Torque transfer in optical tweezers due to orbital angular momentum

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ABSTRACT

We describe two methods to optically measure the torque applied by the orbital angular momentum of the trapping beam in an optical tweezers setup. The first decomposes the beam into orbital angular momentum carrying modes and measures the power in each mode to determine the change in angular momentum of the beam. The second method is based on measuring the torque transfer due to spin angular momentum and the linear relationship between rotation rate and applied torque to determine the orbital angular momentum transfer. The second method is applied to measuring the transfer efficiency for different particle–mode combinations. We present the results of these experiments and discuss some of the difficulties encountered.

Keywords: Optical tweezers, Orbital angular momentum

1. INTRODUCTION

The idea that light carries momentum is one that is greatly appreciated by those in the optical micromanipulation field, as momentum transfer is the mechanism behind the three dimensional trapping of microscopic dielectric particles. Light that carries angular momentum has also been widely utilised for micromanipulation. Optical angular momentum comes in two flavours: spin angular momentum associated with the polarisation of the light’s electric field, and orbital angular momentum which arises from the phase distribution of the light’s wavefront. Both have been used to apply torque to a particle trapped in optical tweezers, yet measurement of this torque has proved more difficult. Ideally, this torque should be measured optically as this allows for the effects of the environment surrounding the trapped particle to be studied. Such a measurement technique exists for the spin component of the optically applied torque and was first demonstrated for optically trapped glass rods, and then used to make microviscosity measurements inside a micelle using a spherical birefringent probe particle. A technique to measure the torque applied in optical tweezers by orbital angular momentum has proved much more elusive. In this paper we present two methods that attempt to capture such a technique. The first takes a direct approach by attempting to measure the orbital angular momentum carrying modes of the laser beam after it has passed through the optical trap. The second technique relies on the linear relationship between the spin angular momentum transfer and the rotation rate in order to measure the orbital component.

Measurement of orbital angular momentum, or at least detection of orbital angular momentum carrying modes, has been of interest in the field of quantum information and computation. The interest lies in the possibility of multidimensional entanglement due to infinite number of spatial modes with different associated angular momentum. Computer generated holograms allow these modes to be separated and optical fibres can be used to measure the hologram’s output. The orbital angular momentum states of single photons produced by parametric down-conversion have been measured in this way. The experiments showed that orbital angular momentum is conserved during down conversion and that the orbital angular momentum states are entangled. Orbital angular momentum states have also been detected using interferometric techniques. Dove prisms in each arm of the interferometer are used to introduce a phase shift in the beam so that the photons interfere when recombined such that photons with odd values of \( l \) (the azimuthal index of the mode which is associated with the mode’s angular momentum) are sorted into one output port, while even modes are sorted to the other output port. Further sorting can be achieved by cascading additional interferometers. An effect called the rotational

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Doppler shift between two beams with different azimuthal indices can also be used to quantify orbital angular momentum. The two beams are rotated with respect to one another and a beat frequency manifests itself as the rotation of an off-axis dark vortex. The beat frequency is proportional to the azimuthal charge of the vortex (the value \( l \)) and thus the orbital angular momentum of the beam. This technique could in principle be used to measure the orbital angular momentum of an arbitrary beam, however it would require different frequencies and their amplitudes to be distinguished accurately. The rotating optical element required to introduce the time varying phase shift between the two beams would make such a measurement difficult due to noise in the beat signal.

Using a similar technique to the single photon experiments, measurement of the orbital angular momentum of an arbitrary beam has been demonstrated. In these experiments the torque applied to a elongated phase object by a paraxial laser beam was determined by measuring the power in the forward scattered azimuthal modes. In this case the two-fold symmetry of the phase object was taken advantage of to decrease the number of possible modes that the power could be scattered in to. In the present paper the same trick is used by choosing a two fold symmetric particle to trap and rotate. Further complications are introduced by the optical tweezers setup, due to the highly converging and diverging nature of the laser beam used for optical trapping. These issues are addressed in this paper.

2. A DIRECT ORBITAL ANGULAR MOMENTUM MEASUREMENT TECHNIQUE

To measure the orbital angular momentum content of an arbitrary light beam, the beam must be split into a number of different components, each with a known angular momentum. The power of each of the components will therefore determine the beam’s orbital angular momentum composition. The components of choice are the Laguerre–Gauss (LG) modes which have their orbital angular momentum determined by their azimuthal index \( l \). In order to make this modal decomposition, a hologram is used to split the beam into modes with different values of \( l \). The complication with such a method is that there are a large number of LG modes that can potentially compose the beam. This is unlike the spin angular momentum case, where there only exists two modes: left or right circular polarisations (or horizontal and vertical polarisations). Dealing with this complication requires the use of multiple holograms. We use two fold symmetric particles to eliminate the possibility of odd LG modes in order to simplify this problem somewhat.

![Figure 1. Operation of a LG_{02} off-axis hologram. In the first case (a) a Gaussian beam is incident, while in the second (b) a LG_{02} beam is incident on the hologram. The zeroth and first orders of the output are depicted.](image-url)

Here we will describe the function of the LG holograms as they form the basis of the orbital angular momentum measurement. The holograms used are off-axis phase holograms as depicted in figure 1. For the case of a Gaussian
Figure 2. Operation of a LG\(_{02}\) on-axis hologram (a), with a LG\(_{02}\) incident on it. The output here is supposed to represent a Gaussian combined with other higher order modes. This is not what the output would actually look like because of interference. It would look more like an LG\(_{02}\) mode with the dark spot (due to the phase singularity) off-centred. The second case (b) depicts the functions of two particles that are trapped by the beam and rotating. The third case (c) shows an LG\(_{04}\) hologram in the beam path, that would act as an analysing hologram.

Let us now consider the function of an on-axis hologram, figure 2(a). Instead of the Gaussian being diffracted at an angle, it overlays the other modes generated by the hologram. However, the Gaussian is the only mode that has intensity in the centre of the laser beam. Therefore the intensity of the centre of the beam tells us the strength of the Gaussian, which is a measure of the efficiency of the hologram. Two beads that are trapped and rotating can also be thought of as an on-axis hologram, figure 2(b). Measuring the strength of the Gaussian tells us the efficiency of the two beads in generating a certain mode, which in turn tells us the orbital angular momentum transferred to the particles due to this mode conversion. In order to test if other modes have been generated by the two particles, an analysing hologram can be used, figure 2(c). In this case a LG\(_{04}\) off-axis hologram is depicted and would allow for any LG\(_{04}\) component generated by the particles to be detected and quantified. This scheme is used to attempt to measure the orbital angular momentum transferred to two particles trapped in optical tweezers.

2.1. Experimental Setup

The setup used for this experiment is shown in figure 3. In particular, output 2 was used for this experiment, the other outputs are used for experiments described later in this paper. The Gaussian output from a NdYAG laser was incident on a phase hologram that generates a LG\(_{02}\) in the first order. This beam was then sent to a standard optical tweezers setup: a beam expanded followed by a dichroic to couple the laser light into a high
Figure 3. Setup used for the experiments in this paper. The base optical tweezers system is depicted with different outputs that were used for the two different experimental sections in this paper. Output 1 was used measurements of the laser’s polarisation while output 2 was used for measurements of the modal composition of the trapping beam.

numerical aperture (NA = 1.3) objective. A condenser (NA = 1.4) collected the laser light and a dichroic sent the laser light to the output port. In this experiment the laser beam is then incident on a phase hologram and a camera images the output of the hologram which is displayed on a rotating screen. The object that is trapped in the optical tweezers is two polystyrene beads (1 µm in radius), that were held together by the trapping laser beam and were pressed against the microscope slide in order to prevent them from aligning vertically in the trap.

2.2. Results and Discussion

The orbital angular momentum transferred from the trapping laser beam to the trapped particles was determined by measuring the strength of the LG mode components of the beam after it is transmitted through the microscope. Figure 4 shows the output when no particles are trapped (a), and when two particles are trapped and rotating (b). From (a) we see that the trapping beam was an LG\(_{02}\) mode, as this was the only channel in which a Gaussian component was observed. The output when the particles are trapped looks quite complicated. This is not necessarily an issue as we are only interested in the central intensity of the mode as this represents the Gaussian component of the beam. The asymmetry of the individual modes is indicative of aberration or misalignment in the optical system as they are time-averaged pictures.

In order to do a more quantitative analysis of the output modes, the central intensities (corresponding to the centre of the modes) are read from the images. The difference in intensity with and without a particle trapped is shown in figure 5. In this plot we see that the power does scatter to the expected Gaussian and LG\(_{04}\) modes. However, the power in the LG\(_{03}\) mode is unexpected. In fact due to the power in this mode, this measurement suggests that the beam is gaining angular momentum from the particles, which is of course impossible. The most likely cause of such an error is that the system was slightly misaligned between the measurements with
Figure 4. Output modes from the microscope. The central intensity of each mode signifies whether a certain LG mode was present in the laser beam coming from the microscope. It can be seen that when no particle was present in the trap (a) then only an LG\(_{02}\) component existed in the beam. However, power is scattered into other modes when a particle is present (b).

Figure 5. Central intensities of the output modes from the microscope. They are given by the central intensity of the modes with the particles trapped minus the central intensity when the trap is empty.
and without the particle. This really illustrates the weakness of using this method to measure orbital angular momentum, and that is the very careful alignment that is required. One possible solution to this problem would be to use a spatial light modulator (SLM) to generate the analysing hologram. This would allow for the different orders to be measured without having to physically move the hologram, instead the SLM would just have to change its display.

3. A LESS DIRECT MEASUREMENT TECHNIQUE

Due to the difficulties of measuring orbital angular momentum via the modal decomposition method previously described, we present a second technique for quantifying angular momentum transfer in optical tweezers. This technique is based on an existing technique to measure spin angular momentum transfer,\(^2\) and takes advantage of the linear relationship between the rotation rate of a trapped particle and the torque applied to it, in order to measure the orbital angular momentum transfer. The spin component is determined by measuring the change in polarisation of the laser beam as it is transmitted through the trapped particles. This is done by measuring the power in the two possible ‘modes’: left circularly polarised and right circularly polarised. The torque on the particle is given by:

\[
\tau_{\text{spin}} = \frac{\Delta\sigma P}{\omega}
\]

where \(\Delta\sigma\) is the change in the degree of circular polarisation, \(P\) is the laser power and \(\omega\) is the optical angular frequency. In the optical tweezers setup the optically applied torque is equal to the viscous drag torque and, therefore, proportional to the rotation rate, due to the steady rotation of the particle and the low-Reynolds-number Newtonian fluid that surrounds the particle. The optical torque is equal to the sum of the torque due to spin angular momentum and the torque from the orbital component:

\[
\tau_{\text{total}} = \tau_{\text{orbital}} + \tau_{\text{spin}} = \Omega K
\]

where \(K\) is a constant of proportionality and \(\Omega\) is the rotation rate. The torque due to the spin angular momentum can be measured and so can the rotation rate. Therefore varying the torque due to the spin component allows us to measure the torque due to orbital angular momentum.

3.1. Experimental Setup

The optical tweezers setup, figure 3, has been described earlier in this paper. In this experiment output 1, depicted in the setup diagram, was used. The output laser light from the microscope was first deflected by a glass slide to photo-detector 3, which measured the rotation rate of the trapped particle. Only a very small amount of light was deflected so that it does not affect the change in polarisation measurement. The photodetector measured the intensity of a section of the beam so that the signal varied with the rotation of the trapped particle. The beam transmitted through the glass slide was then directed to a circularly polarised component detection system, which consisted of a quarter-wave plate followed by a polarising beam splitter and two photodetectors. The quarter-wave plate meant that one detector measured the left circularly polarised component, and the second measured the right circularly polarised component of the laser beam. The trapped particles are the same polystyrene beads used in the earlier experiments in this paper.

3.2. Results and Discussion

The change in polarisation and rotation rate were measured for three different polarisations. The results for this are shown in figure 6. The slope and intercept of this plot yield the orbital angular momentum transferred to the trapped particle. Please refer to Parkin et al.\(^10\) for a more detailed description of this result. It should be mentioned here that the orbital component transferred was five times greater than the spin component, which demonstrates the advantage of using orbital angular momentum for rotating asymmetric objects. The same measurement was done for a series of different LG mode trapping beams for different numbers of beads. The results for these experiments are shown in figure 7. This series of experiments is a measure of the transfer efficiency for different particle–mode combinations. The idea is that the trapped particles act as microholograms with an azimuthal index equal to the number of beads. It was expected that the particles would most efficiently couple to the Gaussian mode (with an azimuthal index of zero). Although it is difficult ascertain such behaviour...
from the entire set of results of these experiments, close inspection does indeed reveal some consistent behaviour. If one considers just the torque transfer for the LG\_04 trapping beam, then we see that the most efficient transfer is to 4 beads, followed by 3 beads and then 2 beads. This is expected if coupling to the Gaussian mode is most efficient. Another consistency lies in the transfer efficiency for the 3 beads trapped in the three different modes. In this case the LG\_03 provides the most efficient transfer. The other results become quite reasonable if one considers that, while coupling to the Gaussian mode should be most efficient, its also possible that coupling to modes with low azimuthal indices (LG\_01 and LG\_0\_1 for example) will be relatively efficient. In order to really test this possibility, more particle–mode combinations would need to be investigated.
4. CONCLUSION

We have presented two methods to optically measure the orbital angular momentum transferred to a particle in optical tweezers. The modal decomposition technique produced some interesting results that showed the potential of this technique to directly measure into which modes the power is coupled. However, it also illustrated the difficulties associated with such techniques, as the alignment of all of the elements in the setup is critical. Due to these difficulties a second method was devised based on an established technique to measure spin angular momentum. The technique produced a result for the orbital angular momentum and was applied to studying the transfer efficiency using different LG modes to rotate different numbers of beads. The beads were expected to act as microholograms, therefore certain combinations of beads number and azimuthal indices were expected to achieve efficient transfer. Unfortunately this was not as clear as we had hoped in the results, but we have suggested that extending the data set would help to resolve this issue. One conclusion that we can clearly draw from these results is that orbital angular momentum is more efficient in applying torque to asymmetric objects than its spin counterpart. We believe that orbital angular momentum measurement could be applied to the study of torques in microscopic biological systems and micromachines, as the asymmetry of the objects involved lends itself to rotation by orbital angular momentum.

REFERENCES