On Wavelet Analysis of Auditory Evoked Potentials

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Abstract

Objective: To determine a preferred wavelet transform procedure for multi-resolution analysis (MRA) of auditory evoked potentials (AEP).

Methods: A number of wavelet transform (WT) algorithms, mother wavelets, and pre-processing techniques were examined by way of critical theoretical discussion followed by experimental testing of key points using real and simulated auditory brainstem response (ABR) waveforms. Conclusions from these examinations were then tested on a normative ABR dataset.

Results and Conclusions: The results demonstrate that optimal AEP WT MRA is most likely to occur when an over-sampled discrete wavelet transformation (DWT) is used, utilising a smooth (regularity $\geq 3$) and symmetrical (linear phase) mother wavelet, and a reflection boundary extension policy.

Significance: This study demonstrates the practical importance of, and explains how to minimise potential artefacts due to, four inter-related issues relevant to AEP WT MRA, namely: shift-variance, phase distortion, reconstruction smoothness, and boundary artefacts.
1. Introduction

The wavelet transform (WT) is a mathematical tool capable of deconstructing a signal into its component scales (frequencies), and then detailing how each scale evolves over time. This provides simultaneous access to time, amplitude, and scale (frequency) information, and therefore the ability to conduct efficient multi-resolution analysis (MRA) (Jawerth and Sweldens, 1994; Hess-Neilsen and Wickerhauser, 1996; Unser and Aldroubi, 1996; Blinowska and Durka, 1997).

Because of its MRA abilities, the WT has proven to be a valuable tool for the analysis of many electrophysiological potentials. Earlier examples of this success can be found in the electroencephalogram (EEG) (Schiff et al., 1994; Kalayci and Ozdamar, 1995), somatosensory evoked potentials (SEP) (Thakor et al., 1993), respiratory evoked potentials (Lim et al., 1995), and electrocardiogram (ECG) (Meste et al., 1994; Li et al., 1995). Because of these successes, the WT is now being widely applied to the MRA of auditory evoked potentials (AEP).

A typical AEP MRA would proceed by decomposing the signal into individual scales of interest. In this way, different versions of the original signal can be created that illustrate different features of the original signal (essentially different frequency bands). These features can then be analysed individually, free from interference from other features.

Figure 1 shows an example of a six level, dyadic, DWT decomposition of an ABR signal. Figure 2 shows an example ABR signal and its reconstructed ABR DWT signals at wavelet approximation scale A6, and detail scales D6, D5 and D4 (using the Daubechies 5 mother wavelet, as shown in Figure 3). Figure 2 clearly illustrates that A6 highlights the low frequency component of peak V, while D6 and D5 highlight the medium and high frequency components of peaks I, III, and V respectively.

The potential benefit of WT and MRA of AEP is reflected by a growing number of reports in the literature (Samar et al., 1995; Samar et al., 1999; Wilson, 2002). Examples include improved waveform morphology and detection (including automatic detection) for auditory ABR (Hanrahan, 1990; Bertrand et al., 1994; Popescu et al., 1999), auditory middle latency response (AMLR)
(Bertrand et al., 1994), auditory late latency responses (ALLR) (Hoppe et al., 2001) and the auditory P300 event related potential (Heinrich et al., 2001; Quian Quiroga 2000; Quian Quiroga et al., 2000, 2001a,b; Quian Quiroga and Zuijtelaar, 2002; Demiralp et al., 2001; Demiralp and Ademoglu, 2001). The WT has also been used to improve I-V inter-wave latency accuracy in ABR (Wei et al., 1998), de-noise single event related potential trials (Effern et al., 2000; Quian Quiroga and Garcia, 2003), analyse gamma band responses in P300 (Gurtubay et al., 2001), analyse passive auditory processing in children (Kolev et al., 2001), and automatically discriminate between awake and unresponsive states during propofol anaesthetic sedation for AMLR (Huang et al., 1999; Kochs et al., 2001). Finally, measures such as wavelet entropy have been shown offer additional understanding of brain dynamics (Quian Quiroga et al., 2001a, Rosso et al., 2001, Yordanova et al., 2002, 2003).

With the growing use of the WT in AEP analysis, however, comes a growing potential for its misuse. This paper aims to highlight some of the potential pitfalls of WT analysis with respect to the MRA of AEP signals in particular.

Part one of the paper examines a number of factors relevant to the selection of a WT algorithm and a mother wavelet. Each issue is discussed separately by way of a critical theoretical discussion, followed by experimental testing of key points using real and simulated AEP waveforms.

Based on these results, part two of the paper will apply a sample of preferred WT algorithms to a normative set of ABR waveforms. For consistency, each algorithm will use the same (preferred) mother wavelet and pre-processing protocol.

All of these results will then be used to highlight the desirable properties of a preferred WT procedure for MRA of AEP signals.

It should be noted that it is not a goal of this paper to propose a specific WT algorithm or a mother wavelet for all AEP applications. Rather, it aims to highlight the various properties that must be considered before tackling a specific AEP WT application.
1.1. Properties of AEP Signals

It is worthwhile at this point to define some of the properties of AEP signals that should be considered when deciding exactly how to perform MRA of AEP signals using the WT:

1. The frequency spectrum of AEP signals often contains more than one primary frequency component that is of clinical interest (Boston, 1981). For example, the frequency spectrum of the auditory brainstem response (ABR) consists of three main components, each of which contributes differently to different ABR waves in the time domain (Boston, 1981; Pratt et al., 1989).

2. AEP signals are transient (non-stationary) in nature and present with variable peak morphology, both within a single AEP signal and between different AEP signals (de Weerd, 1981). In addition, even “normative” waveforms can contain peaks that are missing in some “normal” subjects.

3. AEP signals are often over-sampled, that is they are sampled at a frequency much greater than the theoretical minimum of (greater than) twice the highest frequency of the input signal. There are a number of advantages to over-sampling which are well known, namely increased time resolution (greater accuracy when measuring peak latencies) and improved signal to noise ratio of the analogue to digital conversion process (as the quantisation noise is spread over the whole band, not just band of interest (Oppenheim et al., 1999, section 4.9).

4. AEP signals are smooth in appearance. That is, they are continuous in both value and gradient (at least). This smoothness is due both to their physiological source (as the synchronous firing of large collections of neurons) and the result of small variations in waveform latency during the ensemble averaging process (Woody, 1967; de Weerd, 1981).

5. AEP signals are captured in the presence of ongoing EEG activity that is normally considered as additive noise (Boston, 1981). In addition, there is a potential for muscle artefacts to be present in the waveforms, further distorting waveform morphology.
In this paper we shall illustrate the discussion primarily with reference to ABR waveforms. Because of the above properties are common to all AEP signals, however, the general principles illustrated in this paper are applicable to most (if not all) AEP signals.

1.2. The Wavelet Transform

For the purposes of this paper, only a brief, conceptual overview of the WT will be provided. For a full description, the reader is referred to the standard texts of Daubechies (1992), Strang and Nguyen (1996) and Mallat (1999).

Briefly, a wavelet (sometimes called a mother wavelet) can be thought of as a mathematical function with certain properties such as integration to zero and various degrees of compact support (amplitude decays to zero as time tends to infinity), smoothness, and symmetry. These features make wavelets ideal for analysing transient signals such as AEPs. Figure 3 shows examples of some commonly used mother wavelet functions.

A signal is analysed, or decomposed, by a mother wavelet by applying a WT. Conceptually, MRA using the WT can be described as follows:

1. A mother wavelet is chosen for the analysis.
2. The wavelet is placed at the start of the input signal.
3. The degree of correlation between the wavelet to this section of the signal is calculated (via the convolution integral).
4. The wavelet is shifted (translated) in time, and step 3 repeated until the whole signal has been analysed.
5. The mother wavelet is then scaled (dilated).
6. Steps 2 to 5 are repeated until the entire signal has been analysed at all desired scales.

In this way, wavelet analysis can be thought of as applying multiple matched filters that are looking for shifted and scaled versions of themselves within the input signal. If the WT finds a good match, then the identified component produces a large wavelet coefficient. Clearly, one of the keys
to wavelet analysis is the appropriate selection of a mother wavelet that in some sense “matches”
the features of interest in the input signal. This is one of the topics addressed later in this paper.

The outline of the WT given above is purely at the conceptual level. The reader should note
that there are a number of algorithms available for actually implementing the WT, or
approximations to it. For example, within the Mathworks Matlab© Software package one has a
choice of the Wavelet toolbox (as used in this paper) or the Wavelab Toolbox (Buckheit and
Donoho, 1995). In addition, Appendix B, of Mallat (1999), provides and extensive list of the
available freeware wavelet toolboxes.

In this paper, we shall refer to the WT in its broadest sense, that is, as a mathematical tool
for MRA. However, in accordance with Daubechies (1992), to further delineate the available
implementations of the WT we shall distinguish:

1. The continuous wavelet transform (CWT) as the discrete-time wavelet transform applying a
   (suitably sampled) continuous mother wavelet. In the CWT, the dilation parameter can vary
   continuously, but the translation parameter is discrete (as all the signals considered in this paper
   are discrete-time, i.e., sampled waveforms).

2. The discrete wavelet transform (DWT) as the discrete-time wavelet transform applying a
   discrete-time mother wavelet at (non-zero) integer scales only, i.e., both the dilation and
   translation parameters of the mother wavelet are discrete.

To further distinguish the different implementations of the DWT, we shall reserve reference
to the DWT as an orthogonal (or bi-orthogonal) DWT. That is, a critically sampled DWT with the
mother wavelet applied only at dyadic scales. We shall then refer to the redundant varieties of the
DWT by pre-pending a suffix that highlights the type of redundancy introduced.

In this paper we only consider dyadic decompositions of the input signal, where the mother
wavelet is dilated by a factor of two at each level, as is shown in Figure 1. This results in what is
known as a constant Q decomposition, where the central frequency and bandwidth of the wavelets
halve at each scale. The dyadic decomposition has desirable time-frequency properties and so is
commonly applied, however alternative decomposition strategies, known as wavelet packets, can also be designed whose time-frequency properties may be adapted to the input signal (Coifman et al., 1992). Although wavelet packets are not specifically considered in this paper, the issues we address are applicable.

Whilst there are numerous reports in the literature documenting the (apparently) successful application of the WT to AEP MRA, the WT can have significant problems dependent upon (in order of importance):

1. Selection of an appropriate WT algorithm, particularly with respect to the potential problem of shift variance.
2. Selection of an appropriate mother wavelet for the analysis, with particular emphasis on smoothness of the reconstructed waveforms and the potential problem of phase distortion. Note: the selection of the WT algorithm and the mother wavelet are co-dependent as not all mother wavelets are suitable for implementation by all WT algorithms. However, for the sake of clarity, they are treated separately (as far as possible) in this study.
3. Pre-processing that should be undertaken prior to wavelet analysis, particularly with respect to boundary extension policies.

These issues are well known, and discussed, in the engineering literature (Simoncelli et al., 1992; Unser and Aldroubi, 1996; Mallat, 1999; Kingsbury, 2000), but appear to be less well known by the practitioners of AEP signal analysis. Therefore, it is one of the primary aims of this paper to offer a (non-mathematical) tutorial as to the appropriate usage of the WT specifically for MRA of AEP signals.

2. PART ONE - Wavelet Considerations for AEP MRA

2.1. Consideration One - Selection of an Appropriate WT Algorithm

The DWT is (by far) the most widely used WT algorithm for multi-resolution AEP analysis (e.g., Bertrand et al., 1994; Samar et al., 1999; Huang et al., 1999; Quian Quiroga 2000; Quian
Quiroga et al., 2000, 2001a,b; Quian Quiroga and Zuijtelaar, 2002; Demiralp et al., 2001; Hoppe et al., 2001; Kochs et al., 2001).

The advantage of the DWT is its computational efficiency and, depending on the mother wavelets used, its (bi-) orthogonality. In particular, the DWT provides a sparse time-frequency representation of the original signal (the wavelet coefficients) that has the same number of samples as the original signal.

The disadvantage of the DWT is its shift variance (i.e., not shift invariant) (Simoncelli et al., 1992; Unser and Aldroubi, 1996; Mallat, 1999; Kingsbury, 2000). Shift variance results from the use of critical sub-sampling (often referred to as down-sampling) in the DWT. In this way, every second wavelet coefficient is discarded at each decomposition level. This is done both to reduce the amount of data that has to be analysed, and to enforce the implicit time-frequency uncertainty of the analysis (as the analysis becomes more certain about the frequency components of the signal, it becomes less certain about when they occur in time).

By critically sub-sampling, however, the resulting wavelet coefficients become highly dependent on which coefficients remain. This can lead to small shifts in the input waveform (e.g., due to changes in the latencies of the AEP waves) causing large changes in the wavelet coefficients, large variations in the distribution of energy between coefficients at different scales, and possibly large changes in the reconstructed waveforms (e.g., Figure 4).

Another way of describing this phenomenon is to consider the frequency response of the mother wavelets. As no realisable wavelet filter (i.e., with compact support) can have an ideal “brick-wall” frequency response (i.e., the attenuation in the stop band will be finite), aliasing will be introduced. That is, when the WT sub-bands (which nominally have half the bandwidth of the original signal) are sub-sampled by a factor of two, the Nyquist criteria is violated and frequency components above (or below) the cut-off frequency of the filter are aliased into the wrong sub-band (see Oppenheim et al., 1999, chapter 4).
It should be noted that the aliasing introduced by the DWT cancels out when the inverse DWT (IDWT) is performed using all of the wavelet coefficients (i.e., when the original AEP signal is reconstructed). This makes a DWT followed by an IDWT shift invariant only when all of the wavelet coefficients are used to perform the IDWT. As soon as coefficients are not included in the IDWT, e.g. a single sub-band is extracted, the aliasing no longer cancels out and the output is no longer shift invariant.

In justifying their use of DWT in AEP MRA, most authors cite its (bi-) orthogonality (e.g. Bertrand et al., 1994; Samar et al., 1995; Huang et al., 1999). Whilst an orthogonal transform has been shown to be important for data compression and fast calculations, such importance has not been shown for MRA (Mallat, 1996). In fact, to make the DWT orthogonal, critical sub-sampling must be used. In fact, if any sub-sampling is performed, then some degree of shift variance will occur.

The lack of reference to shift-variance in the AEP WT MRA literature has serious implications. For example, Huang et al., (1999) describe a method of predicting depth of anaesthesia based on the magnitude of just four wavelet coefficients from the A4 band of an auditory middle latency response DWT. As their method utilised a critically sampled DWT, however, the values of these four coefficients would have been sensitive to latency changes of the input waveforms, i.e., they would have been shift variant. Perhaps to mitigate this problem (by reducing the sub-band aliasing), the authors choose to utilise a much higher order mother wavelet (a 20th order Daubechies) than would normally be selected. In a similar study, Kochs et al., (2001) selected three DWT coefficients from the D4 sub-band. Their use of a much lower order (3rd order) Daubechies mother wavelet would have made the potential for shift variance even greater.

In one of the few examples of AEP WT MRA research that directly mentions the problem of shift invariance, Bertrand et al., (1994) describe WT protocols and filtering applications using ABR and AMLR signals as examples. Whilst mentioning the problem, they appear to understate the complex relationship between input signal shifts and changes to wavelet coefficients. In addition,
their claim that the problem does not exist with time-locked signals appears to be ill-founded, as evoked potential signals are only approximately time-locked intra-patient. There will always be shifts in the input signal due to both latency differences inter-patient and variations in stimulus levels intra-patient. These shifts will lead to shift variance. Whilst not mentioning shift variance directly, Effern et al., (2000) utilise the DWT to analyse auditory P300 signals, but perform latency corrections using a Woody filter (Woody, 1967). Whilst the Woody filter would have reduced the shift variance problem, it would not have eliminated it.

In seeking solutions to the problem of shift variance, several options can be considered. Common to them all is an attempt to eliminate or minimise the amount of sub-band aliasing that occurs by a combination of relaxing the critical sub-sampling criteria and reducing the transition bandwidth of the mother wavelet.

The first, and the simplest, way of making the DWT shift invariant is not to perform any sub-sampling at all. This we shall refer to as an over complete discrete wavelet transform (OCDWT), although it is probably more commonly referred to as the algorithme à trous (Mallat, 1999). Because there is no sub-sampling of data, the mother wavelet has to be dilated (by inserting zeros) at each level of the transform. Obviously, the OCDWT is shift invariant and can be used with any of the mother wavelets conventionally used with the DWT. However, the OCDWT requires additional computation, memory, and significant boundary padding as the mother wavelet doubles in length at every decomposition level. Also, the OCDWT is only strictly shift-invariant under circular convolution, i.e., when periodic boundary extension has been used. The absence of critical sub-sampling need not occur at every WT level, however, and approximate shift invariance could occur if critical sampling is stopped at levels above those of interest to the AEP MRA.

Another WT that performs no sub-sampling, and is therefore shift invariant, is the CWT (Mallat, 1999, Daubechies, 1992). For the CWT, the mother wavelet is a continuous function, e.g. either a first or second derivative of a Gaussian. An advantage of the CWT is that it can be applied at any scale directly, without the iterations required by the DWT. In addition, there is no need to
perform the inverse transform (in fact, the inverse transform often does not exist) as the AEP MRA
analysis is performed on the wavelet coefficients directly. This means that for a small number of
analysis scales, the CWT may be more computationally efficient than the DWT (assuming that
signal is over-sampled and the IDWT will be performed to reconstruct specific sub-bands in the
DWT analysis).

A more complex way of minimising shift variance is to build two wavelet decomposition
trees (with alternate phase sub-sampling), one for a mother wavelet with even symmetry and the
other for the same mother wavelet, but with odd symmetry. In this way, the dual tree complex
wavelet transform (DTCWT) (Kingsbury, 2000) measures both the real (even) and the imaginary
(odd) components of the input signal (hence the name complex wavelet transform). The DTCWT is
again approximately shift invariant, and offers both the magnitude and phase information of the
signal. As two decompositions have to be performed, however, computational memory
requirements are twice that of the DWT.

An alternative decomposition methodology which is not, strictly speaking, a WT (though it
is related, see Daubechies (1992), section 8.3.5) is the Laplacian pyramid (LP) (Burt and Adelson,
1983). The input signal is initially smoothed with a Gaussian filter and then critically sampled by a
factor of two. This signal is then up-sampled with a nearest neighbour interpolating filter and this
approximation of the original signal subtracted from the original signal. This error signal (which is
at the same sampling frequency as the original signal) then defines the detail information lost during
the smoothing, critical sampling (down-sampling), and up-sampling process. This process can be
iterated a number of times to produce a sub-sampled low pass signal and a number of error signals
equal to the number of levels of iteration. The original signal can be reconstructed by iteratively
interpolating the low-pass signal and then adding the error (detail) signal. The reason this transform
is called the Laplacian pyramid is that the error signals, which are effectively a difference of
Gaussian signals (with the second Gaussian having half the bandwidth of the first), are
indistinguishable from the Laplacian (second derivative) of a Gaussian (Marr and Hildreth, 1980).
Additional methods for obtaining sub-sampled shift invariant transforms are via the wavelet transform modulus maxima (Mallat, 1999, section 6.2.; Mallat and Zong, 1992) and the power shiftable discrete wavelet transform (PSDWT, Simoncelli et al., 1992). However, neither of these transforms are investigated further in this study.

2.1.1. Methods

To examine the issue of shift variance in ABR WT MRA in more detail, we took a single pre-recorded ABR signal from the sample collected in part two of this paper (and as shown in Figure 2), and sequentially delayed it by one sample point (i.e., shifted it to the right by sequentially adding a zero sample point to the beginning of the trace) to create a further 63 sequentially shifted ABR signals. We then performed a six level wavelet decomposition on each ABR signal and reconstructed the waveforms at levels A6 and D6 only. Obviously, if the WT was shift invariant, the reconstructed signals will be identical except for the sequential shifts introduced into the original signal. We used the DWT with a 5th order bi-orthogonal spline wavelet (bior5.5), the OCDWT with zero to six levels of critical sub-sampling (see section 3.1.3 below) with a bior5.5 wavelet, the CWT with a Mexican hat wavelet, the DTCWT with a near-symmetric wavelet, and the LP with a length 9 binomial filter. It should be noted that as it was not possible to use the same mother wavelet in all the wavelet transform algorithms, we chose mother wavelets that were either symmetrical or approximately symmetrical (to minimise phase distortion), and that were of approximately the same support length (between 9 and 13 taps) (to give filters that would introduce similar amounts of aliasing in to each sub-band).

To determine the degree of distortion introduced by the sequential shifts the mean absolute error (MAE) at all corresponding points on the shifted waveforms was measured. If the WT was truly shift invariant, it would return a MAE of zero.

2.1.2. Results and Discussion
Figure 4 shows the reconstructed ABR WT D6 waveform series plotted for each type of WT used. Table 1 shows the mean absolute error for both the A6 and D6 reconstructions. Figure 5 shows the MAE for reconstructions from a generalisation of the DWT and OCDWT where the level of critical sub-sampling is varied from zero (the OCDWT) to six (the DWT).

These results clearly show that the DWT is extremely shift variant with both the amplitudes and latencies of the ABR A6 and D6 peaks varying unpredictably as the input signal is shifted. Of the other WTs tested, they ranked from complete shift invariance to relatively large shift variance in the order of CWT, OCDWT (symmetric extension and fully sampled), LP and DTCWT. The greater MAE seen in all WT D6 scales compared to the A6 scales was because the D6 sub-band has aliasing present both from the lower (A6) and the higher frequency bands (D5).

Finally, Figure 5 shows that for the OCDWT (symmetric extension), both the A6 and D6 sub-bands are approximately shift invariant when the OCDWT is critically sub-sampled to level 4 and then fully sampled at levels 5 and 6. This shows that there is very little aliasing introduced into a decomposition level (in this case level 6) provided that the level above it is not critically sub-sampled (in this case level 5).

2.1.3. Conclusion

These results clearly show that the critically sampled DWT is a poor choice for multi-resolution AEP signal analysis because it is highly shift variant. However, to better identify the preferred alternative, we will directly compare a number of WT algorithms on a normative ABR dataset in part two of this paper.

2.2. Consideration Two - Selection of an Appropriate Mother Wavelet

The second potential issue with the application of the WT is related to the selection of the mother wavelet. The general topic of mother wavelet selection is discussed in some detail in Section
7.2.1 of Mallat (1999), and there is an approachable mathematical perspective presented in Unser and Blu (2003).

Choosing the best mother wavelet for AEP MRA can be difficult because many wavelet properties cannot be jointly optimised. For example, if we select a Daubechies mother wavelet because they are optimal in the sense of having minimum support length for a given number of vanishing moments, then we have to accept that they are not optimal in the sense of symmetry and smoothness.

Because of this, it is unlikely that one truly optimal mother wavelet will be found for all AEP MRA applications. It therefore becomes critical that each practitioner choose (or design) their wavelet based on the special requirements of their particular application. As a minimum, however, two essential features should always be considered when selecting an appropriate mother wavelet for the MRA of AEP signals - symmetry (linear phase) and smoothness (regularity).

**Symmetry (linear phase):** Linear phase is generally a desirable property of a digital filter as it means the filter has constant group (or phase) delay, i.e., all the frequency components of the signal are delayed by the same number of samples as they pass through a filter (Oppenheim et al., 1999, Section 5.1.2). This means that if a number of in-phase frequency components pass through a linear phase filter, they will be in-phase at the output. This is not the case for a filter with a non-linear phase response, and *phase distortion* will be introduced.

The phase response of a filter, and therefore a wavelet, is defined by its symmetry properties. If a mother wavelet has either even or odd symmetry (either symmetrical or anti-symmetrical), then it will have linear phase.

In AEP MRA, we are primarily interested in morphology and latency of peaks and so all filtering operations should preserve phase (Madhavan et al., 1986; Pratt et al., 1989). The location and amplitude of peaks is highly dependent upon the relative phase of the frequency components in the signal and so using mother wavelets with a linear phase is critical.
There is also an additional benefit of using mother wavelets with linear phase: as the mother wavelet is symmetrical, the number of multiplications in the convolution integral can be halved (by adding signal samples prior to multiplication by the filter coefficient). With the trivial exception of the orthogonal Haar wavelet (a B-spline of degree zero), only bi-orthogonal mother wavelets can be designed to have linear phase.

*Smoothness (regularity):* The regularity of the mother wavelet essentially defines how smooth (differentiable) the wavelet functions are, and therefore how smooth their reconstructed approximations will be. If we assume that the shape of the mother wavelet should match the features of interest in the AEP signal (which are ideally smooth peaks, Section 1.1), then the mother wavelet should be smooth and have no discontinuities.

In general, mother wavelet smoothness is proportional support length, i.e., greater support length implies a smoother wavelet. However, it should also be considered that there is an implicit trade off in time-frequency analysis between resolution in the time domain (which increases with decreasing support length) and resolution in the frequency domain (which increases with increasing support length).

Therefore, for the analysis of AEP signals, which have time varying features, it is important to select a smooth wavelet with minimal support length. In addition, wavelets with compact support have reduced computational complexity (as shorter wavelets have fewer multiplications in the convolution integral) and reduced length of any boundary extension (see Consideration 3).

Another important property of a mother wavelet, which is directly related to regularity, is the number of vanishing moments. In general, a mother wavelet with \( n \) vanishing moments will estimate the multi-scale \( n^{th} \) order derivatives of the signal. Therefore, wavelets with a larger number of vanishing moments can, in general, represent signals with fewer non-zero coefficients, thus providing a sparse representation suitable for data compression and fast calculations (Mallat, 1999).
The provision of a sparse representation may not, in itself, be an important issue, however, as it will be dependent upon the actual application. Therefore, in relation to MRA of AEP signals, we assume that if a mother wavelet with adequate smoothness is selected, then it will also have an appropriate number of vanishing moments.

In the AEP literature it is rare to find a discussion of the reasons why a particular mother wavelet was selected for a specific application. For example, Huang et al., (1999) examined auditory middle latency responses as measures of anaesthesia depth using a non-symmetrical mother wavelet that was one tenth as long as the input signal. In a similar study, Kochs et al., (2001) used the same non-symmetric filter, but selected one with compact support. Both papers give no indication of why the Daubechies mother wavelet, or how the support length (order), were selected. Hoppe et al., (2001) examined an automatic sequential recognition method for cortical evoked potentials using a number of different mother wavelet functions, but found no significant differences in their results. Finally, Effern et al., (2000) examined a method for single trial analysis of event related potentials, but did not specifically state which mother wavelet they used.

Of all the wavelet families available, the quadratic B-splines Unser et al., (1992) have probably been the most widely used for AEP analyses (e.g. Popescu et al., 1999; Quian Quiroga 2000; Quian Quiroga et al., 2000, 2001a,b; Quian Quiroga and Zuijtelaar, 2002; Başar et al., 2001; Demiralp, 2001). The desirable properties of these wavelets include almost optimal time-frequency uncertainty properties (Unser et al., 1992), reasonable compact support, regularity with minimal oscillations, and a visual match to the AEP signal peaks of interest. However, they do have the disadvantage of widely varying support lengths on the analysis (length 20) and synthesis (length 8 and 4) sides of the filterbank.

2.2.1. Wavelets as Matched Filters

Given our conceptual description of the wavelet transform as a correlation of a signal with a mother wavelet (Section 1.2), it would seem to make intuitive sense to select a mother wavelet that
is, in some way, matched to the features of interest in the signal. However, this should only be seen as a conceptual goal because:

1. It is debatable how one quantifies the degree of similarity between a signal and a mother wavelet. The obvious choice of correlation leads to matched filtering, which is only optimal at detecting a known signal buried in stationary white noise (Woodworth, 1983). As outlined in Section 1.1, the morphology of AEP waveforms is generally not known \textit{a priori} (even for normative waveforms) and the background noise is neither stationary nor white (Boston, 1981). In addition, performing matched filtering using the WT further complicates the issue as the mother wavelet has to match the signal at multiple (commonly only dyadic) scales.

2. While matched filtering is an acceptable approach for detecting waves in AEP, there are at least four issues which should be considered when designing a matched filter. First, the research will not be reproducible unless the exact filter coefficients are published. Second, the intrinsic non-linearities of the auditory pathway means that it is debatable whether a matched filter designed at one stimulus intensity and/or on one subject can be sensibly applied at a different intensity levels and/or on different subjects. Third, in order to design the matched filter in an unbiased way we require a set of data on which to design the filter and another (distinct) set on which to evaluate its performance. Fourth, a matched filter has to be designed for each wave we a trying to detect in the AEP, which becomes computationally undesirable, especially for the seven Jewett waves of the ABR.

Therefore, in general, it is doubtful whether a mother wavelet designed to match a specific target in an AEP signal will be any more reliable than a mother wavelet chosen from a pre-defined family (provided that they have similar properties, such as vanishing moments and regularity). Samar et al., (1999) make exactly this point when discussing the matched Meyer wavelet in relation to other “smoother, more physiologically natural wavelets.”

\textit{2.2.2. Methods}
To examine the importance of mother wavelet symmetry (phase linearity) - we created a simulated “ABR” signal similar in nature to a typical ABR. This signal, shown in Figure 6, consisted of the sum of two in-phase sine waves of frequencies 310 and 490Hz. We then windowed this signal using a Blackman window (Oppenheim et al., 1999), and performed a six level DWT decomposition using an orthogonal mother wavelet with non-linear phase - a 5th order Daubechies wavelet (see Daubechies, 1992, pages 194-202) - and a bi-orthogonal mother wavelet with linear phase - a 5th order spline wavelet (see Daubechies, 1992, pages 271-280). These wavelets were selected as they have the same number of vanishing moments and approximately the same support lengths. The signal was then reconstructed using only the wavelet coefficients from D6 scale (which had a low frequency cut-off of 267Hz and a high frequency cut-off of 533Hz).

To examine the effects of mother wavelet smoothness - we performed a six level DWT decomposition on the ABR waveform shown in Figure 2 with various (linear phase) mother wavelets. We then compare the smoothness of the D6 reconstruction, which in this case should highlight peaks I, III, and V. In order to illustrate (solely) the effect of increasing wavelet smoothness, we use mother wavelets from a single family (bi-orthogonal splines) of increasing order (and therefore of increasing smoothness) and the (infinitely regular) discrete Meyer wavelet.

2.2.3. Results and Discussion

Figure 6 shows the reconstructed D6 waveforms from the non-linear phase, 5th order Daubechies wavelet, and the linear phase, 5th order bi-orthogonal wavelet. The effects of the non-linear phase response on the amplitudes and latencies of the recovered signal are clear, i.e., the reconstruction from the mother wavelet with non-linear phase has either peaks missing or incorrect amplitudes. Therefore, the D6 reconstruction from the non-linear phase wavelet cannot be taken as an accurate rendition of that particular signal component.

Figure 7 shows an original ABR signal (top), and its D6 reconstructions using the different bi-orthogonal spline wavelets (middle) and the (infinitely regular) discrete Meyer wavelet (below).
It can be seen that as the order of the mother wavelet increases, the smoothness of the reconstruction also increases (remembering that for the bi-orthogonal splines, their minimum regularity (at the knots) is \((N_r - 2)\), where \(N_r\) is the order of the reconstruction wavelet). Therefore, we can conclude that a minimum regularity of three was required for MRA of these ABR signals. In addition, Figure 7 clearly illustrates the similarity between the reconstruction of 5\(^{th}\) order spline and the (infinitely regular) discrete Meyer wavelet. This result would indicate that, provided you select a mother wavelet of adequate smoothness, there is unlikely to be any significant difference between reconstructed AEP WT waveform shape between different wavelet families (e.g., Hoppe et al., 2001).

2.2.4. Conclusions

We concluded that symmetry (linear phase) and smoothness (regularity \(\geq 3\) for ABR) are critical features of a mother wavelet if it is to be used for AEP MRA. These properties are met by the bi-orthogonal spline wavelets (Daubechies, 1992), as they have linear phase and the ability to trade-off support length and regularity (and therefore vanishing moments). In addition, unlike the closely related B-splines (Unser et al., 1992), they can be designed to be compactly supported on both the analysis and synthesis sides. Finally, with respect to the ABR, the bi-orthogonal spline wavelets have also been found to be correlated with peaks I-V (Ochoa et al., 1999). As a result of these conclusions, only bi-orthogonal spline wavelets were used in part two of this paper.

2.3. Consideration 3 - Pre-processing Undertaken Prior to Wavelet Analysis

The final issue, which is not unique to wavelet analysis, and is often not mentioned at all in reports in the literature, regards the pre-processing that is performed on the AEP before the MRA. Typical examples of pre-processing operations are pre-filtering, windowing, and boundary extension (although boundary extension is only notionally a pre-processing step as it is frequently implemented as part of the WT algorithm).
Unlike the Fourier transform, the WT does not require windowing of the data. This is because the mother wavelet, by definition, has finite support and decays smoothly to zero.

Consideration must be given, however, to what assumptions are made about the signal beyond the boundaries of the data, i.e., before the first and after the last sample of interest. When analysing the edges (i.e., the beginning or end) of a signal, the WT must make an assumption about what the signal looks like beyond these edges. If these assumptions are incorrect, then sudden changes in the wavelet coefficients can occur and spurious waves can appear in the reconstructed AEP signals.

In principle, the optimal solution to the boundary extension problem would be to begin recording the signal before the region in which the WT will be applied (i.e., using a pre-stimulus delay), and to continue to measure the signal beyond the region in which the WT will be applied. That is, to extend the signal with the actual signal values (e.g., Quian Quiroga et al., 2001a, b). Both of these options allow the AEP signal to be recorded for longer, but risk contamination from earlier and later waveforms that are not of direct interest. For example, the use of data preceding the ABR signal could allow any stimulus artefact to contaminate the ABR WT (as the statistics of the stimulus will be very different from those of the response it evokes). In addition, even a modest wavelet filter of length 9 (taps) requires approximately 9ms of preceding and following data (assuming a 6 level dyadic wavelet decomposition and a sampling rate of 30kHz). For these reasons, it is often appropriate to adopt a suitable boundary extension policy.

The three most commonly used boundary extension policies are zero padding, odd periodic (i.e., circular convolution) and even periodic (i.e., symmetric or boundary reflection). Zero padding assumes that the signal is zero outside the region of interest (and clearly adds the least additional computation). Odd periodic assumption adds copies of the signal to the beginning and end of the original signal to produce a series of signals one after the other. Even periodic assumption adds reversed copies of the signal to the beginning and end of the original signal to produce a series of signals mirroring each other, one after the other.
Of these boundary extension policies, the odd periodic is probably the most common as it is the default policy of most standard WT software packages. In fact, most AEP WT reports do not state what boundary extension policy was applied (e.g. Huang et al., 1999; Demiralp et al., 2001; Kochs et al., 2001); exceptions include Hoppe et al., 2001.

2.3.1. Methods

To demonstrate the effect on reconstruction morphology of zero padding, odd periodic and even periodic, the same ABR signal used in consideration one (section 2.1.1) was pre-processed using these three methods, and then decomposed using a (six level) wavelet decomposition using a 5th order Daubechies wavelet. The signals were then reconstructed at the A6 and D6 scales only.

2.3.2. Results and Discussion

Figure 8 shows the reconstructed A6 and D6 waveforms for each of the three pre-processing techniques. It can clearly seen that both the odd periodic and zero padding boundary extension policies introduce either a false peak at the beginning of the reconstructions (odd periodic only) or a false increase in the amplitude of the final peak at the end of the A6 reconstructions (both odd periodic and zero padding). This is likely to have been caused by the step change introduced to the signal at both \( t = 0 \) and \( t = 15 \) ms by the odd periodic technique, and at 15 ms by the odd periodic and zero padding techniques.

2.3.3. Conclusions

These results suggest that the most sensible boundary extension policy for AEP MRA is the even periodic (symmetric or boundary reflection) policy.

3. PART TWO - WT Analysis of a Normative AEP Sample
While the conclusions of part one of this paper hold significant practical value, they require confirmation using a real sample of AEP signals. As a result, part two of this paper investigated the effect of zero to six levels of sub-sampling on the shift variance properties of the OCDWT, and compared it to the shift variance of the DTCWT. The mother wavelet and pre-processing policy were, as far as was possible, held constant to control for variance caused by these sources. This allowed the variance of the resultant reconstructed WT waveforms to be attributed to the choice of WT, and to be used to conclude a final preferred WT protocol for AEP MRA.

3.1. Methods

3.1.1. Subjects

Twenty female subjects, aged between 18 and 30 years, were conveniently sampled from the general, adult population of Gauteng, South Africa. All participants demonstrated pure tone audiometry thresholds of 20 dBHL or better at 250, 500, 1000, 2000, 4000 and 8000 Hz; normal acoustic immittance measurements as defined by Northern (1980) bilaterally; no self reported history, past or present, of auditory dysfunction, neural dysfunction, or head injury that could effect the ABR results; no current medication that could effect the ABR; and ABR waveforms considered within normal limits by standard normative databases (Hall, 1992).

3.1.2. Procedure

All testing was conducted in sound treated and electrically shielded booths at the WITS Speech and Hearing Clinic within the Department of Speech Pathology and Audiology of the University of the Witwatersrand, Johannesburg, South Africa. Subjects initially filled out a pre-test questionnaire and underwent pure tone audiometry and acoustic immittance testing using a two channel Grason-Stadler Incorporated GSI 10© diagnostic audiometer and GSI 33© acoustic immittance meter respectively. If pre-ABR selection criteria were met, ABR testing was conducted

To elicit the ABR waveforms, alternating click stimuli (driven by an electrical square wave of 100 µs duration) were presented to each subject’s test ear via EAR 3A insertphones (giving a 0.8 ms delay for onset of the ABR waveform) at 90 dBnHL. The non-test ear was masked using white noise at 50 dBnHL. All levels were as indicated by the Biologic EP software. No attempt was made to measure the actual dBSPL at the eardrum.

To record the ABR waveforms, disposable silver, silver-chloride electrodes were placed on each subject’s high forehead (non-inverting), test ear earlobe (inverting) and non-test ear earlobe (ground). Impedances were maintained at <5 kohms as measured by the Biologic “impedance check” function. Other recording parameters were set to 2048 averages, 150 000 amplifier gain (sensitivity or artefact level ± 16.3 µV), 30-3000 Hz bandpass filter, 50 Hz notch filter, 512 point analogue to digital conversion ratio (the maximum allowable on the Biologic Evoked Potential software), and 15.0 ms time epoch (giving a sampling frequency of 34 133 Hz). To remove possible stimulus artefact, the first 20 sample points were preset (“blocked”) to zero.

Two ABR waveforms were recorded from the right ear from each subject. These waveforms were labelled the “raw” ABR waveforms.

3.1.3. Signal processing

The raw ABR signals were taken from the Bio-logic Evoked Potential system in ASCII format and transferred to a personal computer running Mathworks Matlab© Software (Version 6.5). Each pair of raw ABR signals was arithmetically averaged to give a single ABR signal. This averaged ABR signal was then baseline shifted to have an initial amplitude of zero µV. A six level, dyadic, wavelet decomposition was then applied to each averaged ABR signal (illustrated for the DWT in Figure 1). For the purposes of this experiment, the resulting wavelet coefficients were then used to reconstruct the ABR detail signal at wavelet scale D6 only.
In order to investigate the effect of sub-sampling on the shift variance properties of the WT, two types of wavelet algorithms were used.

The first algorithm was a generalisation of the DWT and OCDWT algorithms, where the critically sampled DWT was applied to the first $M$ levels of an $L$-level wavelet decomposition and then the fully sampled OCDWT was applied to the remaining $(L - M)$ levels. This process can be seen as sub-sampling the original ABR waveform with the same filter as is subsequently used for the over complete wavelet analysis. Alternatively, it be seen as a generalisation of the OCDWT, as it produces the conventional DWT when $M = L$ and the fully sampled algorithme à trous when $M = 0$. The mother wavelet used for this algorithm was the 5th order bi-orthogonal spline wavelet (bior5.5) with symmetric length 9 and 11 (tap) filters.

The second algorithm was the dual tree complex wavelet transform (Kingsbury, 2000). The DTCWT (by definition) uses specially designed mother wavelets and so comparison with the other WT algorithm will be slightly confounded by the difference in mother wavelets used. However, the DTCWT has a redundancy of two and therefore offers a compromise between the critically sub-sampled DWT and the fully sampled OCDWT. The mother wavelet used for this algorithm was the near-symmetric b wavelet (near_sym_b) with length 9 and 13 (tap) filters.

We chose to exclude the CWT from the comparison because it is clearly shift invariant (see Figure 4). Also, because the Mexican hat wavelet has no inverse transform, it cannot be directly compared with the other discrete transforms. Such a comparison would have required a different peak detection algorithm than the one used for the OCDWT and DTCWT (such as finding zero crossings of a derivative estimation wavelet).

### 3.1.4. Data Collection and Analysis

A simple algorithm calculated the latencies (time from stimulus onset to wave peak maxima in ms) of the individual reconstructed ABR WT D6 waves corresponding in latency to the original
ABR waves I, III and V. These waves were labelled “D6I, III and V” and their standard deviations for the entire sample were calculated.

To compare the effects of critically sub-sampling the OCDWT, the standard deviation of the reconstructed D6 wave V latencies were compared between each level of critical sub-sampling.

To directly compare the shift variance properties of the OCDWT to the DTCWT, the same level of redundancy (number of wavelet coefficients) had to be ensured. As a result, the D6 wave I, III and V latency standard deviations from the DTCWT (with a redundancy of two) were compared to the matching latency standard deviations obtained from the OCDWT that had been critically sub-sampled in the first two levels of the decomposition only (therefore also having a redundancy of two).

To graphically illustrate the shift variant properties of the fully sampled OCDWT and the critically sampled DWT, all 20 ABR waveforms and their reconstructed ABR WT D6 waveforms were simultaneously plotted for visual comparison.

3.1.5. Ethics

Unconditional ethical clearance was given to conduct the study by the University of the Witwatersrand Committee for Research on Human Subjects (Medical) (protocol number M980134). Informed consent was obtained from each subject before testing.

3.2. Results and Discussion

Figure 9 shows the standard deviation of the latency of peak D6V measured from the D6 reconstructions from each patient as the number of levels that are critically sub-sampled in the OCDWT was increased. This standard deviation gives an estimate of the inter-patient variability. In a normative database we would want it to be as small as possible so that we can detect abnormal peak latencies with maximum sensitivity.
When the OCDWT was fully sampled (0 levels critically sub-sampled), it was very close to being truly shift invariant and therefore all of the variation observed in the measurement of peak D6V can be concluded as having been due to inter-patient variation only. As the number of levels that were critically sub-sampled increased, however, additional variation in the measurement of peak D6V was introduced. This additional variation was unwanted as it had nothing to do with the observed inter-patient variability, but was solely due to an artefact of the wavelet analysis methodology, i.e., shift variance. It can be seen that for reliable measurement of peak D6V latency, we can critically sub-sample only the first 4 levels of the decomposition. This result was almost identical to that presented in Figure 5 and illustrates the efficacy of the illustrative example using an artificially shifted ABR in predicting an effect that occurs on a set of real ABR waveforms. Also note that similar increases in peak latency variation were also observed for peaks I and III (not shown).

Table 2 shows the standard deviation of the measured latencies of the reconstructed ABR WT D6 peaks I, III, and V, for both the OCDWT critically sub-sampled in the first two levels of decomposition and the DTCWT (both having a redundancy of two). It can be seen that the OCDWT showed less than half the peak latency variation of the DTCWT. The redundancy of two therefore provided more than adequate shift invariance for the OCDWT, but was probably inadequate for the DTCWT. In fact, from the results presented in Figure 9, critically sub-sampling of the OCDWT can continue to level 3, giving a redundancy of less than 1.4, whilst still providing enough shift invariance for the purposes of AEP MRA.

Figure 10 confirms graphically that the fully sampled OCDWT produces far less inter-patient variation than does the critically sampled DWT. In addition, a comparison of the upper and lower plots shows the efficacy of OCDWT analysis (D6 reconstruction) in reducing inter-patient variability even compared to the original time domain signals.

4. Conclusions
In this paper we have highlighted the following properties as being critical for the successful wavelet analysis of AEP - the WT sub-sampling strategy, which define its shift variance properties; the symmetry of the mother wavelet, which defines its phase distortion properties; the regularity of the mother wavelet, which defines the smoothness of waveform reconstruction; and the boundary extension policy, which defines its response and the start and end of the sampling period. It has been demonstrated that a WT which has (at least approximately) shift invariance, linear phase, a regularity of at least three, and a reflective boundary extension policy, produces the most accurate and reliable estimates of peak amplitudes and latencies present in AEP WT MRA. Of the WT examined in this study, we recommend the OCDWT for AEP WT MRA.
References


Fig. 1.
Fig. 2.
Fig. 3.

Daubechies 5

Symlet 4

Biorthogonal 3.5
DWT  

CWT  

DTCWT  

OCWT (fully sampled)  

OCDWT (critically sampled to level 4)  

LP  

Fig. 4.
Fig. 5.
Fig. 6.
Fig 7.
Fig. 8.
Fig. 9.
Fig. 10.
FIGURE LEGENDS

Figure 1: An example ABR DWT decomposition tree showing frequency ranges and numbers of coefficients at each scale (ABR sampled at 34,133 Hz, giving 512 samples over a 15 ms period).

Figure 2: Example ABR DWT signals rebuilt from the decomposition tree shown in Figure 1. ABR and ABR DWT scale markers are shown on the right.

Figure 3: Daubechies 5, symlet 4 and biorthogonal 3.5 wavelets.

Figure 4: D6 reconstruction, discrete wavelet transform (DWT); continuous wavelet transform (CWT); dual tree complex wavelet transform (DTCWT); over-complete discrete wavelet transform (OCDWT) fully sampled; OCDWT critically sampled to level 4 and fully sampled at levels 5 and 6; and Laplacian Pyramid (LP).

Figure 5: Effect of critical sub-sampling in the OCDWT on mean absolute error (MAE).

Figure 6: Simulated ABR of windowed sinusoids of 310 and 490 Hz (left) and a comparison of the D6 reconstruction of the simulated ABR for both a non-linear phase (Daubechies 5) wavelet (top) and linear phase (5th order bi-orthogonal B-spline) wavelet (bottom).

Figure 7: ABR waveform (top) and D6 reconstructions using various (linear phase) mother wavelets.

Figure 8: A6 (top) and D6 (bottom) reconstruction with various boundary extension policies (5th order bi-orthogonal B-spline mother wavelet).
Figure 9: Standard deviation of the latency of peak V measured from the D6 reconstruction.

Figure 10: Plots of all 20 ABR traces (upper), all D6 reconstructions using the critically sampled DWT (middle), and all D6 reconstructions using the fully sampled OCDWT (lower). All ABR waveforms were sampled at 34 133Hz, giving 512 samples over a 15ms period.
Table 1

Shift invariance properties of various wavelet transform algorithms.

<table>
<thead>
<tr>
<th>Decomposition</th>
<th>Shift Invariant</th>
<th>MAE A6</th>
<th>MAE D6</th>
</tr>
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<tbody>
<tr>
<td>DWT</td>
<td>No (multiples of sub-sampling lattice only)</td>
<td>3.7 x 10^{-2}</td>
<td>6.7 x 10^{-2}</td>
</tr>
<tr>
<td>OCDWT</td>
<td>Approximate (symmetric extension)</td>
<td>1.6 x 10^{-6}</td>
<td>1.7 x 10^{-6}</td>
</tr>
<tr>
<td>CWT</td>
<td>Yes</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>DTCWT</td>
<td>Approximate</td>
<td>3.5 x 10^{-3}</td>
<td>4.0 x 10^{-3}</td>
</tr>
<tr>
<td>LP</td>
<td>Approximate</td>
<td>6.3 x 10^{-4}</td>
<td>6.7 x 10^{-4}</td>
</tr>
</tbody>
</table>
Table 2

Shift invariance properties of redundancy 2 wavelet transforms.

<table>
<thead>
<tr>
<th>Wavelet Transform</th>
<th>Wave Latency Standard Deviations</th>
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<tbody>
<tr>
<td></td>
<td>D6I</td>
</tr>
<tr>
<td>DTCWT</td>
<td>0.434</td>
</tr>
<tr>
<td>OCDWT</td>
<td>0.182</td>
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