^{recirc} = q_{air}^{lower} + q_{air}^{plung}$$  \[3.2\]

The combination of the equations [3.1] and [3.2] yields:

$$q_{air}^{CD} - q_{air}^{AB} = q_{air}^{upper} + \frac{Q_{air}^{inlet}}{W}$$  \[3.3\]

This equation suggests that the air entrainment above an aerator is only function of the air discharge provided by the air supply system and the net air entrainment through the upper air-water interface of the jet. These parameters are function of the fluid properties, the spillway and aerator geometry, the geometry of the air inlets, the flow properties, and the undewater cavity properties.

The air discharge supplied by the air inlets is usually studied as a function of the subpressure in the cavity and the flow conditions for a given aerator geometry. This relationship is called the air demand of the aerator and will be detailed in the chapter 6. The subpressure in the cavity is obtained by combining the air demand and the pressure loss characteristic of the air supply system (chapter 6). Hence the jet trajectory calculations (chapter 5) will provide: 1- the position of the impact point, 2- the geometry of the cavity beneath the nappe and 3- the angle of the jet with the spillway surface.
We are not able to measure the net quantity of air entrained through the upper free-surface of the jet. However in the aeration region a method to estimate the quantity of air entrained within the flow from air concentration and velocity measurements will be presented in the chapter 7 and experimental results obtained on spillway model will be shown. From these informations an estimate of the air entrainment through the upper surface may be obtained in the free surface aeration region but no information is available in the fully aerated region.

Hence the equations [3.1] & [3.2] should be used rather than the equation [3.3]. The interactions between the air discharge $Q_{air}^{inlet}$, the plunging jet entrainment $q_{air}^{plung}$, the air recirculation $q_{air}^{recirc}$, the lower nappe entrainment $q_{air}^{lower}$ and the upper nappe entrainment $q_{air}^{upper}$ are unknown.
The processes occurring in the impact region remain unclear. It must be emphasized that the air concentration and velocity measurements are not accurate because of the high turbulence and rapid redistributions of air concentration and velocity occurring in this jet in the impact region.

Because the air entrainment in the impact region is not known, it is difficult to estimate the quantity of air entrained within the flow at the start of the downstream flow region and hence the flow conditions at this position. However experimental data obtained at the start of the downstream flow region will be presented in chapter 8 and the air entrainment in this region will be developed from the analysis of the continuity and energy equations using the same method as that developed by WOOD (1985).¹

### 3.2 Parameters

The relevant parameters needed for any dimensional analysis come from the following groups:

A- Fluid properties

These consist of: 1- the pressure above the flow P₀ (Pa), 2- the density of water ρₘ (kg/m³), 3- the density of air ρₐir (kg/m³), 4- the dynamic viscosity of water μ (N.s/m²), 4- the surface tension of the water σ (N/m), 5- the vapour pressure at the experiment temperature Pᵥ (Pa), and 6- the acceleration of gravity g (m/s²) (g = 9.8050 m/s² in Christchurch, New-Zealand).

B- Spillway and aerator geometry

These consist of: 1- the spillway slope α, 2- the spillway width W (m), 3- the roughness of the spillway surface kₘ (m), 4- the offset height tₘ (m), 5- the groove length Lₙ (m), 6- the duct area below the spillway surface A₇ (m²), 7- the angle between the ramp and the spillway φ, and 8- the ramp length L_ramp (m).

C- Geometry of the air inlets

D- Flow properties

These consist of: 1- the velocity distribution, 2- the air concentration distribution at the end of the approach flow, 3- the distribution of the axial component of the turbulent velocity, and 4- the distribution of the lateral component of turbulent velocity.

Normally the distributions of these parameters are replaced by their mean value: 1- the water depth of the flow in the approach zone d (m), 2- the flow velocity V (m/s), 3- the root mean square of axial component of turbulent velocity u' (m/s), and 4- the root mean square value of lateral component of turbulent velocity v' (m/s).

E- Underwater cavity properties

These consist of: 1- the water jet length L_jet (m), 2- the difference between atmospheric pressure and air pressure beneath the nappe AP (Pa), and 3- the air discharge through the ducts Q_air_inlet (m³/s).

¹Air Water Flows
I.R. WOOD
21st Congress IAHR, Aug. 1985, Melbourne, Australia.
F. Downstream flow properties

These consist of: 1- the air concentration distribution downstream of the aerator C, 2- the flow depth, and 3- the velocity distribution.

The study on spillway model uses the atmospheric pressure as pressure above the flow and when the vapour pressure $P_v$ is small compare to the atmospheric pressure and the pressure in the cavity ($P_0 - \Delta P$), the parameters $P_0$ and $P_v$ may be neglected. LAALI (1980)\(^1\) performed experiments with variations of the pressure above the flow and his results will be discussed later.

ERVINE and PALVEY (1987) showed that the lateral component of turbulence intensity $v'$ is proportional to the axial turbulence intensity $\frac{v'}{V}$ and for an axisymmetric jet their results fitted: $\frac{v'}{V} = 0.38 \times \frac{u'}{V}$. For a two-dimensional plane jet it is reasonable to assume that there will be a similar relationship and this enables to replace $u'$ and $v'$ by the independent parameter $u$.

The parameters required to design an aerator are:
- the air discharge $Q_{air, inlet}$ ($m^3/s$),
- the air concentration near the floor downstream of the aerator $C_b$,
- the difference between the atmospheric pressure and the pressure in the cavity beneath the nappe $\Delta P$ (Pa), and
- the water jet length $L_{jet}$ (m).

Each of these design parameters is a function of the initial independent parameters:

$$L_{jet}, Q_{air, inlet}, \Delta P, C_b = f(q_w, p_{air}, \beta, \sigma, \alpha, \omega, k_t, \omega, L_t, A_d, \delta, L_{ramp}, V, d, w)$$ \hspace{1cm} [3.4]

### 3.3 Dimensionless Numbers

The variables above give the following dimensionless numbers:

- the nondimensional air discharge $\beta_{inlet} = \frac{Q_{inlet}}{Q_w}$
- the air concentration at the bed $C_b$
- the length of the jet $\frac{L_{jet}}{d}$
- the dimensionless geometric variables $\frac{k_s}{d}, \frac{l_s}{d}, \frac{L_t}{d}, \frac{A_d}{d} = \frac{d}{W}, \frac{d}{L_{ramp}}$
- the Froude number $Fr = \frac{V}{\sqrt{g \times d}}$
- the Reynolds number $Re = \frac{\rho_w \times V \times d}{\mu}$
- the Weber number $We = \frac{V}{\sqrt{\sigma \times \rho_w \times d}}$
- the Euler number $Eu = \frac{\Delta P}{\rho_{air}}$

---

\(^1\)Scoulement Ventilé. Étude de l'entraînement d'air. Cas d'une cavité formée entre un jet plan et une paroi solide.

A.R. LAALI

- the density ratio \( \frac{\rho_{\text{air}}}{\rho_w} \)
- the turbulence intensity \( T_u = \frac{U'}{\bar{V}} \)

Any combination of these numbers is also dimensionless and may be used to replace one of the combinations. It will be shown later that it is convenient to replace the Euler number by a pressure gradient number \( P_N \) defined as

\[
P_N = \frac{\Delta P}{\rho_w \cdot \bar{g} \cdot d}
\]

This number is obtained from the others by the relation:

\[
P_N = \left( \frac{P_f}{R_e} \right) \cdot \frac{\rho_{\text{air}}}{\rho_w}
\]

For the study of cavity at the rear of a hydrofoil other workers [LAALI and MICHEL (1984)] have used, in place of the air discharge \( \dot{m}_{\text{inlet}} \) and the pressure gradient \( P_N \), 1- the volume air flow coefficient defined as

\[
C_{QV} = \frac{\dot{m}_{\text{inlet}}}{V} \cdot [W \cdot (t_2 + \tau)]
\]

and 2- the relative subpressure defined as

\[
\Sigma_{\text{gmsa}} = \frac{\Delta P + \rho_w \cdot \bar{g} \cdot d}{\frac{1}{2} \cdot \rho_w \cdot \bar{V}^2}
\]

The coefficient \( C_{QV} \) is equal to the ratio of the mean air velocity in the cavity over the flow velocity in the jet. For vapour cavity \( (P_{\text{cavity}} = P_f) \) the number \( \Sigma_{\text{gmsa}} \) equals the cavitation number \( \sigma_v \). The significance of these numbers will be discussed later but both coefficients may be deduced from the others:

\[
C_{QV} = \beta_{\text{inlet}} \cdot \frac{d}{t_2 + \tau}
\]

\[
\Sigma_{\text{gmsa}} = 2 \cdot \left( \frac{P_N + 1}{P_f^2} \right)
\]

Studies are performed on geometrically similar models and it is convenient to use a slice model. Side effects may appear due to the boundary layers on the side walls. However if these boundary effects are assumed small, the problem becomes a two-dimensional study. The nappe subpressure is usually controlled by valves on the air inlet systems and this enables the underpressure to be treated as an independent parameter. In addition the density ratio is almost constant.

From these considerations the relationship [3.4] is rewritten in terms of dimensionless parameters:

\[
\left( \frac{L_{\text{jet}}}{d} , \beta_{\text{inlet}} , C_0 , R_e , W_e , \alpha , \frac{k_2}{d} , \frac{t_2}{d} , \frac{L_2}{d} , \frac{A_d}{d} , \frac{L_{\text{inlet}}}{d} , \frac{L_{\text{tamp}}}{d} , T_u , P_N \right) \quad \text{-- [3.5]}
\]

\[\footnotesize{1} \text{ Air Entrainment in Ventilated Cavities : Case of the Fully Developed "Half-Cavity"}\]
A.R. LAALI and J.M. MICHEL
3.4 Air Entrainment Similitude

There are a multitude of phenomena that may be important in an air entraining flow and in most cases it will only be possible to model the most dominant mechanism. WOOD [1986] suggests that, when there is excess of transport capacity for the entrained air in the flowing fluid in both the model and the prototype, only the entrainment process need to be modeled and in this case simple Freude number modelling is possible.

For the case of two-dimensional plunging jet the entrainment is by vortices with axes perpendicular to the flow direction and studies CASTELEYN et al. [1977]; ERVINE and AHMED [1982] showed that \( Q_{air} = K \cdot (V \cdot V_C)^n \) where \( V \) is the jet velocity, \( K \) a constant and \( V_C \) the velocity at which air entrainment commences.

For a vertical jet the characteristic velocity \( V_C \) is given by
\[
V_C = \frac{4}{\pi} \left( \frac{\mu \cdot g}{\sigma \cdot \rho_w} \right)^{1/3} \left( \frac{L_t}{g \cdot \mu^2} \right)^{1/3} \left( \frac{\rho_w}{\sigma^2 \cdot \rho_w} \right)^{1/3}
\]

where \( \frac{2}{\sigma^2} \cdot g \) is a length scale based on the fluid properties and the acceleration of gravity,
\( \frac{2}{\mu \cdot g} \) is a velocity scale based on all the fluid parameters and \( Z = \frac{8}{\sigma^2 \cdot \rho_w} \) is called the liquid parameter.

Experimental results obtained by ERVINE et al. (1980) are plotted (fig. 3-7) and this suggests that the critical velocity does not depend on the diameter of the jet and hence does not depend on the larger eddy sizes. This implies that the critical velocity depends on an eddy scale determined by the fluid properties and for constant fluid properties the equation [3.6] becomes:
\[
V_C = \frac{4}{\pi} \left( \frac{\mu \cdot g}{\sigma \cdot \rho_w} \right)^{1/3} \left( \frac{L_t}{g \cdot \mu^2} \right)^{1/3} \left( \frac{\rho_w}{\sigma^2 \cdot \rho_w} \right)^{1/3}
\]

---

1Air Water Flow in Hydraulic Structures
I.R. WOOD
Lectures notes, University of Canterbury, New Zealand, 1986.

2Air Entrainment in Siphons-Scale Model Tests and an Extrapolation
J.A. CASTELEYN, P.A. KOLKMAN and P. VAN GROEN

3A Scaling Relationship for a Two-Dimensional Vertical Drop Shaft
D.A. ERVINE and A.A. AHMED

4Effect of Turbulence Intensity on the Rate of Air Entrainment by Plunging Water Jets
D.A. ERVINE, E. MCKEOGH and E.M. ELSAWY
ERVINE's results for a circular jet diameters of 6, 9, 14 and 25 mm show that for the larger turbulent intensities $Tu > 3\%$, $V_c$ is almost constant (fig. 3-7).

The angle $\xi$ of the jet with the liquid surface must also be important (fig. 3-6). Photographs of rollers at the end of a water jet show that the roller surface is almost horizontal. The angle is obtained from: $\xi = \alpha + \delta$ where $\alpha$ is the spillway slope and $\delta$ the angle between the water jet and the spillway surface. Equation [3.7] becomes:

$$V_c = \left( \frac{\mu * \sigma^2}{2 * \mu * \sigma^2 + \frac{1}{2} * P_w} \right) \left( \frac{\mu * \sigma}{\mu * \sigma^2 + \frac{1}{2} * P_w} \right)^{\frac{3}{2}}$$

[3.8]

**Quantity of air entrained**

For a vertical jet, WOOD (1986) considers a turbulent flow characterized by the turbulent scales of measures $u'$ and $L_t$. Considering the pressure variation $\Delta P$ between the upper and lower surface of the jet the dimensional analysis yields:

$$\frac{q_{air}}{(V - V_c)^2} = \left[ \frac{V - V_c}{\mu * \sigma^2} \right] \left[ \frac{\mu * \sigma}{\mu * \sigma^2 + \frac{1}{2} * P_w} \right] \left( \frac{P_w}{\mu * \sigma^2 + \frac{1}{2} * P_w} \right)^{\frac{1}{2}} \left( \frac{L_t}{\mu * \sigma^2 + \frac{1}{2} * P_w} \right) \left( \frac{\Delta P}{\mu * \sigma^2 + \frac{1}{2} * P_w} \right)^{\frac{1}{2}}$$

[3.9]

where $q_{air}$ is the air discharge per unit width. If the turbulent velocities and the scale of the turbulent eddies are related to the mean velocity $V$ and the characteristic depth $d$, and if the velocities are sufficiently large for

$$1 - \frac{V}{V_c} \approx 1$$

[3.10]

the dimensionless air discharge $\beta = \frac{q_{air}}{q_w}$ becomes $\beta = f(Re, Fr, We, Z)$. Or since $Z = \frac{We^6}{Fr^2 * Re^4}$, we get $\beta = f(Re, Fr, We, Z)$. For large velocities implied by the condition [3.10], the influence of the Reynolds number $Re$ is not likely to be important and the liquid parameter $Z$ is a constant. Hence:

$$\beta = f(Fr, We)$$

[3.11]

For plunging jet entrainment simple hydraulic modeling is possible as long as the condition [3.10] is satisfied on both the model and the prototype. In the other cases studies must be related to equation [3.9].

In the particular case of spillway aerator, plunging jet entrainment occurs at the intersection of the water jet and the rollers and additional parameters must be included: 1- the angle of the water jet with the roller surface $\xi$, 2- the effect the spillway floor on the rate of air entrainment and 3- the aeration of the air-water interfaces of the jet.

The study of air entrainment on spillway aerator is complex because of the interaction between different air entrainment processes (fig. 3-8): 1- air entrainment through the upper (1a) and lower (1b) free surfaces of the water jet called nappe entrainment, 2- plunging jet entrainment (2) at the intersection of the jet and the rollers, and 3- air recirculation (3) in the cavity below the jet.

Nappe entrainment occurs by high intensity turbulent eddies close to the jet free surface. ERVINE and FALVET (1987) suggest that some important processes in turbulent jets in the
atmosphere are dependent on Weber and Reynolds numbers and great difficulties appear in modelling free jets. Studies must then be related to equation [3.5].

The study of air entrainment on spillway chute aerators requires consideration of all the processes (plunging jet, nappe entrainment, recirculation,...) and the interactions between them.

The analysis of the mechanisms of air entrainment has shown that the main parameters of air entrainment above an aerator are the quantity of air entrained through the upper free-surface of the jet and the air discharge supplied by the air inlets.

At the present time the only way of estimating the air demand of an aerator is obtained from a large scale model of a slice of the spillway. For low Froude numbers results may be affected by three-dimensional effects in the approach flow region (chapter 4). However the results shown for low Froude numbers are similar to other experiments and therefore it is not possible to see this kind of effect.

Fig. 3-8 - Air entrainment above an aerator