Air Bubble Entrainment by Breaking Waves and Associated Energy Dissipation

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Abstract

A simple mathematical model (air bubble model) that describes energy dissipation due to air bubble entrainment is proposed and applied to a series of laboratory experiments for plunging jet flows (steady) and surf zone waves (unsteady). This leads to a formulation wherein the rise velocity of bubbles is included. For the unsteady case, i.e. air entrainment by wave breaking, some parameters of the model have been estimated from the experiments and expressed in terms of local wave height and distance. Results obtained through the air bubble model are summarized in the following.

Experiments in vertical circular plunging jets (steady) were performed for both in freshwater and seawater, which highlighted the distribution of void fraction, that follows closely analytical solutions derived by Chanson (1997). In addition, various properties of void fraction field were emphasized. Three scale models were used with freshwater for identical Froude numbers in the experiments, which highlighted significant scale effects when Weber number is less than 1000. Similar experiments were also performed with freshwater and seawater and the results showed lesser air entrainment in seawater plunging jets. The pseudo-bubble chord sizes obtained from the experimental data were in the range from less than 0.5 mm to more than 1.0 mm.

The rate of energy dissipation due to entrained air was investigated by applying the air bubble model for three typical phenomena. The results demonstrated that the ratios of energy dissipation due to air bubble entrainment with respect to total energy loss were 25%, 1.4% and (2-4)% for hydraulic jump, 2-D vertical plunging jet and vertical circular jet, respectively.

Experiments on unsteady air bubble entrainment by wave breaking were conducted in a wave channel. Maps of the evolution of the void fraction distribution in surf zone generated by various sizes breaking waves were presented. A significant fraction of the
potential energy of entrained air was measured from the void fraction distributions provided by breaking waves. Measurements showed high void fraction up to 19% in plunging breakers at still water surface whereas 16% in spilling breakers. The ratio of energy dissipation due to entrained air to total energy loss was found (18-22)% and (17-19)% for spilling and plunging breaker, respectively.

The characteristics of time averaged wave parameters (e.g. potential energy, kinetic energy, energy flux, radiation stress) for regular waves were discussed taking into account the air bubble effects. Analytical solutions were sought and explicit expressions were obtained for wave parameters under sinusoidal waves. Effects of the air entrainment on density, pressure and velocity fields were also discussed in detail. The conservation equations for energy and momentum were solved numerically using finite difference methods. Boundary conditions were used at the breaking point.

Scale effects were discussed based on laboratory air entrainment in 2-D wave flume, which was believed to occur in small size models. The data were in good agreement with the basic assumption for vertical distribution of void fraction both in spilling and plunging breakers. The results of the air bubble model were compared with experimental data and found to give good agreement between them for the wave height and wave set-up. Water level rise by entrained air was determined and found significant effects on surf zone hydrodynamics. In addition, wave run-up was measured and discussed.
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# Table of Contents

Figure Captions .......................................................... 9  
List of Tables .............................................................. 16  
List of Symbols ............................................................. 17  

**CHAPTER 1: General Introduction**  
1.1 Presentation .......................................................... 23  
1.2 Outline of the dissertation .......................................... 31  

**CHAPTER 2: Measurements of Void Fraction in Circular Plunging Jet**  
2.1 Introduction .......................................................... 33  
2.2 Experiments and observations .................................... 35  
  2.2.1 Apparatus ......................................................... 35  
  2.2.2 Instrumentation .................................................. 35  
  2.2.3 Air-water flow regions .......................................... 35  
  2.2.4 Observations ..................................................... 35  
  2.2.5 Experimental flow conditions .................................. 35  
  2.2.6 Quantity of entrained air discharge .......................... 35  
2.3 Experimental results: void fractions and bubble count rates 45  
  2.3.1 Distributions of void fractions ................................ 45  
  2.3.2 Distributions of bubble count rates .......................... 45  
  2.3.3 Comparison between data and analytical solution ........ 45  
  2.3.4 Vertical distribution of void fraction ........................ 45
2.4 Discussion

2.4.1 Characteristics of maximum void fraction and bubble count rate

2.4.2 Penetration depth

2.4.3 Distributions of chord length

2.5 Scale effects

2.6 Conclusions

CHAPTER 3: Energy Dissipation by Air Bubble in Steady Case

3.1 Introduction

3.2 A simple model formulation

3.3 Application of model

3.3.1 Hydraulic jump

3.3.1.1 VOID FRACTION DISTRIBUTIONS

3.3.1.2 VOLUME OF ENTRAINED AIR

3.3.1.3 ENERGY DISSIPATION RATE

3.3.2 Two-dimensional vertical plunging jet

3.3.2.1 VOID FRACTION DISTRIBUTIONS

3.3.2.2 VOLUME OF ENTRAINED AIR

3.3.2.3 ENERGY DISSIPATION RATE

3.3.3 Vertical circular plunging jet

3.3.3.1 VOID FRACTION DISTRIBUTIONS

3.3.3.2 VOLUME OF ENTRAINED AIR

3.3.3.3 ENERGY DISSIPATION RATE

3.4 Discussion

3.5 Conclusions
CHAPTER 4: Measurements of Void Fraction and Rate of Energy Dissipation in 2-D Wave Breaking

4.1 Introduction

4.2 Experiments and observations

4.2.1 Wave flume
4.2.2 Instrumentation
4.2.3 Wave generation
4.2.4 Observations of entrained air

4.3 Experimental results: spilling and plunging breakers

4.3.1 Response of probe
4.3.2 Data analysis
4.3.3 Void fraction measurements
4.3.4 Volume and potential energy of entrained air
4.3.5 Comparison of wave energy and potential energy of air bubbles
4.3.6 Energy dissipation rate

4.4 Conclusions

CHAPTER 5: Modeling of Surf Zone Hydrodynamics Considering Air Bubbles

5.1 Introduction

5.2 Basic assumptions

5.2.1 Vertical distribution of void fraction
5.2.2 Averaging procedure
5.2.3 Correction term & boundary condition

5.3 Proposed air bubble model

5.3.1 Evaluation of wave parameters

5.3.1.1 DENSITY AND PRESSURE FIELD
5.3.1.2 STATIC ENERGY
5.3.1.2 POTENTIAL ENERGY
5.3.1.3 KINETIC ENERGY
5.3.1.4 ENERGY FLUX
5.3.1.5 RADIATION STRESS

5.4 Basic equations

5.4.1 Energy balance equation
5.4.2 Momentum balance equation

5.5 Conclusions

CHAPTER 6: Verification of the Air Bubble Model

6.1 Introduction
6.2 Numerical solutions

6.2.1 Transition zone
6.2.1 Numerical scheme

6.3 Results and discussion

6.3.1 Determination of empirical coefficient $k_1$ and $C_0$
6.3.2 Void fraction distributions
6.3.3 Determination of free parameter ($\alpha$)
6.3.4 Wave height and set-up
6.3.5 Water level rise by air bubble
6.3.6 Wave run-up

6.4 Plunging breaker-scale effects in air entrainment
6.5 Conclusions

CHAPTER 7: General Conclusions

References
Figure Captions

1.1 Air entrainment and detrainment cycle.

1.2 Photographs of (a) waterfall (Photograph by Dr. Chanson) and (b) hydraulic jump (Photograph by Prof. G.R. Mckay, Australia).

1.3 Breaking waves. (a) Spilling breaker (Tweed Heads, Australia, courtesy of Dr. Chanson), and (b) plunging breaker (Terasawa beach, Japan).

2.1 Sketch of water pool.

2.2 Sketch of (a) conductivity probe and (b) response of probe tip.

2.3 Air water flow regions of a vertical circular plunging jet.

2.4 Photograph of bubble clouds in vertical circular plunging jet. (a) Model 2 ($d_0 = 12.5$ mm, Freshwater), $x_1 = 50$ mm, $V_1 \sim 3.0$ m/s, $Fr_1 = 9.0$. (b) Model 4 ($d_0 = 12.5$ mm, Seawater), $x_1 = 50$ mm, $V_1 \sim 3.0$ m/s, $Fr_1 = 9.07$.

2.5 Signal processing technique. Upper: Raw signal output: A voltage of 0.6 V occurs when no air bubbles are touching the probe tip. Lower: Square wave represents the algorithm output. Each pulse corresponds to an air bubble encounter.

2.6 Local averaged void fraction distributions- Model 2 ($d_0 = 0.0125$ m, Freshwater), $x_1 = 0.10$ m, $V_1 = 2.51$ m/s, $Fr_1 = 7.40$ (Run Cir-1).

2.7 Dimensionless air bubble frequency distributions in plunging jet flow- Model 2 ($d_0 = 0.0125$ m, Freshwater), $x_1 = 0.10$ m, $V_1 = 2.51$ m/s, $Fr_1 = 7.40$ (Run Cir-1).

2.8 Dimensionless distributions of void fraction $C$ and dimensionless bubble count rate $f_{ab} = F*r_1/V_1$ in vertical circular jets. Comparison between experimental data and Eq. (2.7). (a) Model 2 (Freshwater), and (b) Model 4 (Seawater).
2.9 Void fraction distributions in the vertical direction - Model 2 \( (d_0 = 12.5 \text{ mm}, \) Freshwater), \( r/r_1 = 1.11 \).

2.10 Experimental results of maximum void fractions and dimensionless bubble count rates in vertical circular jet flows for \( Fr_1 = 9 \) and \( x_1/d_0 = 4 \). (a) Dimensionless locations of maximum void fractions and maximum bubble count rates, (b) Maximum void fractions \( C_{\text{max}} \), and (c) Maximum dimensionless bubble count rate \( F_{\text{max}}^* r_1/V_1 \).

2.11 Relationship between void fraction and dimensionless bubble count rate at a given cross-section in vertical circular plunging jet flows - Model 2 \( (d_0 = 0.0125 \text{ m}, \) Freshwater), \( x_1 = 0.05 \text{ m}, Fr_1 = 9.0 \) (Run Cir-5).

2.12 Maximum penetration depth \( D_p/d_0 \) as a function of jet length \( x_1/d_0 \) - Model 2 \( (d_0 = 0.0125 \text{ m}, \) Freshwater).

2.13 Pseudo-bubble chord length distributions. (a) Comparison of chord length between Model 2 and Model 4 for \( x_1/d_0 = 4 \) and \( Fr_1 = 9 \). (b) Model 2 (Freshwater) and (c) Model 4 (Seawater).

2.14 Distributions of void fraction \( C \) for identical inflow condition \( (x_1/d_0 = 4.0, Fr_1 \sim 9.0) \): Model 1 (Run BM44_2), Model 2 (Run Cir-5) and Model 3 (Run Run-3).

2.15 Distributions of dimensionless bubble count rate \( f_{ab} = F^* r_1/V_1 \) for identical inflow condition \( (x_1/d_0 = 4.0, Fr_1 \sim 9.0) \): Model 1 (Run BM44_2), Model 2 (Run Cir-5) and Model 3 (Run Run-3).

2.16 Dimensionless air discharge versus Weber number. Comparison between Model 1, Model 2 and Model 3: (a) \( (x-x_i)/r_1 \sim 1.6 \), (b) \( (x-x_i)/r_1 \sim 2.4 \), and (c) \( (x-x_i)/r_1 \sim 4.0 \).

2.17 Dimensionless turbulent diffusivity versus Weber number. Comparison between Model 1, Model 2 and Model 3: (a) \( (x-x_i)/r_1 \sim 1.6 \), (b) \( (x-x_i)/r_1 \sim 2.4 \), and (c) \( (x-x_i)/r_1 \sim 4.0 \).
3.1 Sketch of (a) water level rises by entrained air and (b) entrained and detrained air bubbles through free surface.

3.2 Sketch of hydraulic jump.

3.3 Air concentration distribution (After Chanson, 1997); $Fr_1=6.05$, $x_1=0.89$ m, $d_1 = 0.017$ m, $V_1 = 2.47$ m/s.

3.4 Horizontal distribution of entrained air volume per unit area; $Fr_1 = 6.05$.

3.5 Energy dissipation rate per unit length along $x$-direction; $Fr_1 = 6.05$.

3.6 Sketch of 2-D vertical supported plunging jet.

3.7 Void fraction distribution (After Cummings and Chanson, 1997); $x_1=0.0875$ m, $d_1 = 0.010$ m, $V_1 = 2.39$ m/s.

3.8 Vertical distribution of air volume per unit area, $Fr_1 = 7.63$.

3.9 Energy dissipation rate per unit length as a function of depth, $Fr_1 = 7.63$.

3.10 Sketch of vertical circular plunging jet.

3.11 Local averaged void fraction distributions-Cir-1 (Model 2, Freshwater); $x_1 = 0.10$ m, $V_1 = 2.51$ m/s.

3.12 Vertical distribution of entrained air volume per unit length-Model 2 ($d_0 = 12.5$ mm, Freshwater).

3.13 Total air volume of entrained air as function of (a) jet length, and (b) nozzle velocity-Model 2 ($d_0 = 12.5$ mm, Freshwater).

3.14 Total air volume of entrained air as function of impact velocity-Model 2 ($d_0 = 12.5$ mm, Freshwater).

3.15 Total energy dissipation rates versus impact velocity-Model 2 (Freshwater, $d_0 = 12.5$ mm).

3.16 Ratio of energy dissipation rate due to air bubbles to total energy dissipation rate as functions of (a) jet length, and (b) nozzle velocity-Model 2 ($d_0 = 12.5$ mm, Freshwater).
3.17 Comparison of the ratio of energy dissipation rate due to entrained air to total energy dissipation rate, $D_a$, between three typical phenomena for nearly identical inflow conditions (HJ: Hydraulic jump, VCPJ: Vertical circular plunging jet and V2DPJ: Vertical 2-dimensional plunging jet).

3.18 Transfer of momentum and air-water entrainment process at (a) hydraulic jump and (b) vertical plunging jet.

A-1 Rise velocity of air bubbles in water as a function of bubble size (Haberman and Morton, 1954)

4.1 Sketch of 2-D wave flume.

4.2 Accuracy of capacitance wave gauge in bubbly water: superelevation as a function of the depth-averaged void fraction. P0, P1 and P2 are the wave gauge, pointer gauge below foam and pointer gauge above foam, respectively.


4.4 Sketch. (a) Output signal from a probe during the passage of a bubble; (b) response of the wave gauge and probe.

4.5 Typical time-series of voltage output from the probe and wave gauge. (a) 2 cm above the still water level, (b) at still water level and (c) 2 cm below the still water level. Solid blue lines represent the output from the probe showing bubble pulses, dashed lines represent the still water level and red lines represent wave profile ($x-x_b = 0.50$ m; $H_0/L_0 = 0.076$: spilling breaker).

4.6 Typical time-series (a) 2 cm above the still water level, (b) at still water level and (c) 4 cm below the still water level. Solid blue lines represent the output from the probe showing bubble pulses, dashed lines represent the
still water level and red lines represent wave profile \((x-x_b = 0.70 \text{ m}; H_o/L_o = 0.024):\) plunging breaker).

**4.7** Time averaged void fraction over one wave period versus number of waves measured at \(x-x_b = 0.50 \text{ m} (H_o/L_o = 0.076):\) spilling breaker. (a) 2 cm above the still water level, (b) at still water level and (c) 2 cm below the still water level.

**4.8** Time averaged void fraction over one wave period versus number of waves measured at \(x-x_b = 0.70 \text{ m} (H_o/L_o = 0.024):\) plunging breaker. (a) 2 cm above the still water level, (b) at still water level and (c) 4 cm below the still water level.

**4.9** Vertical distribution of mean void fraction during breaking event. (a) Spilling breaker \((H_o/L_o = 0.076),\) and (b) plunging breaker \((H_o/L_o = 0.024).\)

**4.10** Void fraction distributions as functions of a dimensionless horizontal distance. (a) Spilling breaker \((H_o/L_o = 0.076),\) and (b) plunging breaker \((H_o/L_o = 0.024).\)

**4.11** Void fraction distribution as a function of dimensionless breaking duration time. (a) Spilling breaker \((H_o/L_o = 0.076),\) and (b) plunging breaker \((H_o/L_o = 0.024).\)

**4.12** Vertical distribution of dimensionless duration of braking event. (a) Spilling breaker \((H_o/L_o = 0.076),\) and (b) plunging breaker \((H_o/L_o = 0.024).\)

**4.13** Air volume distribution as a function of distance in the shoreward direction. (a) Spilling breaker, and (b) plunging breaker.

**4.14** Potential energy due to air bubbles as a function of distance in the shoreward direction. (a) Spilling breaker, and (b) plunging breaker.

**4.15** The volume per unit width \(V_a \text{ }^T (\text{m}^2)\) of air in the tube formed by the plunging wave crest versus wave height.
4.16 Comparison between normalized potential energy increased due to air bubbles and wave energy. (a) Spilling breaker \(H_0/L_0 = 0.076\), and (b) plunging breaker \(H_0/L_0 = 0.024\); \(E_b\) represents the wave energy at breaking point.

4.17 Ratio of potential energy dissipation rate due to entrained air: Spilling breaker (SP), and plunging breaker (PL).

5.1 Vertical distributions of void fraction by Eq. (5.1) \((C_0 = 0.15)\).

5.2 Definition of the coordinate and static water level rises.

5.3 Definition sketch of 2-D wave breaking. B.P. denotes the breaking point, R1 represents the outer surf zone (rapid transitions of wave shape), and R2 is the inner surf zone (rather slow change in wave shape). This sketch is also applicable for spilling breakers.

5.4 Vertical velocity component and correction term versus depth.

5.5 Typical vertical distribution of (a) density and (b) static pressure in air-water mixture.

5.6 Dimensionless static energy as a function of (a) \(C_0\) and (b) \(k_1h\).

5.7 Dimensionless potential energy as a function of \(C_0, k_1h\) and \(k_1H\).

5.8 Dimensionless kinetic energy as a function of \(C_0, k_1h\) and \(kh\).

5.9 Variation of dimensionless energy flux with \(C_0, k_1h\) and \(kh\).

5.10 Variation of dimensionless radiation stress with \(C_0, k_1h, k_1H\) and \(kh\).

6.1 Relationship between non-dimensional parameter \(k_0\) and dimensionless distance. (a) Spilling breaker and (b) plunging breaker.

6.2 Relationship between non-dimensional parameter \(C_0\) and dimensionless distance. (a) Spilling breaker and (b) plunging breaker.

6.3 Comparison of Eq. (5.1) with experimental data of void fraction- spilling breaker \((H_0/L_0 = 0.076)\). The solid line (—) are from Eq. (5.1) and hollow symbol (○) show the experimental data.
6.4 Comparison of Eq. (5.1) with experimental data of void fraction- plunging breaker ($H_0/L_0 = 0.024$). The solid line (—) is from Eq. (5.1) and hollow symbol (○) show the experimental data.

6.5 Comparison the rate of energy dissipation due to wave and air bubble for spilling breaker. (a) $H_0/L_0 = 0.062$ and (b) $H_0/L_0 = 0.076$.

6.6 Comparison the rate of energy dissipation due to wave and air bubble for plunging breaker. (a) $H_0/L_0 = 0.024$ and (b) $H_0/L_0 = 0.032$.

6.7 Comparison between model results and experimental data of (a) wave height and (b) wave set-up for $H_0/L_0 = 0.062$ ($\alpha = 4.0$, Spilling breakers).

6.8 Comparison between model results and experimental data of (a) wave height and (b) wave set-up for $H_0/L_0 = 0.076$ ($\alpha = 4.0$, Spilling breakers).

6.9 Comparison between model results and experimental data of (a) wave height and (b) wave set-up for $H_0/L_0 = 0.024$ ($\alpha = 3.9$, Plunging breakers).

6.10 Comparison between model results and experimental data of (a) wave height and (b) wave set-up for $H_0/L_0 = 0.032$ ($\alpha = 4.3$, Plunging breakers).

6.11 Wave height, wave set-up and estimated water level rise due to air bubbles as a function of horizontal distance. (a) Spilling breaker ($H_0/L_0 = 0.076$) and (b) plunging breaker ($H_0/L_0 = 0.024$).

6.12 (a) Wave run-up height of regular waves on gentle beaches, (b) width of swash zones versus wave steepness.

6.13 Comparison of air entrainment in prototype and in laboratory. (a) Plunging breaker at Terasawa beach and (b) laboratory model of a plunging breaker on a sloping beach.
## List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Experimental flow conditions of circular plunging jets</td>
<td>42</td>
</tr>
<tr>
<td>2.2</td>
<td>Froude and Reynolds number, air volume discharge &amp; turbulent intensity</td>
<td>44</td>
</tr>
<tr>
<td>2.3</td>
<td>Characteristics of air-water flow measurements in circular plunging jets</td>
<td>48</td>
</tr>
<tr>
<td>3.1</td>
<td>Measurements of potential energy and energy dissipation rate</td>
<td>74</td>
</tr>
<tr>
<td>3.2</td>
<td>Measurements of potential energy and energy dissipation rate</td>
<td>78</td>
</tr>
<tr>
<td>3.3</td>
<td>Measurements of potential energy and energy dissipation rate</td>
<td>85</td>
</tr>
<tr>
<td>4.1</td>
<td>Wave breaking experiments: characteristics of wave breaking</td>
<td>94</td>
</tr>
<tr>
<td>5.1</td>
<td>Wave transformation model using energy balance</td>
<td>134</td>
</tr>
</tbody>
</table>
List of Symbols

Main symbols

- $A$: wave amplitude
- $C$: void fraction
- $C_{max}$: maximum void fraction
- $C_0$: void fraction at still water level
- $c$: wave celerity
- $c_g$: wave group velocity
- $ch_{ab}$: air bubble chord length
- $D$: energy dissipation rate per unit area
- $D_a$: ratio of energy dissipation due to entrained air
- $D_{air}$: energy dissipation rate per unit area by air bubble
- $D_p$: penetration depth of air bubbles
- $D_t$: turbulent diffusivity of air bubble in air-water flow
- $D^\#$: dimensionless turbulent diffusivity
- $d$: mean water depth
- $d_{ab}$: air bubble diameter
- $d_0$: nozzle diameter
- $d_1$: upstream flow depth
- $d_2$: downstream flow depth
- $E$: local wave energy
- $E_b$: wave energy at the breaking point
- $E_d$: rate of energy dissipation due to entrained air
- $E_d^T$: total rate of energy dissipation due to entrained air
\begin{itemize}
\item $E_{da}$ energy dissipation rate per unit length
\item $E_H$ rate of energy dissipation through head loss
\item $E_j$ rate of upstream energy transport
\item $EF$ energy flux due to wave
\item $EF'$ change in energy flux due to entrained air
\item $F$ number of bubble per second impact at probe tip
\item $F_{max}$ maximum bubble frequency
\item $Fr_1$ Froude number at impingement point
\item $f_{ab}$ dimensionless air bubble frequency
\item $g$ acceleration of gravity
\item $H$ local wave height
\item $H_b$ wave height at breaking point
\item $H_0$ wave height at offshore point
\item $h$ still water depth
\item $h_b$ still water depth at breaking point
\item $h_G$ distance of center-of-gravity of air volume from free surface
\item $I_0$ modified Bessel function of the first kind of order zero
\item Iribarren number
\item $i$ integer
\item $k$ wave number
\item $k_0$ empirical coefficient
\item $k_1$ wave decay factor
\item $KE$ kinetic energy due to wave
\item $KE'$ change in kinetic energy due to entrained air
\item $L_0$ wave length at offshore point
\item $L_R$ geometric scale ratio
\item $M$ dimensionless potential energy
\item $m$ beach slope
\end{itemize}
$N$ dimensionless potential energy

$O$ dimensionless kinetic energy

$P$ dimensionless energy flux

$p_{air}$ atmospheric pressure

$p_D$ dynamic pressure

$PE$ potential energy of air-water in the presence of wave

$PE_0$ potential energy of water in the absence of wave

$p$ pressure of air-water mixture

$p_a$ pressure of air

$p_w$ pressure of water

$p_0$ static pressure

$Q$ dimensionless radiation stress

$Q_w$ water discharge

$Q_a$ air discharge

$Q_{ad}$ down-going air discharge

$Q_{au}$ up-going air discharge

$q$ water discharge per unit width

$R$ air bubble radius

$R_u$ run-up

$R_e$ Reynolds number

$r$ radial distance

$r_{50}$ radial distance where $V=0.5*V_f$

$r_a$ radial distance from jet centerline to inner edge of void fraction distribution

$r_b$ radial distance from jet centerline to outer edge of void fraction distribution

$r(C_{max})$ distance normal to the flow direction where $C=C_{max}$

$r(F_{max})$ distance normal to the flow direction where $F=F_{max}$

$r_1$ jet radius at impingement point

$S$ scale
$SE$ static energy of air-water

$SE_0$ static energy of water

$S_{xx}$ radiation stress due to wave

$S_{xx}'$ change in radiation stress due to entrained air

$T$ wave period

total measuring time

$T_u$ turbulent intensity

t time

$t_{ch}$ bubble chord time

$t_r$ bubble rise time

$u$ velocity of air-water in the $x$-direction

$u_a$ velocity of air in the $x$-direction

$u_w$ velocity of water in the $x$-direction

$V$ velocity

$V_a$ air volume per unit length and width

$V_a^T$ total air volume

$V_{au}$ up-going air volume

$V_{ad}$ down-going air volume

$V_0$ nozzle flow velocity

$V_e$ onset velocity of air entrainment

critical velocity

$V_l$ upstream flow velocity

impact flow velocity of plunging jet

$V_2$ downstream flow depth

$We_1$ Weber number at impingement point

$w$ velocity of air-water in the $z$-direction

$w_a$ velocity of air in the $z$-direction
\( w_r \) bubble rise velocity

\((w_r)_{Hyd} \) bubble rise velocity in hydrostatic pressure

\( w_w \) velocity of water in the \( z \)-direction

\( w' \) correction term of \( w \)

\( x \) distance along the flow direction
distance along the still water surface of channel

distance between wave maker and breaking point

distance between nozzle and impingement point
distance between channel intake and upstream flow condition
denotes the position in the surf zone

denotes the position in the surf zone

distance measured normal to the flow direction

distance measured perpendicular to the channel bottom
distance measured perpendicular to water level rise

**Greek symbols**

2 free parameter

\( \Delta E_{c_g} \) energy flux between sections \( x_1 \) and \( x_2 \)

\( \Delta H \) head loss

\( \Delta h \) water level rise due to entrained air

\( \Delta L \) distance of transit zone

\( \Delta PE \) time averaged potential energy due to entrained air

\( \Delta SE \) change in static energy due to entrained air

\( \Delta t \) duration of breaking event

\( \Delta Y \) width of swash zone

\( \Delta \tau \) total time of air bubble encounter
\[ \Delta \eta \] superelevation of water surface
\[ \eta \] elevation of water surface
\[ \eta_+ \] elevation of water surface above still water surface
\[ \eta' \] wave set-up
\[ \tilde{\eta}_b \] wave set-up at breaking point
\[ \mu_w \] dynamic viscosity of water
\[ \theta \] wave direction
\[ \rho \] density of air-water mixture
\[ \rho_a \] density of air
\[ \rho_w \] density of water
\[ \sigma \] surface tension
\[ \omega \] angular frequency

**Abbreviations**

- **B.P.** breaking point
- **H** horizontal
- **PL** plunging
- **SP** spilling
- **T.Z.** transit zone
- **V** vertical
- \( \partial / \partial x \) partial derivative with respect to \( x \)
- \( \partial / \partial z \) partial derivative with respect to \( z \)
1

General Introduction

Summary

The introductory chapter begins with an overview of the mechanism of air entrainment and detrainment through free surface of fluids at rest and moving. The concept of energy dissipation by air bubbles is defined and its applications and significance for both plunging jet flows (steady) and surf zone waves (unsteady) are discussed.

1.1 Presentation

On Earth, the dependence of life on air and water is absolute. We live within the atmosphere and are affected by environmental problems (e.g. weather and increasing carbon dioxide). In this respect, the ocean plays a vital role for the dissolution of carbon dioxide and release of oxygen to the atmosphere contribute to the balance between these gases (Chanson, 1997). Air and water are constantly interacting in the atmosphere, at the sea surface and on the continents and there are constantly exchanges of fluxes of momentum and heat. These processes emphasize the importance of ocean dynamics as well as atmospheric dynamics. In the ocean, some part of wave energy may be transferred to the entrained air bubbles during wave breaking and dissipated, with the bubbles escaping at the free surface.
Air bubbles entrained by breaking waves play an important role in the transport of mass and energy across the air-sea interface (Melville, 1992). Study of wave transformation in shallow waters and related problems have been one of the most important subjects in which coastal engineers have been deeply concerned throughout the last few decades. A main issue is the estimate of wave energy dissipation in the surf zone. Many ideas have been proposed in recent studies. Between them two mechanisms have received considerable attention from last two decades: the surface roller concept and energy flux difference model. Svendsen (1984) presented a solution considering surface roller theory, which gives good agreement for wave height variation from breaking point to, inshore, but the wave set-up is not favorable. Dally et al. (1985) used a heuristic expression for energy dissipation. Their model was calibrated and verified, using laboratory data, with good results for the wave decay and the maximum set-up values for some test cases; but it does not describe correctly the distribution of set-up/set-down across the surf zone. All the models are fairly capable of predicting wave height variations but the set-up. There is some controversy that although the wave height is decreasing after wave breaking, the momentum might not (Dally et al., 1985, Fig. 11 & Svendsen, 1984, Figs. 14 and 15). Dally et al. (1985) suggested that there is no energy dissipation before the curl of the breaking wave touches down and air is entrained. Both a literature review and personal observations indicate that air bubbles have significant effects in the surf zone but up to present, there is no energy dissipation model considering explicitly the air entrainment process except Führbötter’s model. Führbötter (1970) developed an energy dissipation model considering air bubble effects but it was not validated with the experiments.

This thesis focuses on the description and quantification of the entrainment of air in the surf zone and its effects on the rate of energy dissipation. The major objectives of the thesis are:

(1) investigate the air entrainment, detrainment and their significance,
(2) develop a simple model and apply it to steady and unsteady cases,
(3) quantify the energy transformation and dissipation by air bubbles in plunging jet flow, and
(4) construct a modeling of surf zone hydrodynamics based on entrained air.
A brief overview of the major objectives is presented below.

(i) Air entrainment and detrainment

![Figure 1.1: Air entrainment and detrainment cycle.](image)

The above sketch (Fig. 1.1) presents the cycle of air entrainment and detrainment process through a free surface at breaking waves. Air entrainment may occur naturally or artificially. It is observed in coastal, hydraulic and chemical engineering applications (e.g. Fig.1.2 and Fig. 1.3). In penetrating the water surface, the droplets drag air into the water. The literature on this subject deals for a large part with theoretical and experimental studies of the various aspects of air entrainment. A major part of this study will be addressed to this topic and consequently energy dissipation processes. Except compression and surface tension effects, a large portion of wave energy or upstream energy is stored at first by the static energy of the air bubbles, which are driven
into water and afterward, it is dissipated by turbulence (Fürhböter, 1970). The region where energy dissipation takes place is not necessarily near the region of air supply. It depends on the water flow conditions and transport capacity. The air may be transported over large distances in the surf zone (Fig. 1.3(b)).

(ii) Air bubble model
Most existing energy dissipation models in surf zone are based primarily on four main assumptions (Massel, 1996):

(i) dissipation is equivalent to dissipation in a bore connecting two regions of uniform flow (Battjes and Janssen, 1978),
(ii) dissipation is proportional to the difference between the local energy flux and the stable energy flux (Dally et al., 1985),
(iii) dissipation is controlled by the presence of a surface roller (Svendsen, 1984)
(iv) the breaking wave height is saturated, i.e. the wave height is proportional to the local water depth and the proportionality coefficient is assumed to be constant across the surf zone,

An air bubble model is developed based on physical observations and the cycle of entrainment and detrainment sketched in Fig. 1.1. The concept of air bubble model is based upon the first law of thermodynamics, which states that the net energy (e.g. potential energy, heat) supplied to the system equals the increase in energy of the system plus the energy that leaves the system as work is done. The air bubble model includes two parameters that are well defined and have physical meanings. The choice of the model based on air entrainment in plunging jet flows and surf zone waves can ensure that energy dissipation is quantitatively and qualititively comparable to that of the existing models. This issue is discussed in detail in chapter 3 and chapter 6.
(iii) *Energy transformation and dissipation by air bubbles in plunging jet flow*

Coastal engineers base their thinking on the premise that there are some similarities of air entrainment between plunging jet and plunging breakers. In this respect, the study is restricted only for plunging jets in this section. Three types of air entrainment for plunging jets are included in this study: a 2-D vertical plunging jet, a vertical circular plunging jet and a hydraulic jump. A brief discussion is given below about vertical plunging jets and hydraulic jump.

![Figure 1.2](image)

**Figure 1.2:** Photographs of (a) waterfall (Photograph by Dr. Chanson) and (b) hydraulic jump (Photograph by Prof. G.R. Mckay, Australia).

**Air entrainment at vertical plunging jets**

When a water jet impinges a pool of water at rest, air bubbles may be entrained and carried away below the pool free surface (Fig. 1.2(a)): this process is called plunging jet entrainment discussed in chapters 2 and 3. The air entrainment process is a function of...
the impact velocity. In a vertical plunging jet, air bubbles start to be entrained when the jet impact velocity $V_I$ exceeds a critical value. Plunging jet flow situations are encountered in nature. There are some similarities of air entrainment at plunging jet flow and in plunging breaking waves (e.g. Chanson and Lee, 1997).

**Air entrainment at hydraulic jump**

The hydraulic jump is characterized by the development of large-scale turbulence, surface waves and spray, energy dissipation and air entrainment (Fig. 1.2(b)). It is a limiting case of a plunging jet in a horizontal channel. The study is primarily interested to investigate the rate of energy dissipation by air bubble entrainment in hydraulic jump, because there is some similarity between the front face of a spilling breaker and a hydraulic jump (Fredsoe and Deigaard, 1992). In a hydraulic jump the roller is stationary and visual observation suggests that the maximum roller height is about 10 to 20% larger than the downstream flow depth (Chanson, 1997).

**(iv) Air entrainment by breaking waves in surf zone**

Wave breaking on beaches is a conspicuous, often impressive natural phenomenon. The surf zone is characterized by some air entrainment which is highlighted by the “white waters”. The sloping bottom affects the breaking process while a great amount of air bubble is entrained into water downstream of the breaking point. The entrained air bubble distribution is highly variable and in general the bubble density decreases with depth and increases with sea-state (Crawford and Farmer, 1987). Furthermore, the present study suggests a similar conclusion in laboratory experiments (chapter 4 and 5). Recently, similar results also measured by Stanton and Thornton (2000) in the field. The emphasis is on surf zone modeling of the energy dissipation by air bubbles due to wave breaking.
Breaking waves in the field

In the surf-zone, the most prominent stage of wave breaking is the initial overturning motion of the wave crest that creates spray and white water, sometimes with the forward projection of a jet of water. During this process, some energy is stored into air bubbles and some are transferred from the organized wave motion to a wider region resulting in wave height decay. Horikawa and Kuo (1966) suggested that the entrained air bubbles play main role of energy dissipation at least at initial stages.

Breaking waves are mainly of four types (i.e. spilling, plunging, collapsing and surging). This classification is based on the physical appearance and mathematical definition (Galvin, 1968, 1972). Although air entrainment occurs primarily in all types of wave breaker but the study focuses only on spilling and plunging breakers (Fig. 1.3). These issues are addressed in chapter 4, 5 and 6 where it will be shown that air entrainment may account for a significant fraction of the energy lost in spilling and plunging breakers.

(a) (b)

Figure 1.3: Breaking waves. (a) Spilling breaker (Tweed Heads, Australia, courtesy of Dr. Chanson), and (b) plunging breaker (Terasawa beach, Japan).
Breaking waves in the laboratory

Observations and experiments suggest that the type of wave breaking is a function of the incident wave steepness and the beach slope. Most quantitative knowledge of the subject derives from laboratory experiments with controlled breaking waves. The importance of energy dissipation and water level rise by entrained air which have significant bearing on this thesis are discussed.

Rapp and Melville (1990) reported that up to 40% of the initial energy contained in the wave field could be dissipated by breaking where they measured it taking the difference in energy flux between upstream and downstream measurements. Comprehensive reviews of different models to describe breaking waves in shallow waters can be found in Dally et al., 1985; Svendsen, 1984; Battjee, 1988. Most importantly, these models do not address the effects of air bubble entrainment. Recently, Aoki et al. (2000) proposed that the water level rise caused by the unsteady air bubble entrainment at a plunging breaker result in energy transfer from short waves to long-period waves near the shoreline. The present study also regards the water level rise due to entrained air as has significant effects on wave set-up.

Several researchers performed some important investigations on energy dissipation due to air bubbles in the surf zone. These are the following:

(i) the entrained air bubbles induce firstly a rise in water level associated with an energy transfer into potential energy (Führböter, 1970).

(ii) a large fraction (30-50%) of the wave energy lost is expended in entraining the bubble plume (Lamarre and Melville, 1991).

(iii) enhancing energy and mass transfers by bubble entrainment (Merlivat and Memery, 1983).

(iv) volume of entrained air correlates with the energy dissipation (Loewen and Melville, 1994).

(v) plunging breakers induce the highest degree of aeration (Hall, 1990)
1.2 Outline of the dissertation

The main effort has been made to describe the surf zone hydrodynamics (e.g. potential energy, kinetic energy, energy flux, radiation stress, energy dissipation rate and mean water level) based on a new idea that consider air bubble effects. In order to understand the role of air bubbles in the surf zone (unsteady), first performed comprehensive experiments in steady plunging jet flows. Since the origin of air entrainment at the ocean surface is turbulent and complicated, an averaging technique has been used. The content of this thesis is divided into six chapters. An outline of each chapter is briefly described in the following.

Chapter 2 is devoted to a comprehensive description of void fraction measurements, void fraction measuring devices and air-water properties in plunging jet flows. The experiments are conducted for three geometric scales based upon the Froude similitude and discusses the scale effects. For one identical scale, experiments were reproduced with freshwater and seawater. In addition, this chapter discusses the penetration depth and bubble chord times.

Chapter 3 introduces a simple model that enables to estimate the energy transformation and dissipation rate. The model is tested for experimental data of three typical phenomena of air entrainment in free surface flows: a hydraulic jump, a 2-D vertical and vertical circular plunging jet. The results are compared between the three phenomena. A relationship between entrained air volume and water falling heights are also presented in this chapter.

Chapter 4 presents an extensive set of measurements of air entrainment in 2-D laboratory breaking waves for both spilling and plunging breakers. The analysis of the air pulse in the surf zone has been performed. The vertical and horizontal distributions of void fraction are described in detail. The measurements demonstrate that the volume of entrained air and energy dissipation rate is a function of wave steepness.
Chapter 5 describes surf zone wave parameters (e.g. potential energy, kinetic energy, energy flux, radiation stress), which is developed considering air bubble effects. The model presented in chapter 3 is extended for unsteady cases with the help of averaging technique. Energy balance equation and momentum balance equation are described in terms of air bubble effects.

Chapter 6 discusses the applicability of the model and the scale effects based on laboratory data. The physical parameters of the air bubble model are developed based on experimental data. The reliability of basic assumption of void fraction distributions is shown by the experiments. The breaking characteristics of the model, with regard to the wave height and wave set-up, are compared with the experimental data. In addition, the run-up results are also presented.

Chapter 7 concisely summarizes the findings of this study.
Measurements of Void Fraction in Circular Plunging Jet

Summary

Measurements of the local data of void fraction and bubble frequency were performed using a conductivity probe. Three scale models were used and detailed air-water measurements were performed systematically for identical Froude numbers. Similar experiments were performed with freshwater and seawater. The results show (i) lesser air entrainment in seawater plunging jets for identical inflow conditions, (ii) the distributions of void fraction follow closely analytical solutions of the diffusion equation as developed by Chanson (1997), (iii) air entrainment process depends on the jet impact velocity, (iv) more fine bubbles were detected in seawater than in freshwater, (v) penetration depth was found to be a sensitive to falling water jet height, (vi) bubble chord times were measured and the pseudo-bubble chord sizes were obtained to be in the range of less than 0.5 mm to more than 10 mm, and (vii) significant scale effect was seen for the model with Weber number less than 1000.

2.1 Introduction

In hydraulic structures, a vertical plunging jet often a primary cause of air entrainment (Sene, 1988). Air entrainment occurs often in nature and is also encountered in many
industrial operations. It can occur during the pouring and filling one liquid into another. The pouring of liquids, breaking waves at the ocean surface, waterfalls, hydraulic jumps, are a few examples of the many situations that are readily observed to cause the phenomenon. Past studies on plunging jet flows showed that air bubbles start to be entrained when the jet impact velocity $V_1$ exceeds a characteristic velocity $V_e$ which is a function of the jet turbulence: i.e., $V_e = f(T_u)$ (McKeogh, 1978; Cummings and Chanson, 1999). Bin (1993) noted that the mechanism of air entrainment depends on a number of parameters including the flow rate, jet surface turbulence and jet geometry. The process of air entrainment can be visualized as air pockets first being trapped between water surface and inflow, and then carried downstream by the mean flow.

Numerous studies were conducted with circular plunging jets (Bin 1993). However, to investigate the rate of energy dissipation and other characteristics, void fraction distributions have been presented in this chapter for vertical circular plunging jets both in freshwater and in seawater. The measurements of void fraction distribution are not novel in freshwater but in seawater.

In free-surface flows, gravity effects are important and most laboratory studies are based upon the Froude similitude (Chanson, 1999). The entrainment of air bubbles and the mechanisms of bubble breakup and coalescence are dominated by surface tension effects implying the need for Weber similitude (Wood 1991, Chanson 1997). For geometrically similar models, it is impossible to satisfy simultaneously Froude and Weber similarities with the same fluids in model and prototype. In small size models based upon a Froude similitude, the air entrainment process may be underestimated (Chanson et al., 2002).

This chapter gives a thorough account of basic air entrainment characteristics, scale effects and various properties of void fraction fields. In this respect, three geometric scales are selected and similar experiments are conducted based upon the Froude similitude.
2.2 Experiments and observations

2.2.1 Apparatus

Experiments in vertical circular plunging jet flows were conducted in two flumes with four configurations called Models 1, 2, 3 and 4. Model 1 was located at the University of Queensland, Australia. The receiving channel was 0.3 m wide, 1.8 m deep with glass walls and 3.6 m long. The circular PVC pipe was 3.5 m long with inside diameter 0.025 m for Model 1. On the other hand, Models 2, 3 and 4 were located at Toyohashi University of Technology, Japan. The water pool was 2.0 m long 0.10 m wide and 0.74 m with deep steels and glasses wall (Fig. 2.1). A straight cylindrical nozzle (PVC) of 1.0 m height was used with inside diameter 0.0125 m for Models 2 and 4 and almost same height but inside diameter 0.0068 m was used for Model 3.

Figure 2.1: Sketch of water pool.
The Models were designed to be geometrically similar based upon a Froude similitude with undistorted scale. The geometric scaling ratio was $L_R = 2.0$ between model 1 and models 2 and 4 whereas it was 3.66 between models 1 and 3.

In the measurements, freshwater and seawater were used. Seawater was collected on the Enshu coast (Pacific Ocean, Fig.1.3 (b)). The water supply comes from receiving pool by pumps. Flow rate of water was measured using a cylindrical glass and the water depth by pointer gage. The pump ups the water to the top and was made to generate waterfall by sending into PVC pipe top end. Thereby, a water level was always constant in a tank. The discharge was measured at every one hour and error was less than 2%.

### 2.2.2 Instrumentation

A Kanomax™ System 7931 single-tip L-shape conductivity probe (inner electrode $\varnothing$: 0.1 mm) was used in Models 2, 3 and 4 (Fig. 2.2(a)). An air bubble excites the conductivity probe. The measurement principle of conductivity probes is based upon the difference in electrical resistivity between air and water. The resistance of water is one thousand times lower than the resistance of air. When the probe tip is in contact with water current will flow between the tip and the supporting metal; when it is in contact with air no current will flow. The typical response of the probe when a bubble passes is illustrated in Fig. 2.2(b). The bubble pulse analysis will be explained in more detail in the chapter 4.

The local void fraction is defined as the time that the probe tip is in air with respect to the total measuring time:

$$C = \sum \frac{\Delta t}{T}$$  \hspace{1cm} (2.1)

where $i$ indicates an individual bubble, $\Delta t$ is time that probe tip is in air and $T$ is the total measuring time.
Figure 2.2: Sketch of (a) conductivity probe and (b) response of probe tip.

The displacement of the probe in the direction normal to the jet direction was controlled by fine adjustment traveling mechanisms. Measurements were recorded with scan rate of 2 kHz per channel. In models 2 and 3, raw probe outputs were recorded at 25 kHz for 2.6 seconds to calculate bubble chord time distributions. The void fraction and bubble count rate were calculated by the Kanomax™ analog integrator during five minutes in Models 2 and 3. The probe set up at some installation depth was scanned horizontally at intervals of a minimum of 0.2 mm so that it may pass along the central point of a jet.
Measurements were taken on the pool centerline. The impact velocity \( V_I \) and water jet diameters \( d_i \) were deduced from the following equations:

\[
Q_w = \frac{\pi}{4} V_0 d_0^2 \tag{2.2}
\]

\[
Q_w = \frac{\pi}{4} V_i d_i^2 = \frac{\pi}{4} V_0 d_0^2 \tag{2.3}
\]

\[
\frac{V_1^2}{2g} = \frac{V_0^2}{2g} + x_i; \quad \text{[Bernoulli equation]} \tag{2.4}
\]

### 2.2.3 Air-water flow regions

Air-water flow field is characterized by three regions: developing flow region (DFR), redistribution flows region (RFR) and a fully developed flow region (FDFR) (Fig. 2.3).

![Air water flow regions of a vertical circular plunging jet.](image)

**Figure 2.3:** Air water flow regions of a vertical circular plunging jet.
In the developing flow region, the air content is zero on the jet centerline. The region where air content is rapidly redistributed from zero void fractions to maximum void fraction on the jet centerline is defined as redistribution flow region. And the immediate downstream flow region of RFR is known as fully developed flow region. In this region the air content decreases from maximum void fraction to zero void fraction on the jet centerline. In Fig. 2.3, from impinging point to point A, point A to point B, and point B to rest of region are known as DFR, RFR and FDFR respectively.

Photographs of air entrainment in vertical circular plunging jet are presented in Fig. 2.4 for identical inflow conditions. In naked eyes, it is seen from Fig. 2.4 that the bubble motion roughly vertical and the rise bubbles are scattered near the free surface.

(a)

**Figure 2.4:** Photograph of bubble clouds in vertical circular plunging jet. (a) Model 2 \((d_0 = 12.5 \text{ mm, Freshwater})\), \(x_j = 50 \text{ mm, } V_j \sim 3.0 \text{ m/s, } Fr_j = 9.0\). (b) Model 4 \((d_0 = 12.5 \text{ mm, Seawater})\), \(x_j = 50 \text{ mm, } V_j \sim 3.0 \text{ m/s, } Fr_j = 9.07\).
For identical inflow photograph (Fig. 2.4) it is seen visually that the seawater experiments appeared to entrain more fine bubbles than the freshwater plunging jets. Such a trend is reasonably consistent with chord time measurements. Chord time results (section 2.4.3) suggest that the lesser large-size bubbles were entrained in seawater compared to the freshwater. During the experiments, it was observed that a large number of tiny bubbles were seen in the entire flume. These tiny bubbles were strongly affected by large recirculation eddies and their rise velocity appeared very small (Chanson et al., 2002).

2.2.4 Observations

Observations of the bubbles were made through the glass sidewall of the pool by both inspection and using high-speed digital video camera (Sony). For \( V_l > V_e \), air entrainment was visible and Fig 2.4 presents underwater photographs of the bubbly flow region. Larger air packets were entrained below the air cavity with the stretching and breakup of the cavity tip. Visual observations showed predominantly entrained bubble sizes between 0.5 and 5 mm. Such millimetric size bubbles have a nearly constant bubble rise velocity.

It was also observed that the lower limit of bubble swarm fluctuates continuously, but time average gives the maximum penetration depth. In the present experiments, high-speed video camera revealed that the bubble clouds had a diameter of about 3.5-12 cm in fresh water, whereas it was larger in seawater and the maximum diameter found in redistribution flow region. Typical photographs of bubble cloud are shown in Fig. 2.4. Moreover, it was observed that the air bubbles were more tightly packed at the cloud center than at the edges (Fig. 2.4).

An example of output signals is shown in Figure 2.5, which is the result of the cloud detection algorithm for a typical L-shape probe signal. The upper graph shows the raw data with the conductivity probe and lower graph (square wave) shows the algorithm output.
Figure 2.5: Signal processing technique. Upper: Raw signal output: A voltage of 0.6 V occurs when no air bubbles are touching the probe tip. Lower: Square wave represents the algorithm output. Each pulse corresponds to an air bubble encounter.

2.2.5 Experimental flow conditions

It was mentioned that at a plunging jet air bubbles are entrained when the jet impact velocity $V_i$ exceeds a critical velocity $V_e$, called onset velocity or inception velocity. The present results of air entrainment inception conditions are summarized in Table 2.1, where the onset velocity for air entrainment $V_e$ is defined as the mean jet velocity at impact.
McKeogh (1978) showed first that the inception conditions are functions of the free-falling jet turbulence. Inception of air bubble entrainment is not a precise condition (Cummings and Chanson 1999). For example, entrainment of bubble less than 0.2 mm diameter is very difficult to detect visually. A jet may entrain one of a few bubbles only every few minutes. During the measurements of inception velocity, a longer investigation period (nearly 3 minutes) was selected because of occasional entrapment of fine bubbles. It was observed consistently that the inception velocity $V_e$ increased with increasing jet length $x_1$ for given experiments. Such a result consistent with circular jet data (McKeogh, 1978). The turbulent intensity $T_u$ decreases with increasing $x_1$ because of no friction on free falling jet. It must be emphasized that the data depend critically upon the definitions of air entrainment inception and of the jet turbulence.

**Table 2.1: Experimental flow conditions of circular plunging jets**

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Run</th>
<th>$Q_w$</th>
<th>$d_0$</th>
<th>$V_e$</th>
<th>$x_1$</th>
<th>$V_1$</th>
<th>$d_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>L/s</td>
<td>mm</td>
<td>m/s</td>
<td>m</td>
<td>m/s</td>
<td>mm</td>
</tr>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(6)</td>
<td>(7)</td>
<td>(8)</td>
<td>(9)</td>
</tr>
<tr>
<td>Model 1</td>
<td>BM35_1</td>
<td>1.6</td>
<td>25</td>
<td>1.580</td>
<td>0.10</td>
<td>3.50</td>
<td>23.9</td>
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<tr>
<td></td>
<td>BM4_2</td>
<td>1.9</td>
<td></td>
<td></td>
<td></td>
<td>4.10</td>
<td>24.2</td>
</tr>
<tr>
<td></td>
<td>BM44_2</td>
<td>2.0</td>
<td></td>
<td></td>
<td></td>
<td>4.40</td>
<td>24.3</td>
</tr>
<tr>
<td>Model 2</td>
<td>Cir-1</td>
<td>0.27</td>
<td>12.8</td>
<td>1.420</td>
<td>0.10</td>
<td>2.51</td>
<td>11.70</td>
</tr>
<tr>
<td></td>
<td>Cir-2</td>
<td>0.27</td>
<td></td>
<td>1.027</td>
<td>0.05</td>
<td>2.31</td>
<td>12.22</td>
</tr>
<tr>
<td></td>
<td>Cir-3</td>
<td>0.27</td>
<td></td>
<td>0.750</td>
<td>0.025</td>
<td>2.20</td>
<td>12.51</td>
</tr>
<tr>
<td></td>
<td>Cir-4</td>
<td>0.35</td>
<td></td>
<td>1.027</td>
<td>0.05</td>
<td>2.89</td>
<td>12.46</td>
</tr>
<tr>
<td></td>
<td>Cir-5</td>
<td>0.37</td>
<td></td>
<td>1.027</td>
<td>0.05</td>
<td>3.10</td>
<td>12.49</td>
</tr>
<tr>
<td>Model 3</td>
<td>Run-1</td>
<td>0.060</td>
<td>6.83</td>
<td>0.734</td>
<td>0.0273</td>
<td>1.82</td>
<td>6.53</td>
</tr>
<tr>
<td></td>
<td>Run-2</td>
<td>0.074</td>
<td></td>
<td></td>
<td></td>
<td>2.15</td>
<td>6.62</td>
</tr>
<tr>
<td></td>
<td>Run-3</td>
<td>0.080</td>
<td></td>
<td></td>
<td></td>
<td>2.30</td>
<td>6.65</td>
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<tr>
<td>Model 4</td>
<td>Sea-2</td>
<td>0.29</td>
<td>12.8</td>
<td>1.038</td>
<td>0.05</td>
<td>2.46</td>
<td>12.24</td>
</tr>
<tr>
<td></td>
<td>Sea-4</td>
<td>0.35</td>
<td></td>
<td></td>
<td></td>
<td>2.89</td>
<td>12.46</td>
</tr>
<tr>
<td></td>
<td>Sea-5</td>
<td>0.38</td>
<td></td>
<td></td>
<td></td>
<td>3.13</td>
<td>12.49</td>
</tr>
</tbody>
</table>
Although jet turbulence was not measured in Models 2 and 3, the results are consistent with the observations of McKeogh (1978) and Ervine et al. (1980). The results demonstrate also that the inception conditions are identical in freshwater and seawater for an identical experiment: i.e., \( V_e = 1.0 \, \text{m/s} \) for \( d_0 = 12.5 \, \text{mm} \) and \( x_1 = 50 \, \text{mm} \). The entire flow conditions and air entrainment inception conditions of freshwater and seawater are summarized in the Table 2.1.

### 2.2.6 Quantity of entrained air discharge

The quantity of entrained air discharge \( Q_a \) was calculated from the distributions of void fraction \( C \) and air-water velocity \( V \):

\[
Q_a = 2\pi \int_0^\infty C \* V \* r \, dr \tag{2.5}
\]

where \( r \) is the radial distance normal to flow direction and \( C \) and \( V \) were measured below the impingement point (i.e. \( x > x_1 \)). Although air-water velocity was not measured in the present study, it will take same shape as in monophasic shear flows (Brattberg and Chanson, 1998). For accurate estimate of quantity of entrained air discharge, an order of magnitude of the air-water velocity was assumed \( V = V_1 / 2 \) at the location \( r_{50} \) (Brattberg and Chanson 1998, Chanson et al. 2002) where \( r_{50} \) is the distance normal to the flow direction. The air discharge was measured by Eq. (2.5) and summarized in Table 2.2 (column 5).

The dimensionless quantity of entrained air \( (Q_a / Q_w) \) was also calculated using the empirical formula (Eq. (2.6)) of van de Donk (1981) depending upon the ratio of water jet length and nozzle diameter. The estimated values of \( Q_a / Q_w \) from Eq.(2.6) were 0.22, 0.13-0.34 and 0.22 corresponding to Model 1, Model 2 and Model 3, respectively. The measurements of entrained air discharge by Eq. (2.6) were found larger than measured by Eq. (2.5).
\[
\frac{Q_a}{Q_w} = 0.09 \times (x_1/d_0)^{0.65}
\]

(2.6)

where \(x_1/d_0 = 2.5\sim 100\); \(d_0 = 0.01 \sim 0.1\) m and \(V_o = 1\sim 10\) m/s.

The value of \(Q_a/Q_w\) was also determined from the best fit-data with an analytical solution (Eq. (2.7)) which is given in Table 2.3 (column 7). Some other results such as jet turbulence intensity \(T_u\), Reynolds number \(R_e\), impact Froude number \(Fr_1\) are summarized in Table 2.2.

**Table 2.2: Froude and Reynolds number, air volume discharge & turbulent intensity**

<table>
<thead>
<tr>
<th>Run</th>
<th>(x_1) (m)</th>
<th>(Fr_1)</th>
<th>(R_e)</th>
<th>(Q_a) (L/s)</th>
<th>(Q_a/Q_w)</th>
<th>(T_u) (%)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BM35_1</td>
<td>0.10</td>
<td>7.2</td>
<td>39808</td>
<td>0.131</td>
<td>0.081</td>
<td>0.39</td>
<td>Freshwater (surface tension, (\sigma = 0.055) N/m.)</td>
</tr>
<tr>
<td>BM4_2</td>
<td>0.10</td>
<td>8.4</td>
<td>45741</td>
<td>0.294</td>
<td>0.154</td>
<td>0.46</td>
<td></td>
</tr>
<tr>
<td>BM44_2</td>
<td>9.0</td>
<td>49543</td>
<td>0.330</td>
<td>0.165</td>
<td>0.96</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cir-1</td>
<td>0.10</td>
<td>7.40</td>
<td>12683</td>
<td>0.046</td>
<td>0.170</td>
<td></td>
<td>Freshwater (surface tension, (\sigma = 0.073) N/m.)</td>
</tr>
<tr>
<td>Cir-2</td>
<td>0.05</td>
<td>6.67</td>
<td>12683</td>
<td>0.025</td>
<td>0.092</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cir-3</td>
<td>0.025</td>
<td>6.27</td>
<td>12683</td>
<td>0.007</td>
<td>0.026</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>Cir-4</td>
<td>0.05</td>
<td>8.26</td>
<td>16545</td>
<td>0.042</td>
<td>0.120</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cir-5</td>
<td>0.05</td>
<td>9.00</td>
<td>17397</td>
<td>0.051</td>
<td>0.137</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Run-1</td>
<td>0.0273</td>
<td>7.19</td>
<td>5388</td>
<td>0.001</td>
<td>0.016</td>
<td>N/A</td>
<td>Freshwater (surface tension, (\sigma = 0.073) N/m.)</td>
</tr>
<tr>
<td>Run-2</td>
<td>0.0273</td>
<td>8.43</td>
<td>6556</td>
<td>0.003</td>
<td>0.040</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>Run-3</td>
<td>9.00</td>
<td>7043</td>
<td>0.004</td>
<td>0.050</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sea-2</td>
<td>7.19</td>
<td>12683</td>
<td>0.012</td>
<td>0.040</td>
<td></td>
<td></td>
<td>Seawater (surface tension, (\sigma = 0.076) N/m.)</td>
</tr>
<tr>
<td>Sea-4</td>
<td>0.05</td>
<td>8.34</td>
<td>16545</td>
<td>0.028</td>
<td>0.080</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>Sea-5</td>
<td>9.07</td>
<td>17397</td>
<td>0.045</td>
<td>0.011</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: \(Fr_1 = V_1/\sqrt{g \cdot r_1}\); \(R_e = \rho_w \cdot r_0 \cdot V_o / \mu_w\) and N/A = Not Available.
2.3 Experimental results: void fractions and bubble count rates

2.3.1 Distributions of void fractions

According to Chanson (1997), as shown in Fig. 2.3, it turns out that air bubbles spread below the expansion of a dislocation. Therefore, under the water surface, to a certain depth, air bubbles do not reach on the central line of the jet, so void fraction here serves as zero mostly. This domain is called the development flow region.

Figure 2.6: Local averaged void fraction distributions- Model 2 \((d_0 = 0.0125 \text{ m, Freshwater}), x_1 = 0.10 \text{ m}, V_1 = 2.51 \text{ m/s, } Fr_1 = 7.40 \) (Run Cir-1).

Figure 2.6 shows the void fraction distributions at several locations below the impingement point, which are plotted as functions of the radial distance normal to the jet. The void fraction distributions are right-and-left symmetry bordering on the centerline of a jet and mostly zero concentration on the centerline of a jet for \(x-x_1 = 0.01\)
m and 0.015 m which turns out that these sections are in the development flow region. The maximum void fraction has reached 33% at \( x-x_1 = 0.01 \) m. Below the development flow region the peak value of void fraction is reduced with the increase in \( x \) and it becomes flat when \( x-x_1 = 0.075 \) m. It is also seen that there are significant void fraction at the jet’s centerline \( (r = 0) \) below the development flow region. The above tendencies were almost the same as the other experimental cases.

### 2.3.2 Distributions of bubble count rates

Air bubble frequency or bubble count rates defined as the number of bubbles detected per second were also recorded with the conductivity probe during the experiments. Typical results of bubble count rate in dimensionless form are presented in Fig. 2.7 as a function of dimensionless radial distance.

**Figure 2.7:** Dimensionless air bubble frequency distributions in plunging jet flow—Model 2 \( (d_o = 0.0125 \) m, Freshwater), \( x_f = 0.10 \) m, \( V_f = 2.51 \) m/s, \( Fr_f = 7.40 \) (Run Cir-1).
Figure 2.7 shows that the maximum value of air bubble frequency is reduced with the depth increasing. The local air bubble frequency are zero at the jet centerline in the range $0 \text{ m} < x-x_l < 0.035 \text{ m}$ because only water was present there, whereas at deeper position $(x-x_l > 0.075 \text{ m})$ the significant air bubble frequency are present at the centerline. Overall, the tendencies of bubble frequencies or bubble count rates are almost the same as distributions of void fraction (Fig. 2.6). Distribution of dimensionless bubble count rates become flatter with depth $(x-x_l)$ increasing below the developing flow region. Some information on the air-water flow structure derives from the behavior of void fraction and air bubble frequency as discussed in section 2.4.

2.3.3 Comparison between data and analytical solution

Applying a superposition method, Chanson (1997) obtained an analytical solution to the diffusion equation in the case of vertical circular plunging jet:

$$C = \frac{Q_a}{Q_w} \frac{1}{4*D^*} \frac{1}{x-x_l} \frac{1}{r(C_{max})} \frac{1}{r(C_{max})} \exp\left(-\frac{1}{4*D^*} \frac{r}{x-x_l} \frac{1}{r(C_{max})} \frac{r}{x-x_l} \frac{1}{r(C_{max})} + 1\right) \frac{r}{r(C_{max})} \frac{r}{r(C_{max})} I_0\left(\frac{1}{2*D^*} \frac{r}{x-x_l} \frac{r}{r(C_{max})} \frac{r}{r(C_{max})}\right)$$

where $x$ is the longitudinal distance measured from the nozzle, $x_l$ is water jet length, $r$ is the normal direction, $r_l$ is the impact water jet thickness, $Q_a$ is the volume of air flux, $D^*$ is a dimensionless diffusivity ($D^*=D_t/(V_l r_l)$) and $I_0$ is the modified Bessel function of the first kind of order zero (see review of Chanson, 1997).

The values of $D^*$ and $Q_a/Q_w$ in Table 2.3 were determined so that the analytical curves show best fitted to the experimental data. The value of $r (C = C_{max})$ was also measured from the data corresponding to each section (e.g. $x-x_l = 0.01 \text{ m}, 0.015 \text{ m} \text{ and } 0.025 \text{ m}$ etc.).
Table 2.3: Characteristics of air-water flow measurements in circular plunging jets

<table>
<thead>
<tr>
<th>Run</th>
<th>$V_1$ (m/s)</th>
<th>$Fr_1$</th>
<th>$x-x_1/r_1$</th>
<th>$C_{max}$</th>
<th>$F_{max}N_1/V_1$</th>
<th>$Q_a/Q_w$</th>
<th>$D^f$</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>BM35_1</td>
<td>3.50</td>
<td>7.2</td>
<td>1.67</td>
<td>0.23</td>
<td>0.32</td>
<td>0.108</td>
<td>3.4E-3</td>
<td>Fresh water</td>
</tr>
<tr>
<td>BM4_2</td>
<td>4.10</td>
<td>8.4</td>
<td>1.65</td>
<td>0.36</td>
<td>0.38</td>
<td>0.178</td>
<td>5.2E-3</td>
<td></td>
</tr>
<tr>
<td>BM44_2</td>
<td>4.40</td>
<td>9.0</td>
<td>1.64</td>
<td>0.39</td>
<td>0.42</td>
<td>0.193</td>
<td>5.0E-3</td>
<td></td>
</tr>
<tr>
<td>Cir-2</td>
<td>2.31</td>
<td>6.67</td>
<td>1.64</td>
<td>0.21</td>
<td>0.32</td>
<td>0.160</td>
<td>7.0E-3</td>
<td>Fresh water</td>
</tr>
<tr>
<td>Cir-4</td>
<td>2.89</td>
<td>8.26</td>
<td>1.61</td>
<td>0.33</td>
<td>0.40</td>
<td>0.240</td>
<td>6.5E-3</td>
<td></td>
</tr>
<tr>
<td>Cir-5</td>
<td>3.10</td>
<td>9.00</td>
<td>1.60</td>
<td>0.36</td>
<td>0.40</td>
<td>0.280</td>
<td>6.5E-3</td>
<td></td>
</tr>
<tr>
<td>Run-1</td>
<td>1.82</td>
<td>7.19</td>
<td>1.69</td>
<td>0.04</td>
<td>0.05</td>
<td>0.030</td>
<td>8.0E-3</td>
<td>Fresh water</td>
</tr>
<tr>
<td>Run-2</td>
<td>2.15</td>
<td>8.43</td>
<td>1.66</td>
<td>0.11</td>
<td>0.11</td>
<td>0.070</td>
<td>4.5E-3</td>
<td></td>
</tr>
<tr>
<td>Run-3</td>
<td>2.30</td>
<td>9.00</td>
<td>1.65</td>
<td>0.12</td>
<td>0.14</td>
<td>0.075</td>
<td>4.7E-3</td>
<td></td>
</tr>
<tr>
<td>Sea-2</td>
<td>2.46</td>
<td>7.19</td>
<td>1.67</td>
<td>0.15</td>
<td>0.21</td>
<td>0.110</td>
<td>6.0E-3</td>
<td>Seawater</td>
</tr>
<tr>
<td>Sea-4</td>
<td>2.87</td>
<td>8.34</td>
<td>1.65</td>
<td>0.22</td>
<td>0.28</td>
<td>0.159</td>
<td>6.0E-3</td>
<td></td>
</tr>
<tr>
<td>Sea-5</td>
<td>3.13</td>
<td>9.07</td>
<td>1.64</td>
<td>0.24</td>
<td>0.32</td>
<td>0.193</td>
<td>6.3E-3</td>
<td></td>
</tr>
</tbody>
</table>
(a) Model 2 \((d_0 = 0.0125 \text{ m}, \text{Freshwater}), x_I = 0.05 \text{ m}, V_I = 3.10 \text{ m/s}, Fr_1 = 9.0\) (Cir-5)
(b) Model 4 \((d_0 = 0.0125 \text{ m, Seawater}), x_1 = 0.05 \text{ m, } V_I = 3.13 \text{ m/s, } Fr_I = 9.07 \text{ (Sea-5)}\)

**Figure 2.8:** Dimensionless distributions of void fraction \(C\) and dimensionless bubble count rate \(f_{ab} = F^* r_I / V_I\) in vertical circular jets. Comparison between experimental data and Eq. (2.7). (a) Model 2 (Freshwater), and (b) Model 4 (Seawater).
For a vertical circular plunging jet, the void fraction distributions follow closely an analytical solution of the diffusion equation. Figure 2.8 compares the experimental data with Eq. (2.7) in the developing flow region for identical inflow conditions in freshwater and in seawater. In the developing flow region, the distributions of void fraction exhibits smooth in freshwater and scattered in seawater but experimental data and analytical solutions of the advective diffusion equation shows good agreement in all the sections \((x-x_f = 1 \text{ cm}, x-x_f = 1.5 \text{ cm} \text{ and } x-x_f = 2.5 \text{ cm})\).

### 2.3.4 Vertical distribution of void fraction

![Graph showing void fraction distributions](image)

**Figure 2.9:** Void fraction distributions in the vertical direction - Model 2 \((d_0 = 12.5 \text{ mm, Freshwater}), r/r_f = 1.11\).

Figure 2.9 shows the vertical distributions of void fraction at \(r/r_f = 1.11\) in circular plunging jets corresponding to several impact velocities. The concentration of air for
circular plunging jets gradually decreases in the developing flow region (DFR) and fully developed flow region (FDFR) as shown in Fig. 2.9 with depth increasing, whereas it increases in the redistribution flow region (RFR). To our knowledge, little information is available on the flow characteristics in the redistribution flow region. Vortex system by plunging jet might induce substantial modifications of the velocity field in the RFR. It must be noted that more studies are required in the redistribution flow region. Note that DFR, RFR, and FDFR are defined in Fig. 2.9 based on experimental data and a sketch in Fig. 2.3 for a circular plunging jet.

2.4 Discussion

2.4.1 Characteristics of maximum void fraction and bubble count rate

Figure 2.10 shows experimental results obtained for identical inflow conditions (i.e., \(x_1/d_0 = 4, Fr_1 = 9\)) in freshwater (Models 1, 2 and 3) and seawater (Model 4). The results highlighted maximum void fraction and dimensionless bubble frequency in the developing shear layers. Consistently, the maximum bubble count rate occurs in the inner shear region, i.e. the distance \(r (F_{max})\) is smaller than \(r (C_{max})\) at which the void fraction shows maximum (Fig.2.10 (a)). The data in Fig. 2.10(a) suggests that except for Model 3 the location of \(C_{max}\) and \(F_{max}\) increase as the depth \((x-x_i)\) increases. The location of maximum air content and bubble frequency satisfy the following relationship in developing flow region, which was previously observed in circular plunging jets (Manasseh and Chanson 2001) and in two-dimensional jets (Chanson and Brattberg 1998):

\[
0 < \frac{r(F_{max})}{r_i} < \frac{r(C_{max})}{r_i} \tag{2.8}
\]
(a) Dimensionless locations of maximum void fractions and maximum bubble count rates

(b) Maximum void fractions $C_{max}$
(c) Maximum dimensionless bubble count rate $F_{\text{max}}^* r_1/V_1$

Figure 2.10: Experimental results of maximum void fractions and dimensionless bubble count rates in vertical circular jet flows for $Fr_1 = 9$ and $x_1/d_0 = 4$. (a) Dimensionless locations of maximum void fractions and maximum bubble count rates, (b) Maximum void fractions $C_{\text{max}}$, and (c) Maximum dimensionless bubble count rate $F_{\text{max}}^* r_1/V_1$.

It is seen that the maximum void fraction and dimensionless bubble count rates decreases exponentially with distance $x-x_1$ immediately downstream of the impingement point (Fig. 2.10(b), (c)). Interestingly, Brattberg and Chanson (1998) showed that the air bubbles are broken up into smaller-size bubbles immediately downstream of the entrapment point ($x-x_1 = 0$) for 2-D vertical plunging jet, whereas vertical circular plunging jet explores the opposite results. Although reason is not clear, there is a possibility that the larger perimeter of shear layer in circular plunging jet results in very
fast air bubbles shifts outwards of the jet centerline (in the horizontal direction). More study is required in the development flow region.

**Figure 2.11:** Relationship between void fraction and dimensionless bubble count rate at a given cross-section in vertical circular plunging jet flows-Model 2 ($d_0 = 0.0125$ m, Freshwater), $x_1 = 0.05$ m, $Fr_1 = 9.0$ (Run Cir-5).

Figure 2.11 represents typical relationships between dimensionless bubble count rate and void fraction. Although this relationship is found to be unique in self-aerated chute flows, the relationship of void fraction and bubble count rate exhibits a characteristic shape in the developing region of plunging jets flows. For a given void fraction, greater bubble count rate is observed in the inner developing flow region.
2.4.2 Penetration depth

The maximum penetration depth was measured vertically from impingement point by high-speed video camera, which is shown in Fig. 2.12. The lower end of bubble clouds (Fig. 2.4) continuously moves a little vertically. In the experiments the time averaged penetration depth was determined. The measurement was compared with empirical relationship of Sande and Smith (1975). The empirical formula is

\[ D_p = 1.20 V_t^{0.77} d_0^{0.625} x_1^{-0.094} \]  

(2.9)

where \( D_p \) is the maximum penetration depth, \( d_0 \) is the nozzle diameter, \( x_1 \) is the water jet height.

Figure 2.12 shows that the penetration depth increases in the domain where water jet height is small but experimental results are found to be larger than those by Eq. (2.9).
The judgement of the penetration depth depends much on observer, because there are few air bubbles at the bottom of bubble cloud. The data in Fig. 2.12 suggests that for \( x_1/d_0 \geq 12 \), the dimensionless penetration depth is almost constant for constant nozzle diameter. Ohkawa *et al.* (1987) also measured the similar trend of the dimensionless penetration depth.

### 2.4.3 Distributions of chord length

The bubble chord time is defined as the duration of a bubble on the probe sensor. Chord time data were calculated from the raw signal scanned at 25 kHz for 2.6 seconds at 8 locations per cross-section. The results are presented in terms of pseudo-bubble chord length \( ch_{ab} \) defined as:

\[
ch_{ab} = V_j \cdot t_{ch}
\]

where \( t_{ch} \) is the bubble chord time and \( V_j \) is the jet impingement velocity.

(a) \( Fr_j = 9, \ (x-x_1)/r_1 = 4 \): Model 2 (Freshwater, Cir-5) and Model 4 (Seawater, Sea-5)
(b) $Fr_1 = 9$, Model 2 (Freshwater, run Cir-5)

(c) $Fr_1 = 9$, Model 4 (Seawater, run Sea-5)

Figure 2.13: Pseudo-bubble chord length distributions. (a) Comparison of chord length between Model 2 and Model 4 for $x_1/d_0 = 4$ and $Fr_1 = 9$. (b) Model 2 (Freshwater) and (c) Model 4 (Seawater).
Typical examples of some experimental results of pseudo-bubble chord length distribution are shown for identical inflow conditions with freshwater and seawater in Figure 2.13. Each figure shows the normalized probability function of pseudo-chord length \( ch_{ab} \) where the histogram column represents the probability of chord length in 0.5 mm intervals. The last column (i.e. > 10) indicates the probability of chord lengths exceeding 10 mm. Each histogram describes all bubbles detected in a cross-section (i.e. 8 locations) at depths \((x-x_f) = 10, 15 \) and 25 mm.

The experimental results show that the probability of bubble chord length is the largest for bubble sizes between 0 and 2 mm. Figure 2.13(a) shows that the probability of pseudo-chord length is the highest between 0.1 to 1.2 mm in seawater and between 0.25 to 2 mm in freshwater. Moreover, it is seen from Figs. 2.13(b) and 2.13(c) the existence of large chord-length bubbles at \( x-x_f = 1 \) cm (close to the impingement point). These large bubbles may be large air pockets entrapped at impingement, which is subsequently broken up by turbulent shear (Chanson et al., 2002). The seawater plunging jet flows contain comparatively a greater number of fine bubbles than freshwater plunging jet flow for identical inflow conditions. This is caused possibly by the combination of lesser entrainment of large-size bubbles (in freshwater) and greater entrapment of fine bubbles (in seawater).

### 2.5 Scale effects

Figures 2.14 and 2.15 show a comparison of void fractions and bubble count rates corresponding to one inflow Froude number, respectively. In plunging jets, experimental data observed in the two largest models (i.e. Models 1 & 2) are basically identical at each cross-section for \( Fr_1 \sim 9.0 \) (e.g. Fig. 2.14 & Fig 2.15).
Figure 2.14: Distributions of void fraction $C$ for identical inflow condition ($x_1/d_0 = 4.0$, $Fr_1 \sim 9.0$): Model 1 (Run BM44_2), Model 2 (Run Cir-5) and Model 3 (Run Run-3).
Figure 2.15: Distributions of dimensionless bubble count rate $f_{ab} = F^* r_1 / V_1$ for identical inflow condition ($r_1/d_0 = 4.0$, $Fr_1 \sim 9.0$): Model 1 (Run BM44_2), Model 2 (Run Cir-5) and Model 3 (Run Run-3).
Figures 2.14(a), (b) and (c) show a comparison of void fraction distributions for \((x-x_1)/r_1\) ~ 1.60, 2.40 and 4.10 respectively for one inflow Froude number \((Fr_1 \sim 9.0)\). For all the cases, experimental data observed in the two largest models (i.e. Model 1 & Model 2) are basically identical at each cross-section. The significant differences are seen for Model 3 at all the cross-section (Fig. 2.14). Figure 2.15 also shows a notable differences for bubble count rates at each cross-section for Model 3. Similar results of void fraction and bubble count rates were also observed for \(Fr_1 \sim 7.0\) and 8.0. Some differences were noted for the lowest Froude number \((Fr_1 \sim 7.0)\).

For all investigated flow conditions (Table 2.3), significantly less air was entrained in Model 3 in comparison with Model 1 and Model 2. This is clearly seen in Figs. 2.14 and 2.15 in terms of both void fraction and bubble count rate. It is believed that, in Model 3, air entrainment was affected by scale effects.

Scale effect is discussed in terms of the dimensionless air discharge ratio \(Q_a/Q_w\) and the dimensionless turbulent diffusivity \(D^\#\) based upon Weber number \(We_1\). For identical fluids in three models, the Froude similitude implies that the Weber number differs between experiments and that the surface tension-dominated processes may not be properly scaled. Figure 2.16 and 2.17 show \(Q_a/Q_w\) and \(D^\#\) as functions of Weber number \(We_1\), where \(Q_a/Q_w\) and \(D\) are determined so that the analytical curves of \(C\) are best fit to experimental data.

The figures imply that the smallest model experiments with \(We_1 < 1000\) were strongly influenced by scale effects because \(Q_a/Q_w\) and \(D^\#\) show different values from those of Model 1 and Model2. It is concluded that air entrainment at vertical circular plunging jets is affected by scale effects for \(We_1 < 1000\) where \(We_1\) is the inflow Weber number. The present results may be applied to air entrainment at plunging breaking waves in chapter 6.
Figure 2.16: Dimensionless air discharge versus Weber number. Comparison between Model 1, Model 2 and Model 3: (a) \((x-x_1)/r_1 \sim 1.6\), (b) \((x-x_1)/r_1 \sim 2.4\), and (c) \((x-x_1)/r_1 \sim 4.0\).
Figure 2.17: Dimensionless turbulent diffusivity versus Weber number. Comparison between Model 1, Model 2 and Model 3: (a) \((x-x_1)/r_1 \sim 1.6\), (b) \((x-x_1)/r_1 \sim 2.4\), and (c) \((x-x_1)/r_1 \sim 4.0\).
2.6 Conclusions

In this chapter, it has been attempted only to explore the basic physics of air bubbles and build intuition of void faction fields precisely for vertical circular plunging jet (steady case) in freshwater and seawater. Three scale models were used with jet nozzle diameters of 6.8, 12.5 and 25 mm and the results are also used for discussing scale effects that affects air entrainment process. A study of air entrainment inception conditions showed that the onset velocity $V_e$ is identical for freshwater and seawater. In seawater, significantly less air is entrained than in freshwater, leaving all inflow parameters equal.

A conductivity type void meter was used specifically to measure void fraction and bubble frequency distributions under the impingement point. The void fraction profile follows closely analytical solution of diffusion equation. The maximum void fraction and bubble count rates decreases with increasing depth in the developing flow region.

In addition, it appears that the penetration depth was found to be sensitive to falling water jets. Distributions of pseudo-bubble chord sizes ranged from less than 0.5 mm to more than 10 mm for freshwater and seawater, and the averaged pseudo-chord sizes are between 4 and 6 mm for all water solutions.

For the void fraction, which can be represented in term of $Q_a/Q_w$ and $D^*$, comparison between the results of three different scale models highlight significant scale effects when $We_j < 1000$. 
Energy Dissipation by Air Bubble in Steady Case

Summary

A simple model is presented in this chapter that enables to estimate the energy transformation and dissipation by air bubbles quantitatively for three typical phenomena of air entrainment through free surface: a hydraulic jump, a 2-D vertical plunging jet and a vertical circular plunging jet into water. The model is related to rise velocity and two physical parameters that are well defined by experimental data. The averaged rate of energy dissipation by air bubbles obtained from the experimental data were 25%, 1.4% and (2-4)% with respect to total energy loss for the hydraulic jump, 2-D vertical plunging jet and vertical circular plunging jet, respectively.

3.1 Introduction

In the previous chapter, the void fraction data were presented and their various characteristics were described only for circular plunging jets. This chapter gives a thorough account of the volume of entrained air, energy transformation and dissipation rate for a hydraulic jump and vertical plunging jets. Air entrainment is associated with a rise in water level caused by liquid displacement upwards, which implies an increase in the potential energy. The upstream energy or
kinetic energy is first stored as the potential energy caused by air bubbles, which are entrained into the water, and later the rising bubbles dissipate it. Many papers can be found that deal with the air bubble in plunging jet flows (steady). However, the number of papers on the surf zone hydrodynamics (unsteady) is rather limited, especially when dealing with the air-water flow field. The reason for this may be the complexity of phenomena involved. The importance of air entrainment caused by breaking waves is readily understood when we observe in the surf zone. To quantify precisely the energy dissipation rate by entrained air in unsteady (surf zone) phenomena, investigation on plunging jet flows (steady) will be useful.

Over the years, several researchers (e.g. Chanson and Brattberg 2000; Cumming and Chanson 1997; Bonetto and Lahey 1993) have given an overview of the past research efforts on air entrainment in plunging jet flows and pointed out the characteristics of entrained air in plunging jet flows. However, energy dissipation by air bubbles has not been quantified. To our best knowledge, there is only one paper on energy dissipation by air bubbles in plunging liquid jet bubble column (Evans et al., 1992). They measured the energy dissipation rate per unit volume knowing the length of mixing zone and their study was restricted only for plunging liquid jet. For quantitative and qualitative analysis of energy dissipation due to air entrainment in plunging jet flows (including, 2-D vertical plunging jet, vertical circular plunging jet and hydraulic jump), a simple mathematical model will be developed here.

The model to be presented is an “air bubble model” in the sense of the first law of thermodynamics, which states that the net energy (e.g. potential energy, heat) supplied to the system equals the increase in energy of the system plus the energy that leaves the system as work is done.

The aim of this chapter is to investigate the contribution of entrained air to energy transformation and dissipation in steady cases.
3.2 A simple model formulation

Entrained air bubbles and their detrainment through water surface are sketched in Fig. 3.1. The water level rises by $\Delta h$ above the initial still water depth $h$ because of air bubble entrainment. The water level rise $\Delta h$ is expressed with the following formula, when the $z$-axis is taken vertically from the seabed and a void fraction distribution of air bubbles is expressed as $C(z)$:

$$
\Delta h = \int_0^{h+\Delta h} C(z) \, dz
$$

(3.1)

The increase in potential energy $\Delta PE$ due to air bubbles entrainment can be expressed as the following formula, if the density of air is disregarded:

$$
\Delta PE = \int_0^{h+\Delta h} \rho_w (1 - C) g z \, dz - \int_0^h \rho_w g z \, dz
$$

$$
= \rho_w g \int_0^{h+\Delta h} C(h + \frac{\Delta h}{2} - z) \, dz = \rho_w g V_a h_G
$$

(3.2)

where $\rho_w$ is the water density, $h_G$ is the distance of center-of-gravity of air volume measured from the water surface and $V_a$ is entrained air volume. In Eq. (3.2), $\Delta h$ is taken as sufficiently small value compared with $h_G$. This shows that the potential energy increment is proportional not only to the amount of air bubbles but also to the distance of center-of-gravity of air volume.

The amount of entrained air bubbles rises in an action of lift, passes through the water surface soon, and is emitted into air. That is, in the detrainment process, air bubbles will be released in atmosphere. Now it can be stated that the energy dissipation rate of the potential energy is proportional to the rate of rise of air bubbles. Supposing the rise
velocity \( w_r \) is constant and uniform, energy dissipation rate \( E_d \) will be given by the following formula:

\[
E_d = \rho_v g \int_0^{h+\Delta h} C \frac{dz}{dt} dz = \rho_v g V_a w_r,
\]

(3.3)

Figure 3.1: Sketch of (a) water level rises by entrained air and (b) entrained and detrained air bubbles through free surface.

Thus the rate of energy dissipation will be independent of the penetration depth of air bubbles, and will be proportional both to the amount of air bubbles and to the rise velocity of air bubble. Now what is necessary is just to think that in a steady state, air bubbles must be supplied continuously to the flow, and the dissipated energy must be compensated at the same rate of \( E_d \). According to visual observation, although the rate of rise of air bubbles is not necessarily uniform in the upward direction, the rise velocity is roughly assumed to be constant. However, how to give of rise velocity is a problem for
unsteady case. The rise velocity \( w_r \) (see Appendix in this chapter) in Eq. (3.3) is a function of bubble size and ranges 10 to 40 cm/s for intermediate bubble diameters between 0.2 to 20 mm (Wood, 1991; Chanson, 1997). In the present study \( w_r = 0.25 \) m/s is used, because the average bubble diameter was found nearly 2.5 mm to 3 mm in the experiments.

### 3.3 Application of model

The above model, which is stated in Eq. (3.3), is applied to experimental data of three typical phenomena of air entrainment through free surface: a hydraulic jump, a 2-D vertical and a vertical circular plunging jet. In each case a strong mixing process takes place.

#### 3.3.1 Hydraulic jump

![Figure 3.2: Sketch of hydraulic jump.](image)
Hydraulic jumps are the most puzzling flow phenomena that occur in nature under steady one-directional flows. It is characterized by the energy dissipation and air entrainment, which is defined by Fig. 3.2. In the simplest view, the flow in a hydraulic jump has many similarities to the broken face of a spilling breaker.

3.3.1.1 VOID FRACTION DISTRIBUTIONS
Experimental data of void fraction distribution reported by Chanson (1997) for hydraulic jumps are analyzed in detail. Fig. 3.3 shows vertical distributions of void fraction at several sections of flow with different $x$ coordinate obtained by Chanson where maximum air content decays in the flow direction.

![Figure 3.3: Air concentration distribution (After Chanson, 1997); $Fr_1=6.05$, $x_1=0.89m$, $d_1=0.017m$, $V_1=2.47m/s$.](image-url)
3.3.1.2 VOLUME OF ENTRAINED AIR

The volume of entrained air over a unit length of \( x \), \( V_a \), is computed from the measured distribution of void fraction \( C(y) \) by using the following definition:

\[
V_a = \int_0^{y_0} C(y) \, dy
\]  

(3.4)

where the choice of \( C = 0.70 \) is the pseudo-free-surface threshold criterion and \( y_{0.7} \) is the air-water reference depth.

![Graph showing the horizontal distribution of entrained air volume per unit area; \( Fr_j = 6.05 \).](image)

The results (Fig.3.4) indicate that air volume increases drastically from impinging point to the re-circulating region and it decreases subsequently the rest of region in the downstream direction. The reason of this is explained by Chanson and Brattberg (2000). They reported, in the roller of hydraulic jump, the momentum direction changes 180-
degree. It is thought that the change in momentum direction causes a momentum transfer leading to the higher air entrainment resulting air volume increasing.

3.3.1.3 ENERGY DISSIPATION RATE

The rate of total energy dissipation in a stationary hydraulic jump is given by:

\[
E_d = \rho_w g q \Delta H = \rho_w g (V_1 d_1) \frac{(d_2 - d_1)^3}{4d_2 d_1}
\]

where \( q = V_1 d_1 \), \( \Delta H = (d_2 - d_1)^3 / 4d_2 d_1 \). Here \( q \) is the water discharge per unit width, \( \Delta H \) is the head loss, \( d_1 \) is the upstream flow depth, and \( d_2 \) is the downstream depth.

![Energy dissipation rate per unit length along x-direction](image)

**Figure 3.5:** Energy dissipation rate per unit length along x-direction; \( Fr_1 = 6.05 \).

Using air volume per unit area and depth of gravity center of air, the potential energy dissipation rate \( E_d \) per unit length can be calculated from Eq. (2.3) corresponding to each
section in Fig. 3.4 and plotted in Fig. 3.5. Note that this is a conservative estimate, since the measurement was performed for air contents $C < 0.70$.

Note that Fig. 3.4 and Fig. 3.5 shows the similar trend, because in Eq. (3.3) energy dissipation rate is proportional to entrained air volume. Finally, to estimate the total energy dissipation rate due to air bubbles, $E_d^T$, the curve in Fig. 3.5 is integrated over the jump length in $x$-direction, which is to be compared with the rate of total energy dissipation through head loss, $E_H$ in Eq. (3.5).

**Table 3.1: Measurements of potential energy and energy dissipation rate**

<table>
<thead>
<tr>
<th>Ref.</th>
<th>$x-x_1$ (m)</th>
<th>$h_G$ (m)</th>
<th>$V_a$ (m)</th>
<th>$E_d$ (J/(m²·s))</th>
<th>$E_d^T$ (J/(m·s))</th>
<th>$E_H$ (J/(m·s))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chanson</td>
<td>0.00</td>
<td>0.006</td>
<td>0.008</td>
<td>19.62</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1997)</td>
<td>0.10</td>
<td>0.015</td>
<td>0.020</td>
<td>49.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.20</td>
<td>0.032</td>
<td>0.019</td>
<td>46.60</td>
<td>20.64</td>
<td>80.5</td>
</tr>
<tr>
<td></td>
<td>0.30</td>
<td>0.034</td>
<td>0.018</td>
<td>44.12</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.45</td>
<td>0.046</td>
<td>0.0097</td>
<td>23.79</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.65</td>
<td>0.028</td>
<td>0.0017</td>
<td>4.17</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The total energy dissipation rate due to air bubbles $E_d^T$ is presented in Table 3.1, which suggests that around 25% of the total energy loss be dissipated through the potential energy of air bubbles.

### 3.3.2 Two-dimensional vertical plunging jet

When a water jet impinges a pool of water at rest, air bubbles may be entrained and carried away below the pool free surface. This process is called plunging jet entrainment sketched in Fig. 3.6 (After Chanson, 1997).
The mechanism of air entrainment depends upon the impact velocity. In a vertical plunging jet, air bubbles start to be entrained when the jet impact velocity $V_i$ exceeds a critical value.

3.3.2.1 VOID FRACTION DISTRIBUTIONS

In two-dimensional plunging jet flows, Cummings and Chanson (1997) measured air concentration distributions that follow closely analytical solutions of the diffusion equation. In the present section, it is estimated potential energy and energy dissipation rate for 2-D plunging jet. Note that the data of Cummings and Chanson (1997) are insufficient for the estimate of energy dissipation rate because the void fraction in deeper position was not measured. The scales of the Fig. 3.7 are not clear; it is taken from the reference by scanning. In Fig. 3.7, the maximum void fractions are 12%, 8%, 5% and 3% corresponding to $x-x_i = 10, 50, 100$ and 200 mm, respectively.
3.3.2.2 VOLUME OF ENTRAINED AIR

The volume of entrained air per unit depth was calculated from the distributions of air concentration $C(y)$ (Fig. 3.7) by using the definition:

$$V_a = \int_0^\infty C(y) \, dy$$

(3.6)

where $y$ is the distance normal to jet support.

The data in Fig. 3.8 show a change in the transit region of developing flow and redistribution flow. Air volume increases in the redistribution flow region and subsequently decreases in the rest of region. The reason is explained in section 3.3.3.2 in detail.
3.3.2.3 ENERGY DISSIPATION RATE

The upstream energy transport rate for 2-D vertical plunging jet per unit width is defined at the impact point as

$$ E_j = \frac{1}{2} \rho \cdot V_i^2 \cdot q = \frac{1}{2} \rho \cdot V_i^2 \cdot (dV_1) $$

(3.7)

To estimate the potential energy dissipation rate, the experimental data presented in Fig. 3.7 were used. Using the same calculation technique as made in the hydraulic jump, we calculated the potential energy dissipation rate per unit depth using Eq. (3.3) knowing the air volume from Fig. 3.8 and plotted it in Fig. 3.9.
Figure 3.9: Energy dissipation rate per unit depth as a function of depth, \( Fr_1 = 7.63 \).

The total potential energy dissipation rate, \( E_d^T \), was determined from Fig. 3.9 by integrating the curve along the depth. The results are listed in Table 3.2.

### Table 3.2: Measurements of potential energy and energy dissipation rate

<table>
<thead>
<tr>
<th>Ref.</th>
<th>( x-x_1 ) (m)</th>
<th>( h_G ) (m)</th>
<th>( V_a ) (m/s)</th>
<th>( E_d ) (J/(m².s))</th>
<th>( E_d^T ) (J/(m.s))</th>
<th>( E_j ) (J/(m.s))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cummings</td>
<td>0.01</td>
<td>0.01</td>
<td>0.00218</td>
<td>5.35</td>
<td></td>
<td>9.2</td>
</tr>
<tr>
<td>and Chanson</td>
<td>0.05</td>
<td>0.05</td>
<td>0.0020</td>
<td>4.91</td>
<td></td>
<td>9.2</td>
</tr>
<tr>
<td>(1997)</td>
<td>0.10</td>
<td>0.10</td>
<td>0.0022</td>
<td>5.40</td>
<td>0.92</td>
<td>68.26</td>
</tr>
<tr>
<td></td>
<td>0.20</td>
<td>0.20</td>
<td>0.0015</td>
<td>3.68</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From Table 3.2, we have found the potential energy dissipation rate estimated through air bubbles is 0.92 J/(m.s), which is only 1.4% of the upstream energy flux.
3.3.3 Vertical circular plunging jet

Figure 3.10 shows the ideal sketch of vertical circular plunging jet. Void fraction distributions at different cross sections below the impingement point and its various properties were measured by the authors in a circular plunging jet, which is presented in detail in chapter 2. In this section, air volume and energy dissipation rate will be discussed.

![Sketch of vertical circular plunging jet](image)

**Figure 3.10:** Sketch of vertical circular plunging jet.

### 3.3.3.1 Void Fraction Distributions

Figure 3.11 exhibits void fraction profiles, which are plotted as functions of the dimensionless radial distance normal to the jet axis.
Figure 3.11: Local averaged void fraction distributions-Cir-1 (Model 2, Freshwater); $x_1 = 0.10$ m, $V_1 = 2.51$ m/s.

3.3.3.2 VOLUME OF ENTRAINED AIR

The volume of entrained air per unit depth, $V_a$, was calculated from the distribution of void fraction $C(r)$ (Table 3.3) using the following Eq. (3.8):

$$V_a = \int_0^{2\pi} \int_{r_a}^{r_b} r C(r) \, dr \, d\theta$$  \hspace{1cm} (3.8)

where $r_a$ and $r_b$ are the distance from the jet’s centerline to the inner and outer edges of the void fraction distribution, respectively.

The measurements were performed up to deeper position where the existence of air bubbles was found almost zero and this point was almost identical with the maximum penetration depth. Experimental results (Fig. 3.12) indicate that the volume of entrained air per unit length increases with increasing jet impact velocity. Similar result was estimated by Bonetto and Lahey, (1993). In Fig. 3.12, the volume of air per unit depth, $V_a$, increases in the deeper position rapidly and somewhat near the free.
surface. As yet, we are not fully aware of this region (below the development flow region), but it is believed that more air bubbles are redistributed in this region due to vortex (e.g. Wood, 1991; Chanson, 1997).

Figure 3.12: Vertical distribution of entrained air volume per unit length - Model 2 ($d_0 = 12.5$ mm, Freshwater).

According to the continuity of air-water flow, the downward air discharge must be equal to the upward air disappear at any depth, because the system is steady. Practically, in the developing flow region it was measured only the down-going air bubbles. From Fig. 2.3 it is seen that many upward air bubbles are scattered near the free surface and it is very difficult to measure those bubbles. Avoiding this complexity, it can be measured the total amount of upward air in the following way:

$$V_{ua} = \frac{V_{w}}{w_{w}}*V_{ad} \quad \text{(Since, } Q_{ad} = Q_{ua}) \quad (3.9)$$
where $V_{au}$ and $V_{ad}$ are the up-going and down-going air volume, respectively.

The value of the ratios $\frac{V_{au}}{V_{ad}}(=\frac{V_l}{w_r})$ may reach about 10 near the water surface.

However, in the following discussion, it is not made any correction on this up-going air bubbles because it is difficult to estimate the value of the ratio and the lower limit of the correction.

**Figure 3.13:** Total air volume of entrained air as function of (a) jet length, and (b) nozzle velocity-Model 2 ($d_0 = 12.5$ mm, Freshwater).

The total amount of entrained air, $V_a^T$, for each run (Table 3.3) is plotted against $x_I$ and $V_0$ (Fig. 3.13). The effect of water jet height on the volume of entrained air was studied for one nozzle velocity ($V_0 = 208.5$ cm/s). The results show a linear increase of air volume with increasing water jet height. On the other hand, volume of entrained air increases with increasing nozzle velocity $V_0$ for a constant jet height ($x_I$).
= 5 cm). Overall, we have seen from Fig. 3.13 that the volume of entrained air $V^T_a$ is the strong function of water jet height and nozzle velocity.

![Graph showing the relationship between $V^T_a$ (m$^3$) and $V_i$ (m/s).]

**Figure 3.14:** Total air volume of entrained air as function of impact velocity - Model 2 ($d_o = 12.5$ mm, Freshwater).

Figure 3.14 shows the total air volume due to entrained air is related to the impact velocity. The relationship between total air volume and jet impact velocity is almost linear.

### 3.3.3.3. ENERGY DISSIPATION RATE

From the calculated data of air volume shown in Fig. 3.12, it was estimated the potential energy and energy dissipation rate per unit length for each section using Eq. (3.2) and Eq. (3.3), respectively. The measured potential energy per unit length was plotted and total energy dissipation rate over the depth was determined by integrating...
it. The rate of total potential energy dissipation, $E_d^T$, corresponding to each run is plotted against the impact velocity in Fig. 3.15.

**Figure 3.15:** Total energy dissipation rates versus impact velocity-Model 2 (Freshwater, $d_0 = 12.5$ mm).

Figure 3.15 show that energy dissipation rate is strongly dependent on jet impact velocity. As the impact velocity increases, energy dissipation rate also increases. The plot of energy dissipation rate takes same curve as the air volume $V_a^T$, because energy dissipation rate is proportional to entrained air volume (Figs. 3.14 and 3.15), because the rise velocity was assumed to be uniform.

All the calculated values of circular plunging jet are listed in Table 3.3. The calculated air volume (Table 3.3, column 6) and energy dissipation rate (Table 3.3, column 7) are presented for inflow velocities ranging from 2.20 to 3.10 m/s in Table 3.3.
Table 3.3: Measurements of potential energy and energy dissipation rate

<table>
<thead>
<tr>
<th>Run No.</th>
<th>$x_1$ (m)</th>
<th>$V_0$ (m/s)</th>
<th>$V_j$ (m/s)</th>
<th>$d_j$ (m)</th>
<th>$V_{aj}^T$ (m$^3$)</th>
<th>$E_d^T$ (J/s)</th>
<th>$D_a$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 2</td>
<td>Cir-1</td>
<td>0.10</td>
<td>2.085</td>
<td>2.51</td>
<td>0.01170</td>
<td>0.0000134</td>
<td>0.033</td>
</tr>
<tr>
<td></td>
<td>Cir-2</td>
<td>0.05</td>
<td>2.085</td>
<td>2.31</td>
<td>0.01222</td>
<td>0.0000095</td>
<td>0.023</td>
</tr>
<tr>
<td></td>
<td>Cir-3</td>
<td>0.025</td>
<td>2.085</td>
<td>2.20</td>
<td>0.01251</td>
<td>0.0000058</td>
<td>0.014</td>
</tr>
<tr>
<td></td>
<td>Cir-4</td>
<td>0.05</td>
<td>2.72</td>
<td>2.89</td>
<td>0.01246</td>
<td>0.0000167</td>
<td>0.040</td>
</tr>
<tr>
<td></td>
<td>Cir-5</td>
<td>0.05</td>
<td>2.92</td>
<td>3.10</td>
<td>0.01249</td>
<td>0.0000235</td>
<td>0.057</td>
</tr>
</tbody>
</table>

The upstream energy transport rate for vertical circular plunging jet is defined as

$$E_j = \frac{1}{2} \rho_w Q_w V_1^2$$

(3.10)

**Figure 3.16:** Ratio of energy dissipation rate due to air bubbles to total energy dissipation rate as functions of (a) jet length, and (b) nozzle velocity-Model 2 ($d_0 = 12.5$ mm, Freshwater).
In the steady state, the whole of this energy is lost within the water pool. It is denoted
the ratio of $E_d$ to $E_j$ as $D_a$:

$$D_a(\%) = \frac{E_d}{E_j} \times 100$$  \hspace{1cm} (3.11)

That means $D_a$ is the ratio of energy dissipation rate due to air bubbles to total energy
dissipation rate.

The value of $D_a$ corresponding to each case is shown in Table 3.3. Energy dissipation
rate due to entrained air is only about 2~4% of total upstream energy flow rate for $V_I$
= 2.20~3.10 m/s. From the experiments it was difficult to explain the whole energy
dissipation of upstream energy through the potential energy dissipation due to air.

Figure 3.16 shows the relationship between ratio of energy dissipation due to
entrained air, $D_a$, and water jet height, $x_j$, and nozzle velocity, $V_0$. It is clear that $D_a$
decreases as the water jet height increases with constant nozzle velocity ($V_0 = 208.5$
cm/s). On the other hand, $D_a$ is not related to the nozzle velocity, $V_0$, for fixed water
jet ($x_j = 5$ cm).

\section*{3.4 Discussion}

Air volume, potential energy and energy dissipation rates due to entrained air are
estimated for three steady phenomena. In redistribution flow region of 2-D and
circular vertical plunging jet, the actual air volume $V_a$ is almost equivalent to $V_{ad} +$
$V_{au}$, whereas in the developing flow region $V_a$ may be only $V_{ad}$. For this reason air
volume distributions are peaky in redistribution region. Since potential energy
dissipation rate per unit length $E_d$ is proportional to air volume, $E_d$ may possibly
become larger than that estimated in the previous section.

The ratio of energy dissipation through potential energy transferred by the entrained
air to total energy dissipation rate, $D_a$, estimated from the data analysis was about
25% in the case of hydraulic jump, 1.4% in the case of 2-D vertical plunging jet and (2-4)% in the case of circular plunging jet.

**Figure 3.17:** Comparison of the ratio of energy dissipation rate due to entrained air to total energy dissipation rate, $D_a$, between three typical phenomena for nearly identical inflow conditions (HJ: Hydraulic jump, VCPJ: Vertical circular plunging jet and V2DPJ: Vertical 2-dimensional plunging jet).

It is observed that $D_a$ becomes larger in the case of hydraulic jump although the impact velocity is almost identical to other two cases (Fig. 3.17). The probable reason of this will be the orientation of the roller. Figure 3.18 shows that in roller of hydraulic jump, the momentum direction changes 180-degree compared to 90-degree change in the vertical plunging jet (Chanson and Brattberg, 2000). The change in momentum direction causes a momentum transfer leading to the higher energy dissipation rate.
On the other hand, vertical circular plunging jet leads the energy dissipation rate almost two times higher than 2-D vertical plunging jet in the case of $V_1 = 2.31$ m/s. The reason for this may be the difference of perimeter. The length of perimeter through which air is entrained is larger in the case of circular plunging jet than 2-D vertical plunging jet.

### 3.5 Conclusions

Our investigation allowed a quantitative comparison among energy dissipation between three typical air entrainment phenomena. A simple model is proposed and the energy dissipation rate is calculated using the model. In the model the rise velocity is used as a constant value. Ratios of energy dissipation rate due to air bubbles to total energy dissipation rate $D_a$ are obtained about 25%, 1.4% and (2-4)% for hydraulic jump, 2-D vertical plunging jet and vertical circular jet, respectively. Although the upstream velocities are almost the same for all the cases, the ratios of energy dissipation rate show significant difference among the three phenomena.
Measurements of Void Fraction and Rate of Energy Dissipation in 2-D Wave Breaking

Summary

The void fractions in two-dimensional (2-D) wave breaking are measured by a conductivity probe and various properties of void fraction field are shown. The experimental results show that void fraction distribution are the function of water depth and probe immersion time (duration of breaking event). The maximum void fraction is found about 19% near the still water surface in the case of plunging breaker, whereas it is about 16% in the case of spilling breaker. An estimate of about (18-20)% are obtained for the ratio of potential energy dissipation rate due to air bubbles to the total energy dissipation rate for plunging breakers, whereas around (19-23)% for the spilling breakers.

4.1 Introduction

In the preceding chapter, the rate of energy dissipation by air bubble that describes in plunging jet flow (steady) was presented.

In surf-zone, wave breaking is an efficient mechanism for the dissipation of surface-wave energy (Melville and Rapp, 1985). Air bubbles are entrained through sea surface at wave breaking. Horikawa and Kuo (1966) suggested that the air bubbles are largely responsible to dissipate the energy at least at the initial stages. During the
stormy weather, the sea-surface becomes almost white by air bubbles, which is always visible to the naked eye. Air entrainment is associated with a rise in water level caused by liquid displacement upwards, which implies an increase in the potential energy. The ability to predict energy transformation by air bubbles in steady and unsteady flows is essential for various hydraulic problems. In order to understand the energy transformation and dissipation quantitatively for unsteady case, 2-D wave breaking is considered as a typical phenomenon of air entrainment through free surface.

In turbulent air-water flows, the bubble rise velocity is affected by the turbulence (Chanson, 1997). Most discussions suggested that bubble rise velocity becomes slower due to high-level of turbulence and it can be expressed as the proportional to square root of the vertical pressure gradient (see Appendix, chap. 3). This problem is still somewhat subject to discussions however. No study has proved conclusively either an increase or a decrease of bubble rise velocity.

In contrast to the plunging jet flows, studies on surf-zone air bubbles are very limited yet, probably because the flow fields of broken wave are very complicated after mixing the air bubbles. It is almost unclear how these air bubbles affect the fluid motion in the surf zone. Only a small number of studies investigated the characteristics of surf-zone air bubbles. Führböter (1970) suggested that the entrained air caused a transfer of energy into potential energy, but he did not give quantitative discussion of the transferred energy. Hwung et al. (1992) estimated the potential energy ingredient by air bubbles measuring void fraction distribution in a wave flume for both spilling and plunging breakers. Lamarre and Melville (1991) have done excellent works. They measured a set of void fraction for 2-D and 3-D wave breaking and reported that up to (30-50)% of the energy dissipated by breaking is used in entraining the air bubbles against their buoyancy.

In this chapter, the experimental facilities, instrumentation and the experimental procedure are first described and then measurements of the void fraction. This
chapter focuses mainly on the amount of wave energy that is transferred into potential energy and dissipated by air bubbles through free surface.

4.2 Experiments and observations

4.2.1 Wave flume
The experiments were performed in a wave channel of 20 m long, 0.80 m wide and 0.60 m deep filled with fresh water to a depth of 0.40 m, which is shown in Fig. 4.1. The flume was horizontal and the two sidewalls along the breaking zone of the flume were made of glass panels, supported by a metal frame.

Figure 4.1: Sketch of 2-D wave flume.

A sloping bottom (1V: 9.5H) was installed at 9.65 m distant from a wave generator. The wave generator was a piston type with a vertical flat plate moving horizontally in sinusoidal motion. The working section was 13 m long in length.
4.2.2 Instrumentation

Water depth and surface elevation were measured using a pointer gauge and three capacitance wave gauges respectively. The wave gauges were calibrated daily by raising and lowering the water level in the wave flume and the relationship between the wave amplitude and the output voltage was linear. The pointer gauge and one of the wave gauges were positioned on the same carriage as the void-fraction probe. The other two wave gauges were positioned at 5 m and 5.30 m from the wave maker to get the information of the incident wave height.

The effect of air bubbles on the wave gauge was tested in separate experiments. Air was introduced at the bottom end of a vertical cylinder installed in a still water tank. Visual observations showed that the foam was confined to a region above the still water level. Tests performed with void fractions ranging from 0 to 0.05, showed that the wave gauges recorded reasonably accurately the rise in water level induced by the air bubbles. The error was of the same order of magnitude as the bubble foam thickness formed at the water surface in the cylinder, although the output of the wave gauge tended to correspond to the level above the foam (Fig. 4.2).

The same L-shape conductivity probe as used for the experiments of vertical plunging jets was used for 2-D wave breaking. The probe, wave gauges and pointer gauges were fixed on a trolley system and displaced in the horizontal and vertical directions. The probe tip was set up in the opposite direction to the wave propagation, and its small dimensions allowed it to respond to individual bubbles. The void-fraction probe surveyed the breaking region with a grid spacing of 5 to 25 cm increments along channel and 2 cm increments in depth. At a location close to the free surface, the probe was not always immersed. Measurements were carried out along the centerline of the wave channel and all the data were recorded with the scan rate of 1 kHz per channel.

Additional measurements (e.g. plunge point, penetration depth and air packet) were performed using a high-speed digital video camera.
4.2.3 Wave generation

A summary of the characteristics of waves breaking is given in Table 4.1 for three tests in both cases of spilling and plunging breakers. In the tests, wave periods were $T = 1.12$ s and $T = 1.8$ s for spilling and plunging breakers, respectively.

The surf similarity parameter (Iribarren and Nogales, 1949) is given as

$$I_0 = \frac{m}{\sqrt{H_a/L_0}}$$  \hspace{1cm} (4.1)
where, \( m \) is the beach slope, \( H_0 \) and \( L_0 \) are the wave height and wavelength at deep water. The typical value of \( I_0 \) in the range 0.45 – 3.2 suggests plunging waves, whereas spilling waves correspond \( I_0 < 0.45 \).

**Table 4.1: Wave breaking experiments: characteristics of wave breaking**

<table>
<thead>
<tr>
<th>Test (1)</th>
<th>( H_0 ) (m)</th>
<th>( T ) (s)</th>
<th>( H_0/L_0 )</th>
<th>( H_b ) (m)</th>
<th>( H_b/h_b )</th>
<th>( I_0 )</th>
<th>Br. Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>SP-1</td>
<td>0.110</td>
<td></td>
<td>0.056</td>
<td>0.117</td>
<td>1.08</td>
<td>0.443</td>
<td></td>
</tr>
<tr>
<td>SP-2</td>
<td>0.122</td>
<td>1.12</td>
<td>0.062</td>
<td>0.133</td>
<td>1.03</td>
<td>0.421</td>
<td>Spilling</td>
</tr>
<tr>
<td>SP-3</td>
<td>0.150</td>
<td></td>
<td>0.076</td>
<td>0.149</td>
<td>0.91</td>
<td>0.380</td>
<td></td>
</tr>
<tr>
<td>PL-1</td>
<td>0.125</td>
<td></td>
<td>0.024</td>
<td>0.180</td>
<td>1.01</td>
<td>0.677</td>
<td></td>
</tr>
<tr>
<td>PL-2</td>
<td>0.145</td>
<td>1.80</td>
<td>0.028</td>
<td>0.198</td>
<td>1.05</td>
<td>0.627</td>
<td>Plunging</td>
</tr>
<tr>
<td>PL-3</td>
<td>0.166</td>
<td></td>
<td>0.032</td>
<td>0.207</td>
<td>1.07</td>
<td>0.586</td>
<td></td>
</tr>
</tbody>
</table>

### 4.2.4. Observations of entrained air

The visualized entrained air bubbles during the breaking process are shown in Fig. 4.3 through video images and sketches. Air bubbles are entrained at the plunging point, not exactly at the breaking point. Similar phenomena also occur in a spilling breaker. After a wave has broken as a spilling or plunging breaker a transition occurs. In spilling breaker, the surface roller grows and air bubbles are kept in the roller from breaking point to some distance and then entrains into water (Fig. 4.3). It can be observed that the plunging jet generated a splash up of water, which continues the breaking process and creates large coherent vortices that could reach the bottom. At the plunging point, some air bubbles propagated inward direction with wave propagation near the free surface and some are spread backward direction near the bottom, which may be due to the water stagnation and advection. As is typical for plunging waves, a large air tube is produced from the initial impact at breaking and hit undisturbed water at a second plunge point, with the cycle starting again with another splash up, shown in Fig. 4.3 (Photograph). Galvin (1969) called this second
plunge points the splash touchdown point and gave measurements of the distance between the first two plunge points.

**Figure 4.3:** Sketches- Upper: spilling breaker. Lower: plunging breaker. B.P and T.Z. denote breaking point and transit point, respectively.
On the other hand, overturning or splash up did not necessarily occur in spilling breakers (Fig.4.3, sketch-Upper). Breaking was visible by the appearance of aerated water near the top of the wave. The volume of the plume is increased and traveled with the front face wave as a surface roller, which subsequently entrained into water after some distances from the breaking point (Fig. 4.3, Photograph).

### 4.3 Experimental results: spilling and plunging breakers

#### 4.3.1 Response of probe

Elaborate measurements of air bubble entrainment in the surf zone were performed for spilling and plunging breakers. The output from the probe was sampled at a sampling frequency of 1 kHz. The large positive voltage pulse was produced each time a bubble impacted the probe. A sketch of air pulses and wave profile are shown in Fig. 4.4(b), which was used to determine the void fraction. When a bubble hits a probe tip, the output of the void meter rises like the region between 1 to 2 in Fig. 4.4(a). Region between 2 and 3 shows the tip is completely inside the bubble where the output is constant. As the tip touches the bubble-water surface again which is indicated by the region between 4 and 4, the response is much faster than the region between 1 and 2 due to surface tension.

(a)

![Output](image.png)

<table>
<thead>
<tr>
<th>Time</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>air</td>
<td>water</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 4.4: Sketch. (a) Output signal from a probe during the passage of a bubble; (b) response of the wave gauge and probe.

4.3.2 Data analysis

The typical responses of the probe and wave gauges are illustrated in Figs. 4.5 and 4.6. When an air bubble is pierced by a probe tip, the output shows a square pulse. In the measurements the voltage signals from the wave gauges corresponding to the still water level and wave profile and the signals of air bubble pulses were recorded. In data processing, for the output from the probe, negative voltage is set ‘0’ that indicates water and the positive voltage set to ‘1’ that indicates air. The outputs from the wave gauges were multiplied by the calibration coefficients to obtain elevation of water.
Figure 4.5: Typical time-series of voltage output from the probe and wave gauge. (a) 2 cm above the still water level, (b) at still water level and (c) 2 cm below the still water level. Solid blue lines represent the output from the probe showing bubble pulses, dashed lines represent the still water level and red lines represent wave profile ($x - x_b = 0.50$ m; $H_0/L_0 = 0.076$: spilling breaker).
Figure 4.6: Typical time-series (a) 2 cm above the still water level, (b) at still water level and (c) 4 cm below the still water level. Solid blue lines represent the output from the probe showing bubble pulses, dashed lines represent the still water level and red lines represent wave profile ($x - x_b = 0.70$ m; $H/L_0 = 0.024$: plunging breaker).

Figure 4.5 shows the time series of surface elevation and pulses of air bubbles obtained from the wave gauge and void-fraction probe with the probe being located at 2 cm, 0 cm and – 2 cm above the still water surface for the case of spilling breaker, whereas in Fig. 4.6 the probe was located at 2 cm, 0 cm and – 4 cm above the still
water surface for the case of plunging breaker. In these figures, down crossing of the water surface with the output signal from the probe indicates that the void-fraction probe comes out of the water. When this occurs, large excursions with the value of ‘1’ in the void fraction signal are generated. Figures 4.5 and 4.6 show that swapping between air and water does not always coincide between the signals of surface profiles and void probe. The probable reason for this may be electric response or transfer delay between the void probe and wave gauge. Time delay can occur during processing of raw data to square pulse by void meter. The data were edited to eliminate these transitions with the help of surface profiles. For the probe located close to the free surface the time of probe immersion (duration of breaking event), $\Delta t$ (see Fig. 4.4(b)) was not long enough but many pulses of air bubbles were found as shown in Figs. 4.5 and 4.6.

### 4.3.3 Void-fraction measurements

Calculation technique of void fraction from the output of the probe is presented below. In the duration of breaking event or the period during which the probe is immersed under water, $\Delta t$ (see Fig. 4.4(b)), the total time of air bubble encounter, $\Delta \tau$, is given as

\[
\Delta \tau = \sum_{\Delta \tau_i} \Delta \tau_i
\]  

(4.2)

where $\Delta \tau_i$ is the period that an air bubble takes when it passes the probe tip and $i$ denotes a number of air bubbles detected during $\Delta t$.

The air concentration or void fraction for one wave period $T$ is obtained from
Figure 4.7: Time averaged void fraction over one wave period versus number of waves measured at $x-x_0 = 0.50$ m ($H_0/L_0 = 0.076$: spilling breaker). (a) 2 cm above the still water level, (b) at still water level and (c) 2 cm below the still water level.

Figures 4.7 and 4.8 present the measurements of time averaged void-fraction during one wave period for spilling and plunging breaker, respectively. Approximately 570 waves were extracted from 10 minutes data recording for each depth in spilling.
breaker, whereas it were 350 to 420 waves from 10 to 12 minutes data for plunging breaker.

**Figure 4.8:** Time averaged void fraction over one wave period versus number of waves measured at $x-x_b = 0.70$ m ($H_0/L_0 = 0.024$: plunging breaker). (a) 2 cm above the still water level, (b) at still water level and (c) 4 cm below the still water level.

The void fraction $C$ and the duration of breaking event $\Delta t$ were averaged over 570 waves for spilling breaker and 350 to 420 waves for plunging breaker. It was seen that $\Delta t$ was approximately $1/8$, $1/4$ and $1/2$ of the wave period ($T=1.8$ sec) in plunging breaker for the presented graphs Fig. 4.8(a), (b) and (c), respectively, whereas it was
1/4.5, 1/2.5 and 1/1.7 corresponds to Figs. 4.7(a), (b) and (c), respectively for spilling breaker. Averaging $C$ and $\Delta t$ over all the individual waves gives mean void fraction and breaking duration at several locations inside the surf zone.

Figure 4.9: Vertical distribution of mean void fraction during breaking event. (a) Spilling breaker ($H_0/L_0 = 0.076$), and (b) plunging breaker ($H_0/L_0 = 0.024$).

In Fig. 4.9, the mean void fractions during breaking event are presented at several locations, where $x-x_b$ denotes the distance measured from the breaking point. Comparison of these figures indicates that there are some differences in void fraction distribution between spilling and plunging breakers especially near the still water level. The void fractions are higher for some cases in spilling breakers than in plunging breakers for $z = +2$ cm (Fig. 4.9). Although breaking duration was smaller near the free surface the maximum air concentrations were detected. Figure 4.9(a) suggests that the maximum void fraction was found at $x-x_b = 0.50$ m, where $x-x_b$ is the distance from the breaking point (Fig. 4.3b). This position is close to the end point of the surface roller. It was observed that at the same location penetration depth
was also maximum. Figure 4.9(b) shows that void fraction has its maximum at $x - x_b = 0.60$ m and in fact, this position is close to the plunge point. Moreover, the data clearly show the significant void fraction at second plunging point ($x - x_b = 0.80$ m) due to splash-up cycles. The plunging point was confirmed by high-speed video visualization. The data of void fraction consistently decays exponentially with the depth. Wu (1988) and Stanton et al. (2000) found similar trend of void fraction distributions for the large-scale experiments and field measurement. Further, experimental results (Fig. 4.9) indicate that the maximum void fraction is around 20% near the still water level in the case of plunging breaker, whereas it is around 16% for spilling breaker. Similar results were found by Hwung et al., (1992) for both of the breakers. They measured maximum void fraction 18% for plunging breaker and 12% for spilling breaker. Comparing Fig 4.9(a) to (b), it is seen that plunging breaker induces a deeper penetration of air bubbles.

![Figure 4.10: Void fraction distributions as functions of dimensionless horizontal distance. (a) Spilling breaker ($H_0/L_0 = 0.076$), and (b) plunging breaker ($H_0/L_0 = 0.024$).](image-url)
Figure 4.10 shows that the void fraction distribution in horizontal direction for several depths. The figures show clearly that from breaking point to some distance the void fractions are almost zero for both cases and all the profiles have peaks which are almost identical with the end point of the surface roller and with the plunge point for spilling and plunging breaker, respectively. The void fraction distributions decrease subsequently from the peak point in both cases.

![Figure 4.10: Void fraction distribution in horizontal direction for several depths.](image)

**Figure 4.11:** Void fraction distribution as a function of dimensionless breaking duration time. (a) Spilling breaker \((H_0/L_0 = 0.076)\), and (b) plunging breaker \((H_0/L_0 = 0.024)\).

The relations between the void fraction \(C\) and the time of immersion or the duration of breaking event, \(\Delta t\) at different elevations are illustrated in Fig. 4.11. The void fraction decreases as the dimensionless duration of breaking event \(\Delta t/T\) increases since fewer bubbles are injected over \(\Delta t\) in the deeper position (Figs. 4.5(c) and 4.6(c)). Figure 4.11(a) shows that void fraction decays linearly as the duration of breaking event increases, whereas the exponential relationship can be seen in several cases in Fig. 4.11(b). It could be argued that spilling and plunging breakers differ due
to structure of turbulence. For the plunging breaker, the void fraction field is dominated by the turbulence more significantly than spilling breaker, especially near the free surface.

Figure 4.12 shows the vertical distribution of the dimensionless duration of breaking event. Comparing Fig. 4.12(a) and (b), it is seen that the duration of breaking event (probe immersion time) increases with increasing depth and the distance from the breaking point. The duration of breaking event becomes almost equal to the wave period near $z = -0.05 \text{ m}$. In the range of $z = 0.02 \text{ m}$ to $-0.05 \text{ m}$, the duration of breaking event becomes larger in the inner surf zone (near the shoreline) in both of the breakers.

![Figure 4.12](image)

**Figure 4.12:** Vertical distribution of dimensionless duration of braking event. (a) Spilling breaker ($H_0/L_0 = 0.076$), and (b) plunging breaker ($H_0/L_0 = 0.024$).

### 4.3.4 Volume and potential energy of entrained air

The time averaged volume of entrained air and potential energy increase due to entrained air bubbles per unit length and width were computed from the void fraction mappings (Fig. 4.9) using the following Eq. (4.4) and Eq. (4.5) respectively:
where $\eta_*$ is the instantaneous water surface elevation.

\[
V_a = \int_{z_h}^{z_*} C(z) \frac{\Delta t(z)}{T} \, dz
\]  

(4.4)

\[
\Delta PE = \rho_a g \int_{z_h}^{z_*} \left\{ \frac{1}{T} \int_{t_0}^{t_*} C(z) (\eta_* - z) \, dt \right\} \, dz
\]  

(4.5)

**Figure. 4.13:** Air volume distribution as a function of distance in the shoreward direction. (a) Spilling breaker, and (b) plunging breaker.

Figure 4.13 shows the variation of entrained air volume per unit area. Fig. 4.10 shows that from breaking point to some distance (transit zone, Fig. 4.3), the void fraction is almost zero indicating no contribution of air volume and potential energy in this domain (Figs. 4.13 and 4.14). It should be pointed out that the distribution of $\Delta PE$ is found similar type configurations to air volume for steady case, because potential energy of entrained air is proportional to entrained air volume (chapter 3). In the case of 2-D wave breaking, the trends of air volume are not exactly same as potential
energy (Figs. 4.13 and 4.14). Since the entrained air volume is not only the function of depth, but also the duration of breaking event, the center-of-gravity of entrained air might be varied each location. The maximum air volume and maximum potential energy are found near the plunging point and the end point of surface roller. Figs. 4.13 and 4.14 show that after transit zone to the position of maximum penetration (Fig. 4.3), $V_a$ and $\Delta PE$ rapidly increases and subsequently decreases from plunging point to rest of inner zone.

![Graph](image)

**Figure. 4.14:** Potential energy due to air bubbles as a function of distance in the shoreward direction. (a) Spilling breaker, and (b) plunging breaker.

It is also observed that air packet was conserved up to $\Delta L/L_0 \approx 1/12$ in plunging breakers, whereas surface roller was conserved up to $\Delta L/L_0 = 1/9$ in spilling breakers, where $\Delta L$ is the length of transit zone (Fig. 4.3). Lamrre and Melville (1991) observed that the volume of air enclosed in the initial air packet is conserved for up to $1/4$ of a wave period after breaking. However, the judgements of this issue may vary
observer to observer. It was also seen that the air packet was not created exactly from the breaking point (Fig. 4.3).

From video recordings of the 2-D breaking events from the side of the channel the size of the “tube” of air initially enclosed by the plunging breakers was measured. In Fig. 4.15 the volume of air in the tube obtained from the video images, $V_a^T$, is plotted versus deep-water wave height. The figure suggests that volume of entrained air increase almost linearly as the wave height increases.

![Graph showing $V_a^T (m^2)$ vs $H_0 (m)$](image)

**Figure. 4.15:** The volume per unit width $V_a^T (m^2)$ of air in the tube formed by the plunging wave crest versus wave height.

### 4.3.5 Comparison of wave energy and potential energy due to air bubbles

In Fig. 4.16 the increased potential energy $\Delta PE$ is plotted as a function of horizontal distance, normalized by the wave energy at the breaking point $E_b$. The local wave energy is also plotted for comparison.
Figure 4.16: Comparison between normalized potential energy increased due to air bubbles and wave energy. (a) Spilling breaker \((H_0/L_0 = 0.076)\), and (b) plunging breaker \((H_0/L_0 = 0.024)\); \(E_0\) represents the wave energy at breaking point.

The potential energy due to entrained air per unit area increases significantly after the transition zone of spilling and plunging breaker. The maximum potential energy, which corresponds to the maximum air volume, is found just after transition zone for both breakers. Figure 4.16(a) suggests that around 6\% of local wave energy is used to merge the air bubble in the case of spilling breaker at \((x-x_b)/L_0 = 0.25\). In the same way, Fig. 4.16(b) shows the relationship between local wave energy and potential energy for plunging breaker. It is seen from the figure that maximum potential energy of 11\% of the local wave energy is found near the plunging point at \((x-x_b)/L_0 = 0.11\). Investigations were also performed for other two cases for both spilling and plunging breakers and results are mentioned in the conclusion section in this chapter.
4.3.6 Energy dissipation rate

It was observed that the air entrainment occurs from plunging point to stable wave condition\(^{(1)}\) in the surf zone. It is of keen interest to quantify the energy dissipation rate by means of representative quantities of air bubbles in this domain. In this respect, total energy dissipation rate by air bubbles is first measured using Eq. (4.6) knowing potential energy or air volume as it was done for plunging jets in chapter 3.

\[
E'_{da} = \int_{x_1}^{x_2} E_{da}dx = \int_{x_1}^{x_2} \frac{\Delta PE}{h_g} - w_r dx = \rho_a g \int_{x_1}^{x_2} V_a w_r dx
\]

(4.6)

where \(x_1\) and \(x_2\) represent the offshore and onshore limits of aerated zone. The rise velocity \(w_r\) is assumed 0.25 m/s, because still we have no references either an increase or a decrease in rise velocity in the surf zone (Chanson 1997).

On the other hand, the shoreward energy flux for waves of amplitude \(A\) in water of depth \(h\) is \(\frac{1}{2} \rho_a g A^2 (gh)^{1/2}\). If it is supposed that this is being lost over a distance in the shoreward direction, then the energy dissipation rate becomes:

\[
\Delta EC_g = \frac{1}{2} \rho_a g \left[ \{ A^2 (gh)^{1/2} \} - \{ A^2 (gh)^{1/2} \} \right]_1^{x_2}
\]

(4.7)

where subscripts 1 and 2 represent the position of \(x_1\) and \(x_2\), respectively.

Finally, the ratio of potential energy dissipation rate due to air bubbles entrained by wave breaking to the total energy dissipation rate, \(D_a\), can be expressed as

\[
D_a(\%) = \frac{E_{da}}{\Delta EC_g} \times 100
\]

(4.8)

\(^{(1)}\) See Fig. 1. (Dally et al., 1985)
Figure. 4.17: Ratio of potential energy dissipation rate due to entrained air to the total energy dissipation rate: Spilling breaker (SP), and plunging breaker (PL).

As shown in Fig. 4.17 three data sets for plunging breaker from the surf zone give the ratios of energy dissipation $D_a$ of the order of 18-20%. Similarly the contribution of energy dissipation by air bubbles in spilling breaker is estimated and shows 19-23%. Although it is difficult to comment about $D_a$ from Fig. 4.17, roughly it is observed that the ratio of energy dissipation due to entrained air bubble reduces with the wave height in plunging breaker, whereas increases in spilling breaker.

4.4 Conclusions

In the present chapter it has been investigated precisely various properties of void fraction fields for 2-D wave breaking (unsteady case). In the experiments, it was observed that the aerated area grew rapidly from the end of transit zone to the plunging point and the end point of roller, reached a maximum, and subsequently
decreased. The maximum void fraction was found about 19% near the still water surface in plunging breaker, whereas it was around 16% in spilling breaker. The distribution of void fraction $C$ decreases and the duration of breaking event increase exponentially with the depth. The relationship between $C$ and $\Delta t/T$ was found almost linear. The potential energy $\Delta PE/E_b$ of entrained air was measured from the void fraction distributions and it was nearly 11-7% of local wave energy close to the plunge point corresponding to three plunging breakers. On the other hand, about 7-4% of local wave energy was used to merge the air bubble in the case of spilling breakers near the end point of surface roller. The ratio of potential energy dissipation rate due to air bubbles to the total energy dissipation rate $D_a$ was around 20% both for plunging and spilling breakers. Further studies of the ratio of energy dissipation due to air bubbles should be investigated based on many experiments.
5

Modeling of Surf Zone Hydrodynamics Considering Air Bubbles

Summary

The air bubble model is introduced to predict energy dissipation due to air bubbles entrainment into water in the surf zone. Wave breaking induces a modification of wave shapes and containing a large number of air bubbles into water, resulting in the flow field of air-water complicated. An averaging technique is used to describe the influence of the entrained air bubbles. The characteristics of time averaged wave parameters for regular waves are discussed taking the air bubble effects into account.

5.1 Introduction

In shallow water, the sloping bottom causes the breaking process, where a great amount of air bubble is entrained into water near the breaking point. It is believed that such air bubbles are responsible for energy dissipation in the surf zone. The representative types of breaking wave are spilling breakers and plunging breakers. With the mixing of air bubbles, the flow fields of broken waves are turbulent and complicated. The injected air bubbles beneath a breaking wave are rapidly broken up by turbulence, producing an initial size spectrum proportional to \((radius)^{-10/3}\) (Garrett et al., 1999). This tiny air bubbles are not always visible to the naked eye, yet they play a very important role in the surf zone. Overall, it is convinced that air bubbles
have significant effects in the surf-zone dynamics, such as wave energy dissipation, wave set-up, run-up and long wave generation. To model all these processes accurately, the characteristics of air bubbles in the surf zone have to be known in detail.

In the past few decades, a number of works were conducted to make clear the internal mechanics of breaking waves in order to elucidate the characteristics of wave energy over the surf zone. Among them, two mechanisms have received considerable attention from last two decades: the surface roller concept and energy flux difference model. Svendsen (1984) presented a solution considering surface roller theory, which is in good agreement with wave height variation from breaking point to inshore, but the set-up is not favorable. Dally *et al.* (1985) used a heuristic expression for energy dissipation. Their model was calibrated and verified using laboratory data, with good results for the wave decay and the maximum set-up values for some test cases; but it does not describe correctly the distribution of set-up/set-down across the surf zone.

In parallel with the above two models, Stive’s (1984) model predictions of wave height performed well but set-up was less concluded with experimental data. In this model he used two empirical coefficients that are assumed constant over the surf zone. Sawaragi *et al.* (1984) modified the momentum balance and energy balance equation including the process of the wave energy dissipation due to the turbulence. They showed good wave height attenuation in comparison with experimental results. The wave set-up was pretty well under the spilling breaker but had a little difference in the case of plunging breaker. Swift (1993) presented a solution based upon simple extension of the analytical solution by Dally *et al.* (1985). Although he showed significant improvement of wave set-up, still transition zone is unclear. Bowen *et al.* (1968) noted that the measured set-down is nearly uniform for a distance after breaking is initiated, in this case up to the point where the curl of the plunging breaker touches down (the first three data point, page 2573, Fig. 2 in Bowen’s paper).

In fact, all the models are fairly capable of predicting wave height variations, but the set-up. There is controversy that although the wave height is decreasing after wave breaking, the momentum might not (Dally *et al.*, 1985, Fig. 11 & Svendsen, 1984,
Fig. 14 and 15). Dally *et al.* (1985) suggested that there is no energy dissipation before the curl touches down and air is entrained.

After a wave has broken as a spilling or plunging breaker, a transition occurs. In this study ‘transition zone’ is defined as a distance between breaking point and air entrainment point. The characteristics of wave parameters with rapid transition of the waves have been very difficult to describe, especially for plunging breakers.

In order to give an overall picture of the dynamical role of air bubble in surf-zone, how the parameters are affected is explained in this chapter. The wave energy dissipation is also approximated by a new proposed model in this context.

### 5.2 Basic assumption

#### 5.2.1 Vertical distribution of void fraction

With respect to the modeling of void fraction profiles under breaking waves, it is possible to treat the cases of both spilling and plunging breakers in the same manner. In the present study the distribution of air bubbles in the vertical direction is considered to take the following exponential form proposed by Wu (1981a):

\[
C(z) = C_0 \exp(k_1 z)
\]  

(5.1)

where \( C(z) \) is the part of the volume locally occupied by bubbles per unit width (time-averaged concentration), \( k_1 \) is a decay parameter characterizing vertical distribution of air bubbles and \( C_0 \) denotes the reference concentration at the mean water surface \( z = 0 \).

The following boundary conditions are automatically satisfied:

\[
C(z) = C_0 \text{ at the surface } z = 0
\]

and

\[
C(z) \rightarrow 0, \text{ for } z \rightarrow -\infty
\]
Figure 5.1: Vertical distributions of void fraction by Eq. (5.1) ($C_0 = 0.15$).

Figure 5.1 shows how the mathematical model predicts the vertical distribution of void fraction due to the wave breaking. Significant decay is seen in void fraction profiles with increasing $k_l$ for fixed reference void fraction $C_0$.

The rise of the free-surface level $\Delta h$ is a function of the amount of entrained air and water depth (Fig. 5.2). The total volume of entrained air into water per unit width is defined as:

$$\Delta h = \int_{z-h}^{0} C(z) \, dz$$

(5.2)

where $z$ is taken upward from the raised water surface.
5.2.2 Averaging procedure

It was mentioned that a great amount of air bubble is entrained at the breaking point but it is impossible to calculate the behavior of every single bubble in the cloud. By applying an averaging procedure all quantities like water pressure and bubble radius become continuous functions of space rather than discrete. This averaging concept was proposed by van Wijngaarden (1968) and Biesheuvel and van Wijngaarden (1984).

It is started with averaging and introducing averaged quantities: pressure $p$, horizontal velocity $u$, vertical velocity $w$, and density $\rho$, where the averaging is taken over the mixture containing many bubbles. The results obtained by Biesheuvel and van Wijngaarden (1984) can be summarized as follows:

**Figure 5.2:** Definition of the coordinate and static water level rise.
Figure 5.3: Definition sketch of 2-D wave breaking. B.P. denotes the breaking point, R1 represents the outer surf zone (rapid transitions of wave shape), and R2 is the inner surf zone (rather slow change in wave shape). This sketch is also applicable for spilling breakers.

\[
\begin{align*}
  u &= (1 - C)u_a + Cu_w \\
  w &= (1 - C)w_a + C(w_a + w_r) \\
  p &= (1 - C)p_w + C p_a \\
  \rho &= (1 - C)\rho_w + C \rho_a
\end{align*}
\]

(5.3a) (5.3b) (5.3c) (5.3d)

where, subscripts ‘a’, ‘w’ denote the air and water, respectively and \( w_r \) denotes the rise velocity of bubbles. For the case of radial motion of a bubble, the internal pressure of air bubble can be expressed as:

\[
p_a = p_w + \frac{2\sigma}{R}
\]

(5.3e)

where \( \sigma \) and \( R \) denote surface tension and air bubble radius, respectively.
Since the air density is much smaller than the water density and can be neglected, the density of the aerated region will generally be less than that of the undisturbed seawater owing to entrained air. If the surface tension of air bubble is left out of account then the above relations reduce to:

\begin{align}
  u &= (1 - C)u_w + Cu_a \quad (5.4a) \\
  w &= (1 - C)w_w + C(w_a + w_r) \quad (5.4b) \\
  p &= p_w \quad (5.4c) \\
  \rho &= (1 - C)p_w \quad (5.4d)
\end{align}

Under further assumption that the horizontal and vertical velocity fields of the water do not change significantly due to the air bubbles entrainment, i.e. \( u_w = u_a \) and \( w_w = w_a \), then above relations are approximated as:

\begin{align}
  u &= u_w \\
  w &= w_w + Cw_r \\
  p &= p_w \\
  \rho &= (1 - C)p_w \quad (5.5)
\end{align}

### 5.2.3 Correction term & boundary condition

The above assumptions do not satisfy the continuity equation. Note that the term \( Cw_r \) is not the time dependent quantities, which might be the reason for the discontinuity. Requiring now that to satisfy the continuity equation, we therefore modify the vertical velocity term in the following manner:

\begin{align}
  w &= w_w + w' \quad (5.6)
\end{align}

where \( w' \) is the correction term.
For steady flow, the continuity equation becomes:

\[
\text{div}(\rho \mathbf{V}) = 0
\]

\[
\frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial z} (\rho w) = 0
\]

(5.7)

i.e.

\[
\frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial z} (\rho w) = 0
\]

(5.8)

where \(\rho\) is the function of \(z\) only.

Finally the first order linear differential equation in terms of \(w^\prime\) can be obtained with the help of \(u, w\) and continuity equation:

\[
\frac{\partial w^\prime}{\partial z} - \frac{C_0 k e^{i\xi}}{(1 - C_0 e^{i\xi})} w^\prime = \frac{C_0 k e^{i\xi}}{(1 - C_0 e^{i\xi})} w_w
\]

(5.9)
which is subject to the boundary condition of vanishing the correction term at the bottom,

\[ w' \to 0 \quad \text{for} \quad z = -h \]

Hence, the final form of correction term can be expressed as:

\[
 w' = \left( -\frac{\pi H}{T \sinh kh} \right) \frac{C_0}{1 - C_0 e^{k_r^*}} \left[ e^{\frac{h}{k}} \left( \frac{k k_2}{k^2 - k_1^2} \cosh(k + z) \right) - \frac{k k_2}{k^2 - k_1^2} e^{-z k} \right] \sin(\omega t - kx)
\]

\( (5.10) \)

5.3 Proposed air bubble model

The energy properties of linear water waves have been studied carefully in the past half-century. The derivation of the integral properties considering air bubbles effect makes it possible to investigate further into different phenomena for wave energy. Three approaches are commonly used to evaluate the wave properties in the surf zone. The first approach is evaluation of wave parameters averaged over the wave period, second approach predicts wave height decay and third is determination of the water level rise in the surf zone.

5.3.1 Evaluation of wave parameters

Key parameters in the modeling of the breaking wave energy decay are potential energy, kinetic energy, energy flux and dissipation rate of wave energy. In this section, it is restricted the study to shallow water waves only. To develop a model considering air bubble effects that describes breaking waves in the nearshore region, it is necessary to derive the expressions of those parameters.
5.3.1.1 DENSITY AND PRESSURE FIELD

Under the wave conditions, density and pressure fields in the vertical direction are determined by the local air concentration. Because of air bubbles entrained, the air water mixture becomes significantly lighter than the water below it. The vertical structure of density (Fig. 5.5(a)) is determined by the void fraction \( C(z) \) for different \( C_0 \) and wave decay factor \( k_1 \).

\[
\rho = (1 - C_0 \exp(k_1 z)) \rho_w
\]

(5.11a)

On the other hand, the pressure force is assumed to be hydrostatic in the absence of waves. The static pressure includes the effect of water level rise by entrained air but is different from triangular distribution (Fig. 5.5(b)). The static pressure including water level rise by air bubbles is given as:

\[
p(z) = p_{atm} - \rho_w g \left\{ z + \frac{C_0}{k_1} (1 - e^{k_1 z}) \right\}
\]

(5.11b)

where \( p_{atm} \) is the atmospheric pressure and the above relation satisfies the boundary conditions that \( p = p_{atm} \) at \( z = 0 \); \( p = p_{atm} + \rho_w gh \) at \( z = -h - \Delta h \).

![Figure 5.5](image)

**Figure 5.5:** Typical vertical distribution of (a) density and (b) static pressure in air-water mixture.
Figure 5.5(b) shows that the static pressure is affected by air bubbles, whereas dynamic pressure may be not.

### 5.3.1.2 STATIC ENERGY

The excess static energy per unit horizontal area due to entrained air bubbles is defined based on the Fig. 5.2:

\[
\Delta SE = SE - SE_0 = \int_{-h-\Delta h}^{h} \rho g z \, dz - \int_{-h}^{h} \rho g z \, dz
\]  

(5.12a)

The 1st term on the right hand side in Eq. (5.12a) can be recognized as total potential energy including air bubble effect and the 2nd term without air bubbles. It is given in a dimensionless form:

\[
M = \frac{\Delta SE}{\rho g h^2 / 2} = C_0 \left[ \frac{2(1 - e^{-k_1 h})}{(k_1 h)^2} - \frac{2(1 - C_0) \frac{e^{-k_1 h}}{1 - C_0 e^{-k_1 h}}} {k_1 h} \right]  
\]

(5.12b)

i.e.,

\[
M = f(C_0, k_1 h)
\]

(5.12c)

**Figure 5.6:** Dimensionless static energy as a function of (a) \( C_0 \) and (b) \( k_1 h \).
Figure 5.6 shows dependence of $M$ on $C_0$ and $k_1h$, which suggests that the excess static energy increases with increasing $C_0$ and decreasing $k_1h$. It is also seen that the increase in static energy due to air are around 7%, 15% and 20% corresponding to $k_1h$ = 1.8, 0.8 and 0.5 respectively for the value of $C_0 = 0.20$.

### 5.3.1.3 POTENTIAL ENERGY

Potential energy of the wave system is defined as the work done to deform a horizontal free surface into the disturbed state. Assuming a sinusoidal wave motion, the increase in potential energy $\Delta PE$ due to air bubbles is defined as the difference between the potential energy of water with air bubbles (second bracket term) and without air bubbles in presence of waves:

$$\Delta PE = PE - PE_0 = \int_{z=0}^{z=\Delta h} \rho_w (1-C_0) g z \, dz - \int_{z=0}^{z=\Delta h} \rho_w g z \, dz$$

The above equation can be expressed in dimensionless form:

$$N = \frac{\Delta PE}{\rho_w g H^2/16} = C_0 \left[ -1 + 16 \left( \frac{1 - e^{-k_1h}}{(k_1H)^2} - \frac{1 - C_0}{(k_1H)^2} k_1h \frac{e^{-k_1h}}{1 - C_0 e^{-k_1h}} \right) \right]$$

i.e.,

$$N = f(C_0, k_1h, k_1H)$$

(a) $k_1H = 3.0$

(b) $k_1H = 3.5$

(c) $k_1H = 2.5$

(d) $k_1H = 3.0$

(e) $k_1H = 3.5$
Figure 5.7 illustrates the relation between $N$ and $C_0$, $k_1h$ and $k_1H$ calculated from Eq. (5.13b). The dimensionless potential energy $N$ increases as $C_0$ increases and $k_1H$ decreases (Figs. 5.7(a), (b) and (d)). $N$ shows a significant change for $k_1H < 2$. Figure 5.7(c) suggests that $N$ becomes flat for $k_1h > 5$ and decreases for $k_1h < 5$ for certain value of $k_1H = 3.0$.

5.3.1.4 KINETIC ENERGY

Surface gravity waves posses kinetic energy due to motion of the fluid. The kinetic energy per unit surface area is obtained by integrating that per unit volume over the depth and averaging it over the wave period:

$$KE' = \frac{1}{2} \rho \int_{h_1}^{h_2} (u^2 + w^2) \, dz$$  \hspace{1cm} (5.14a)
Here, the $z$-integral is taken up to the mean water level: $z = 0$, because the integral up to $z = \eta$ gives a higher order term. For the surface waves on water with air bubbles the kinetic energy $KE'$ is estimated as

$$KE' = \int_{-\eta}^{0} \frac{1}{2} \rho_g \left( 1 - C \right) \left( u_u^2 + (w_u + w')^2 \right) dz \quad \text{(5.14b)}$$

Since simplifying the above expression in analytical form is difficult, a numerical procedure is introduced. In dimensionless form:

$$O = \frac{KE'}{\rho_g g h^2 / 16} = \frac{2k}{\sinh 2kh} \int_{-\eta}^{\eta} \left( 1 - C_0 e^{kh} \right) \left[ \cosh^2 k(h + z) + \sinh k(h + z) \right]$$

$$+ \frac{C_0}{(1 - C_0 e^{kh}(k^2 - k_1^2))} \left[ e^{kh} (k k_1 \cosh k(h + z) - k^2 \sinh k(h + z) - k k_1 e^{-kh}) \right] \right] dz$$

i.e.,

$$O = f(C_0, kh, k_1h) \quad \text{(5.14c)}$$

![Graphs showing the variation of $O$ with $C_0$ for different values of $k_1h$ and $kh$](image)
Figures 5.8(a)-(d) show the dimensionless kinetic energy represented by Eq. (5.14c). In the presence of waves the kinetic energy decreases with air bubble concentration increases (Fig. 5.8(a)) but this effect is almost negligible in comparison with the increase in potential energy. Comparing Fig. 5.8(a) and (b) with Fig. 5.7(a) and (b), it is seen that the kinetic energy decreases 2% by air bubbles, whereas the increase in potential energy is around 15% for \( C_0 = 0.20 \) and \( k_1 h = 0.6 \). It is also seen (Fig. 5.8(c)) that air bubble effects is almost constant for \( kh > 1.2 \) and this range may be deep-water wave conditions. In the range of \( kh < 1.2 \), Fig. 5.8(c) shows \( O \) is affected by the entrained air significantly.

### 5.3.1.5 ENERGY FLUX

Energy flux is defined as the rate at which the energy is transferred. The contributions from the water in the wave motion and from the air bubbles can be found together. Mathematically, mean energy flux including air bubbles effect, \( EF' \) per unit width is defined as:
where, $P_D$ is the dynamic pressure defined:

$$P_D = p + \frac{1}{2} \rho (u^2 + w^2) u \, dz$$

(5.15b)

After simplification, the energy flux becomes:

$$EF' = Ec\left(1 + \frac{kh}{\sinh 2kh}\right) + \frac{2kEc}{\sinh 2kh} \Delta h$$

(5.15c)

The first term on the right hand side corresponds to the energy flux in the absence of air bubbles, $EF$, and the second term is the contribution of air bubbles. The above relation can be written in dimensionless form:

$$P = \frac{EF'}{EF} = 1 + \frac{1}{(\frac{1}{2} + \frac{kh}{\sinh 2kh})} \frac{2k}{k_i} \frac{C_0}{\sinh 2kh (1 - e^{-kh})}$$

(5.15d)

i.e.,

$$P = f(C_0, kh, k)$$

(5.15e)

As can be seen from Figs 5.9 (a) and (b), $P$ increases almost linearly from unity with increase in the value of $C_0$. When the depth becomes shallow, i.e., $k_{ih}$ and $kh$ becomes smaller, energy flux becomes more sensitive to the void fraction. Also the variation of $P$ becomes milder for large $C_0$ (Fig. 5.9(c)).
Figure 5.9: Variation of dimensionless energy flux with $C_0$, $k_1/h$ and $kh$.

5.3.1.6 RADIATION STRESS
Longuet-Higgins and Stewart (1960, 1964) who introduced the term “Radiation stress,” and defined it as “the excess flow of momentum due to the presence of the...
waves.” The flux of momentum is comprised by two contributions: one due to wave-induced velocities of the water particles and another due to the pressure. Introducing the air bubble effects, the time averaged radiation stress in the direction of wave propagation is defined as the time averaged total momentum flux due to the presence of waves minus the mean flux in the absence of waves:

\[
S_{xx}' = \int_{h-h}^{h} (p + \rho u^2) \, dz - \int_{h-h}^{h} (p_0 - \rho g \Delta h) \, dz \quad (5.16a)
\]

After simplification,

\[
S_{xx}' = \int_{h-h}^{h} \rho (u^2 - w^2) \, dz + \int_{0}^{h} p \, dz + \int_{h-h}^{h} \rho g \Delta h \, dz - \int_{h-h}^{h} p_0 \, dz \quad (5.16b)
\]

The last term on the right hand side of Eq. (5.16b) has minor effect compared to the others, so this term can be neglected. Now the final form of radiation stress becomes:

\[
S_{xx}' = \int_{h-h}^{h} \rho (u^2 - w^2) \, dz + \int_{0}^{h} p \, dz + \int_{h-h}^{h} \rho g \Delta h \, dz \quad (5.16c)
\]

After simplification,

\[
S_{xx}' = \frac{2Ek}{\sinh 2kh} \int_{h-h}^{h} (1 - C_0 e^{k1z}) \{ \cosh k (h + z) - \sinh k (h + z) + \frac{C_0}{(1 - C_0 e^{k1z})(k^2 - k_1^2)} \}
\]

\[
\left[ e^{kh} (kk_1 \cosh k (h + z) - k_1^2 \sinh k (h + z)) - kk_1 e^{k1z} \right] dz + \rho \sigma g \left( \frac{H^2}{16} + h \Delta h \right)
\]

Now it can be written in dimensionless form:

\[
Q = \frac{S_{xx}'}{S_{xx}} = f (C_0, kh, k_1h, k_1H) \quad (5.16d)
\]

where \( S_{xx} \) is the radiation stress without air bubble effects.
Due to the presence of air bubbles, radiation stress may vary. Figure 5.10 shows the variation of dimensionless radiation stress with respect to several parameters.

5.4 Basic equations
If the analysis is restricted to a 2-D uniform coast with normally incident waves, a situation that can be represented in a wave flume, water surface elevation and wave height can be determined by simultaneous solution of the energy equation and momentum equation.

### 5.4.1 Energy balance equation

The energy equation expresses the conservation of wave energy flux. It is used to describe the decay of the wave height due to the loss of energy. The most common approach has been to describe wave transformation by the energy flux balanced equation with dissipation function:

\[
\frac{d(EF')}{dx} = -D
\]  

(5.17)

where \( EF' \) is the energy flux including air bubble effects, \( D = \alpha D_{air} \) is the energy dissipation rate per unit surface area, \( D_{air} \) is the energy dissipation rate per unit area due to entrained air and \( \alpha \) denotes a free parameter that may be determined from the experimental data. Various models of energy dissipation have been proposed for computing the wave decay in the surf zone. So far, no model has yet been developed based on air bubble effects. Although Führböter (1970) developed energy dissipation model considering air bubble but the model was not verified with experiment. In this study, the proposed \( D_{air} \) simply treated as “air bubble model” which is stated in section 2.2 (chapter 3).

\[
D_{air} = \rho_w g V_a w_r
\]  

(5.18)

where,

\[
V_a = \frac{C_0}{k_1} \left( \frac{1 - \exp(-k_1 h)}{1 - C_0 \exp(-k_1 h)} \right)
\]  

(5.19)

where \( C_0 \) is the void fraction at still water surface, \( k_1 \) is the decay factor, \( V_a \) is the volume per unit area of entrained air and \( w_r \) is the rise velocity.
Within the surf zone, the energy conservation equation Eq. (5.17) still holds; what is required is the appropriate form for the energy loss in the surf zone. Widely used formulas for computing energy dissipation rate are the bore model (Battjes and Janssen, 1978), surface roller model (Svendsen, 1984) and energy flux model (Dally et al., 1985), which are based on different principles and with different, purposes and outlined in Table 5.1.

Table 5.1: Wave transformation model using energy balance

<table>
<thead>
<tr>
<th>Investigator (1)</th>
<th>Expression of dissipation (2)</th>
<th>Prediction factor (3)</th>
<th>Remarks (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Battjes and Janssen</td>
<td>$D = \frac{\alpha}{4T} \rho_v g \frac{H^3}{h}$</td>
<td>$H, \bar{\eta}$</td>
<td>$\alpha$: Cons.</td>
</tr>
<tr>
<td>(1978)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Svendsen (1984)</td>
<td>$D = [(1 + \frac{\eta_c}{H} \frac{H}{h}){1 + \frac{H}{h} (\frac{\eta_c}{H} - 1)}]^{-1}$</td>
<td>$H, \bar{\eta}$</td>
<td>$\eta_c/H$: Cons.</td>
</tr>
<tr>
<td>Dally et al. (1985)</td>
<td>$D = \frac{K}{h} {E c_g - (E c_g)_s}$</td>
<td>$H, \bar{\eta}$</td>
<td>$K$: Cons.</td>
</tr>
<tr>
<td>Author (2002)</td>
<td>$D = \alpha \rho_v g V_w, r$</td>
<td>$H, \bar{\eta}$</td>
<td>$\alpha$: Cons.</td>
</tr>
</tbody>
</table>

5.4.2 Momentum balance equation

The shoaling, refraction, diffraction and dissipation processes induce the spatial changes in the radiation stress resulting in changes of mean sea level due to wave action. Longuet-Higgins and Stewart (1964) derived equations for wave-induced set-down and set-up by considering horizontal momentum balance of sea level gradient and the gradient of radiation stress which takes the form:

$$\frac{d \bar{\eta}}{dx} = - \frac{1}{\rho_v g d} \frac{d S_{xx}}{dx}$$

(5.20)
where \( d = h + \bar{h} \) is the mean water depth, \( \bar{h} \) is the elevation of mean water level. In Eq. (5.20), the oscillating parts of the set-up and resultant shear stress have been neglected.

Owing to the air bubble entrained, the momentum flux is affected but the bottom pressure is not (i.e. air bubble has no effect on the bottom, see section 5.3.1.1). Thus momentum conservation equation becomes slightly different form:

\[
\frac{d\bar{\eta}}{dx} = -\frac{1}{\rho_w g d} \frac{dS_{ss}'}{dx}
\]

(5.21)

where \( S_{ss}' \) is a radiation tensor component including the contribution of air bubble effects.

It is believed that the water level rise is caused by bubble-induced flow bulking. This water level may contribute to improve wave set-up in the surf zone. The rise in water level due to air bubble is not included in Eq. (5.21). So it is emphasized that the actual wave set up \( \bar{h}' \) that includes water level rise due to entrained air can be written as:

\[
\bar{h}' = \bar{h} + \Delta h
\]

(5.22)

### 5.5 Conclusions

The variations of potential energy, kinetic energy, wave heights and wave set-up in the surf zone have been described theoretically taking air bubble effects into account. It has been focused on some properties of the wave parameters in terms of air bubble effects. A simple model (proposed in Chapter 3) is used for determining the wave height, based on the solution of energy and momentum equations.
Verification of the Air Bubble Model

Summary

Experimental and theoretical results are presented and compared in this chapter, which include the void fraction distribution, wave height and wave set-up. Two parameters were used in the basic assumption of void fraction distribution and were determined by comparing with the experimental results and expressed in terms of local wave height and relative distance from the breaking point. The results of void fraction, wave height and wave set-up showed reasonable agreement between the model and measurements. The water level rise due to entrained air was found less significant. Wave run-up was measured and compared with empirical formula. Scale effects were also discussed.

6.1 Introduction

To demonstrate the performance of the model described in the previous chapter, the experiments stated in chapter 4 were compared with the calculation based on the model. The study concentrates on shoaling of unidirectional regular waves that includes spilling and plunging type of breaking. Characteristically, waves with plunging type breaker dissipate a large portion of their energy in a concentrated manner in the region just shoreward of the breaker line, while spilling breakers dissipate their energy at a slower rate. Although the potential for air bubble
entrainment in plunging breaker is much greater than the spilling breaker (Cokelet, 1977), this chapter discusses both types of wave breakers.

Owing to air bubbles, a system of equations for wave height and wave-induced set-up are modified and solved simultaneously in an iterative manner. To verify the validity of air bubble model, the experiments are conducted in a laboratory flume.

It was mentioned in chapter 3 that air entrainment is associated with a rise in water level. The water level rise included by of entrained air may be related to the wave set-up in the surf zone. However, we do not as yet have enough information regarding the effects of water level rise by air bubble entrainment on the wave parameters (e.g. wave set-up, run-up and long waves generation) in the surf zone. Recently Aoki et al., (2000) proposed that air bubble entrainment might be one of the important causes of energy transfer from short waves to long waves in the surf zone. Chanson et al., (2001) suggested that wave breaking near the coastline is also associated with significant sediment transport and resulting flow becomes a three-phase flow: gas (air), liquid (water) and solid (sediment). It is believed that a change in water level rise by air bubbles may have significant effects on wave set-up.

Up to present, the scale effect is not understood well enough to provide quantification of air entrainment near the surf zone in the field. Since the air bubble entrainment process is improperly scaled by a Froude similitude, most laboratory experiments tend to underestimate its effects, especially on the wave field (Chanson et al., 2001). This issue is further discussed in section 6.4.

This chapter highlights two empirical coefficients and one free parameter that are calibrated from the data of laboratory experiments. The model results are compared with experiments for void fraction, wave height and wave set-up. The scale effect of air entrainment in a plunging breaker is discussed. In addition, the wave run-up is measured and compared with empirical formula.

### 6.2 Numerical solutions
6.2.1 Transition zone

In the transition region (defined in Chapter 4), it was observed that for plunging breaker air is entrained into water at the plunging point not exactly at the breaking point. The entrained air bubble propagates inshore with the wave propagation and some are backward probably due to water stagnation and advection. The physical connection in the transition zone is not well understood for $k_1$ and $C_0$. In order to avoid physically unrealistic predictions from the model, the linear relationship is considered for $C_0$ with the distance.

6.2.2 Numerical scheme

Numerical computation was performed using finite difference approach. The numerical scheme was developed which is capable of describing the one-dimensional transformation of wave height over the sloping bottom considering the effect of set-up in mean water level.

With the help of boundary conditions at the breaking point, the conservation of energy equation Eq. (6.1) and the momentum equation Eq. (6.2) can be solved numerically.

\[
\frac{d(EF')}{dx} = -D \tag{6.1}
\]

and

\[
\frac{d\bar{h}}{dx} = -\frac{1}{\rho wd} \frac{d(S_w')}{dx} \tag{6.2}
\]

To calculate $\bar{h}$, the following information is required:

Step 1: Specification of the deep-water parameters and still water depth.

Step 2: Assumption of first approximation as $\bar{h} = 0$ at all the grid points, wave height and set-up/set-down established.
Step 3: Computation of the mean water depth (new water depth) then new wave height and new wave set-up/set-down.

Step 4: Repetition of step 3 until the update value for the wave set-up/set-down becomes very close to the previous value.

The rise in water level due to air bubble is not included in Eq. (6.2). So it is emphasized that the actual wave set up $\bar{\eta}'$ included water level rise due to entrained air can be written as:

$$\bar{\eta}' = \bar{\eta} + \Delta h$$  \hspace{1cm} (6.3)

### 6.3 Results and discussion

Before comparing the model and experimental results for wave height and wave set-up, some aspects of air-water flow field in the surf zone are discussed, which include empirical coefficients $C_0$ and $k_1$, free parameter $\alpha$ and void fraction distributions $C$.

#### 6.3.1 Determination of empirical coefficient $k_1$ and $C_0$

In Eq. (5.1) the unknown quantities are $k_1$ and $C_0$. The values of these parameters can be estimated by comparison with experimental data. Introducing a new dimensionless parameter $k_0$ defined as

$$k_0 = k_1 H$$  \hspace{1cm} (6.4)

where $H$ is a local wave height.

The parameter $k_1$ for void fraction distribution in the surf zone was determined by fitting a theoretical curve to the experimental data for both spilling and plunging breakers. $k_1$ has smaller value when penetration depth is larger and vice-versa. It would be preferable to choose single value for $k_0$, which gives satisfactory results for all cases, allowing the model to be used on beaches with arbitrary shape. As shown in Fig. 6.1, though $k_0$ varies a little with wave steepness and with the distance from the
breaking point, it shows nearly constant value. Thus Eq. (6.4) suggests that $k_1$ increases with decreasing $H$. It is found from the figures that $k_0 = 4$ for plunging breaker and $k_0 = 3.75$ for spilling breaker which are shown by the dashed lines.

![Figure 6.1](image_url)

**Figure 6.1:** Relationship between non-dimensional parameter $k_0$ and dimensionless distance. (a) Spilling breaker and (b) plunging breaker.

On the other hand, to find a reasonable value or expression of $C_0$ three sets of data are used as in Fig. 6.2 for both spilling and plunging breakers. The parameters $C_0$ was evaluated experimentally from the wave tests in the flume. Although there is a scatter in void fraction distribution for different wave steepness ($H_0/L_0$) especially in spilling breaker, the experimental points all lie along approximately the same line except some scattered data. In Fig. 6.2 the best-fit curves to the values of $C_0$ are shown. From the Fig. 6.2, it may be concluded that the values of $C_0$ depend on the horizontal distance from the breaking point $(x-x_b)/L_0$. 
Figure 6.2: Relationship between non-dimensional parameter $C_0$ and dimensionless distance. (a) Spilling breaker and (b) plunging breaker.

All the data for void fraction are correlated by the following expressions for spilling and plunging breakers, respectively:

**Spilling breaker**

$$C_0 = 0.80 \times \frac{(x-x_b)}{L_0} \quad \text{for} \quad 0 \leq \frac{(x-x_b)}{L_0} \leq 0.20 \quad (6.5)$$

$$C_0 = -0.39 \times \frac{(x-x_b)}{L_0} + 0.238 \quad \text{for} \quad 0.20 \leq \frac{(x-x_b)}{L_0}$$

**Plunging breaker**

$$C_0 = 1.285 \times \frac{(x-x_b)}{L_0} \quad \text{for} \quad 0 \leq \frac{(x-x_b)}{L_0} \leq 0.14 \quad (6.6)$$

$$C_0 = -0.75 \times \frac{(x-x_b)}{L_0} + 0.285 \quad \text{for} \quad 0.14 \leq \frac{(x-x_b)}{L_0}$$
6.3.2 Void fraction distributions

Figures 6.3 and 6.4 show that the reliability of Eq. (5.1) (chapter 5) in describing the vertical distribution of void fraction, in which the data used for comparison for both spilling and plunging breaker. Figures 6.3 and 6.4 show the effect of $k_0/H (= k_1)$ on void fraction distribution, in which several distributions are observed for each test ($H_0/L_0 = 0.076$ and $H_0/L_0 = 0.024$). This indicates that under a given set of wave conditions the void fraction distribution may vary from section to section as the $H$ value may vary. In Fig.6.3, the location $x-x_b = 0.5$ m is identical with the end point of roller and visual observation suggests that air bubbles penetrate maximum at the same location. Figure 6.3 also represents that the region between $x-x_b = 0.7$ m and $0.95$ m corresponds the inner surf zone (defined in Fig. 5.3, chapter 5) and in this region, the results from the model agree well with the data.

\[ \text{Figure 6.3: Comparison of Eq. (5.1) with experimental data of void fraction spilling breaker } (H_0/L_0 = 0.076). \text{ The solid line } (\cdots) \text{ is from Eq. (5.1) and hollow symbol } (\circ) \text{ show the experimental data.} \]

On the other hand, in Fig. 6.4, the positions $x-x_b = 0.6$ m and $0.8$ m are the plunge and splash up points, respectively. At the plunging point ($x-x_b = 0.6$ m) penetration depth becomes the maximum. The measured results are affected just after the transition
region (nearly, $x-x_b = 0.6$ m) due to turbulence, especially in plunging breakers and show a little disagreement near the still water surface.

**Figure 6.4:** Comparison of Eq. (5.1) with experimental data of void fraction—plunging breaker ($H_0/L_0 = 0.024$). The solid line (—) is from Eq. (5.1) and hollow symbol (○) show the experimental data.

### 6.3.3 Determination of the free parameter ($\alpha$)

Figures 6.5 and 6.6 compares the horizontal distribution of the rates of dissipation of wave energy and potential energy due to air bubbles for both spilling and plunging breakers.

It can be seen from figures that the rate of energy dissipation due to air bubble $\alpha D_{air}$ shows good approximation to the real rate of wave energy dissipation for a certain value of $\alpha$ in the inner surf zone, while in the transit zone the agreement is not satisfactory. It is not surprising, because the contribution of energy dissipation due to air was found almost negligible in this domain (chapter 4).

The estimated value of energy dissipation rate due to wave breaking, $D$, are scattered near the end point of surface roller and plunge point. The free parameter $\alpha$ is set equal to the average value in the range $3.0 \sim 4.5$ over the surf zone for both spilling and plunging breakers.
Figure 6.5: Comparison the rate of energy dissipation due to wave and air bubble for spilling breaker. (a) $H_0/L_0 = 0.062$ and (b) $H_0/L_0 = 0.076$.

Figure 6.6: Comparison the rate of energy dissipation due to wave and air bubble for plunging breaker. (a) $H_0/L_0 = 0.024$ and (b) $H_0/L_0 = 0.032$. 
6.3.4 Wave height and set-up

The distribution of wave height and wave set-up for each case was measured at a number of locations. The experiments were conducted in a wave flume with a plain beach of slope $1V:9.5H$. Comparisons will be shown for two cases corresponding to spilling and plunging breakers in the surf zone. Table 4.1 (chapter 4) presented the wave period and the wave height at the start of the slope for each of the cases. The experimental data are compared with model results, which is shown in the Figs. 6.7, 6.8, 6.9 and 6.10, all normalized to their respective breaking and offshore values.

![Figure 6.7](image)

**Figure 6.7:** Comparison between model results and experimental data of (a) wave height and (b) wave set-up for $H_0/L_0 = 0.062$ ($\alpha = 4.0$, Spilling breakers).

Figures 6.7 and 6.8 show the variation of the wave height and wave set-up obtained by the measurements (circles) and calculation by the air bubble model (full line) for spilling breakers. Figs. 6.7(a) and 6.8(a) show the comparison of wave height between model results and the data for $H_0/L_0 = 0.062$ and $H_0/L_0 = 0.076$, respectively. Although the agreement for wave heights are well in the inner surf zone $(x-x_b)/L_0 > 0.2$, a significant discrepancy can be seen in the transit region. The computed wave
height just after breaking is increasing with \( x \) rather than decreasing as seen in the measured results, which indicates that wave energy is not dissipated initially in the model (Fig.6.7 and Fig. 6.8). This is because the position of air entrainment is a little distant from the breaking point, which leads to a delay in the decay of the wave height in the air bubble model. The scatter in measured wave height may be due to wave reflection from the steeper beach slope.

![Graphs showing wave height and set-up comparison](image)

**Figure 6.8:** Comparison between model results and experimental data of (a) wave height and (b) wave set-up for \( H_0/L_0 = 0.076 \) (\( \alpha = 4.0 \), Spilling breakers).

The situation is quite different when the variation in mean water level is considered. The computed variation of wave set-up is shown in Figs. 6.7(b) and 6.8(b) in comparison with the measurements. The well-known horizontal shift between the breaking point and the point where set-up starts is clearly seen in the measurements data both for \( H_0/L_0 = 0.062 \) and \( H_0/L_0 = 0.076 \). The water level rise due to entrained air, \( \eta \), is included in the wave set-up, which is shown by solid line (sky). The agreement is seen to be fairly good in the transit region but there is some discrepancy in the inner surf zone. Figures 6.7 and 6.8 show that after breaking point the
measured data of wave height but those of wave set-up keeps constant for some distance. A similar variation can also be observed in other investigations such as Bowen et al. (1968), Stive and Wind (1982), Svendsen (1984) and Dally et al. (1985). There is contradiction that the wave height starts to decay from the breaking point but wave set-up does not.

![Figure 6.9: Comparison between model results and experimental data of (a) wave height and (b) wave set-up for $H_0/L_0 = 0.024$ ($\alpha = 3.9$, Plunging breakers).](image)

Figures 6.9 and 6.10 show the comparisons for $H_0/L_0 = 0.024$ and $H_0/L_0 = 0.032$ (Plunging breakers). Again, the model overestimates the wave height near the breaking point. It is evident that in a model, the value of $C_0$ was used almost zero near the breaking point indicating weaker initial energy dissipation. On the other hand, the prediction of wave set-up shows good consistent for $(x-x_b)/L_0 < 0.15$ but poor agreement near the shoreline (Fig. 6.9(b) and 6.10(b)). Since in plunging breaker a large portion of air is entrained and turbulence occurs, mean water level increases on steeper slope than milder, which may leads to the wave height scatter. Nevertheless, except some of discrepancies in the vicinity of the breaking point and
shoreline, air bubble model leads to fairly good prediction of the surf zone variations of the wave height and wave set-up.

![Figure 6.10: Comparison between model results and experimental data of (a) wave height and (b) wave set-up for $H_0/L_0 = 0.032$ ($\alpha = 4.3$, Plunging breakers).](image)

**6.3.5 Water level rise by entrained air bubble**

Measured wave height and wave set-up and estimated water level rise due to entrained air is plotted in Fig. 6.11 for both spilling and plunging breaker. Figure 6.11 suggests that wave set-up occurs after some distance from breaking point where air bubbles appears. Interestingly, it has seen that the contribution of air bubbles is almost zero in this domain (chapter 4). This information might be interesting because it is consistent with the Dally’s suggestion. His suggestion was that no energy is dissipated until the curl touches down and “white water” appears.

The contributions of waves and air bubbles are included in measured data of wave set-up. From the measurements, the effects of air bubble was exerted from wave set-
up and plotted in Fig. 6.11. It was seen that a water level rise by air bubble would be a significant percent of wave set-up.

**Figure 6.11**: Wave height, wave set-up and estimated water level rise due to air bubbles as a function of horizontal distance. (a) Spilling breaker ($H_0/L_0 = 0.076$) and (b) plunging breaker ($H_0/L_0 = 0.024$).

### 6.3.6 Wave run-up

In the surf zone wave, run-up $R_u$ is defined as the maximum vertical elevation above the still water level to which the water rises on the beach. For the breaking waves, as yet there is no theoretical development to treat the wave run-up. The wave run-up height itself has been investigated empirically. Hunt (1959) obtained:

$$\frac{R_u}{H_0} = \frac{\tan \theta}{\sqrt{H_0/L_0}}$$

(6.7)

where $\theta$ is the slope angle (i.e., $\tan \theta = m$).

In contrast to the run-up height, the width of the swash zone for breaking waves was measured by Battjes (1974) empirically:
\[ \frac{\Delta Y}{H_0} = 0.4 \tan^2 \theta \frac{H_0}{L_0} \] 

(6.8)

**Figure 6.12:** (a) Wave run-up height of regular waves on gentle beaches, (b) width of swash zones versus wave steepness.

Both of these relations indicate that the dimensionless wave run-up and width of surf zone depends primarily on the incident wave steepness and the beach slope (Fig. 6.12). It was mentioned that water level rises because of air bubble entrainment in the surf zone. It is believed that the increase in water level may have significant effects on wave run-up height. Since there is no theoretical study on wave run-up, it is difficult to explain the air bubbles effect separately.

By observation it was seen that wave run-up was related to the type of breaker. Plunging breakers generate the highest relative run-up than spilling breakers. The reason for this may be plunging breakers dissipate their energy over a shorter surf
zone by air bubbles and transfer a large portion of their energy to forward motion of the water, whereas spilling breakers dissipate their energy over a wider surf zone by air bubbles.

6.4 Plunging breaker-scale effects in air entrainment

Model-prototype similarity is performed usually with the Froude similitude. If the same fluids are used in both the model and the prototype, effects of viscosity and surface tension introduce distortions other than gravity, and scale effects are not negligible altogether.

In the plunging jet flows (chapter 2), it was found for the range of investigated flow condition, Table 2.1, air entrainment at vertical plunging jets was affected by scale effects for $\text{We}_1 < 1000$ where $\text{We}_1 = \rho_w V_i^2 d_i / \sigma$ is the inflow Weber number. The present section applies this result to plunging breaking waves. In this respect, the concept of Chanson and Lee (1997) was used. They showed that there are some similarities between plunging jet and plunging breakers (deeper waters) at impact point and the jet impact velocity may be scaled by $\sqrt{2gH_b}$ where the water jet thickness is approximately $0.05*H_b$.

For example, $V_j = \sqrt{2gH_b}$ (m/s), $d_1 = 0.05 \, H_b$ (m), $\rho_w = 1000$ (kg/m$^3$), $\sigma = 0.073$ (N/m), and with the choice of 1000 for critical Weber number $\text{We}_1$, this gives about 0.27 (m) for $H_b$. Larger and smaller values of $H_b$ are possible for different choices of $\text{We}_1$, but scale effects occur for $\text{We}_1 < 1000$. This yields scale effects may be significant in the laboratory for $H_b < 0.27$ (m).

Figure 6.13 illustrates a comparison between small prototype plunging breakers and a laboratory study.
**Figure 6.13**: Comparison of air entrainment in prototype and in laboratory. (a) Plunging breaker at Terasawa beach and (b) laboratory model of a plunging breaker on a sloping beach.

Chanson and Lee (1997) showed that air entrainment by laboratory studies is underestimated when the wave height at breaking is less than 0.25 to 0.35. Similar conclusion was made by Hall (1990). He noted that entrained air bubbles would not be similar in small-scale physical models because of lack of similarity of the Weber number between field and laboratory. From the above discussion, it is concluded that the entrained air bubbles would not be scaled properly in small-scale physical models.

### 6.5 Conclusions

The two parameters $C_0$ and $k_1$ are experimentally evaluated and related to the horizontal distance and wave height respectively. The model was found sensitive to the free parameter $\alpha$ in the inner surf zone, whereas it was insensitive in the transit zone. The rate of energy dissipation due to entrained air $D_{\text{air}}$ was approximated to the rate of energy dissipation due to wave breaking $D$ in inner surf zone when the value of $\alpha$ was taken in the range $3.5 \sim 4.5$ for both spilling and plunging breakers. From this information, it is concluded that the rate of energy dissipation that can be explained by entrained air ($D_{\text{air}}$) is 20-25% of the rate of total energy dissipation due to wave breaking ($D$).

The experimental data of void fractions in vertical direction were compared with the theory and the results showed good agreement.

Comparison of the results by numerical computations with the experiments yields the following. Firstly, good agreement was seen for wave set-up, but not for wave height, especially in the transit zone. The reason for this might be the existence of fewer air bubbles, which led to less energy dissipation. Secondly, the exclusion of the air bubbles due to wave breaking led to the well-known shift between the break point and the point where set-up in the mean water level was initiated. Hence because of
absence of air bubble effects the radiation stress may keep constant for a while, even though the wave height started to decay (experimentally). Thirdly, the contribution of water level rise due to entrained air on wave set-up was found not so significant. In laboratory wave flumes, air entrainment at plunging breaking waves is affected by the scale effects when the wave height at breaking is less than about 0.30 m. Moreover, wave run up also measured and discussed.
General Conclusions

In this thesis keeping the eyes on the energy dissipation, a simple model (air bubble model) was proposed and verified by the experiments. The unsolved problems in plunging jet flows and in surf-zone dynamics was also discussed in detail.

In shallow water, the sloping bottom causes wave breaking. As urged in the introductory chapter a great amount of air bubble entrained in the breaking process is important for the energy dissipation. Although it is clear that wave breaking is a dominant mechanism by which air is entrained, it is not so easy to quantify the air entrainment itself. With the interest in the unsteady bubble entrainment, this study was first carried out to explore the steady bubble injection phenomena.

Following the first law of thermodynamics, an energy dissipation model (air bubble model) was derived from air-water flow characteristics where the rise velocity was included. All the parameters in the model well defined and have physical meaning. The model was applied to steady and unsteady situations (chapter 3 and 4) and its performances were verified in Chapter 6.

Attention was first paid to explore the basic physics of air bubbles and build intuition of void faction fields precisely for vertical circular plunging jet (steady case) in freshwater and seawater. Three scale models were used with jet nozzle diameters of 6.8, 12.5 and 25 mm and discussions focused on scale effects affecting air entrainment process. A study of air entrainment inception conditions showed that the onset velocity $V_e$ is identical for freshwater and seawater. In seawater, significantly less air is entrained than in freshwater, leaving all inflow parameters equal. Plunging jet study with 3 geometric sizes highlighted scale effects in small size laboratory
experiments, suggesting the effects of entrained air bubbles may be more significant in the field than observed in laboratory wave flume.

A void meter with L-shape probe was used specifically to measure void fraction and bubble frequency distributions under the impingement point. The void fraction profile follows closely analytical solution of diffusion equation. In addition, it appeared that the penetration depth was found to be a function of falling water jets. Distributions of pseudo-bubble chord sizes ranged from less than 0.5 mm to more than 10 mm for freshwater and seawater, and the averaged pseudo-chord sizes were between 4 and 6 mm for all water solutions. The results highlighted significant scale effects when $We_1 < 1000$ in terms of void fraction and bubble count rate. For $We_1 < 1000$, the air entrainment rate was underestimated.

The investigation allowed a quantitative comparison among energy dissipation between three typical steady air entrainment phenomena. Profiles of void fraction were used to estimate the volume of air at different sections. The volume of entrained air was related to plunging jet impact velocity. The ratios of energy dissipation due to entrained air to the total energy loss were around 25%, 1.4% and (2-4)% for hydraulic jump, 2-D vertical plunging jet and vertical circular jet, respectively. Although the upstream velocities were almost same for all the cases, energy dissipation rate due to air bubble entrainment showed significant difference among the three phenomena. The contribution of energy dissipation by entrained air bubble was consistent between hydraulic jump and spilling breaker but there was inconsistency for plunging jet and plunging breaker, possibly because of shallow water depth at breaking waves. That means, the volume of fluid was affected by air entrainment and energy dissipation, to the total volume of water. Since the hydraulic jump laboratory experiments were performed in shallow waters, which is comparatively closer to spilling breakers in shallow waters. On the other hand, plunging jet experiments were performed in a deepwater pool, which almost 5 times more depth of water than plunging breakers.

It has been investigated precisely various properties of void fraction fields for breaking waves (unsteady cases). The maximum void fraction was found 19% near
the still water surface in plunging breaker whereas; it was around 16% in spilling breaker. In 2-D wave breaking, it was observed that the aerated area grew rapidly from the end of the transition zone to the plunging point or roller end point, reached a maximum, and subsequently decreased. The distribution of void fraction is not only the function of the vertical coordinate \( z \), but the duration of breaking event \( \Delta t \) and shows exponentially decay with \( z \) and almost linear with \( \Delta t/T \). The ratio of potential energy of entrained air to total wave energy \( \Delta PE/E_b \) was measured from the void fraction distributions and it was nearly 11-7% which corresponds to plunge point for plunging breakers. On the other hand, it was around 7-4% in the case of spilling breakers. The ratio of energy dissipation by air bubbles was found (18-20)% and (19-23)% for plunging and spilling breaker, respectively. Three data sets have been used to investigate the characteristics of unsteady phenomena for each case.

The physical mechanisms behind the variation of potential energy, kinetic energy, wave heights and wave set-up in the surf zone have been analyzed theoretically considering air bubble effects. It highlighted some properties of the wave parameters due to air bubble effects. The air bubble model (proposed in Chapter 3) was used for determining the wave height and wave set-up based on the numerical solution of energy and momentum equation.

Two parameters \( C_0 \) and \( k_1 \) experimentally evaluated were related to the horizontal relative distance and wave height respectively. The free parameter \( \alpha \) was introduced and determined based on the ratio of energy dissipation due to air bubble to total wave energy loss. For certain values of \( \alpha \), the rate of energy dissipation due to entrained air was good approximated to the rate of energy dissipation due to wave in inner surf zone for both spilling and plunging breakers. The data of void fraction in vertical direction were compared with the theory and results showed good agreement.

The comparison of computational results with measurements showed reasonable agreement for wave set-up whereas wave height was less, especially in the transition zone. The reason for this might be the existence of fewer air bubbles, which leaded to the less energy dissipation. The exclusion of the air bubbles in the transition zone
led to the well-known shift between the break point and the point where set-up in the mean water level was initiated. The absence of air bubble effects the radiation stress may keep constant for a while, even though the wave height started to decay (experimentally). In addition, the contribution of water level rise due to entrained air on wave set-up was found not so significant. Moreover, wave run up also measured and compared with empirical formula and discussed.

The scale effects also affected the air entrainment process in laboratory wave flume and the entrained air bubbles would not be similar to the large-scale physical models. Air entrainment at plunging breaking waves is affected by scale effects when the wave height at breaking is less than 0.30 m.
References


