A SIMPLE MATHEMATICAL MODEL FOR THE EFFECTS OF THE GROWTH OF TOURISM ON ENVIRONMENT 1

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ABSTRACT
While many authors have discussed the evolution of tourism flow, few are concerned with how tourism flow starts to evolve and the mechanism behind that evolution. The purpose of this work is to propose a mathematical model that may assist understand the mechanism. Assuming basic relationships between the resources, number of visitors and their respective rates of growth, we derive a model defined by two Ordinary Differential Equations whose solutions describe the evolution of both variables in time. As a study case for the model, we use data collected at the Juatinga Ecological Reserve, 23.2° S, 44.5° W , in the State of Rio de Janeiro, Brazil. We conclude that the model fits the data well and the values that can be obtained for the parameters characterize qualitatively the type of visitors.

Keywords: mathematical model, tourism, environment, numerical, ordinary differential equations

INTRODUCTION
In the early 16th century the mountains that run almost parallel to the coastal region of Brazil between 23° S and 24° S and 44° W and 46° W were almost completely covered by rainforest. A large portion of this land, mainly in what nowdays is the southern part of the State of Rio de Janeiro, was granted as property to Martin de Sá*. His descendents used it for agriculture, and employed slaves from Africa for the work force. With the end of slavery in 1882, the plantations were no longer as lucrative as before and agricultural activity declined or ceased in many places.

Through the years, the mix of native Indians, African slaves and Portuguese colonizers yielded a new cultural group, the Caícaras, that occupy the whole of the coastal region. Travel within the hills remains a difficult task. Some areas on the coast still can only be reached by boat or several ours of trekking. Consequently, some Caícaras remained in relative isolation from modern world (and western societies) and continued to practice their cultural traditions. Until recently, the Caíara people were organized in clans and based their economy on fishing, hunting and domestic agriculture. Inheriting from the Indian culture, they are natural conservationists, because their own subsistence depends on the good quality of the environment.

The Caíara, however, have not been immune to globalization pressures. In recent decades, their lives have been disturbed, and influenced, by commercial fishing, pollution of coastal waters, the property market and tourism. The first two pressures reduced the Caíarás’ maritime harvests, while the latter two, precipitated emigration and, at least, superficial cultural changes. Those Caíarás that sold their lands either became employees of the new owners at their holiday houses, or migrated, often to the relatively nearby large cities such as São Paulo. This group, in particular, suffered the greatest social transformation, reflected in high incidences of alcoholism and prostitution (Barros, 2002).

Consequences of the growth of tourism for the Caíaras include changes to their traditional lifestyle and their related environment. Understanding the mechanisms of interaction between culture of a traditional society, environment and tourism may help minimize the negative aspects of the impact of tourism and even help plan to use them into the societal benefit. With this purpose, we propose in this paper a simple mathematical model of the relationship between the number of visitors and the resources at a potential tourism place. As a study case for the model, we focus on the impacts of tourism on the Caíara people that remained in one small village, surrounded by largely untouched Atlantic Forest, rivers and beaches, which are attractions for surfers, divers and recreational fishers. The parameters used in the model came from data collected by one of us between the years 1999 and 2002 at the Juatinga Ecological Reserve (Sinay, 2002).

THE MODEL
We denote time by the letter t; the number of visitors by V; combined resources such as the environment, local culture and other elements that could be of interest to tourists with R and changes in these variables with Δ(*), where * in parenthesis will be substituted by the corresponding variable.
The average rate of growth (or decay if negative) of the number of visitors and resources in a time interval \( \Delta t \), are then, respectively
\[
\frac{\Delta V}{\Delta t}, \quad \frac{\Delta R}{\Delta t}.
\]
Assuming that, as the area becomes known to visitors/tourists, word-of-mouth recommendation is the only form of promotion and each tourist has the potential to influence a constant number (c) of friends to visit this destination, we have
\[
\frac{\Delta V}{\Delta t} = cV \tag{1}
\]
However, as tourism flow increases, in small and at least semi-traditional communities, cultural and environmental integrity tend to diminish (WCST, 1995, WTO, 2004, Davis, 1999, van der Duim and Caalders, 2002, Carter, 1999). Therefore, when competition for scarce resources or crowding occurs, tourists’ satisfaction diminishes. Negative word-of-mouth advertising takes then place, making tourism flow decrease to a level where satisfaction is again positive (Sinay, 2002). This self-adjusting social response mechanism can be included in the model by multiplying \( V \) on the right-hand-side of equation (1) by \( kR - V \), or
\[
\frac{\Delta V}{\Delta t} = cV(kR - V). \tag{2}
\]
We choose as a mathematical model of nature’s rate of growth(Begon,1996):
\[
\frac{\Delta R}{\Delta t} = aR(R^* - R), \tag{3}
\]
which is a good approximation for short periods of time. While tourists to the Caïcaras community at Juatinga Ecological Reserve are currently specifically interested in maritime environmental features, for the purpose of the model we extend (3) to include all resources that might be of interest to tourists in a natural or near natural setting. Here, \( R^* \) is a constant, the carrying capacity, ecological or social as determined by the visitors to the place and management. Since the presence of tourists affects environmental setting elements such as the local culture and the environment, we introduce in (3) a term proportional to the number of visitors; thus,
\[
\frac{\Delta R}{\Delta t} = aR(R^* - R) - bV, \tag{4}
\]
If, in equations (2) and (4) the instantaneous rates rather than the average rates are considered, we have the system of Ordinary Differential Equations
\[
\frac{dR}{dt} = aR(R^* - R) - bV. \tag{5}
\]
\[
\frac{dV}{dt} = cV(kR - V). \tag{6}
\]
Finally, measuring the resources using \( R^* \) as a unit, the number of visitors in units of the product \( R \times k \), and time in units of \( aR^* \), the following substitutions can be made in the equations
\[
R = R^*r,
\]
\[
V = kR^*v,
\]
\[
t = aR^*t.
\]
This results i:
\[
\frac{dr}{d\tau} = r(1 - r) - \alpha v, \tag{7}
\]
\[
\frac{dv}{d\tau} = \beta v(r - v). \tag{8}
\]
where,
\[
\alpha = \frac{bk}{aR^*}, \quad \beta = \frac{kc}{a},
\]
In addition to equations (7) and (8) being simpler than equations (5) and (6), they show that the behavior of the system resources-visitors, described by the solutions of those equations, does not depend on the five original parameters but only on the two combinations \( \alpha \) and \( \beta \)

**STATIC SOLUTIONS AND THEIR STABILITY**

Straightforward calculations show that the time-independent solutions of equations (7-8) are
\[
r = r_0 = 0, \quad v = v_0 = 0; \]
\[
r = r_1 = 1 \quad v = v_1 = 0; \]
\[
r = r_2 = 1 - \alpha \quad v = v_2 = 1 - \alpha, \quad \text{provided } \alpha \leq 1.
\]
We interpret these solutions as scenarios in the following way:
Scenario 1: \( r = r_0, v = v_0 \).
There are no resources and, therefore, there is no interest for tourism.

Scenario 2: \( r = r_1, v = v_1 \).
The place is yet to be discovered by tourists.

Scenario 3: \( r = r_2, v = v_2 \).
A sustained situation exists, at the cost of not having fully developed resources, where there is a constant number of visitors.

We say that a solution \((\hat{r}, \hat{v})\) of an Ordinary Differential Equation is asymptotically stable to small perturbations if every other time-dependent solution \((r(\tau), v(\tau))\) of it, which, at time \(\tau=0\) is close to \((\hat{r}, \hat{v})\), will tend to it as \(\tau \to \infty\). If \((r(\tau), v(\tau))\) moves far away from \((\hat{r}, \hat{v})\) as \(\tau \to \infty\) we say that \((\hat{r}, \hat{v})\) is unstable. When the first (or linear) approximation to the perturbed solution is periodic, we call \((\hat{r}, \hat{v})\) a center. In this case, \((r(\tau), v(\tau))\) may, or may not, be periodic and investigation of higher order approximations is necessary.

The mathematical theory of linear stability is well known (Stoker, 1950) and permits us to conclude that the solution \((r_0, v_0)\) is unstable. Depending on the location of its initial values \((r(0), v(0))\), a solution \((r(\tau), v(\tau))\) may tend to \((r_1, v_1)\), or go away from it. Solutions with this type of instability are called saddle points. The stability of \((r_2, v_2)\) requires a more detailed analysis.

Let
\[
\alpha^\pm = \frac{\left(1 \pm \sqrt{\beta}\right)^2}{1 + \left(1 + \sqrt{\beta}\right)^2}, \quad \alpha_c = \frac{1 + \beta}{2 + \beta},
\]  

(9)

Then, if \(\alpha \leq \alpha_c\), \((r_2, v_2)\) is stable and any solution \((r(\tau), v(\tau))\) starting near it goes exponentially to the equilibrium point. If \(\alpha < \alpha < \alpha_c\), time-dependent solutions spiral down toward \((r_2, v_2)\) in the plane \(r - v\). At \(\alpha = \alpha_c\) the static solution is a center. On the interval \(\alpha_c < \alpha < \alpha^*\) solutions spiral out of \((r_2, v_2)\), and if \(\alpha^* < \alpha\leq \alpha_c\), time-dependent solutions go away exponentially from the equilibrium point.

Using Hopf’s bifurcation theorem (Sinay, 1996) it is possible to prove that \(\alpha = \alpha_c\) is a bifurcation point where the solution \((r_2, v_2)\) splits into a periodic one which exists for \(\alpha < \alpha < \alpha_c\).

The four different behaviors of the solutions are exemplified in Figures 1 to 4, where we chose \(\beta = 0.3316\) fixed (see Table 4) and an \(\alpha\) in each one of the described intervals.

\[ \begin{align*}
\alpha &= 0.15, \beta = 0.3316 \\
\alpha &= 0.5, \beta = 0.3316 \\
\alpha &= 0.15, \beta = 0.3316 \\
\alpha &= 0.5, \beta = 0.3316 \\
\end{align*} \]

**Figure 1.** Directions Field
The static solution \(r = v = 0.85\) is stable. The time-dependent solutions go exponentially toward it.

**Figure 2.** Directions Field
The static solution \(r = v = 0.5\) is stable. The time-dependent solutions spiral toward it in the \(r-v\) plane.
The static solution \( r = v = 0.35 \) is unstable. The time-dependent solutions spiral out of it in the \( r - v \) plane.

The static solution \( r = v = 0.2 \) is unstable. The time-dependent solutions get away from it exponentially.

ADJUSTMENT OF THE MODEL TO DATA

From 1999 to 2002 the first author visited several times the Juatinga Ecological Reserve, 23.2° S, 44.5° W, collecting the data shown in Table 1.

<table>
<thead>
<tr>
<th>date</th>
<th>02/23/99</th>
<th>04/23/00</th>
<th>10/12/00</th>
<th>12/31/00</th>
<th>03/04/01</th>
<th>11/02/01</th>
<th>11/15/01</th>
<th>02/19/02</th>
</tr>
</thead>
<tbody>
<tr>
<td>tents</td>
<td>50</td>
<td>90</td>
<td>110</td>
<td>200</td>
<td>240</td>
<td>25</td>
<td>75</td>
<td>175</td>
</tr>
<tr>
<td>days</td>
<td>0</td>
<td>425</td>
<td>172</td>
<td>80</td>
<td>63</td>
<td>243</td>
<td>13</td>
<td>96</td>
</tr>
<tr>
<td>accumulated</td>
<td>0</td>
<td>425</td>
<td>597</td>
<td>677</td>
<td>740</td>
<td>983</td>
<td>996</td>
<td>1092</td>
</tr>
</tbody>
</table>

Table 1. Field Data

In order to have a completely calibrated model, it would have been necessary to make a long and expensive follow up of the Caicaras’s cultural values and quality of the environment. We circumambulated this problem considering that the model presented here is not a forecasting tool but rather an instrument that could help to understand the interaction between resources and tourism and, therefore, that reality could be approximated assuming that all the variables included in the category resources can be represented by the measure of the area covered by forest, in the sense that the largest the devastation of it, the greatest the loss of cultural values of an originally conservationist people. Thus, we fixed as \( R^* \) an initial area of 100,000 m², of which 99,500 m² were covered by forest by February 23, 1999. Given the characteristics of the region, tropical, with moderate to high temperatures, large indexes of humidity and fertile soil, we also assumed that, when nature is left alone, fact represented by \( v \equiv 0 \) in equations (7-8), the forest can grow from 500 m² of coverage to 99,500 m² in five years. In this particular case, \( v \equiv 0 \) equation (8) is identically satisfied and equation (7) can be solved explicitly, allowing to determine the value of the parameter \( a \). It can be observed in Figure 5 (Sinay, 2002) that large variations in the number of visitors took place during the year 2000 and later. Since not only the number of tourists changed drastically, but also their behavior, we divided the data in the four periods shown in Table 2.

<table>
<thead>
<tr>
<th>Date</th>
<th>Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>02/23/1999</td>
<td>597</td>
</tr>
<tr>
<td>03/04/2001</td>
<td>740</td>
</tr>
<tr>
<td>11/02/2001</td>
<td>983</td>
</tr>
<tr>
<td>02/19/2002</td>
<td>1092</td>
</tr>
</tbody>
</table>
We then solved equations (7) and (8) numerically on each period (Press, 1989), varying the values of the parameters in such a way as to minimize the differences between the data and the calculated number of tents. Since the solution of this problem is not unique, we imposed the restriction that, in the average, each tent, including the necessary circulation corridors among all of them, would need an approximately 10m² area. We thus obtained the values shown in Table 3, where we include the previously determined value of the parameter $a$ and the respective time-independent solutions in variables with dimensions. Table 4 contains the parameters $\alpha$ and $\beta$ for the dimensionless variables as well as, for each calculated value of $\beta$, the corresponding $\alpha'$, $\alpha''$, $\alpha_c$.

**DISCUSSION OF RESULTS AND CONCLUSIONS**

In order to discuss the results it is first necessary to give meanings to the different parameters used in the model. We have already seen that the parameter $a$ in equation (5) is related to the rate of growth of the resources, the greater $a$ is, the faster $R$ increases or decreases depending on whether it is smaller or greater than $R^*$ respectively. The parameter $b$ measures the capacity of destruction or modification of $R$ that the visitors have. The greater $b$ is the slower the growth of $R$ is and vice-versa. The parameter $c$ has the same meaning the parameter $a$ has, but for the number of visitors. Finally, the parameter $k$ can be understood as a threshold of dissatisfaction. When, in equation (6), the ratio $V/R$ is greater than $k$, the right-hand-side of the equation is negative, and therefore, so is the rate of growth of $V$, i.e., $V$ decreases. Since $V/R$ is the average of visitors per square meter of forest area, $k$ is the maximum acceptable average for a given type of tourist. When there are too many of them for an available area, the rate of growth of $V$ becomes negative. Then, the passage of $V/R$ from less than to larger than $k$ can be understood as the mechanism that triggers the negative word-of-mouth advertisement.

| Table 3. Adjusted parameters and static solutions with dimensions |
|---|---|---|---|---|---|---|
| $a$ | $R^*$ | $b$ | $c$ | $k$ | $R$ | $V$ |
| $5.800 \times 10^{-8}$ | $100.00 \times 10^0$ | $9.8 \times 10^{-2}$ | $7.450 \times 10^{-6}$ | $2.564 \times 10^{-3}$ | $9.567 E+0$ | $2.453 E+0$ |
| $5.800 \times 10^{-8}$ | $100.00 \times 10^0$ | $3.4 \times 10^{-1}$ | $1.055 \times 10^{-4}$ | $2.520 \times 10^{-3}$ | $8.523 E+0$ | $2.148 E+0$ |
| $5.800 \times 10^{-8}$ | $100.00 \times 10^0$ | $3.6 \times 10^{-1}$ | $1.470 \times 10^{-4}$ | $1.650 \times 10^{-3}$ | $1.000 E+0$ | $1.650 E+0$ |
| $5.800 \times 10^{-8}$ | $100.00 \times 10^0$ | $3.6 \times 10^{-1}$ | $5.400 \times 10^{-4}$ | $1.920 \times 10^{-3}$ | $8.808 E+0$ | $1.691 E+0$ |

Table 4. Dimensionless parameters and stability boundaries

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\alpha^*$</th>
<th>$\alpha^+$</th>
<th>$\alpha_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.332E-02</td>
<td>3.316E-01</td>
<td>1.525E-01</td>
<td>7.129E-01</td>
<td>5.711E-01</td>
</tr>
<tr>
<td>1.477E-01</td>
<td>4.584E+00</td>
<td>5.656E-01</td>
<td>9.080E-01</td>
<td>8.481E-01</td>
</tr>
<tr>
<td>1.024E-06</td>
<td>4.183E+05</td>
<td>4.968E-01</td>
<td>5.032E-01</td>
<td>5.000E-01</td>
</tr>
</tbody>
</table>

We must now observe that, in Table 4, the four calculated values of $\alpha$ are less than the corresponding $\alpha'$ and, therefore, the time-independent solutions $(r_1, v_1)$ are stable and they attract nearby time-dependent solutions. The parameters obtained for the first set of data, corresponding to the first 597 days, characterize one type of visitors. Figure 6 and Figure 7 show that the data is fitted very well for the model and, coherent with the stability property of the corresponding time-independent solution, the data follows a pattern that leads to the equilibrium point $R=95667$, $V=245$ in rounded numbers. Table 1 shows that from 10/12/2000 to 12/31/2000, less than three months, the number of tourists almost doubled. This was consequence of an article published in one of the newspapers of major circulation in the city of Rio de Janeiro. This is characterized by the corresponding value of the parameter $c$, which is more than ten times greater than the corresponding one for the first set of data (see Table 3). Once again, the data follows a pattern that leads to the new equilibrium point $R=85228$, $V=215$.

Clearly, the large increase in the number of visitors had a negative effect on the environment, infrastructure and local inhabitants, documented with photographs.
not included here for lack of space. The sharp drop in the number of tourist from
the second to the third set of data was, according to the model, due to a corre-
sponding very large drop in the level of tolerance to overcrowding and decrease
of the environmental quality. Table 3 shows that the parameter k for the third set
of data is about 10,000 times smaller than the one for the second set, while the
parameters b and c are almost the same. Also, see Figure 8, the cleared area is
roughly ten times the original 500 m². The result of this was a return to a num-
ber and type of visitors similar to the one of the first set of data, fact observed in
the field and also described by the model because the parameters b, and k are of
the same order of magnitude for both sets, being c larger for the fourth set.

First two sets of data as functions of time

Figure 6. Evolution in time during the first 740 days of the numerical solutions
of equations (7) and (8).

Visitors - Resources

Figure 7*. Phase-Portrait of the numerical solutions of equations (7) and (8)
during the first 740 days.

Resources-Visitors

Figure 8. Phase-Portrait of numerical solutions and field data.

This, together with the behavior depicted in Figure 8 seems to indicate that the
process was starting all over again, probably, according to the model, in poorer
environmental conditions, fact also observed in the field.

* EP stands for Equilibrium Point
We conclude that the model can fit the data very well and the values that can be obtained for the parameters characterize qualitatively the type of visitors. Much research remains to be done and validation of the model with data from other places is highly desirable. In the mean time, we believe that joint efforts made by government authorities and non-government organizations, using the model for closely checking the number of visitors and their profile, could avoid a disastrous situation, since the repetition of the process described above, in cycles with environmental quality decaying from one to the next would lead to the equilibrium point zero resources and zero visitors.

REFERENCES


Acknowledgment: The research for this article was partially supported by the Brazilian agency Coordenação de Aperfeiçoamento de Pessoal de Nível Superior – CAPES, through the fellowship granted by the process 1672029.