A General Correlation for Turbulent Velocity Profiles of Dilute Polymer Solutions

Une Relation Générale pour les Profils de Vitesse Turbulente dans les Solutions de Polymère Dilué

1. Introduction

The writer would like to congratulate the authors for their complete and comprehensive study of turbulent velocity profiles of dilute polymer solutions. He wishes to draw their attention to the similarity between the mechanisms of drag reduction due to the injection of polymers and the reduction of friction factor due to air entrainment in high velocity open channel flows.

2. Drag reduction

The drag reduction observed by the authors is caused by the introduction of small quantities of linear high molecular weight polymers. VIRK (1975) suggested that the interactions between turbulent bursts and macromolecules of polymer in the elastic layer are responsible for the drag reduction.

In self-aerated flows, the presence of light particles (i.e. air bubbles) in the flow layers next to the bottom is expected to reduce the shear stress between the flow layers. The re-analysis of the data of JEVDJEVICH and LEVIN (1953), STRAUB and ANDERSON (1958) and AIVAZYAN (1986) shows a reduction of the friction factor when the average air concentration increases. The results are presented in figure 1 where the ratio $f_e/f_D$ is plotted as a function of the average air concentration $C_{mean}$, $f_D$ being the non-aerated friction factor calculated using the Colebrook-White formula and $f_e$ being the aerated flow friction factor.

2. Velocity distribution

CAIN and WOOD (1981) showed that the velocity distribution of self-aerated flows on Aviemore dam obtained by CAIN (1978) can be approximated by:

$$\frac{V}{V_{90}} = \left(\frac{y}{Y_{90}}\right)^{1/n}$$

(29)

where the exponent $n$ for the roughness of the Aviemore dam ($k_s = 1$ mm) is : $n = 6.0$, $y$ is the distance normal to the bottom, $Y_{90}$ is the characteristic depth where the air concentration is 90% and $V_{90}$ is the velocity at $Y_{90}$. Aviemore's measurements were made in gradually varied flows with the mean air

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1Natural aeration occurring at the free surface of high velocity flows is referred as free surface aeration or self-aeration.
concentration in the range 0 to 50%, and the equation (29) does not depend on the mean air concentration. Hence the presence of air does not greatly affect the velocity distribution.

Neglecting the law of the wake, the power law (eq. (29)) may be rewritten as a logarithmic distribution:

\[
\frac{V}{V^*} = \frac{1}{K} \ln \left( \frac{y}{d} \right) + \frac{n+1}{n} \sqrt{\frac{8}{f_D}}
\]

non-aerated flow (30a)

or

\[
\frac{V}{(V^*)_e} = \frac{1}{K} \ln \left( \frac{y}{Y_{90}} \right) + \frac{n+1}{n} \sqrt{\frac{8}{f_e}}
\]

aerated flow (30b)

where \((V^*)_e\) is the shear velocity of aerated flow, and \(K\) the Von Karman constant \((K = 0.41)\). Using the notation of the authors, the combination of these equations leads to:

\[
(U^+)_e = U^+ + \Psi(f_D, C_{mean})
\]

(31)

where \((U^+)_e\) is the normalized local velocity of aerated flows and \(\Psi\) is a function of the friction factor and the mean air concentration:

\[
\Psi = \frac{1}{K} \ln (1 - C_{mean}) + \frac{n+1}{n} \sqrt{\frac{8}{f_D}} \left( \frac{n}{n+1} \sqrt{\frac{f_D}{f_e}} \frac{V_{90}}{U_w} - 1 \right)
\]

(32)

where \(U_w\) is the flow velocity, and \(f_e/f\) and \(V_{90}/U_w\) are function of \(C_{mean}\) only (CHANSON 1992). For the measurements on Aviemore dam, the Darcy coefficient \(f_D\) is in the range 0.021 to 0.023, and the equation (32) may be estimated as:

\[
\Psi = 55.9 * C_{mean}^{3.31}
\]

(33)

The equation (31) implies : \(\alpha = 1, \beta = 0\) and \(\Gamma = \Psi\), but the correlation between the drag reduction and the modification of the velocity distribution in self-aerated flows is not yet clear as discussed by CHANSON (1992).

3. Air Concentration distribution

An interesting aspect of the drag reduction in self-aerated flows is the interaction between the drag reduction, the velocity profile, and the air concentration distribution next to the channel bottom.

A re-analysis of the air concentration measurements of CAIN (1978) obtained on prototype spillway and CHANSON (1988) obtained on spillway model shows consistently the presence of an air concentration boundary layer (CHANSON 1989). Some of the data are presented on figure 2 and compared with a simple model of air bubble diffusion developed by WOOD (1984) outside of the air concentration boundary layer:

\[
C = \frac{B'}{B' + \exp(-G' \cos \alpha \cdot y^2)}
\]

for \(y > \delta_{ac}\) (34)

where \(G'\cos \alpha\) and \(B'\) are function of the mean concentration only (WOOD 1984, CHANSON 1989) and \(y' = y/Y_{90}\). In the air concentration boundary layer the air concentration distribution may be estimated as:

\[
C = k \sqrt{\frac{y}{\delta_{ac}}}
\]

for \(y < \delta_{ac}\) (35)

where \(\delta_{ac}\) is the air concentration boundary layer thickness, and \(k\) is a constant that satisfies the continuity between the equations (34) and (35):

\[
k = \frac{B'}{B' + \exp(-G' \cos \alpha \cdot \left( \frac{\delta_{ab}}{Y_{90}} \right)^2)}
\]

(36)

where \(\delta_{ab} = 10 \text{ to } 15 \text{ mm (CHANSON 1989).} \)
The presence of air bubbles in the flow layers next to the spillway surface is expected to play a major role in the alterations of the turbulence. By analogy with dilute polymer solutions, the air concentration boundary layer might play a role similar to the elastic sub-layer in the drag reduction processes. In the flow layers next to the floor, the air bubble sizes are smaller than 1 mm, and the bubbles act as rigid spheres. The air bubbles that are larger than the small eddies sizes, are expected to block or to interact with the turbulent eddies. Further the writer believes that such rigid air bubbles behave as macro-molecules of polymer by blocking the turbulence bursting processes in the elastic sub-layer as suggested by VIRK (1975).

With respect of the study of dilute polymer solutions, the writer wonders if the distribution of polymer molecules is homogeneous across the pipe or if a similar particle boundary layer was observed.

**Notations**

- $B'$: integration constant of the air concentration distribution
- $C$: air concentration defined as the volume of air per unit volume;
- $C_{\text{mean}}$: mean air concentration defined as $(1 - C_{\text{mean}}) \times Y_{90} = \delta_a$;
- $D_H$: hydraulic diameter (m);
- $d$: equivalent clear water depth (m);
- $f_D$: Darcy coefficient for non-aerated flows;
- $f_e$: Darcy coefficient of aerated flows;
- $G'$: integration constant of the air concentration distribution;
- $K$: Von Karman universal constant;
- $k_s$: roughness height (m);
- $n$: exponent of the power law velocity distribution;
- $q_w$: water discharge per unit width (m$^2$/s);
- $U_{\text{av}}$: average flow velocity $U_{\text{av}} = q_w / d$;
- $(U^+)_{\text{e}}$: normalized local velocity of aerated flows;
- $V$: velocity (m/s);
- $V_{\text{*}}$: shear velocity of non-aerated flows (m/s);
- $(V^+)_{\text{e}}$: shear velocity of aerated flows (m/s);
- $Y_{90}$: characteristic depth where the air concentration is 90%;
- $y$: distance from the bottom measured perpendicular to the channel surface (m);
- $\delta_{ab}$: air concentration boundary layer thickness (m);

**References / Bibliographie**


Fig. 1 - Drag reduction in self-aerated flows

<table>
<thead>
<tr>
<th>Ref.</th>
<th>(k_s) (mm)</th>
<th>(k_s/D_H)</th>
<th>Re</th>
<th>(C_{mean})</th>
<th>Comments</th>
</tr>
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<tbody>
<tr>
<td>(I)</td>
<td>10 to 20</td>
<td>0.015 to</td>
<td>8.3E+4</td>
<td>0.58 to</td>
<td>Prototype. Wide channel (W = 5.75 m). Stone lining.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.035 to</td>
<td>3E+7</td>
<td>0.66</td>
<td></td>
</tr>
<tr>
<td>(II)</td>
<td>0.71</td>
<td>3E-3 to</td>
<td>4.7E+5</td>
<td>0.15 to</td>
<td>Spillway model (W = 0.457 m). Artificial roughness.</td>
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<tr>
<td></td>
<td></td>
<td>1.6E-2 to</td>
<td>2E+6</td>
<td>0.73</td>
<td></td>
</tr>
<tr>
<td>(III)</td>
<td>0.1 to</td>
<td>5E-4 to</td>
<td>1.7E+5</td>
<td>0.21 to</td>
<td>Prototype and large spillway model (W = 0.25 to 6 m).</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.04</td>
<td>to</td>
<td>0.54</td>
<td></td>
</tr>
</tbody>
</table>

Note: (I) JEVDJEVICH and LEVIN (1953)  
(II) STRAUB and ANDERSON (1958)  
(III) AIVAZYAN (1986)
Fig. 2 - Air concentration distribution in self-aerated flows

<table>
<thead>
<tr>
<th>Ref.</th>
<th>$U_w$ (m/s)</th>
<th>$C_{mean}$</th>
<th>$f_e$</th>
<th>$(V_*)_e$ (m/s)</th>
<th>$\sqrt{f_e/8} \cdot U_w$</th>
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<td>(I)</td>
<td>17.7</td>
<td>0.43</td>
<td>0.0165</td>
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<tr>
<td>(II)</td>
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<td>0.40</td>
<td>0.0129</td>
<td>0.401</td>
<td></td>
</tr>
</tbody>
</table>

Note: (I) CAIN (1978)  
(II) CHANSON (1988)