Air Concentration Distribution in Self-Aerated Flow

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Discussion by

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The authors presented interesting new data. For completeness on the subject, the writer must state that the authors omitted two important studies. These are the works of WOOD (1984) and CAIN (1978).

Air concentration distribution

Downstream of the point of inception of air entrainment, WOOD (1984) developed a simple model to represent the turbulent diffusion of the entrained air within the flow. The model gives the shape of the air concentration distribution for all mean air concentrations:

\[
C = \frac{B'}{B' + \exp(-G' \cos \alpha \ y'^2)}
\]  

(A1)

where \(B'\) and \(G'\) are functions of the mean air concentration only (table A1), \(y' = y/Y_{90}\) and \(Y_{90}\) is the characteristics depth where \(C = 90\%\). The mean air concentration \(C_{\text{mean}}\) is defined in terms of \(Y_{90}\) and \(d_w\):

\[
(1 - C_{\text{mean}}) Y_{90} = d_w
\]  

(A2)

where \(d_w\) is the equivalent clear water flow depth.

Although equation (A1) was initially developed for fully-developed (or equilibrium) aerated flows, the equation was validated with model and prototype data in both the developing and fully-developed aerated flow regions (WOOD 1984, 1985, 1991, CHANSON 1993). An example is shown on figure A1. Note that, next to the channel bottom, the air content profile departs from equation (A1) as discussed by CHANSON (1994).

Equation (A1) is a simple expression based upon a physical analysis, taking into account the turbulent diffusion process and the buoyancy effects. It is more meaningful than the empirical fitting proposed by the authors (eq. (1)).

<table>
<thead>
<tr>
<th>(C_{\text{mean}})</th>
<th>(G' \cos \alpha)</th>
<th>(B')</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
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<td>0.721</td>
<td>1.5744</td>
<td>1.8641</td>
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</tbody>
</table>

Note: \(^{(a)}\) computed from STRAUB and ANDERSON’s (1958) data
Model and prototype data

Several researchers (e.g. ISACHENKO 1965, CAIN 1978, XI 1988) performed also air concentration measurements in the developing and fully-developed flow regions. The writer re-analysed several sets of data (CHANSON 1993) and, in each case, these data matched very closely equation (A1) (e.g. fig. A1). Further, the work of CAIN (1978) provides unique field data.

CAIN (1978) performed experiments on a prototype spillway (i.e. Aviemore dam spillway, New Zealand - $\alpha = 45$ deg., smooth concrete) downstream of the inception point of air entrainment. Some results are shown on figure A1. CAIN's (1978) work, "free" of scale effects, is an important milestone in the analysis of self-aerated flows. It is regrettable that the authors ignored CAIN's (1978) data.

List of symbols

- $B'$: integration constant of the equilibrium air concentration distribution;
- $C_{\text{mean}}$: depth averaged air concentration defined as : $(1 - Y_{90}) C_{\text{mean}} = d_w$;
- $d_w$: equivalent clear water depth (m) defined as:
  $$d_w = \int_{C=0\%}^{C=90\%} (1 - C) \, dy$$
- $G'$: integration constant of the equilibrium air concentration distribution;
- $q_w$: water discharge per unit width (m$^2$/s);
- $Y_{90}$: characteristic depth (m) where the air concentration is 90%;
- $y'$: dimensionless depth: $y' = y/Y_{90}$.

List of references

Fig. A1 - Air concentration distributions on Aviemore dam spillway (CAIN 1978) - Comparison with equation (A1) - $\alpha = 45$ degrees, $q_w = 2.23 \text{ m}^2/\text{s}$

![Graph showing air concentration distributions with data points and equation (A1) predictions.]

- DATA - $C_{\text{mean}} = 0.27$
- DATA - $C_{\text{mean}} = 0.36$
- DATA - $C_{\text{mean}} = 0.43$
- DATA - $C_{\text{mean}} = 0.50$

- --- EQ. (A1) - $C_{\text{mean}} = 0.27$
- --- EQ. (A1) - $C_{\text{mean}} = 0.36$
- --- EQ. (A1) - $C_{\text{mean}} = 0.43$
- --- EQ. (A1) - $C_{\text{mean}} = 0.50$