Integrated Assessment to Quantify
Size-Based Grade Engineering®
Operating Strategies and Economic Impacts

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Abstract

The global mining industry is focused on improving unit metal productivity and energy efficiency to fulfil increasing demand for natural resources. These are currently being impacted by the dramatic decrease in average grade of mine ore bodies. Lower head grades require greater comminution and grinding energy intensity to achieve the liberation target required for downstream processes. Size based Grade Engineering® aims to increase feed grades by removing low grade uneconomic material through screening prior to energy intensive grinding. Two size based strategies are assessed in an open cut Cu-porphyry deposit in the current study: preferential grade by size deportment and differential blasting for grade. The former refers to a natural based rock phenomena whereby a significant metal proportion preferentially departs into specific size fractions after breakage. Differential blasting aims to change blast product fragmentation to induce grade by size deportment through the exploitation of spatial grade heterogeneity. This characteristic relates the presence of high and low grade areas within a certain production volume originally assigned to a single processing destination based on its average grade. These size based separation drivers are subsequently exploited through a Grade Engineering circuit. This comprises a set of screens and crushers, with a configuration and operating settings defined a Grade Engineering recipe.

While assessments at a strategic level indicate that application of size based Grade Engineering strategies are able to add significant value over the life of mine, the effective deployment of these techniques at production level present significant challenges to the standard operating philosophy. The additional operating flexibility driven by the ability of dynamically exploiting size based levers through the appropriate Grade Engineering recipe need to be properly managed. This ensures that size based Grade Engineering benefits can be achieved at the production environment. An integrated value driven methodology has been developed to manage complexity incurred by the dynamic approach by incorporation in process models coupled with stochastic optimisation. This allows the optimum Grade Engineering recipe to be determined maximising the value per unit of time that can be drawn from a production volume. This framework addressed two fundamental characteristics pertaining to production scale evaluations. Firstly the non-linear interaction between rock based attributes and operating parameters through the integration of JKMRC performance models within the Grade Engineering circuit. Secondly, the presence of inherent process uncertainty and its impact in process optimisation. The introduction of
uncertainty in the stochastic optimisation problem enables the assessment of risk and operating robustness, both essential in robust decision-making processes. This was achieved by integrating multiple and diverse methodologies developed in this thesis, encompassing:

- The development and application of a mathematical model to describe preferential grade by size deportment through a response ranking (RR). This parameter has been extensively employed to characterise the aforementioned phenomenon across different geological style deposits (i.e. stock work vein hosted, Cu breccia porphyry and Cu volcanic porphyry) and sample size scales (i.e. drill core and run of mine, ROM samples) described in this work.

- First order assessment of operating impact associated with exploitation of preferential grade by size deportment through a novel visualisation method. This clearly reveals that size based separation opportunity is not merely a function of RR magnitude but also relies on head grade, proportion of material upgraded in the undersize (i.e. referred as mass pull) and defined material processing destination.

- The development of a coarse liberation model to integrate RR into equipment performance models, essential in process optimisation. This allows to take into account the interaction between particle size distribution and RR. This approach comprised extensive ROM sample characterisation through a novel preferential grade by size characterisation test and sophisticated data analysis techniques.

- The derived attributes pertaining to preferential grade by size validation at industrial scale were employed to characterise the likely production uncertainties associated with an eventual Grade Engineering application. A linkage between RR values and inherent geological variability can be determined. Information gathered during the trial also provided inputs in the further stochastic optimisation assessment.

- The assessment of the impact of modified mill feed particle size distribution due to application of Grade Engineering strategy through simulation. This was achieved by employing a factorial design approach coupled with mass simulation capabilities embedded within the Integrated Extraction Simulator (IES), a new cloud based process simulator.

- A stochastic optimisation methodology encompassing application of the simple average approximation technique coupled with a customised genetic algorithm. This enables the appraisement of impact from changes in available mill capacity upon value drawn from optimised Grade Engineering recipe. An operating robustness
factor was developed to mimic relationship between maximised value and confidence level of achieving a defined operating constraint. Interactions between the two distinctive size based strategies as well as synergies with throughput based approaches (i.e. Mine to mill) can be simultaneously assessed from value, risk and operating robustness perspective.
Declaration by author

This thesis is composed of my original work, and contains no material previously published or written by another person except where due reference has been made in the text. I have clearly stated the contribution by others to jointly-authored works that I have included in my thesis.

I have clearly stated the contribution of others to my thesis as a whole, including statistical assistance, survey design, data analysis, significant technical procedures, professional editorial advice, and any other original research work used or reported in my thesis. The content of my thesis is the result of work I have carried out since the commencement of my research higher degree candidature and does not include a substantial part of work that has been submitted to qualify for the award of any other degree or diploma in any university or other tertiary institution. I have clearly stated which parts of my thesis, if any, have been submitted to qualify for another award.

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Publications during candidature

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Contributions by others to the thesis

This thesis forms part of a larger research project, Grade Engineering® sponsored by the mining industry and the Australian government through the Cooperative Research Centre for Optimising Resource Extraction (CRC ORE). The conception and design of the Grade Engineering® concept took place by Dr. Alan Bye, Dr. Steve Walters, Dr. Luke Keeney and many others, of whose individual contributions I am not fully aware.

The work in this thesis would not have been possible without the contributions from colleagues. These contributions include:

1) Dr. Steve Walters and Mr. Patrick Walters for their very initial input in the developing of progressive crushing test.

2) Integration Extraction Simulator (IES) Team, who calibrated and transferred the comminution models embedded in JKSimMet to IES. Mr. Nick Beaton, Mr. Greg Shapland, Mr. Rob Watkins and Mr. Oliver Kloiber-Deane.

3) Mr. Jon Rutter from Newcrest Mining (former employee) provided access to Lihir site to conduct one of the very first integrated preferential grade by size characterisation program. Those learnings where fundamental in the further development of preferential grade by size sampling protocols at production scale.

4) Taaf Bachman and Cecilia Arrué from Anglo American Copper provided drill core samples and access to site to conduct the preferential grade by size industrial scale validation.

Statement of parts of the thesis submitted to qualify for the award of another degree

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I would like to thank my colleagues at JKMRC. I do believe that this institution is the best research mineral processing centre globally. Although the centre is undergoing changes as a result of the mining downturn; I’m confident that sooner than later, JKMRC will be better shaped and prepared for the new challenges ahead. Thanks to Professor Alice Clark, JKMRC director for your mentorship.

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This experience has turned me in a different person, technically well-equipped, more focused, but more importantly a better person.

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ANZSRC code: 091405 Mining Engineering, 50%

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Abbreviations Contained in Thesis

Au: Gold

Axh: Impact hardness, metallurgical parameter obtained typically from JKMRC drop weight test

BMWi: Bond mill work index, metallurgical parameter obtained from the bond ball mill grindability test.

CLM: Coarse liberation matrix, matrix which contains how preferential grade by size response changes as function of the parent size and its size reduction.

CRC ORE: Cooperative Research Centre for Optimising Resource Extraction

Cu: Copper

CW: Cumulative weight passing at a defined size, measured as fraction (0-1) or percentages (0-100%)

JKMRC: Julius Kruttschnitt Mineral Research Centre

PCT: Progressive crushing test, test employed to develop coarse liberation matrix

ROM: Run of mine

RR: Response Ranking, parameter extensively used across this PhD thesis, it is employed to measure the extent of preferential grade by size deportment phenomenon

SAG: Semi-Autogenous Grinding

Upg: Metal upgrade, in this thesis is calculated through the ratio between the grade of the undersize and feed sample grade.
Chapter 1  Introduction

This Chapter introduces the context, aims and the structure of the current thesis.

1. Addressing Mining Productivity Challenges through Grade Engineering.

The mining industry is facing several technical, economic, social and environmental challenges affecting profitability and therefore unit metal productivity. The drivers behind the obvious dramatic decline in mining productivity have been higher prices and to a large extent the depletion of near surface, high-grade ore bodies (Garay and Shwarz, 2015; Topp et al., 2008). Lower head grades generate more mineable waste and increased processing tonnage to produce equivalent metal, jeopardizing mining productivity. An immediate consequence of decreasing run of mine head grades is the considerable increase in energy consumption and therefore processing costs (Noergate and Haque, 2010; Norgate & Jahanshahi, 2010; Norgate et al., 2007). Lower head grades require more comminution and grinding energy to achieve a target size (microns) associated with a liberation range adequate for downstream separation processes, like flotation. Figure 1.1 illustrates the relationship between total mining energy per unit of Cu and Cu head grade (left side, Figure 1.1), which is related with a defined year (right side, Figure 1.1). This clearly suggests that mining industry is approaching an exponential energy-grade relationship, where a slight decrease in feed grade leads to a significant increase in energy consumption.

As head grades continue to decline, production costs per unit of metal produced will continue to rise making this strategy vulnerable to lower commodity prices. To overcome this trend, the mining industry needs to focus on finding new technologies and operational strategies to increase extraction and energy efficiency (Walters, 2016; Napier-Munn, 2015; Bearman, 2013).

Size based Grade Engineering® is a methodology aims to increase run of mine head grades through removal of coarse uneconomic material prior to inefficient and costly grinding through screening (Walters, 2016). Two size based Grade Engineering levers are extensively investigated in this work (Figure 1.2), preferential grade by size deportment and differential blasting for grade. Preferential grade by size deportment refers to a “natural” based rock property whereby a significant proportion of metal preferentially deports into specific size fractions after breakage. Analysis conducted by
Carrasco (2013) indicated that this response is highly variable and geomeallurgical characterisation is required to embed a derived performance attribute into resource block models for further exploitation.

![Graph showing energy consumption and Cu grades](image)

*Figure 1-1. Cu feed grades impact upon total energy consumption (Bascur, 2011) combined with feed Cu grades global trend over time (Mackezte, 2011)*

Differential blasting aims to change blast product fragmentation to induce grade by size deportment through the exploitation of deposit spatial grade heterogeneity characteristic. This relates the presence of spatial high grade and low grade discrete clusters within a certain production volume originally assigned to a single destination (e.g. waste, leach, and mill) based on its average grade. In differential blasting for grade high levels of energy are applied to high grade pockets and low energy is imported to low grade zones, allowing high and low grade cluster fragmented rock to be separated based on their different particle size distributions, via screening. The only published example of the application of differential blasting for grade is the trial carried out by CRC ORE at the Mogalakwena PGE open pit operation in South Africa in 2011 (Todoir and Bye, 2012; Ziemski, 2011).

These two coarse size based Grade Engineering responses are subsequently exploited through a Grade Engineering circuit, comprising a set of screens and crushers suitably configured. This circuit produces multiple processing streams with different metal content (i.e. grade) and particle size distributions. Grade Engineering aims to maximise operational value by diverting the post screening streams to the optimum economic processing destination (e.g. Mill, Leach, Waste). The proportion
of material and grade is a dynamic feature of size based Grade Engineering that can be modified to suit variable operational parameters such as mill or mining constraints over time.

Although size based Grade Engineering provides an additional level of flexibility there is an increased degree of complexity that needs to be properly managed for effective operational implementation.

2. Integrated size-based Grade Engineering Assessment

Previous size based separation studies were focused on developing tools to identify and map size based Grade Engineering attributes (Carrasco, 2013) and to demonstrate conceptually the feasibility of implementation (Todoir and Bye, 2012; Ziemski, 2011). These studies placed less emphasis on the operational and economic impacts of size based Grade Engineering within current operating practices. The current thesis is focused on the development of an integrated methodology to assess size based separation techniques by means of mathematical optimisation to determine the optimum Grade Engineering circuit configuration (known as “recipe” in batch manufacturing terminology) to maximise value per unit of time that can be drawn from a production volume. This novel framework encompasses:

1) The development of a methodology to mathematically describe preferential grade by size deportment response through a Response Ranking (RR) parameter.

Figure 1-2. Two size based Grade Engineering levers assessed in this work.
2) A coarse liberation model that is able to predict preferential grade by size deportment responses across varying particle size distributions.

3) The impact of size based Grade Engineering techniques on comminution performance (i.e. throughput) through the utilisation of the mass simulation capabilities embedded in the Integrated Extraction Simulator (IES).

4) The development of a simulation platform to analyse the interplay between the two size based Grade Engineering levers and Grade Engineering circuit performance. The last mimicked through the industry accepted JKMRC process models (Napier-Munn et al., 1996).

5) The utilisation of stochastic optimisation coupled with circuit simulation to assess size based Grade Engineering techniques from value, risk and operating robustness perspectives. This enables the determination of optimum processing path sequences (i.e. Grade Engineering circuit configuration) as well as crusher side setting and screen apertures to maximise value per unit of time. The optimum combination of operating parameters is referred to as a “recipe” in the batch manufacturing industry (ANSI/ISA95, 2005). The uncertainty/variability required to conduct stochastic process optimisation have been modelled using information from an Industrial Grade Engineering validation trial, where various sources of uncertainty were extensively investigated.


The integrated methodology developed in this thesis has been underpinned by the production control module embedded to Manufacturing Execution System (MES) or Collaborative Production Management (CPM) solutions (Engell and Harjunkoski, 2012; Harjunkoski et al., 2009; ARC, 2009; ANSI/ISA95, 2005). Within production control, the operating set points (known as “recipe”) of the material batches defined by production planning and scheduling are determined (Figure 1.3). This spans the characterisation of the process through detailed models, circuit simulation and optimisation at steady state conditions and often within a narrow time horizon. As it is observed in Figure 1.3, typically the amount of fine details and problem complexity (e.g. unavoidable process
nonlinearities) increases towards the control layer, as the models get closer to physical processes, whilst the time horizon simultaneously decreases.

The methodology developed in this work represents a step forward towards the capabilities definition of MES mining systems, spanning data integration and data analysis to effectively exploit the economic and operating benefits associated with process flexibility.

![Figure 1-3. PhD focus, recipe definition (left) compared with the level automation pyramid (right).](image)

4. **Research Aims**

Conduct a size based Grade Engineering economic and operating assessment in the context of production control.

- Integrate preferential grade by size deportment and differential blasting for grade with equipment performance models to enable its interaction with operating parameters to render process optimisation.

- Understand key variables associated within the exploitation of preferential grade by size deportment and its operating impact.
• Assess impact of modified mill feed particle size distribution upon comminution performance due to the application of size based Grade Engineering techniques.

• Characterise and integrate the likely size based Grade Engineering process uncertainty/variability within the economic as well as operating assessment to determine entailed operating robustness and risk in addition of value.

5. **Statement of Originality**

• Novel methodologies to characterise preferential grade by size deportment with application in resource block model population (i.e. geometallurgical as well as production scale characterisation) and process optimisation due to its integration with equipment performance models.

• Value driven decision support system to quantify size based coarse separation through screening strategies and the associated economic and operating impact. This integrates: ore characteristics, equipment specifications and circuit and processing options.

• First use of stochastic optimisation to simultaneously assess the additional operating flexibility provided by sized based coarse separation techniques from a risk, operating robustness and value perspective.

6. **Research Hypotheses**

• Differential blasting for grade and preferential grade by size deportment can be effectively described by a mathematical function which can be effectively embedded within current available equipment performance models to conduct process optimisation.

• Cut-off grades, proportion of material upgraded through screening need to be taken into account in addition to preferential grade by size response to conduct an economic as well as operating appraisal.
• Process simulation allows to assess the impact on comminution performance due to changes in mill feed particle size distribution due to the application of size based Grade Engineering.

• The introduction of uncertainty/variability enables the assessment of size based Grade Engineering operating strategies from a risk and operating robustness perspective in addition to value.

7. Statement of Sustainability Contribution

The application of coarse size based separation operating techniques seek to improve unit metal productivity through the early rejection of low grade coarse uneconomic material prior highly intensive and inefficient grinding. The integration of the diverse methodologies developed in this PhD encompassing: characterisation, process modelling, process simulation and optimisation aimed to support the effective operating application of size based Grade Engineering strategy.

8. Organisation of the Thesis

This thesis has been partially structured in the form of papers and Chapters 3 to 8 are published or submitted papers, which comprise the body of the thesis. Figure 1.4 outlines the themes addressed in the body of the PhD thesis. Each box describes the methodology in the top and the main outcome in the bottom.

Chapter 1: This Chapter introduces the context, aims and the structure of the current thesis.

Chapter 2: It is focused on the literature review spanning the investigation of limited published literature pertaining to size based coarse separation Grade Engineering levers and methodologies and tools to conduct an integrated size based Grade Engineering assessment from an operational perspective.
Chapter 3 (1 in Figure 1.4, Carrasco et al., 2014): Describes the methodology to mathematically describing preferential grade by size deportment through a single parameter. Samples are screened in defined size fractions to determine the proportion of mass as well as grade within each size class. The methodology is based on fitting an upgrade metal undersize (i.e. grade of the undersize screen to screen feed ratio, Upg) and cumulative undersize weight curve (i.e. proportion of material in undersize relative to feed mass, CW) (Figure 1.5). The fitting parameter is employed to measure the extent of preferential grade by size initially through a “K” parameter later defined as a Response Ranking, “RR”. Carrasco et al., (2014) applied this methodology to major Au-Cu Australian mine across different sample size scales (i.e. drill core, blast hole, Semi-Autogenous mill feed belt cuts) depicting the remarkable Upg and CW curve stability. This behaviour is further supported by the application of this methodology to a significant CRC ORE geological data base comprising of more than 2000 samples from different geological deposit styles. Essentially the RR factor is the slope of Upg and CW in log-log space (Eq.1). RR values are extensively employed in Chapter 5 onwards.

Figure 1-4. PhD Thesis Body Structure.
Chapter 4 (2 in Figure 1.4, Carrasco et al., 2016a): While Paper 1 is purely focused on defining a novel methodology to mathematically characterise preferential grade by size deportment, this paper discusses the operating significance of exploiting this response through screening by means of a novel visualisation method; the Grade by Size exploitation diagram (Figure 1.6). This diagram enables rapid assessment of the impact of critical operating parameters and rock based attributes upon size based separation ore body amenability. This encompasses: the proportion of mass in the undersize (i.e. referred as “mass pull”, concept extensively employed across the PhD thesis), head grade, RR and cut-off grade.

The area inside defined by the mass pulls, provides the operational limits for exploitation of preferential grade by size deportment. These limits are sensitive to changing mass pull making this size based Grade Engineering attribute a dynamic operational lever providing additional operating flexibility. Changes in cut-off grade moves the area horizontally without affecting the shape of the exploitation region, but affecting the proportion of the resource amenable for size based separation when preferential grade by size deportment characterisation data (i.e. RR and grade) is overlaid. Although this methodology can be employed to conduct a first order size based coarse separation
evaluation, it does not render a detailed economic impact. Furthermore this methodology does not account for the possible interactions of the aforementioned variables. Changes in mill feed particle size distribution post screening can significantly alter comminution performance, leading to variations in revenue and therefore cut-off grades. The understanding of this effect coupled with the potential variations in RR due to changes in particle size distribution and the likely size based Grade Engineering process variability/uncertainty comprise the main focus of the following chapters (i.e. Chapter 5, Chapter 6 and Chapter 7).

Chapter 5 (3 in Figure 1.4, Carrasco et al., 2016b). This work seeks to characterise the extent and nature of the likely size based Grade Engineering process uncertainty/variability. Outcomes of this analysis will be fed to the further stochastic optimisation assessment, described in Chapter 8 (6 in Figure 1.4). This work decouples the different sources of uncertainties operating during a preferential grade by size industrial scale trial from world class porphyry deposit. The area identified as amenable to size based coarse separation was selected based on the preferential grade by size characterisation program comprised of the geometallurgical protocols developed in Carrasco (2013) coupled with mathematical characterisation outlined in Chapter 3 (1, Figure 1.4) and Chapter 4 (2, Figure 1.4).

The industrial trial comprised the screening of ~40,000 tons of Run of Mine (ROM) material during 28 days where 4 size fractions were periodically sampled using a front end loader (Figure 1.7).
Samples were sent to a metallurgical laboratory for assaying to determine daily RR production scale values (Eq. 1). The methodology appraises three sources of uncertainty, spanning:

1) Screen efficiencies during the trial.
2) Sampling errors linked to assays obtained at each size fraction (gauged by means of fundamental sampling error).
3) Mathematical modelling of preferential grade by size responses through RR parameter.

These sources of uncertainty are compared in terms of effect on RR variability associated with the inherent geological variability. This analysis enables to understand the robustness of the model employed to mathematically mimic preferential grade by size responses at production scale as well as the factors that could jeopardize the RR estimation. The large source of uncertainty (i.e. error modelling, geological variability, sampling errors, screen inefficiencies) determine the nature of the uncertainty modelled and propagated within the value driven stochastic optimisation evaluated extensively described in Chapter 8. In addition, the data gathered during the trial is employed to predict uncertainty in grade as well as blasting particle size distribution, key inputs in further stochastic optimisation assessments.

![Figure 1-7](image.png)

*Figure 1-7. Illustration of the screen trial, depicting size fractions in inches. The fines, -6” of the 2nd screen were recycled to 1st screen.*

Chapter 6 (4 in Figure 4, Carrasco et al., 2016c). This work describes a methodology (Figure 8) to develop a coarse liberation model based on preferential grade by size deportment (RR). This model predicts a RR value as function of particle size distribution and the size reduction process. The interaction between RR values and changes in particle size distribution is essential in production implementation and process optimisation where more detailed process models are required. This methodology involved the extensive run of mine grade by size sample characterisation from three different geological style deposits (i.e. stock work vein hosted, Cu breccia porphyry, Cu volcanic
porphyry). Samples were characterised through the novel progressive crushing test in conjunction with sophisticated statistical data analysis techniques (i.e. ANOVA test, Monte Carlo Simulation, error propagation). The information drawn from progressive crushing test enables the generation of a coarse liberation matrix (CLM), spanning RR per parent size and the evolution of this response (RR) as size reduction increases per sample tested. The statistical meaningful differences of the RR values within CLM are analysed by talking into account model and mass balance associated errors. A statistically robust coarse liberation model can be developed. The present work adds an additional dimension, grade, to the current comminution models focused on understanding how new particles are created within a wider range of size reduction equipment.

![Diagram of Methodology to develop a coarse liberation model based on RR.](image)

Chapter 7 (5 in Figure 4, Carrasco et al., 2017). This work aims to understand the impact of modified mill feed particle size distributions upon comminution performance (i.e. throughput), critical in the integrated Grade Engineering assessment. Novel metallurgical simulation software, the Integrated Extraction Simulator (IES) was employed. The widely accepted JKMRC comminution models (Napier-Munn et al., 1996) were transferred to IES to simulate the comminution performance of the Cu porphyry deposit circuit under assessment (Figure 9). The mass simulation capabilities (i.e. ability to run multiple simulations in a relative short period of time) embedded in IES enable a factorial design analysis to assess multiple operating scenarios representing the possible size based Grade Engineering strategies and dynamic processing rock
attributes. Results across the multiple processing scenarios tested indicated that the change in SAG mill energy was strongly influenced primarily by changes in mill feed particle size distribution. Nevertheless, ball mill energy consumption did not appear to be significantly changed (this is mainly controlled by the mill ball load, Napier-Munn et al., 1996), indicating that SAG mill performance controls comminution circuit throughput. The SAG gross energy is drastically reduced when the Grade Engineering streams are fed to comminution circuit. The approach employed was to incrementally increase the comminution mill throughput until reaching the base case SAG gross energy (Fig. 10). This ascertains the additional throughput achieved by exploiting the mill energy available related with the Grade Engineering modified particle size distribution feed. As an example, the anticipated SAG feed tonnage increase for a 20% undersize mall pull, at medium blasting fragmentation and a mill feed Axb of 25, is depicted in Fig. 10. In this example an increase of total mill throughput of 12% is observed. This analysis enables the development of a size based Grade Engineering throughput model which takes into account SAG mill F20 (size fraction 20% passing) with impact hardness (Axb) and grindability (BMWi). This model is embedded in the objective function employed in the value driven assessment by means of stochastic optimisation extensively described in Chapter 8 (6 in Figure 4).

Figure 1-9. SABC comminution circuit under assessment (SAG-ball mill with pebble crusher)
Figure 1-10. SAG mill gross power feed tonnage rate versus SAG feed tonnage with 20% mass pull.

Chapter 8 (6 in Figure 4, Carrasco et al., 2016d). Chapters 3 and 4, provide the scientific foundation of size based coarse separation concepts mainly through the thorough study of preferential grade by size deportment responses. This work has the purpose of integrating the outcomes of Chapters 5, 6 and 7 to develop a methodology to conduct a size based Grade Engineering integrated assessment to manage the additional level of operating flexibility to ensure its effective operational deployment. This allows the optimum Grade Engineering circuit configuration and operating settings to be determined that maximises value per unit of time that can be drawn from a production volume under a set of user defined constraints. The introduction of uncertainty in the stochastic optimisation problem enables the assessment of the risk and operating robustness, both essential in robust decision-making processes.

Data from a Grade Engineering Industrial trial (Chapter 5, 3 in Figure 4) has been employed to model the likely production scale uncertainties of the inputs entailed in the integrated evaluation encompassing stochastic optimisation. The Coarse Liberation model developed in Chapter 6 (4 in Figure 4) has been integrated with the Whiten’s crusher model (Whiten, 1974) to predict mass as well as grade per size fraction. Both, size based Grade engineering uncertainty and coarse liberation model were embedded in a simulation platform whereby JKMRC equipment models were coded in Matlab® to predict the interaction between grade by size responses and screen and crusher performance. The objective function (i.e. defined as value per unit of time), employed in the
stochastic optimisation takes into account the size based Grade Engineering impact upon comminution performance through the model developed in Chapter 7, (5 in Figure 4).

Different size based Grade Engineering scenarios were employed to test the methodology. Those comprising distinctive size based Grade Engineering attributes (i.e. spatial grade heterogeneity and preferential grade by size). A customised Genetic algorithm coupled with sample average approximation technique is then employed to determine the Grade Engineering recipe that maximises the defined objective function (1 in Figure 11). A sensitivity analysis is conducted by changing the capacity of what is very often identified as the bottleneck within the current mineral processing circuit as the available comminution capacity. This enables a determination to be made of the impact of a defined operating mode upon the value that size based Grade Engineering is able to deliver in conjunction with an associated operating recipe (2 in Figure 11). The chance constrained approaches allow an assessment of the relationship between user defined operating constraints and the objective function. Those constraints are often associated with safety and product quality, which sometimes are equally important as the objective functions. This interaction (optimal-constraints) can be obtained by determining the optimal solution for different confidence levels, representing the probability (reliability) of complying the inequality constraints. The shape of the curve describes the robustness of the solution, which is crucial for decision making. The steeper the curve the less robust is the optimum value drawn from the optimisation. This relationship has been modelled through an operating robustness factor for rapid comparison among different operating scenarios. This can be combined in the value (i.e. objective function defined), risk (standard deviation of the objective value shown as error bars) and operating robustness (parameter obtained by performing the robustness analysis, 3 in Figure 2) diagram depicted in Figure 12. This novel methodology is called Ore Logic®, a value driven decision support tool to aid size base Grade Engineering operational deployment.

Chapter 9: Conclusions. This chapter outlines, the conclusions pertaining to the value driven integrated size based Grade Engineering assessment within the context of production control.

Chapter 10: Recommendations for further work. Recommendations for further work in an important emerging area of research and development are presented.

Appendices: Background to Gy’s sampling theory, review of different size based Grade Engineering sampling strategies at production scale and digital information is provided.

There is confidentiality issues in providing raw grade by size data to public domain, since there are important economic benefits through Grade Engineering by size as it will be depicted in Chapter 8.
9. Project Background

The current research has been supported by the Cooperative Centre for Optimising Resource Extraction (CRC ORE). CRC ORE is an Australian government supported research initiative, which seeks to transform resource extraction and the way it is evaluated by developing innovative techniques to upgrade ore between mining and concentration. Anglo American and Newcrest
mining companies are particularly acknowledged for data and samples provided, both were essential in the development of the diverse methodologies described in this PhD thesis.

10. References.


Walters, S.G., 2016.Driving Productivity by Increasing Feed Quality Through Application of Innovative Grade Engineering® Technologies. Grade Engineering White paper, retrieved from:


Chapter 2  Literature Review

The primary aim of this chapter is to determine the methodologies required to develop an integrated size based Grade Engineering assessment in the context of production control. A review of the limited published information available pertaining to differential blasting for grade and preferential grade by size deportment is provided. The integration of characterisation, the impact upon comminution performance and the use of optimisation under uncertainty to manage the additional Grade Engineering operating flexibility are extensively discussed.

1. Minerals Industry Context

The mining industry is facing several technical, economic, social and environmental challenges affecting mining profitability and sustainability (Carrasco, 2013; Bearman, 2012; Franks et al., 2012; Prior et al., 2012; Topp et al., 2008). The need of meeting metal demand at higher operating costs coupled with volatility in commodity prices are an increasingly important concern. The mining industry has reacted to this complex scenario by reducing the cut-off grade material which converts more resources into reserves, increasing revenues but at higher production costs with marginal profitability improvement. This trend has been supported by high commodity prices which also have exaggerated capital intensity and labour inputs. This in turn has led to significant decrease in mining productivity (Figure 2.1).

Figure 2.1. Multifactor Mining Productivity for Chile and Australia (year 2000=1) (Castillo et al., 2015 and ABS, 2016)
The core reason is higher prices make it economically viable to mine deposits that would otherwise be uneconomic through resource depletion (Topp et al., 2008). However, lower head grades generate more mineable waste and increase processing tonnage to produce equivalent metal. As head grades continue to decline, production costs will continue to rise making this strategy unsustainable in the long term.

Decrease of mine head grades is a function of depletion of near surface, high-grade ore bodies which are not being replenished by exploration discoveries, and also a reaction to technologies that can support larger scale material movement and mineral processing. The net result is that for most metals while feed grades have declined over time, annual metal production has increased (Figure 2.2).

![Figure 2.2: Copper production and run of mine copper grades since 1900 and introduction of mining technologies (Mackenzie, 2011).](image)

The ability to exploit economics of scale have enabled profitable utilisation of increasingly lower grade ores, evidence suggests limits to this type of exponential growth (Prior et al., 2012; Rendu, 2006). Bartos (2007) points out that bulk open pit mining has been prompted by improvements in haulage technology with direct cost savings related to haul truck size between 1960 and 2005. The trend indicates that further increase in truck size will not provide a significant economic advantage (Figure 2.3).
Early (prior to comminution and grinding) gangue rejection has been identified as a feasible technical alternative whereby metal productivity and efficiency can be improved (Carrasco, 2013; Bearman, 2012; Logan and Krishnan 2012; Bamber, 2008a; Bamber et al., 2008b; Bamber et al., 2006a; Bamber et al., 2006b; Burns and Grimes 1986). Size based Grade Engineering® is a concept that aims to improve unit metal productivity and energy efficiency per unit of metal by increasing feed grades to concentrator via screening.

2. Literature Review Structure

Figure 2.4 depicts the literature review structure. First a size based Grade Engineering overview is provided. The associated characterisation methodologies available are then discussed. The potential impact of size based Grade Engineering upon comminution performance is appraised from a Mine to Mill perspective. Ultimately, the extensively employment of optimisation under uncertainty techniques across highly flexible processing industries and its application to the current context is thoroughly examined.
Size Based Grade Engineering®

Overview

Characterisation

Impact on Comminution Circuit Performance

Optimisation under Uncertainty

Integrated Assessment to Quantify Size Based Grade Engineering Operational Strategies and Economic Impact

Figure 2.4. Literature Review Structure.

3. Leverage Additional Grade Engineering Flexibility

Size based Grade Engineering involves a range of integrated technologies and operating protocols for improving effective feed grades through early coarse rejection of low value components prior to energy intensive and costly processing activities (Walters, 2016). Size based Grade Engineering encompasses two separation levers, preferential grade by size deportment and differential blasting for grade (Figure 2.5). Separation is achieved at minimum mining unit through variable combinations of blasting fragmentation and screening (i.e. through Grade Engineering circuit) intervening between the mine to mill interface.

Preferential grade by size deportment refers to a natural based rock property whereby a significant metal proportion preferentially deports into specific size fractions after breakage (Carrasco, 2013). Figure 2.6 depicts a Semi-Autogenous mill (SAG) feed sample where Au grade significantly varies across the size fractions. Although this sample is defined as waste (feed grade 0.26 ppm < grade cut-off 0.3 ppm) there are size fractions that can be classified as ore.
Differential blasting aims to change blast product fragmentation to “induce” grade by size deportment through the exploitation of spatial grade heterogeneity characteristic. This relates the presence of spatial high grade and low grade discrete clusters within a certain production volume originally assigned to a single destination (e.g. waste, leach, and mill) based on its average grade. In differential blasting for grade high levels of energy are applied to high grade pockets and low energy is imported to low grade zones, allowing high and low grade cluster fragmented rock to be separated based on their different particle size distributions, via screening. Figure 2.7 illustrates Cu grade variation by blast hole at bench scale in a Cu porphyry deposit. All material was assigned to the comminution circuit (i.e. mill). However, within the same volume there are areas that should have been sent to different processing destination based on its grade and defined operating grade-cut-off. Green clusters describe material assigned to leach, blue clusters describe waste grade rock, and the red/orange clusters describe mill grade ore.
Differential blasting for grade and preferential grade by size deportment are further exploited through a Grade Engineering circuit. Figure 2.8 depicts a generic Grade Engineering circuit. This circuit comprises a grizzly screen, double screen deck, depicted as two screen devices (S1 and S2) and a crusher (C). The flexibility provides the option to change operating settings (screen aperture, crusher setting side) as well as selecting the optimum treatment pathways within the circuit. Material can be ultimately diverted to three possible processing destinations, waste (W), leach (L) and mill (M). This size based separation strategy can be used to dynamically manage short term production constraints and bottlenecks over life of mine with less reliance on major capital investments. However, flexibility also entails operating complexity that need to be adequately controlled for an effective operating deployment.
4. Process Assessment Framework from Production Perspective

The economic/performance assessment within the process industry (certainly in mining), involves essentially three steps, characterisation, process modelling and simulation and optimisation. This framework remains practically unaltered across different decision levels within the industry (Engell and Harjunkoski, 2012; Harjunkoski et al., 2009; Scholten, 2007). However, the degree of resolution of the inputs practically determine the tools and methodologies required in the simulation and optimisation steps (Goodwin et al., 2008; Biegler and Grossmann, 2004). As the decision making involves long time frames (i.e. strategic), the system can be modelled using simple linear representations of production process (e.g. plant capacity and unit ratios). As the decision making gets closer to production, it becomes crucial to take into account nonlinearities in mining and beneficiation operations (Wassick, 2009). Thus, process models selected need to have enough level of resolution to capture the non-linear interaction between rock type processing parameters (such as impact hardness, Axb, Napier-Munn et al., 1996) and equipment operating settings. Researches from JKMRC have addressed this issue in the minerals industry by the development and successfully application of the population balance-based model approach (Powell and Morrison, 2007; Napier-Munn et al., 1996). This approach mimics size reduction and subsequent valuable recovery at a particle size level. This modelling and simulation scheme enables accurate gauging of comminution equipment performance as well as their complex interaction within the system. The employment of this understanding has clearly being reflected in the development of the Mine to Mill (M2M) concept. This seeks to enhance Semi-Autogenous (SAG) mill performance by customising blasting fragmentation, this strategy is explained in further detail in Section 6.

The exploitation of either preferential grade by size or spatial grade heterogeneity via differential blasting for grade will inevitably lead to changes in mill feed particle size distribution and therefore changes in comminution performance, and therefore it needs to be considered in the size based Grade Engineering assessment.

The additional operating flexibility increases complexity that need to be managed. Industries with significant level of flexibility such as manufacturing, chemical and oil and gas have coped with associated complexity through the development of a robust “processing recipes” as part of the economic evaluation. The concept of a recipe is extensively employed in batch production systems to the set of production tasks (i.e. operating set points) to produce a product with defined properties (Engell and Harjunkoski, 2012; Harjunkoski et al., 2009; Scholten, 2007; Kallrath, 2002; Glismann and Gruhn, 2001).
A clear example of this flexibility successfully implemented in the refining process of the oil and gas industry is discussed. This process can be divided into three areas: crude operation, production and blending. A variety of crude oil can be fed to the production plant, characterised by its flexibility to accommodate a range of flow rates, compositions and physical/chemical properties (density, flash point, etc.) to produce a wide range of saleable products. These are subsequently blended to meet a dynamic product demand. However, variability in feed characteristics are often difficult to quantify and are therefore uncertain (e.g. inconsistencies in the feed stock, coupled with variations in the performance of upstream processes) (Mesfin and Shuhaimi, 2010; Cao et al., 2009). Hence the problem in this flexible production environment is to make the process economically optimal, but still feasible under uncertain feed conditions.

This has been addressed through process optimisation under uncertainty, also referred to as stochastic optimisation. This aims to deliver robust processing decisions and has been extensively applied across process design, operation and control (Gabrel et al., 2014; Sahinidis, 2004) in the aforementioned industries, and to a lesser extent, in mining. Therefore, to conduct a “robust” assessment of Grade Engineering strategy, the inputs uncertainty need to be considered in the operating as well as economic evaluation.

5. **Size Based Grade Engineering Characterisation**

5.1. **Differential Blasting**

Characterisation of spatial grade heterogeneity amenability is primarily ascertained by analysing blast hole data, which provides greater grade resolution than block models. This is because grade variability is smoothed out through geostatistical interpolation techniques such as kriging. A probability plot distribution of blast hole Cu grades by destination for an RL slice of a major Cu porphyry mine illustrates a rapid approach for generating first order statistical characterisation (Walters, 2016). This indicates 20% of assigned mill feed based on blast hole grade distribution would ideally have been sent to dump leach; and 33% of dump leach would ideally have sent to mill (Figure 2.9).
5.2. Preferential grade by size deportment

The Bougainville Cu-Au Mine in Papua New Guinea is the most important published example of production scale application of preferential grade by size deportment (Burns and Grimes 1986). In the mid 1980’s in response to decreasing head grades a comprehensive program of size by size grade assessment was undertaken on specific ore types to ascertain if low grade material could be screened out prior to comminution. However, the outcomes of Bougainville screening project are poorly documented, particularly regarding the methodologies employed to assess the economic and practical feasibility of size based coarse separation strategies. Carrasco (2013) attempted to account this, by developing a methodology focused on ascertaining the magnitude and variability of preferential grade by size deportment employing a geometallurgical approach. This comprised an extensive drill core and blast hole characterisation in conjunction with the thorough data analysis of Semi-Autogenous mill (SAG) feed samples at Telfer Au mine, Newcrest. This comprises in determining the proportion of metal that is recovered within the closest size class where 50% of the
total sample mass is recovered. The cumulative probability distribution of metal (percentage at the defined size class, \(~P50\)) recovered is employed as grouping method, based on the assumption that natural populations follows approximately a straight line (i.e. Gaussian distribution, Figure 2.10). Geological attributes (i.e. texture, mineralisation style) are analysed and compared within the defined groups seeking to understand the likely geological controls. However, the direct application of this technique in process simulation and subsequent production economic optimisation is limited. This is due to that the metric employed to recognise preferential grade by size response is not able to interact with equipment performance models, crucial to understand the operating significance of exploitation of preferential grade by size deportment at production level. Notwithstanding, this framework demonstrated that the protocols can be employed to recognise areas within this deposit amenable to coarse size based preconcentration, this comprehensive methodology has not been yet applied elsewhere.

![Figure 2-10](image)

**Figure 2-10. Metrics employed to identify and rank preferential grade by size (Carrasco, 2013) using P50 as reference a) 50 mm employed in belt cuts; b) 1.18 mm at drilling scale.**

A critical aspect is the understanding of the interaction between fragmentation and preferential grade by size deportment. Particle size distributions can be modified to enhance or suppress the magnitude of preferential grade by size response. Several authors have analysed this problem from a liberation perspective at micro scale (Ozcan and Benzer, 2013; Vizcarra et al., 2010; Hosten and Ozbay, 1998; Fandrich et al., 1997; Petruk, 1988; Berube and Marchand, 1984).
The characterisation comprises the selection of samples which are progressively ground to then conduct liberation in size by size basis; via microscopy technologies (e.g. mineral liberation analysis, MLA). A clear example is provided by Berube and Marchand (1984). This study aimed to investigate the evolution of the mineral liberation characteristics of an iron ore (containing hematite, magnetite and quartz) undergoing grinding (Figure 2.11). Samples are obtained at each grinding step while the effect of the distinctive breakage mechanisms are also assessed.

These methodologies are focused at the liberation (i.e. as the proportion of valuable material surface exposed), which is a critical property in the beneficiation stages subsequent milling (e.g. leaching, flotation). While obvious efforts have been allocated in this area of study, almost no attention has been given to what occurs at a coarse scale (~100 mm), prior to grinding. Work is required to evaluate a progressive crushing approach at coarse scale to determine if this optimal response signature can be conditioned at production scale blasting or primary crushing. This will enable the integration of grade within the current available size reduction models, focused on predicting of mass distribution per size fraction through breakage (Napier-Munn et al., 1996)

Figure 2.11. Size reduction process depicted in Berube and Marchand (1984).
6. Impact of Size based Grade Engineering upon Comminution

The exploitation of either spatial grade heterogeneity or preferential grade by size deportment, through tailor-made blasting fragmentation as well as Grade Engineering circuit (Figure 2.8) will lead to changes in mill feed particle size distribution and therefore changes in comminution performance. Autogenous (AG) and semi-autogenous milling (SAG) are particularly sensitivity to mill feed input variations, such as; rock competency and certainly mill feed particle sizing distribution. Larger ore particles from the feed serve as grinding media. Coarse particles, break the smaller particles, whilst in this process also breakdown into smaller particle, before exiting the mill as a product. Steel balls are charged to the SAG mill to replicate the breakage action of larger ore particles, and also help break the more competent rock lumps, in order to maintain mill throughput.

The Mine to Mill (M2M) strategy is a clear example of manipulating the relationship between plant throughput and feed particle size distribution (Kanchibotla 2000, Morrell and Kojovic 1999, Scott et al., 1999). M2M focuses on enhancing milling throughput by controlling blasting fragmentation. The objective is to increase the proportion of sub-grate size material (~20 mm) and reduce mill residence time, in conjunction with decreasing the material particle content within the critical size range (20-70 mm) which is difficult to break, limiting grinding capacity. The M2M throughput improvements across different deposit styles have ranged from 10 to 20% (Morrell and Kojovic 1999).

The installation of precrushing (i.e. secondary crushing) stages applied to the whole or partial SAG feed stream to improve comminution throughput has also been extensively discussed in the literature (Rose et al., 2015, Siddall and Putland 2007, Putland et al., 2004, Atasoy et al., 2001). This operating strategy is particularly effective in cases where the ore is very competent. Rose et al. (2015), related additional plant capacity benefits by installing an additional crushing unit, namely: a significant decrease of SAG and mill steel ball consumption and improvements in grade and mill liner life. The use of secondary crushing can control the variations in SAG mill throughput which the pebble recycle crushing is not able to address. Putland et al. (2004) proposes the installation of an overflow bin to control the proportion of material sent to secondary crushing. This strategy provides a high degree of flexibility by controlling the blend of primary and secondary crushing products. This ensures the presence of enough grinding media while balancing the energy utilisation between primary and secondary milling. The throughput improvements reported by additional crushing can be moderate (~20%) to significant (~60%) (Siddall and Putland 2007).
Coarse size based separation approaches (i.e. differential blasting for grade and preferential grade by size) offers similarities in producing a mill feed stream with a fine particle size distribution compared with the secondary crushing strategies previously outlined. The application of Grade Engineering to amenable ores offer further economic advantage by improving mill grades through coarse separation preconcentration. Combination of these two optimisation strategies could lead to significant improvements in energy efficiency, and thus unit metal productivity.

7. **Process Optimisation**

The application of mathematical optimisation is an essential piece in economic as well as operating assessment across the different decision layers within the diverse processing industries. This encompasses: supply chain management and associated long term strategies, plant and equipment design as well as real time process control. Process optimisation framework as well as the integration of process uncertainty is thoroughly appraised.

7.3. **Introduction to Mathematical Optimisation**

Mathematical optimisation generally involves two steps, problem modelling and solution method. The former encompasses the appropriate selection of variables, which underpin the constraints and objective function. In strategic planning operating cash flow and net present value (NPV) are commonly employed as objective functions (Newman et al., 2010). At production/tactical planning there are several options, each depending of the operating processing mode, i.e. Maximise revenue per unit of time, minimise energy consumption per unit of product produced, minimise the deviation of the usage of resources (Kallrath, 2002), nonetheless the mathematical scheme still remains. A general formulation of a typical optimization problem is illustrated in Eq.1.

Subject to:

\[
\begin{align*}
\text{Minimize } & \quad f(x) \\
\text{subject to: } & \quad g_i(x) \leq 0; \text{ for } i = 1, \ldots, m.
\end{align*}
\]
\[ h_i(x) = \text{for } i = 1 \ldots, l \]

The set \( X \) can contain continuous variables or integer variables (in this context the Grade Engineering recipe). The function \( \{g_i(x)\} \) refers to the inequality constraints whereas \( \{h_i(x)\} \) the equality constraints together with the feasible set \( X \). The optimization problem can be divided as function of the characteristics of the optimization problem:

**Linear Problems (LP):** when \( f, g_i \) and \( h_i \) are linear functions of \( x \).

**Quadratic Problems (QP):** when \( f \), is quadratic in \( x \) and \( g_i \) and \( h_i \) are linear.

**Mixed Integer Problems (MILP):** when \( f, g_i \) and \( h_i \) are linear and \( x \) can take integer and/or real values.

**Mixed Integer non-linear problems (MINLP):** when \( f, g_i \) and \( h_i \) are (possibly) nonlinear and \( x \) can take integer and/or real values.

These defined the most suitable solution method. LP problems can be easily solved by the simplex algorithms. Mining long term problems lie within this category. However, production type optimisation problems are very often MINLP (Wassick, 2009; Kallrath, 2002).

### 7.4 Optimisation under Uncertainty

Optimisation under uncertainty or stochastic optimisation refers to a collection of methods for minimising or maximising an objective function when uncertainty is present. The structure depicted in Eq.1 still holds, however, instead of using known discrete values, the uncertain data is described in terms of the probability distribution (e.g. Gaussian, log-normal). These uncertain variables are propagated through the process to the output variables. The aim is to integrate the available stochastic information in the optimisation problem.

Stochastic problems can be essentially divided into two different categories those which involved a sequence of decisions over multiple time periods (multistage problems) or a single time period (single stage).
Multistage approach seeks to find an optimal sequence of decisions over a certain period of time. This approach has been extensively used in long term strategic scheduling and planning problems. In mining this approach has received great attention in the last decade (Dimitrakopoulos and Godoy, 2014; Montiel and Dimitrakopoulos 2013). The uncertainty is modelled through geological conditional simulation.

Single stage aims to find a single optimal decision at a given point in time, which is more suitable to the context of the current Grade Engineering assessment. Two strategies are employed to solve single stage problems under uncertainty, two stage programming approach with recourse formulation and probabilistic or chance constrained programming. The first has been appropriately employed to solve planning problems under demand uncertainty while the second approach has been extensively used in production optimisation and process control (Li et al., 2008; Arellano-Garcia, 2006) and thus the method likely employed in this work. In the chance constrained approach, the system’s ability to meet a feasible solution in an uncertain environment is considered (i.e. the system’s reliability). Therefore, this technique enables the quantification of the compromise between profitability and reliability. The stochastic optimisation using chance constrained provides comprehensive information on the economic achievement as a function of the desired confidence level of satisfying process constraints, particularly important in robust decision making process.

7.5. Chance Constrained Optimisation

Chance constrained optimisation is one method of stochastic programming that attempts to reconcile optimisation over uncertain constraints indicating the profitability and reliability of the process which are quite crucial in decision making process (Charnes and Cooper 1959). Chance constraint method has been successfully applied to ascertain optimal design and operation, optimal production planning as well as optimal control of industrial process under uncertainty. This method is particularly effective to analyse the interplay between optimality and flexibility process under uncertain feed conditions (Mesfin and Shuhaimi, 2010; Sahinidis 2004).

Chance constrained problems can be divided according to the properties of the processes, uncertainty and constraint forms (Figure 2.12). The difference between joint and single constrained is that the former requires the reliability in the output feasible region as a whole, while single chance constraint demands the reliability in the individual output region. The use of single or joint chance constrained methods essentially depends of the problem under assessment. If the constraints
are related to the safety considerations of a process operation a joint constrained might be preferred. Single chance constraints are recommended when some outputs constraints are more critical than others.

![Diagram of process, uncertainty, and constraint classifications.]

*Figure 2-12. Classification of chance constrained problems (after Li et al., 2008).*

An important issue in solving probabilistic programming problems is the feasibility analysis. The feasible region is defined by the value of the specific confidence level applied to the constraints. This region will decrease as the confidence level increases. This interaction (optimal-constraints) can be obtained by determining the optimal solution for different confidence levels. Figure 2.13 shows the likely profit profiles in relation to the confidence level assigned. Each of the profiles (A, B, C) could represent a different operating strategy. The strategy A presents a more robust approach since the profit does not change rapidly with changes in the confidence level up to (a). However, between (a) and (b), the profit is compromised, suggesting that for A, the optimum operating point should be close to (a).

![Graph showing profit profiles vs confidence levels.]

*Figure 2-13. Profit profiles vs confidence levels (Li et al., 2008).*
7.6. Methods to Solve Optimisation under Uncertainty (Stochastic Optimisation)

Sample average approximation (SAA) has been employed given its relatively easy numerical implementation and good convergence properties (Shapiro, 2013; Pagnoncelli et al., 2009; Shapiro and Wardi 1996; Robinson 1996). SAA is a two part method that uses Monte Carlo sampling, which transforms the probabilistic optimisation into a deterministic optimisation (Section 5.1). Essentially the profit function \( F(x) \), Eq.(2) is approximated by the expected value \( f(x) \), Eq.(2) of the independent realizations \( \xi_i \), Eq. 2) defined by the probability density distribution utilised to characterise the uncertainty of the optimisation inputs. The right hand side of Eq.3 is deterministic, so deterministic optimisation solution methods can be used to solve the approximate problem.

\[
 f(x) = \max\{E(F(x, \xi_i))\} 
\]

\[
 f(x) = \max\left\{F(x, \xi) \approx \frac{1}{n} \sum_{i=1}^{n} F(x, \xi_i)\right\} 
\]

Figure 2.14 illustrates the classification of global optimisation methods. The exact methods are guaranteed to find an arbitrarily close approximation to a global optimum. The exact methods include Branch and Bound (BB), interval arithmetic and multi-start methods. BB is a kind of method for linear and nonlinear mixed-integer programming. If carried to completion, it is guaranteed to find an optimal solution to linear and convex nonlinear problems. It is the most popular approach in all commercial MILP (Edgar et al., 2001). BB is typically used for determining the optimum mining block sequence in mine scheduling problems (Newman et al., 2010). However, this requires a significant computational effort, since each block requires at least one binary variable representing the decision to mine or not mine the block (Smith 1998). The multi-start methods, on the other hand attempt to find a global optimum by starting the search from many starting points.
The heuristic search methods start with a current solution, then explores all solutions in the neighbourhood of the point to look for the best one. This is followed by repeats if an improved point is found. Metaheuristics algorithms such as Tabu Search (TS), Scatter Search (SS), Simulated Annealing (SA) and Genetic Algorithm (GA) can guide and improve the heuristic algorithm. Although these algorithms employ different techniques to ensure obtaining a global optimum, they share the same framework (Figure 2.15). SA (Kirkpatrick et al., 1983) is motivated by a thermodynamic cooling analogy. In these algorithms a “temperature” parameter is used to adjust the probability of accepting new points even though they do not improve the costs function avoiding being trapped by a local minima (maxima). SA algorithms have been extensively used in stochastic strategic mine planning where conditional geological simulation is used to account for grade variability/uncertainty (Montiel and Dimitrakopoulos, 2013).

GA methods (Goldberg, 1989), on the other hand are motivated by genetics. In this type of algorithm, new trial solutions are generated by crossover (i.e. randomly swapping elements in given vector trial solutions) or by mutation (i.e. randomly adding components to elements of trial solutions).

Nevertheless in the area of process optimisation, GA seems to be the preferred method (2.16). This has been successfully employed in areas of process control and plant design in comminution as well as the aggregate industry (Bengtsson et al., 2009, Svedensten 2007, Husband et al., 2006, Svedensten and Evertsson 2005, Contoni et al., 2000).
8. Conclusions

Size based Grade Engineering encompassing two levers: Preferential grade by size deportment and differential blasting for grade. The former refers to a natural based rock property whereby a significant metal proportion preferentially deports into specific size fractions after breakage. Differential blasting aims to change blast product fragmentation to foster grade by size deportment through the exploitation of deposit spatial grade heterogeneity characteristic. This relates the presence of spatial high grade and low grade discrete clusters within a certain production volume originally assigned to a single destination (e.g. waste, leach, and mill) based on its average grade.
Published literature available is focused on developing tools to identify and map size based Grade Engineering attributes and to demonstrate conceptually the feasibility of implementation. Therefore there is an evident lack of information regarding the economic as well as operating significance of exploiting size based Grade Engineering techniques. This comprises the use of thorough process models to take into account the non-linear interaction between rock based processing attributes and operating parameters. Therefore the methodologies developed by Carrasco (2013) focused on identifying and rank preferential grade by size response cannot be directly employed in this regards. Furthermore, a methodology for characterisation and data analysis to understand the interaction between changes in particle size distribution and preferential grade by size is required. Fragmentation could be potentially modified to enhance or suppress the magnitude of preferential grade by size response.

The deployment of size based Grade Engineering strategies will inevitably change mill feed particle size distributions due to the application of tailor-made blasting fragmentation in conjunction with Grade Engineering circuit operation to either exploit preferential grade by size deportment or spatial grade heterogeneity. Therefore the impact of mill feed grades will be accompanied by changes in comminution performance as several Mine to Mill studies have clearly demonstrated. This can render improvements up to 60% of mill throughput and therefore the possible synergies with size based Grade Engineering techniques need to be considered in the operating and economic assessment.

The exploitation of size based Grade Engineering also provide additional level of operating flexibility by providing the ability to manipulate yield responses and separation combinations to suit dynamic operating modes such as mill or mining constraints over time. Industries with significant level of flexibility (e.g. manufacturing, chemical, oil and gas) have coped with the associated complexity through the development of a robust “recipe” as a key outcome of the economic assessment. This is achieved by the application of optimisation under uncertainty or stochastic optimisation. Uncertainty in this context is employed to refer to inputs variability of the input lack of knowledge.

In the chance constrained approach, the system’s ability to meet a feasible solution in an uncertain/variable environment is considered (i.e. the system’s reliability). Therefore, this technique enables the compromise between profitability and reliability to be appraised. The stochastic optimisation using chance constrained provides comprehensive information on the economic achievement as a function of the desired confidence level of satisfying process constraints, particularly important in robust decision making process.
9. References


Chapter 3  Development of geometallurgical laboratory tests to characterize metal preconcentration by size

1. Abstract

Over the last 30 years the average grade of mined ore bodies has significantly decreased while the proportion of waste removal has in many cases more than doubled. This has been identified as prime reason of a major increase in energy consumption and decrease in productivity across mining operation. Metal coarse preconcentration of ROM feed grades by size rejects low grade gangue/waste material in the size range of 10-100 mm prior to grinding, typically before primary crushing. This relies on propensity of metal to preferentially deport in the fine size fractions after breakage. Metal preconcentration by size has the potential to increase energy efficiency and therefore improve mining productivity on unit of metal basis. Results of an extensive belt cut sampling campaign conducted at a major Australian Au-Cu mine operation show that for some samples between 90% of Au and 80% of Cu is in 40% of the mass at -50 mm fraction. This indicates that low grade coarse material can be removed from the circuit, doubling feed grade at half of the tonnage and therefore improve energy efficiency per unit metal produced. This response, however presents significant variability in the same order as other processing attributes such as impact hardness and flotation recovery. This has led to development of geometallurgical laboratory tests using blast hole chips and drill core facilitating mapping and population of coarse metal preconcentration signatures in long term resources and short term production models. Preferential Cu deportment by size at drilling scale can be recognised, however scales up factors are required to transform laboratory results to production responses. At 50% of mass the drilling products underestimate the belt cut responses by 20-30%. Bulk sampling campaign with blast hole and drill core samples spatially related was also conducted to investigate in more detail the difference between production scale and drilling scale preconcentration responses.

2. Introduction

The mining industry is facing several technical, economic, social and environmental challenges influencing mining profitability and sustainability (Bearman, 2012; Franks et al., 2012; Prior et al., 2012; Topp et al., 2008). The need of meeting metal demand at higher operating costs coupled with volatility in commodity prices are an increasingly important concern. This has turned out in mining companies withdrawing from new development projects and acquisition. Environmental and social regulations are becoming more resilient, demanding mining companies to improve their current sustainability standards (Franks et al., 2012; Prior et al., 2012).

However, population growth is expected to continue increasing urbanisation rates will place an increase demand on commodity production.
The mining industry has reacted to this complex scenario by reducing the cut-off grade material which converts more resources into reserves, increasing revenues but at higher production costs with marginal profitability improvement. This trend has been supported by high commodity prices which also have exaggerated capital intensity and labour inputs. The reason is higher prices make economically viable to mine deposits that would otherwise be uneconomic through resource depletion (Topp et al., 2008). However, lower head grades generate more mineable waste and increased processing tonnage to produce equivalent metal. As head grades continue to decline, production costs will continue rise making this strategy at long term unsustainable.

Decrease of mine head grades is partly a function of depletion of near surface, high-grade ore bodies which are not being replenished by exploration discoveries, and also a reaction to technologies that can support larger scale material movement and mineral processing. The net result is that for most metals while feed grades have declined over time and annual metal production has increased (Figure 3.1) ,(Access Economics, 2008).

![Figure 3-1. Copper production and run of mine copper grades since 1900 and introduction of mining technologies (Mackenzie, 2011).](image)

While the ability to exploit the economics of scale have enabled profitable utilisation of increasingly lower grade ores, evidence suggests limits to this type of exponential growth (Prior et al., 2012, Rendu, 2006). Bartos (2007) points out that the bulk open pit mining approach has been prompted by improvements in haulage technology with direct cost savings related to haul truck size between 1960 to 2005 (Figure 3.2). The trend indicates that further increase in truck size will not provide a significant economic advantage.
Early (prior grinding) gangue rejection has been identified as a feasible technical alternative whereby metal productivity and efficiency can be improved (Bearman, 2012; Logan and Krishnan 2012; Bamber, 2008a; Bamber et al., 2008b; Bamber et al., 2006a; Bamber et al., 2006b). A range of techniques and concepts can enhance and exploit coarse waste rejection and increase effective feed grades to concentrators. These approaches include 1) differential blasting, (Bye 2011); 2) sensor based sorting at particle and feed belt scales (Bearman 2012; Bamber 2008a; Bamber et al., 2008; Bamber, 2006b) and size by size screening to exploit preferential breakage and associated grade deportment (Logan and Krishnan 2012, Burns and Grimes 1986).

This work aims to describe and characterise the last alternative. Metal (grade) preconcentration by size relies on the inherent propensity of metal to preferentially deport into specific particle size fractions after breakage, phenomenon referred as preferential metal deportment by size (Carrasco 2013, Bamber, 2008a; Bamber et al., 2008b; Bamber et al., 2006a; Bamber et al., 2006b, Burns and Grimes 1986).

The Bougainville Cu-Au Mine in Papua New Guinea is the most relevant published example of production scale application of size by size metal/grade deportment. In the mid 1980’s in response to decreasing head grades and the economic outlook a comprehensive program of size by size grade assessment was undertaken on specific ore types to ascertain if low grade material could be screened out prior to comminution. This was prompted by the observation that for vein hosted ores, preferential breakage along mineralized vein structures was often evident after blasting.

Figure 3-2. Relationship between haulage saving costs and truck size between 1960 and 2005. All costs are in 2011 US dollars (after Rendu et al., 2006).
The results of the Bougainville study led to the commissioning of a preconcentration screening plant with a capacity of 35 million tonnes per annum (4000 tph). The <32 mm screened size fraction ~40% of the mass with a Cu-Au upgrade factor of 1.5 (Table 3.1). Preconcentration screening was focussed specifically on marginal grade ores with ~0.2% Cu and ~0.2 g/t Au resulting in an increase in mineable reserves and overall Cu production

Table 3.1. Proportion of reserves that are amenable for preconcentration by size. (after Burns and Grimes 1986)

<table>
<thead>
<tr>
<th>1999 Annual report</th>
<th>Reserves</th>
<th>Cu %</th>
<th>Au ppm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low grade as mined</td>
<td>520 Mt</td>
<td>0.22</td>
<td>0.18</td>
</tr>
<tr>
<td>Screened low grade product</td>
<td>195 Mt</td>
<td>0.34</td>
<td>0.28</td>
</tr>
<tr>
<td>Upgrade factors</td>
<td>38%</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Direct feed</td>
<td>496 Mt</td>
<td>0.45</td>
<td>0.55</td>
</tr>
</tbody>
</table>

Certain results from extensive belt cut sampling campaign conducted on a major Au-Cu mine shows that more than 90% of Au and 80% of Cu can be found in 40% of the mass at -50 mm fraction after blasting and primary crushing (Carrasco 2013).

This feed reflects the potential for doubling feed grade in approximately half the mass in concentrator feed stream. This has a positive impact on comminution performance as particles under 50 mm, commonly known as critical size in SAG mills, often do not broke, increasing the circulating load leading to drop in throughput capacity (Musa et al., 2011). When metal is preferentially deported across size fractions, grade does as well. Particles above 50 mm have lower grade than the finer size fraction, providing additional reason to reject the coarse fraction out the circuit.

Cu preconcentration signatures display same order of variability similar to other processing attributes such as impact hardness and grindability (Carrasco 2013). To enable operational exploitation, grade by size deportment should be mapped in the block model.

A Geometallurgical framework was utilized to develop the grade by size characterisation tests. Geometallurgy aims to provide constrained inputs into resource block models that reflect inherent geological variability and its impact on metallurgical performance (Keeney, 2010; Walters 2009; David, 2007; Bye, 2001). Traditional metallurgical testing often involves large scale composites to represent ‘average’ feed typically for circuit design and equipment sizing purposes.
Geometallurgical testing is designed to deliver much larger numbers of test results that can more effectively define variability and spatial domains. Geometallurgical tests are designed to be lower cost compared to full scale metallurgical testing and use less mass sample mass to avoid compositing due to test requirements (Keeney, 2010; Walters, 2009). The small scale tests to characterise preferential coarse Cu deportment by size are based on drilling products: blast chips and drill core.

3. Methodology

3.1. Preferential Cu grade by size, geometallurgy attribute definition

Work conducted at Bougainville indicated a remarkable variability on metal deportment by size in each ore type defined (Burns and Grimes 1986). The results of initial work were promising, indicating that majority of ore types showed a significant grade increase in finer ROM fractions <32 mm.

To describe preferential grade by size deportment, a diagram of cumulative weight passing (%) against cumulative metal recovery (%) for each size fraction was developed (Figure 3.3). In this plot, a response line at 45° indicates no preferential size by size metal deportment. The more the response moves away from this line stronger the grade by size response. The Bougainville diagram is a useful visualisation approach particularly for comparing limited number of samples. However, it does have limitations. The improvement in grade relative to feed (i.e. upgrade) when the coarse fraction is screening out is not evident on the diagram as well as particle size distribution.

![Figure 3.3. Bougainville diagram displaying average Cu responses for main six ore types (after Burns and Grimes 1986)]
This reason led to seeking an alternate approach to visualise preferential grade by size responses. In this case metal upgrade, which is the grade of screened (recovered) material relative to feed grade and cumulative weight % were considered. An upgrade metal of one means grade of screened material is identical to feed grade, whereas an upgrade of two indicates feed grade can be doubled by screening. This concept has been extensively used to relate quality and quantity of products of separation of given feed quality (Drzymala, 2006).

The following figure (Figure 3.4) illustrates belt cut samples, which cover the dynamic range of Cu grade by size responses.

![Figure 3-4. Upgrade Cu versus Cumulative Weight (%) samples that cover dynamic range of responses.](image)

These visual representations are useful when a limited number of samples need to be compared, however for a larger number of samples, often observed in geometallurgical testing, it is necessary to have a quick method for ranking responses that allows natural populations, with similar grade by size behaviour within the data to be identified.

This resulted in CRC ORE (Cooperative Research Centre for Optimising Resource Extraction) to investigate the extent of grade by size response for several deposits with different geological characteristics (Table 3.2). Intact drill core and coarse reject material typically at 6 mesh (~3 mm) were used in this regard. Samples were sieved at certain size fractions and sent for multi-element assay. Data regarding lithology and alteration provided allowed to confirm the geological variety of samples selected.
The analysis of this information supplied enough confidence to mathematically mimic the shape and the magnitude of preferential Cu grade by size. The parameters of the function are used to rank the preconcentration by size amenability, similar to the A and b parameters, which are employed to describe impact hardness for comminution.

Table 3.2. Cu porphyry CRC ORE data base.

<table>
<thead>
<tr>
<th>Deposit type</th>
<th>Sites Tested</th>
<th>Number of samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cu-Au Porphyry deposit</td>
<td>5</td>
<td>+300</td>
</tr>
<tr>
<td>Cu-Mo Porphyry deposit</td>
<td>5</td>
<td>+400</td>
</tr>
</tbody>
</table>

Figure 3.5 represents the normal probability plot of Cu grade by size ranking response values obtained while Figure 3.6 depicts the average particle size distribution of the belt cuts, with a P80 close to 100 mm. The ranking information was divided to three groups based on populations naturally identified within drilling product response (Section 4). Then, to visualize the upgrade-cumulative weight% behaviour an average response per group was created by weight compositing the samples related.

Figure 3.7 shows the upgrade by weight % per group defined, with A displaying the strongest and C the weakest grade by size response. For the C group when 40% of the mass is accepted, i.e. screening at 30 mm (Carrasco 2013), head grade can be improved by 1.6 whereas for A group the upgrade factor is close to 2.3.
Figure 3-5. Probability normal ranking responses belt cuts data.

Figure 3-6. Average particle size distribution belt cut data %.
4. Geometallurgical tests protocols for recognising preferential metal deportment by size using drilling products

As previously described belt sampling showed significant variability of preferential Cu deportment by size, enabling for coarse preconcentration application through rejection coarse low grade barren material prior to concentrator.

Identification of variable grade by size response and incorporation into mine planning requires that this response attribute should be defined through testing in order to embed into the resource block model. This requires development of a new geometallurgical/metallurgical type laboratory test that can be applied to drill core or blast hole chips. The tests must:

- Be rapid and relatively low cost to provide adequate data that can be used to rank and compare grade by size responses. This supports first-pass mapping and domaining. Lower precision and accuracy is typically offset by larger numbers of spatially related data points to identify trends.

- Provide information to calibrate and scale up models that can be used to transform the results of small scale laboratory tests into parameters suitable for production scale application including circuit optimisation and design.
4.1. Blast hole samples

Blast hole chips are commonly used in grade control as part of short term resource modelling typically 1-2 days before benches are mined. The use of blast hole samples provide an opportunity of using size by size grade distribution as proxy for coarse metal grade by size signature. However, given the large number of blast hole samples per bench and the need to generate valuable information for production in short time, the tests protocols should be relatively low cost and rapid.

The methodology to develop protocols for geometallurgical testing using blast hole samples was divided into two phases:

- Investigation of coarse Cu distribution and mass per size
- Rank and assess Cu deportment by size.

These phases aim to identify whether preferential Cu deportment by size can be recognised, despite the difference in size scale with production (belt cuts).

A procedure for mass sieving was developed based on optimising sample mass per fraction and applying Cu sampling statistics to ensure the integrity of splitting (Carrasco, 2013). The final procedure (considering an initial mass of ~5 kg available) involves an initial split at +/-4.75 mm fraction with sieving of the coarser fraction (+4.75 mm) into 3 sizes classes:

- +9.5 mm
- -9.5+6.7 mm
- -6.7+4.75 mm

The -4.75 mm fraction was then split, taking 50% of the total mass. This percentage was obtained using Gy’s theory (Gy 1982) employing K sampling values proposed for low Cu ore grades (Bartlett and Viljoen 2002). However, it is recommended to estimate K sampling factor for a particular ore, through a sampling experiment (Minnitt et al., 2007), to obtain fundamental sampling error related with metal grade, size, and amount of split mass required per sample (Appendix A provides Gy’s sampling theory background and the utilisation of a sampling nomogram to determine the optimum sampling volume).

The split sub sample taken from the -4.75 mm fraction is sieved at the following fractions:
-4.75+1.18mm
-1.18+0.3mm
-0.3mm

Each of the size fractions was then assayed. The protocol is illustrated in the Figure 3.8 and associated particle size distribution in Figure 3.9.

Figure 3-8. Blast hole protocol for coarse Cu deportment characterization using blast hole samples.

Figure 3-9. Particle size distribution blast hole samples, each line represents an unique sample.

Preferential Cu deportment by size is evident at blast hole scale. Figure 3.10 depicts the blast hole ranking values, where +90% of the data has values greater than 1. Nevertheless, there is one population clearly noticeable that represents the highest Cu deportment signature (located above the~ 80th of the distribution of responses). This population can be related with an area of the
deposit, characterized by stock work veining style embedded in a highly competent rock, whereas the rest of population can be linked with softer weather material. Blast hole sample analysis predicts a lower response to Cu preconcentration by size compared to belt cuts.

![Figure 3-10. Normal probability plot response ranking Cu values belt cut and blast hole samples.](image)

### 4.2. Drill core samples

Diamond drill core sampling is typically available in the feasibility mining phase and is the dominant material available for analysis. Drill samples are mainly used to develop longer term resource models. Drill core provides a more constrained (relative to blast hole data) and coherent opportunity to study the relationships between grade deportment and fundamental geological controls on breakage (Ferrara et al., 1989; Laslett et al. 1990; King 1993; Bocjevski et al. 1998, Esterle et al., 2002; Schneider et al, 2003). This facilitates the ultimate link between Cu deportment by size and the intrinsic rock characteristics, which can be measured using a range of tools available for this purpose, like core scanning devices or visual logging.

Grade by size Cu characterisation using drill core samples was divided into two phases: Investigation and Rank/Assess ranking responses.

Drill core samples were crushed to get 100% passing 3.35 mm (Figure 3.11). This involves, the use of jaw laboratory crusher, setting the close gap setting at 3 mm, drill core is then crushed and sieved, removing the coarse fraction (+3.35 mm), which is crushed again (Figure 3.11). This
process is repeated until 95% of the total mass is located within -3.35 mm fraction. Figure 3.12 depicts the resulting particle size distribution of the samples tested.

There are a number of advantages of using a -3.35 mm feed. In addition to a more optimized mass by size distribution (Carrasco 2013), this top size is commonly used by laboratories to undertake splitting of smaller masses for pulverisation prior to assays. The resulting ‘coarse residues’ are typically stored and would be available as an archive to apply size by size grade deportment testing. A -3.35 mm feed preparation is also a standard feed preparation for Bond Rod and Ball Mill Work Index testing (Bond 1952).

Figure 3-11. Standard crushing 100% passing -3.35 mm protocol drill core samples.

Figure 3-12. Particle size distribution drill core samples, each line represents an unique sample
Samples covering a wider range of geological characteristics, such as vein density, lithology and alteration were selected (Figure 3.13). The samples are crushed and the product is sieved at 6 size fractions (-3.35 +2.36 mm, -2.36 +1.7 mm, -1.7 +1.18 mm, -1.18 +0.6 mm, -0.6 +0.3 mm and -0.3 mm) and then sent for assay. The number of size fractions and associated assay costs as well as in blast hole testing, can be reduced compared to this investigation approach. Preferential Cu deportment at coarse scale can be distinguished on drill core samples (Figure 3.14).

Figure 3-13. Example of mineralisation styles encounter in drill core testing.

These results follow the same trend observed in blast hole and belt cut samples (Figure 3.10). There is a clear group above approximately 80th percentile, which geological characteristics are similar to the blast hole group, stock work carbonate vein embedded in a hard matrix of siltstone. This observation points out that texture and mineralisation have fundamental role in how grade is distributed by size.

Figure 3-14. Normal probability plot ranking values Cu belt cut and blast hole samples.
5. Comparative preferential coarse Cu deportment at different scales

To understand what the differences in ranking mean in terms of the upgrade by cumulative weight% curve, the belt cut ranking data was divided into three groups based on the preconcentration by size populations obtained at drilling scale. Normal probability plots are commonly used to investigate whether the data exhibit a Gaussian distribution. Therefore, changes in inflections and slopes on the probability plot can be used to identify processing populations and then with spatial location, domains. Figure 3.15 depicts how the groups were selected, meanwhile Figure 3.16 and 3.17 illustrates the upgrade by size response curves for drill core and blast hole respectively.

Figure 3-15. Group Cu deportment definition across different scales belt cuts, drill core and blast hole.

Blast hole and drill core responses are remarkable similar, even though it was expected, given the similarities in the ranking and its distribution (Figure 3.15). Those signatures are consistently lower than production scale (Figure 3.18, 3.19, 3.20), where this difference increases at weight recoveries lower than 50%.
Figure 3-16. Upgrade Cu versus Cumulative Weight %, drill core.

Figure 3-17. Upgrade Cu versus Cumulative Weight %, blast hole.
Figure 3-18. Upgrade Cu versus Cumulative Weight % A group belt cuts, drill core and blast hole.

Figure 3-19. Upgrade Cu versus Cumulative Weight % B group belt cuts, drill core and blast hole.
6. Bulk Sampling Testing

Bulk sampling testing was conducted to investigate and validate scale up factors from drilling scale to production. Blast holes were primarily taken covering 5 benches. One bench contained 5 drill core samples of 4 meter each, which is spatially related to the SF SP2 180 (1) bulk sample. Cu deportment by size was ranked using blast hole information, indicating a strong spatial coherence (Figure 3.21). Each bulk sample, of ~200 Kg was crushed and screened at several sizes (Top size 125 mm root series up to 0.3 mm). Each size fraction was sent for multi-element assay. Both bulk samples responses are different (Figure 3.22), as blast hole samples predicted (Figure 3.21), indicating that blast hole data can be used to define short term grade by size responses.

Cu deportment by size was then compared at different scales using blast hole and drill core samples spatially related with SF SP2- 180(1). Average drilling product grade by size response was plotted against the bulk sample response (Figure 3.23). These consistently underestimated the production response across all size fractions. This pattern was already evident even for data that was not spatially related with drilling products (Section 4).

Table 3.3 shows comparison of upgrade by size across the different size scale using 50% weight as reference point to understand the magnitude of the difference.
Figure 3-21. Bulk, Drill core, blast hole blasting sampling campaign. X and Y spatial coordinates.

Figure 3-22. Cu Upgrade versus Cumulative Weight% for Bulk samples tested.
Figure 3.23. Upgrade Cu SF SP2 180 (1) bulk sample and the blast hole and drill core response related.

Table 3.3. Comparison of upgrade by size responses across different scales.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Weight (%)</th>
<th>Cu Upg factor</th>
<th>Scale up factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bulk Sample</td>
<td>50</td>
<td>1.61</td>
<td>1</td>
</tr>
<tr>
<td>Blast Hole Sample</td>
<td>50</td>
<td>1.35</td>
<td>1.18</td>
</tr>
<tr>
<td>Drill Core Sample</td>
<td>50</td>
<td>1.26</td>
<td>1.27</td>
</tr>
</tbody>
</table>

7. Conclusions

 Preferential Cu deportment by size is evident for diverse geological set of samples at drill core and blast hole scale selected at Au-Cu mine and can be identifying using existing laboratory crushing procedures.

Testing protocols can be easily incorporated into routine sample preparation work flows including geochemical assays and selected metallurgical tests such as feed preparation for batch flotation and Bond Work index tests.

The number of sieve sizes used to assess size by size deportment is an important consideration for routine application cost. While large number of size fractions have been used in the current
development study the outcomes indicate that these can be reduced and streamlined for routine application.

Variability of preferential size by size grade deportment resulting from applications of these test protocols using drill core and blast hole chips shows the same dynamic range in variability as belt scale measurements.

A supporting integrated data analysis methodology has been developed which allows direct comparison and ranking of size by size data for different samples types at different size scales (belt cuts-drilling products) and different size distributions.

The data indicates that grade by size response from testing drill core and blast hole chips is consistently lower than the response measured at belt cut and bulk scale.

The spatial coherence of grade by size using blast hole samples indicates the potential of using these samples for populating short term production models, such samples can then be used in addition of the common grade control.

Further work is required to understand the differences between production and drilling in terms grade by size responses, as well as the economic and processing consequences in mine operations.

8. References.


Development of a novel methodology to characterise preferential grade by size deportment and its operational significance

1. Abstract
Over the last 30 years the average grade of ore bodies has significantly decreased while the proportion of waste removal has in many cases more than doubled. This in turn has led to a major increase in energy consumption and decrease in productivity across mining operations. Metal preconcentration at coarse scale (10-100 mm) by screening has the potential to reverse decreasing mining productivity trends through early rejection of uneconomic grade material prior to energy intensive comminution. Metal preconcentration of feed grades using screening exploits the propensity of certain ores to preferentially deport metal into specific size fractions during breakage. This phenomenon is referred as preferential grade by size deportment. The exploitation of preferential grade by size response involves generation of multiple streams with different metal content post screening. Streams can be engineered for different grade characteristics suitable for different processing destination (eg: as waste, leach, and mill). Preferential grade by size data obtained by an extensive belt cut sampling campaign after primary crushing has been used to develop a method to define samples that are amenable for metal preconcentration by size. This amenability changes depending on cut-off grade, magnitude of preferential grade by size response and the proportion of mass contained in individual screen products. Outcomes of this work will support the short term preferential grade by size operational implementation.

2. Introduction
The mining industry is facing a range of economic, technological, social, and environmental challenges all impacting on productivity and sustainability (Bearman, 2012; Franks et al., 2012; Prior et al., 2012). A key components of the economic and technological challenges are an ongoing decrease in feed grades of base and precious metal mining operations together with a need to process more complex ores (Mudd, 2009; Topp et al., 2008; Mudd, 2007). This is partly a function of depletion of near surface, high-grade ore bodies which are not being replenished by exploration discoveries, and increasing reliance on technologies that can support larger scale material movement and mineral processing efficiency. The net result is that, for most metals, while feed grades have declined over time the annual production of metal has dramatically increased as a function of increased demand (Access Economics, 2008).

While the ability to exploit the economics of production scale have enabled profitable exploitation of increasingly lower grade ores, there is evidence suggesting economic limits to this type of exponential growth. This is coupled with constraints on associated infrastructure such as power and
water, together with an increasing requirement to minimize greenhouse emissions and adopt more socially responsible practices (Prior et al., 2012; ABARES, 2011; Franks et al., 2010).

Within Australia, multifactor productivity is used as a measure of the efficiency of capital, labour, materials, services and energy that are utilized to generate a unit of product. Since 2001, according to the aforementioned measure, there has been a consistent decline in productivity in the minerals industry. For mining in Australia it now takes 40% (2000-2001 indexed as 100%) more input to generate a single unit of mineral product (Topp et al., 2008). Similar but less pronounced trends are evident for other countries (Topp et al., 2008).

A significant proportion of the drop in multifactor productivity is attributed to decreasing head grades. Removing the influence of decreasing head grades upon multifactored productivity reflected an overall increase of 2.5% per annum over the previous period of decline (Topp et al., 2008).

The effect of decreasing head grades is to increase energy consumption (Figure 4.1) and therefore unit metal cost of production (Noergate and Haque 2010; Norgate and Jahanshani 2010; Norgate et al., 2007). Lower head grades requires more comminution and grinding to effectively liberate the metal contained in the rock.

![Figure 4-1. Effect of ore grade and grind size on embodied energy copper production during concentrating and smelting (Norgate and Jahanshani 2010).](image)

More than 50% of energy consumed in a typical base and precious metal mining operation can be accounted for in crushing and grinding circuits feeding into conventional flotation recovery (Powell and Bye 2009). As feed grades continue to decrease much of this energy is directed towards inefficient liberation of dominant gangue components at a P80 of generally <150 microns and in some cases <50 microns. To overcome this trend, the mining industry needs to focus on finding
new technologies and operational strategies to increase extraction efficiency and decrease unit metal energy intensity.

For low feed grades early coarse uneconomic material rejection (~10-100 mm) has been identified as an important operational advance which could increase unit metal productivity and efficiency (Carrasco et al., 2014; Bowman and Bearman 2014; Carrasco, 2013; Bearman, 2012; Logan and Krishnan 2012; Bamber, 2008a; Bamber et al., 2008b; Bamber et al., 2006a; Bamber et al., 2006b).

Metal preconcentration of feed grades using screening is based on the propensity of some ores to preferentially deport metal in specific size fractions. Figure 4.2 depicts a belt cut sample where Au grade varies across the size fractions analysed. Although this sample is defined as waste (feed grade 0.26 ppm < cut-off 0.3 ppm) there are size fractions that could be classified as ore. This phenomenon is referred as preferential grade by size deportment. This is a function of the competence difference between the host lithology and mineralisation structure coupled with breakage energy to condition the feed material (Carrasco, 2013).

![Figure 4.2. Belt cut grade by size raw data in an operation with a waste-ore cut-off of 0.3 ppm. There are certain size fractions that can be classified as ore.](image)

Although preferential grade by size deportment is widely recognized and accepted, there is limited published data on the nature and magnitude of preferential deportment response and its potential economic significance.

An extensive Semi-Autogenous Grinding (SAG) mill feed belt cut sampling campaign carried out at Telfer Au-Cu mine in Western Australia indicated that preferential Au-Cu deportment in primary crusher products can generate new reject waste streams by screening at coarse sizes (10-100 mm)
(Bowman and Bearman 2014, Carrasco et al., 2014, Carrasco, 2013). In some cases >90% of the Au is contained in <40% of the mass below 50 mm. This has major implications for increasing unit metal productivity and profitability due to the opportunity of rejecting low grade coarse material prior to comminution. This requires the development of a new set of enabling tools and concepts to facilitate integration of dynamic metal preconcentration by size streams into process control and mine optimisation. The current work presents a novel methodology to analyse preferential grade by size responses within an operational context and provides a framework for economic evaluation.

3. Preferential grade by size deportment ranking

As with any other metallurgical parameter, exploitation and optimisation of preferential grade by size deportment in a production environment require characterisation and quantification.

An example of relationship between particle size distribution and preferential grade by size response is illustrated in Figure 4.3. 40% of the total mass stream is contained at -50 mm fraction at this defined particle size distribution (PSD) (Figure 4.3a). Figure 4.3b depicts the associated preferential grade by size deportment yield response. This is defined as a function that relates metal upgrade and the proportion of mass contained at specific size fractions. Metal upgrade is defined as the ratio between the grade of the size fraction retained and feed grade. For the mass pull shown in Figure 4.3b (-50 mm) accept mass fraction is 1.7 times the feed grade (1.7 metal upgrade).

The shape and the extent of this curve is used to estimate the propensity of metal to preferentially concentrate into finer particles during breakage.

![Figure 4-3. a) Particle size distribution. b) Preferential grade by size deportment response.](image-url)
Based on extensive analysis of preferential grade by size data (Carrasco, 2015) a mathematical model was employed to describe the preferential grade by size deportment curve depicted in Figure 3b, (Eq.1).

\[ U_{pg} = \frac{K}{1 + W \times (K - 1)} \]  

(1)

Where $U_{pg}$: Metal upgrade; $W$: Mass pull; and $K$ which describes the extent of preferential grade by size deportment response. Higher $K$’s display greater preferential grade by size response. $K=1$ means no grade by size response, whereas $0<K<1$, metal is concentrated into the coarse fractions.

This model was selected given its mathematical simplicity (one parameter needs to be fitted, $K$) and the high degree of statistical confidence. Figure 4.4 shows a normal probability plot, the low relative standard deviation (RSD) of $K$ values using the aforementioned model (Eq.1) and Telfer SAG feed mill sampling campaign data as example. A similar approach is used within metallurgical testing to characterise impact hardness, $A_xb$ parameter where $A$ and $b$ values are obtained by fitting an exponential function that relates $t_{10}$ and applied specific energy (Napier-Munn et al., 1996).

![Normal probability plot relative standard deviation K values obtained by using the equation outlined using Telfer Au-Cu mine data.](image-url)
Telfer preferential Au grade by size deportment response (K) was categorised in 4 quartiles due to significant variability of preferential grade by size responses (Figure 4.5). The A group comprises the upper quartile and the group with strongest grade by size footprint with D group the lowest quartile and therefore the lowest preferential grade by size response.

**Figure 4-5. Normal probability plot showing Au ranking response quartiles.**

Average grade by size response curves by quartile based on Au are shown in Figure 4.6, 4.7 and 4.8 for Au, Cu and S respectively, to show the level of statistical confidence as well as the flexibility of the model to describe preferential grade by size responses for Cu and S (Figure 4.7, Figure 4.8).

Cu preferential grade by size is comparable since response curves are very close together (Figure 4.7). Conversely S preferential grade by size curves depicts the similar extent of variability of Au (Figure 4.6) as well as same ranking pattern based on Au responses (from A to D, preferential grade by size responses decrease). This is confirmed by comparing K values for Au and S, displaying a $R^2 = 0.88$ at 95%. This reflects the degree of mineral association.

K values per group/quartile defined based on Au preferential grade by size response as well as RSD can are shown in Table 4.1
Figure 4-6. Upgrade Au versus Cumulative Weight% group A,B,C,D.

Figure 4-7. Upgrade Cu versus cumulative weight% group A,B,C,D.
Table 4.1. Preferential grade by size ranking response (K) per elements and RSD.

<table>
<thead>
<tr>
<th>Ranking based on Au</th>
<th>Au</th>
<th>Cu</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>K</td>
<td>RSD %</td>
<td>K</td>
</tr>
<tr>
<td>A</td>
<td>10.9</td>
<td>1.0</td>
<td>3.9</td>
</tr>
<tr>
<td>B</td>
<td>6.7</td>
<td>1.2</td>
<td>4.3</td>
</tr>
<tr>
<td>C</td>
<td>3.8</td>
<td>1.5</td>
<td>3.5</td>
</tr>
<tr>
<td>D</td>
<td>1.8</td>
<td>1.8</td>
<td>3.4</td>
</tr>
</tbody>
</table>

4. Preferential Grade by Size Exploitation Diagram

Modelling of preferential grade by size response facilitates operational options for implementation of coarse separation/metal preconcentration of ores where a positive economic benefit is evident. Preferential grade by size exploitation provides the opportunity to obtain two or more streams having different metal content, particle size distribution and different processing destinations, compared with single destination feed. Grade of new streams is a function of head grade, K value and user define proportion of mass accepted pull by screening.
The simplest case involves a single cut-off and two possible processing destinations (waste/leaching, mill/leaching, mill/waste). Metal preconcentration by size applied to material containing grade above defined cut-off will have to generate a reject stream (coarse fraction) with grade lower than cut-off. Material with metal content below the defined cut-off will have to produce an accept grade above cut-off to justify the application of metal preconcentration by size.

Figure 4.9 depicts the relationship between grade of the accepted and rejected streams with mass pull (%) for two samples with identical preferential grade by size response (K) but different head grade. Using a waste/ore cut-off grade of 0.3 ppm, sample “a” clearly shows that regardless of selected mass pull, the reject stream cannot generate a grade lower than 0.3 ppm, meaning that coarse separation (metal preconcentration by size) is not a valid option. For sample “b” when mass pulls higher than ~30% are applied, a new reject stream is generated (70% of mass) that can be reclassified as waste while increasing the grade of the accepted stream (~0.8 ppm).

Significant preferential grade by size deportment information coupled with its significant variability makes it unpractical to visualize and thus effectively assess metal preconcentration by size potential using the approach depicted in Figure 4.9. This is highlighted when significant variability in feed grade is presented.

![Figure 4.9](image_url)

*Figure 4.9. Comparison between two samples with different head grade (a=1.1 ppm; b=0.3 ppm) but identical grade by size response.*

The primary objective of metal preconcentration by size is to find the optimum combination among head grade, K and mass pulls that allows for the production of at least two streams with different destinations while maximising net revenue. This will determine the proportion of the ore body that
is amenable for coarse metal preconcentration by size, which can lead to sizing equipment and flowsheet estimation.

The operational limits for the application of metal preconcentration by size can be expressed mathematically by means of a mass balance. For material with grade above cut-off, the reject grade stream is given in Equation 2.

\[
\frac{X_f \times (1 - W \times U_{pg})}{(1 - W)} = X_t \leq \alpha
\]  

(2)

Whereas for material below the cut-off grade:

\[
X_f \times U_{pg} = X_c \geq \alpha
\]

(3)

Where \(X_f\): feed grade; \(X_c\): accept grade; \(X_t\): reject grade; \(\alpha\): cut-off grade

Figure 4.10 intersect these constrains (Eq.2 and Eq.3) and the preferential grade by size deportment characterisation data (Figure 4.5) where ranking response (K) is plotted against Au feed grade.

The right side region in faint red is defined by Eq.2, whereas the left side, in faint blue is characterised by Eq.3. The area contained by these two boundaries (Eq.2 and Eq.3) provides the operational limits for exploitation of preferential grade by size deportment. As K decreases, so does the exploitation grade by size potential. It can be noticed that for a mass pull of 50 % and a cut-off grade of 0.2 ppm more than 90% of the group “A” samples are amenable for metal preconcentration by size while 100% of total preferential grade by size information related with group “D” falls outside of the exploitation region and therefore is not amenable for metal preconcentration by size.

Figure 4.10 can be used to estimate the metal preconcentration by size limits determined by cut-off and mass using mass balance constrains and grade by size characterisation data underneath.
Figure 4.10. Grade by size exploitation diagram, K versus Au grade for 4 preferential grade by size groups at 50 % mass pull and 0.2 ppm cut-off grade.

This limits are sensitive to changing mass pull as Figure 4.11 depicts (20, 50 and 80% mass pulls at cut-off of 0.2 ppm) making preferential grade by size exploitation a dynamic operational lever. Changes in cut-off moves the area horizontally without affecting the shape of the exploitation region but affecting the proportion of the resource that is amenable for metal preconcentration by size when preferential grade by size deportment characterisation data (K and grade) is overlayed.

As the mass pull decreases the amenability zone above cut-off collapses, conversely the amenability zone below cut-off (Figure 4.11). Figure 4.12 displays accept and reject grade stream at given mass pull and K for a sample with a grade higher than the cut-off. Initially, for a defined K value head grade (above cut-off) and mass pull combination the sample is outside of preferential grade by size exploitation area. This is due to grade of reject stream being above the cut-off and therefore its destination has not changed (Figure 4.12). By increasing the mass pulls as Figure 4.9 illustrates, the grade of reject stream decreases and sample is now amenable for metal preconcentration by size.

For samples with head grade lower than cut-off, lower mass pulls, i.e. higher upgrade factors are required to produce a accept grade stream above cut-off (Figure 4.13). In the preferential grade by size exploitation diagram the intersection of K value, Au grade coordinates and mass pull line, defines the mass pull where accepted (for grade material lower than cut-off) and rejected stream (for grade material higher than cut-off) is equal to the cut-off.
Figure 4.14 illustrates the application of preferential grade by size exploitation plot to large bulk samples using preferential grade by size characterisation bulk samples data from a Au/Cu porphyry deposit. This is a different site from where the SAG belt cuts samples were used to develop the plot. Preferential grade by size responses are consistent within the major ore types. Ore type “F”, presents lower grade by size amenability, having the lowest preferential grade by size response (K). Rock type “D” falls outside the grade by size exploitation region, however with the same K response and higher grades, “D” would be potentially amenable for metal preconcentration by size, which indicates that a further sampling campaign around the grade by size exploitation region will be required. This methodology aids sampling strategies for identifying exploitable metal preconcentration by size opportunities.
Figure 4-13. Effect of mass pull upon accept and reject grade stream at given K, head grade (red) below cut-off grade.

Figure 4-14. K ore types bulk sample values versus Au grade for a Au/Cu porphyry

5. Grade by Size Metal Exchange

Preferential grade by size exploitation also allows to metal exchange across different processing destinations. This additional operational flexibility is accentuated for operations with multiple processing destinations. Figure 4.15 illustrates the different metal exchange options for a sample with the opportunity of producing a high grade and low grade stream when 3 destinations are
considered (waste, leach and mill feed). In this example 0.2% Cu grade defines the waste and leach material, while > 0.4% direct mill feed. Depending of the mass pull selected, this sample (head grade 0.3% Cu) can produce a high grade mill feed and leachable stream with lower mass pulls (Region 1, Figure 4.15), an increase in mass pulls will produce a mill feed stream while the reject stream is no longer considered ore (waste) (Region 2, Figure 4.15) meanwhile with mass pulls higher than ~62% will generate a leach as well as waste material (Region 3, Figure 4.16). As it can be noticed the mass pull is a key parameter and determines to a great extent the grade by size “metal exchange” potential.

Figure 4.16 shows the application of preferential grade by size exploitation diagram to compare multiple preferential grade by size responses within multi-cut-off systems. The data underneath was obtained from a drill core preferential grade by size characterisation program showing two main production rock types, “A” and “B”. The use of this plot identifies distinctive preferential grade by size exploitation zones, which support metal exchange by size application: 1,2,3,4 and 5.

![Figure 4.15. Sample capable of producing streams with different processing destinations depending of mass pulls selected. Region 1, mill and leach streams; Region 2, mill and waste streams; Region 3, leach and waste streams.](image)

- **Zone 1**: Area where a given combination of head grade and preferential grade by size response can be upgraded to “Ore” at given mass pull.
- **Zone 2**: Material initially sent to Leach can produce a reject stream classified as “Waste” whereas the accept stream is still leachable material.
- Zone 3, a “zero zone”, where preferential grade by size exploitation does not produce two streams with different processing destination.
- Zone 4, where Leach material can be upgraded by screening, producing a mill feed stream.
- Zone 5, material originally assigned mill destination through preferential grade by size exploitation can produce a reject stream that can be sent to leach.

This diagram can be also be used to envisage the potential for metal exchange opportunity by exploiting preferential grade by size responses. In this example, a significant proportion of rock type “B” is classified as Waste, whereas rock type “A” depicts higher feed grades. This diagram indicates it is feasible to upgrade rock “B” from Waste to Leach while a significant proportion of rock type “A” that is going to mill, can produce a reject stream that can go to Leach, improving feed grades to concentrator.

![Figure 4-16. Preferential Grade by Size Metal Exchange Visualizer.](image)

6. Conclusions

Metal preconcentration by size through exploitation of preferential grade by size deportment provides an additional level of operational flexibility. This can be used to dynamically manage operational bottlenecks over the life of mine with less reliance on major capital investments. This seeks to reject low value material as early as possible within the production chain. This leads to
significant unit metal productivity gains through improving effective feed grades and decreasing energy consumption per unit metal produced as well as reduction of water and reagent consumption.

The extent of preferential grade by size responses has been mathematically characterised by the “K” parameter. The higher K value the stronger the preferential grade by size response and therefore more metal is concentrating into finer particles.

Metal preconcentration by size amenability assessment requires a rigorous preferential grade by size deportment characterisation with a carefully planned sampling campaign to identify the more amenable domain/rock types for preconcentration by size. A bulk sample campaign is required to assess preferential grade by size responses and ultimately metal preconcentration by size potential.

The preferential grade by size exploitation diagram combines processing destination limits and preferential grade by size characterisation information enabling rapid identification of the application of metal preconcentration by size while supporting sampling campaign programs.

Mass pull is a key metal preconcentration by size modifier. Variability of K values coupled with head grades and processing options will require a more customised particle size distributions for a given screen aperture leading to a more rigorous process control specially around blasting fragmentation.

Within the current practice a portion of material is assigned to a fixed processing delivering on the basis of average grade. Metal preconcentration by size enables new feed streams which can be distributed to multiple destinations aiming to maximizing value.

Metal preconcentration will increase unit metal productivity by reducing amount of low grade uneconomic material that is sent to energy intensive and poorly efficient grinding. The exploitation of preferential grade by size deportment response also removes the coarse competent material altering particle size distribution which can potentially exploited to increase mill throughput. Metal preconcentration at coarse scale represents a disruptive challenge to conventional resource to reserve definition and scheduling, requiring a whole of system optimisation approach to understand the value drivers.
7. Acknowledgments

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8. References


Chapter 5      Managing Uncertainty in a Grade Engineering® Industrial Pilot Trial

1. Abstract

Preferential grade by size responses, one of the Grade Engineering® coarse separation levers, aims to remove low grade uneconomic material through screening prior to energy intensive and inefficient grinding. Deposit amenability to this coarse preconcentration technique requires an integrated characterisation program involving a carefully designed sampling strategy. A key aspect within this process is the preferential grade by size industrial pilot trial. This paper outlines the screening of 40,000 tons of Run of mine (ROM) material from a world class Cu-porphyry deposit from an area identified as amenable to coarse preconcentration by size based on geometallurgical characterisation, and investigates three sources of uncertainty upon preferential grade by size pilot trial. Screen efficiency, fundamental sampling errors and the mathematical model employed describing preferential grade by size response are analysed. This methodology recognises the difference between uncertainties associated with preferential grade by size response measurement and variability related to intrinsic geological characteristics. This novel approach aids the optimisation and development of coarse separation control strategies through the understanding of the extent, variability and uncertainty of metal deportment inputs.

2. Introduction

Grade Engineering® refers to a range of integrated strategies aiming to improve feed grades by removing low grade uneconomic material prior to energy intensive and inefficient grinding. Preferential grade by size responses can be used to achieve coarse pre-concentration via screening. This is a natural based rock property whereby a significant metal proportion preferentially deports into specific size fractions after breakage (Carrasco et al., 2016a; Carrasco et al., 2014; Carrasco 2013). This phenomenon can be modelled through a ranking response parameter, “RR”, which mathematically describes the relation between metal upgrade ($U_{pg}$) (i.e. ratio between cumulative undersize material grade and feed grade) and cumulative undersize weight ($CW$) (i.e. proportion of material in the undersize) (Carrasco et al., 2016a; Carrasco et al., 2016b); see Figure 5.1. Samples are screened in defined size fractions to determine the proportion of mass as well as grade within each size class to then determine the $U_{pg}$ and $CW$, both being input variables in Equation 1. Carrasco et al. (2014) summarise the test work protocols and associated data analysis methodology, which has been applied to drill core, blast holes and bulk samples. As Equation 1 illustrates, the RR parameter is essentially the slope of $U_{pg}$ and $CW$ in log-log space, where $\phi$ is a constant factor.
In order to understand the extent of this response, an extensive and careful variability sampling program at drilling scale is required. Either intact drill core or coarse assay reject composites can be employed. However, experience to date clearly indicates that scale up factors need to be applied to transform drill core preferential grade by size responses (typically with a top size of ~3 mm) to production scale (ROM/SAG feed material) (Carrasco et al., 2014; Carrasco 2013). Therefore, the validation of preferential grade by size responses (described through RR) at industrial scale is crucial within the Grade Engineering® assessment. The industrial trial, involves the installation of a screen plant, in which samples are collected to obtain grade by size data to determine an RR value and the extent of scale up factors required. In order to truly identify coarse size based rejection potential, a deep understanding of the several sources of error/uncertainty during its validation at industrial scale is required (Figure 5.2). This paper investigates three sources of error/uncertainty:

1) Screen efficiencies during Grade Engineering® validation trial.
2) Sampling linked to assays obtained at each size fraction investigated during the trial.
3) Mathematical modelling of grade by size responses through RR.

![Figure 5-1.Illustration of preferential grade by size response curve in the case of a sample with a RR value of 100.](image)

\[
RR = \frac{\ln(U_{pg})}{\ln(CW)}
\]  

(1)
3. Preferential Grade by Size Industrial Pilot Trial in a Cu-Porphyry deposit

The promising economic benefits obtained through an economic preferential grade by size assessment in a Cu-porphyry deposit ultimately led to the validation at industrial scale of the RR values initially obtained from core samples in conjunction with bulk characterisation analysis. This involved sampling size fractions to determine Cu grade by size from a ROM sample via screening.

Preferential grade by size validation at an industrial scale trial (Appendix B) comprised the installation of two mobile screen plants, two shovels used to feed material to a screening plant and one front end loader for material handling and sampling. The rock type with the higher RR response observed during the preferential grade by size characterisation program was fed to the screening plant via a static grizzly (1st screen plant, Figure 3) with a capacity of ~1,500 tpd for 28 days, where 4 size fractions were produced. The screen plant installed did not have the characteristics of an industrial production screen (i.e. less capacity and static instead of vibrating grizzly screen). This was reflected in the lower screening efficiencies relative to industrial screening, which are above 80% and in some cases, up to 95% (Bothwell and Mular 2002). A second screen (2nd screen, Figure 3) was installed in order to mimic actual industrial operational performance producing more realistic RR values. Thus, the oversize (Figure 5.3) of 1st screening plant was sent to the 2nd screen in order to reduce the probability of recovering finer particles (entrainment) which would bias the coarse size fraction grade and therefore compromise the grade by size responses (RR).

The second screen then produced two size fractions, a coarse fraction and the finer fraction that was recycled to the feed (fines, -6", Figure 5.3). In order to obtain a representative sample from a coarse fraction large masses are required for acceptable sampling accuracy (Gy, 1982, Appendix A). However, large samples can be avoided by reducing the top size via crushing (Crusher, Figure 5.4).
Figure 5-3. Illustration of the screen trial, depicting size fractions in inches. The fines, -6’’ of the 2nd screen were recycled to 1st screen.

Figure 5-4. Illustration of the interaction among fundamental sampling error, sampled mass and top size drawn from Gy’s sampling theory (Appendix A provides the application of this diagram).

The equipment used for validation of preferential grade by size responses at industrial scale is depicted in Figure 5.5.
The discharge points of a conveyor belt, chute or slurry pipe are the best locations to conduct sampling. The sampling frequency needs to be constant at a constant flow rate to avoid any bias (weighting errors) and increments must be collected by taking a complete cross section, whilst minimising increment and extraction errors (Petersen and Esbensen, 2005; Holmes, 2005; Holmes, 2004; Afewu et al, 1998; Allen, 1981).

Sampling of each fraction (-1”, -3+1”, -6+3” and crushed -4”, Figure 5.3) was completed using a front-end loader (Figure 5.4). The loader partially drove up the sample cone to allow the bucket to be elevated to immediately below the conveyor discharge point. Taking a complete cross-section is important because undesirable particle segregation is a factor that needs to be addressed (grouping/segregation error).

If the stream is sampled at the end of a conveyor belt, particles will segregate mainly into two forms: fines will tend to concentrate in the middle of the belt in the presence of vibrations and the coarser fractions will rise towards the top of a particle bed. Each sample should be taken by collecting the whole stream for a short period of time. The sampler should move in a unidirectional fashion (Figure 5.6b), otherwise an excess of coarse particles is likely to occur (Figure 5.6a).
Figure 5.6. a) Poor sampling practice excess of coarse fractions are collected. b) Good sampling practice

The bucket was perpendicular to the stream to reduce the possible increment delimitation errors (IDE). This was held for a certain period of time defined by the capacity of the screen plant and the volume of material required to obtain a representative sample according to Gy’s sampling theory (Gy 1982). The increment extraction errors (IEE) associated with the potential bucket’s overflow was also considered.

Despite several sampling strategies being put into place to account for lower screening efficiencies, visual inspection during the trial clearly depicted misclassification of material. Although this is expected in normal industrial operation, it is undesirable in this kind of exercise so it motivated further investigation to understand the influence of screen efficiency upon the validation of preferential grade by size responses.

4. Impact of Screen Efficiencies in RR estimation

Screen efficiencies during preferential grade by size validation certainly influence metal deportment response and therefore RR estimation. During the trial each of the samples taken (size fractions, Figure 5.3) were further screened to identify the proportion of particles misreported. This process enabled the estimation of the real proportion of material sampled in each size fraction for assaying as well as the screen efficiency associated with each size fraction sampled. Figure 5.7 depicts the distribution of the differences between RR\textsubscript{ideal} (assuming a 100% screen efficiency) and RR\textsubscript{real} (real screen efficiency observed during the piloting). This difference ranges from 0 (RR\textsubscript{real} = RR\textsubscript{ideal}) to a maximum of 14 RR units. It was found that this difference is strongly related to the screening efficiency associated with the -3+1” size fraction as Figure 5.8 illustrates. Screen efficiencies lower than 80% result in large RR differences up to 20%. Lower screen efficiencies affect the mass sampled within each of the size fractions, leading to changes in the expected fundamental sampling
error. Analysis needs to be conducted to determine the extent of reliability of the samples obtained during the trial, i.e. the degree of uncertainty within each grade by size sample measurement under analysis.

Figure 5-7. Cumulative probability plot difference $RR_{ideal}-RR_{real}$.

Figure 5-8. % $RR$ change and screen efficiency -3+1"
5. Model fitting error estimation

The estimation of the RR factor (preferential grade by size) is obtained by a data fitting process, (i.e. minimisation of least squares ($SST$, Eq.2)) using Eq.1. This model, as any other, has an associated standard error ($SE$, Eq.3) defined by SST as well as the degrees of freedom defined at the number of data points (experimental data) minus the number of parameters used within the model. The model SE is used to determine the uncertainty in the RR due to the fitting process, using a Monte Carlo simulation method. The term “Monte Carlo method” is used to describe a wide range of simulation techniques, all of which are based on the use of random numbers. These methods are used to explore the behaviour of systems which are either too complex or too large to be calculated analytically. The Monte Carlo methodology involves the use of a computer program with a random number generator to conduct many iterations. The results are then examined and a conclusion can be drawn in terms of characteristics of the system. The method employed here is described in detail by Napier-Munn (2014) and applied in Carrasco et al. (2016) to develop a statistically robust coarse liberation model based on the RR concept. The RR is first fitted to the data and the model predictions determined for each point. These predictions are then perturbed using normally distributed random numbers with a mean of zero and a standard deviation equal to the standard error of the original fit (Eq. 3). This process is repeated with a large number (say 1,000) times, and key information such as model predictions and RR values are recorded (Figure 5.9). This methodology was applied to each RR obtained daily during the trial (Carrasco et al., 2016). The significance of the model through chi-square test indicates that the model employed (Eq.1) is statistically significant.

$$SST = \min \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

(2)

where $y_i$ is the experimental value and $\hat{y}_i$ is the value predicted by the fitted model (Eq. 1). The standard error of the fit, $SE$, is:

$$SE = \sqrt{\frac{SST}{n-p}}$$

(3)

where $n$ is the number of data points and $p$ the number of model parameters (1 in this case).
Figure 5-9. Illustration of Monte Carlo analysis to infer confidence interval on model parameters.

It is noteworthy that the error of the RR parameter obtained from a production trial is within the same order of magnitude as the RR error obtained from drill core sample confirming the predictability of the model across different size scales (Figure 5.10).

Figure 5-10. Comparison fitting error (RR units) Bulk samples GE trial and drill core characterisation.

6. **Fundamental Sampling Error (FSE)**

Grade by size data during a preferential grade by size trial was obtained by periodic sampling at each stream depicted in Figure 5.3. This process has an inherent error/uncertainty associated with the size of the sample, which can be estimated using the theory of sampling (TOS) (Petersen et al., 2005). Petersen and Esbensen (2005) discuss how the TOS can be applied to estimate
errors/uncertainties associated with process sampling of moving or stationary streams. There are two approaches, the use of Gy’s sampling theory for independent lots (i.e. defined as the total volume or material to be sampled) and the use of the variogram and its derived auxiliary functions to take into account spatial or temporal correlations of the lots. During the preferential grade by size characterisation trial the shovel as well as 1st screen plant’s location changed as a function of daily operating conditions and logistics (i.e. space availability, muck pile location and safety), and therefore the material sampled each day did not come from a consistent spatial location. It could be assumed, then, that each sample on a daily basis corresponds to an independent event, confirmed by the correlogram diagram in Figure 5.11 (Napier-Munn 2014). The extent of the correlation is measured by the autocorrelation coefficient for a given lag. The series of autocorrelation coefficients, for different lags, (1 day, 2 day, so forth) is called the autocorrelation function. The autocorrelation is not statistically significant (95% of confidence) and therefore Gy’s sampling theory applied to 0-D lots (Petersen and Esbensen 2005) to quantify the uncertainty associated with sampling, particularly through the fundamental sampling error (FSE) (Eq.4). The FSE is essentially the minimum error expected, even when all the incorrect sampling error components have been eliminated.

\[
\sigma_{FE}^2 = C d^2 \left( \frac{1}{M_s} - \frac{1}{M_L} \right)
\]  

(4)

Figure 5.11. Correlogram for daily RR values, dotted lines show 95% confidence intervals (plotted in Minitab)
Where:

\[ \sigma_{FE}^2 \]  Fundamental sampling error variance expressed as relative proportion
C  Sampling constant, characteristic of the material being sampled
D  Largest size of material to be sampled (cm)
M_s  Mass of the sample (g)
M_l  Mass of the lot (g)

Figure 5.12 depicts FSE (%) as a function of particle size, for one day of the industrial scale trial as an example. It can be seen that the error increases as the particle size increases, as expected. Monte Carlo analysis was employed to determine how the uncertainty of the grade per size fraction related to the FSE influences the estimation of RR values. Grade in each size fraction was randomly perturbed within the limits defined by the FSE (assuming a Gaussian distribution), changing the \( U_{pg} \) in Eq.1 and therefore RR in each iteration.

![Figure 5.12: FSE (%) per size fraction of one ROM sample as example.](image)

The error due to fitting and sampling can be compared to understand the degree of uncertainty in the RR estimation. Figure 5.13 shows that the FSE is significantly and consistently higher than the error due to fitting. This analysis indicates that on average with a 95% confidence, a variation higher than \( \pm 1.96 \times 3 \) RR units represents a statistical variation on RR value linked to geological variability, where 1.96 is the standard normal deviate \( (z) \) for a 2-sided 95% confidence interval.
(Table 5.1). Figure 5.14 depicts the expected RR variability including all sources across Grade Engineering® validation.

![Figure 5.13. Error comparison during the Grade Engineering® Trial.](image)

**Table 5.1. Relative standard deviation (%) due to fitting error, FSE and inherent variation during preferential grade by size trial**

<table>
<thead>
<tr>
<th>Fitting Error %</th>
<th>FSE %</th>
<th>Variation Piloting %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.3</td>
<td>3.9</td>
<td>21.7</td>
</tr>
</tbody>
</table>

![Figure 5.14. Relative RR variation (RR daily divided by the me during Grade Engineering® trial (Error bars represent 95% confidence interval based on FSE.](image)
7. Conclusions

There are several sources of error/uncertainty that need to be identified, quantified and addressed during the validation of Grade Engineering® responses (in this particular case, the preferential grade by size response).

Screen efficiencies could mislead preferential grade by size response during the trial. Screening each of the size fractions collected during the trial in a more controlled environment (i.e. metallurgical laboratory) is recommended to understand the extent of this effect upon RR.

Monte Carlo simulation was employed to compare the error associated with RR fitting and that due to FSE. The former was significantly lower, confirming the robustness of the method employed to characterise preferential grade by size response.

In this work Gy’s sampling theory was adequate to quantify the uncertainty associated with sampling (FSE). However in situations where there is a reason to consider the presence of a significant correlation (time, space) across individual increments (sampling volumes), a variogram error analysis and its auxiliary functions is recommended (Petersen et al., 2005) to allow design of a robust sampling strategy to ensure that increment correlations and hidden process variations are taken into account.

In addition to the FSE, there are important sources of error associated with the sampling process itself that need to be considered. Grouping and segregation errors (GSE) and incorrect sampling errors such as increment delimitation errors (IDE) and increment extraction errors (IEE) could compromise grade by size information and therefore the trial validation. These are practical sampling issues for which there are well-known solutions (Pitard, 1993).

Any measurable rock parameter that can be linked to the downstream efficiency process (throughput, recovery, upgradability) has a level of uncertainty that needs to be understood and measured. This allows distinguishing of the inherent rock processing variability associated with changes in geological properties not measurement methods employed.

8. References


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Chapter 6   Methodology to develop a coarse liberation model based on preferential grade by size responses

1. Abstract

Early gangue rejection or metal preconcentration at coarse scale (millimetres) based on size has been identified as a feasible operating alternative whereby energy efficiency and unit metal productivity can be greatly increased. This is achieved by understanding and exploiting ore-specific preferential grade by size responses. Preferential grade by size refers to the propensity of some ores to naturally concentrate metal into specific size fractions during breakage. The magnitude of metal deportment is described through a Ranking Response parameter (RR). This parameter has been used to measure the extent of “liberation at coarse scale”. Mineral Liberation is defined as the measurable rock property that can link with a downstream separation technique which aims to concentrate valuable material to produce a saleable product. Liberation traditionally has been defined at grain scale whereby the efficiency of processes such as flotation is greatly dependent on particle properties at micro scale (microns). However, in size-based coarse separation the efficiency relies on having a processing stream with a strong grade variability across size fractions (i.e. high grade by size response) and therefore a high RR value.

This work aims to develop a model to predict preferential grade by size response, in terms of the RR of ores as a function of particle size distribution and size reduction process. To achieve these aims a novel methodology has been developed comprising a new preferential grade by size characterisation method coupled with Monte Carlo and comparative statistical methods (analysis of variance (ANOVA) and t-test). Six run of mine (ROM) bulk samples from 3 different geological style deposits (stock work vein hosted, Cu-Mo breccia porphyry and Cu-Mo volcanic porphyry) have been utilised in the analysis.

This methodology provides useful insights for the development of an optimum coarse separation circuit flowsheet design for preconcentration prior to energy intensive and inefficient grinding.

2. Introduction

The mining industry is currently facing several energy efficiency and productivity challenges. Grade depletion coupled with high volatility in commodity prices are adding more uncertainty to the current industry outlook (Prior et al., 2012; Topp et al., 2008). Mining cannot continue to rely on the economic benefits that increasing production scale has historically brought to the industry (Rendu et al., 2006). Over the years innovation has proven to be the key instrument whereby the mining industry has been able to cope with production periods characterised by very tight operating margins, either by improving processing efficiency or reduction of operating costs (Jara et al., 2010; Bartos, 2007; Schmitz, 2005).
Early coarse uneconomic material rejection has been identified as a plausible operating alternative that can significantly increase energy efficiency and unit metal productivity (Carrasco et al., 2015; Carrasco et al., 2014; Bowman and Bearman, 2014; Bearman, 2012; Logan and Krishnan, 2012). Size-based preconcentration is based on the propensity of some ores to preferentially deport metal into specific size fractions. This phenomenon is referred to preferential grade by size deportment (Carrasco el al., 2015). Experience to date indicates that this response is highly variable and therefore require characterisation for an effective exploitation (Carrasco et al., 2015; Carrasco et al., 2014). In this work the extent of this natural rock behaviour is measured through a mathematical model (Eq.1) describing the relationship between Ranking Response (RR, dimensionless), metal upgrade (Upg) and cumulative weight (CW) respectively. The particular function used in Eqn.1 will depend on the application. Metal upgrade and cumulative weight are utilised in the model to calculate an RR parameter to measure the extent of preferential grade by size response. Carrasco et al., 2015 depicts the process of describing Upg and CW shape by using a single parameter. Although in the present work a different mathematical function was employed, the methodology is equivalent.

\[
RR = f(Upg, CW)
\] (1)

The concept of mineral liberation is defined as a function of the downstream separation technique aiming at selectively concentrating elements of interest to make a saleable product. "Liberation" is a rock based property that allows certain measurable rock characteristics to be linked with separation process efficiency. For example, in flotation the material is “liberated” to the extent that the mineral surface is sufficiently exposed enabling an effective bubble-particle interaction and therefore a high separation efficiency. In size based coarse separation (Carrasco et al., 2015), the separation technique relies on having a feed with a distinguishable grade across size fractions (i.e. a high grade by size response). RR is therefore used to measure the degree of liberation at coarse scale when size is used as the separation lever. There is undoubtedly a link between these two mineral liberation concepts at micro (microns) and coarse (mm) scale. Comminution will certainly affect both. Several studies have focused on understanding the relationship between liberation at a micro scale (grain size) and size reduction processes (Ozcan and Benzer, 2013; Vizcarra et al., 2010; Hosten and Ozbay, 1998; Fandrich et al., 1997; Petruk, 1988). However almost no attention has been given to what occurs at a coarse scale (mm), prior to grinding.
This work focuses on understanding the interaction between “coarse liberation”, measured by an RR factor, and comminution. Information obtained from a novel preferential grade by size characterisation test is employed to predict RR values as a function of parent particle size and changes in particle size distribution. A set of analytical techniques have been utilised, spanning non-linear regression coupled with Monte Carlo simulation and comparative statistical tools, including analysis of variance (ANOVA) and the t-test.

It is implicit in this analysis that impact breakage is the mechanism being assessed. However, the same framework can be used to assess other breakage mechanisms. Particle bed breakage has been proposed as a breakage mechanism that might increase mineral micro scale liberation (Ozcan and Benzer, 2013; Hosten and Ozbay, 1998; Fandrich et al, 1997) and therefore it could also foster coarse liberation. It seems that this mechanism accentuates the material physical difference of the mineral phases, which promotes preferential breakage (Fandrich et al, 1997).

3. Progressive Crushing Test

Six ROM bulk samples were extensively characterised for preferential grade by size. The aim was to obtain a RR parameter that represents the “global” preferential grade by size bulk sample response (RR) as well as a RR factor per size fraction at given size reduction step (RRi).

The model (Eq.1) describes mathematically the preferential grade by size response for a particulate system. Although this is quite helpful for a rapid domain assessment and inter/intra deposit comparison (Carrasco et al., 2015; Carrasco et al., 2014), for production implementation and process optimisation a more detailed assessment needs to be conducted. Eqn.1 does not consider the interaction between RR and changes in particle size distribution. For instance, for some rock types a further size reduction step might enhance coarse liberation, (i.e. more metal concentrates into finer particles) thus an increase in the initially estimated RR value.

The characterisation of bulk samples for preferential grade by size comprises three steps:

1. Screening material into previous defined size fractions.
2. Obtaining chemical assays in each screened size fraction.
3. Calculating of the RR parameter to quantify the preferential grade by size response.

The source of error within first step (screening) can be easily managed by controlling the screen loading which might neglect screening efficiency. Nevertheless, information obtained within the second step, chemical assays, is more prone to errors, since its resulting error is the sum of the errors (in its variance form) related to the prior sample preparation processes. The management of
chemical assays error is particularly difficult for coarse samples (mm) due to material sampling and its associated sampling statistics requirements (Gy, 1982; Appendix A). For this is usually addressed by reducing its top sizes via crushing to ~3mm (6 Mesh) top size, where fundamental sampling error ($\sigma_{FE}$) is within tolerance limits, typically 5%) (Napier Munn, 2014).

Figure 6.1 depicts a ROM grade by size progressive crushing characterisation. A sample is initially sieved using 6 size fractions, 5”, 3”, 1 ½”, 3/4”, 3/8” and 1/8”. Grade by size results representing the global preferential grade by size responses ($RR_\varnothing$) can be determined by chemical assays per size fraction (once each of size fractions are crushed to an adequate size for splitting) as well as its related mass. This process does not take into account the intermediate size reduction steps. To determine RR per size fraction at each reduction step while avoiding any splitting process during size reduction of coarse material (to manage $\sigma_{FE}$), each initial fraction is crushed and sieved independently (Figure 6.1). The size reduction step during the sample preparation for assays is then exploited to assess changes in preferential grade by size response (RR) due to size reduction at each size fraction. The crusher’s closed side setting (CSS) needs to be appropriately adjusted to produce a product that can be sieved utilising the aforementioned sieves. In order to control the amount of energy that is delivered to each coarse fraction the crusher CSS is adjusted approximately to top size, to avoid over crushing. Therefore, each crusher and sieve operating setting located in the same vertical zone in Figure 6.1 is identical (i.e. CSS and screen aperture). This “cascade” process enables a back calculation of the RR in each reduction step at a given particle size (parent size) by knowing the grade and related mass of the size fraction (mass balance). For example, the head grade of the +5” size fraction is obtained by compositing the assays of their corresponding branch, where in this case the first 16 assays are used (Figure 6.1 and Eq.2). In this case the size reduction steps are not considered. Nevertheless, to calculate the grade by size for a given parent size at any size reduction stage, the intermediate size reduction steps need to be considered. For instance, when the +5” size fraction (parent size) is crushed to 100% passing 1½” (3rd size reduction step, Figure 6.1), the assays and masses are balanced accordingly (Eq.3). Grade by size data sets are then obtained per size fraction at each size reduction step, resulting in 7 discrete data sets (Table 6.1). This information in conjunction with Eq.1 is used to determine the RR related to each grade by size data set. This is in addition to the “global” grade by size responses ($RR_\varnothing$), where grade by size data merely comprises the grade of each parent size originally sieved.

This characterisation method is robust since it does not jeopardize grade by size data analysis due to sampling preparation error, ensuring that RR truly reflects the natural rock propensity to concentrate metal in the finer fractions after breakage.
Figure 6-1. Progressive Crushing Test Work Procedure.

\[ A_{+5''} = \frac{\sum_{l=1}^{16} M_l A_l}{\sum_{l=1}^{16} M_l} \]  
(2)

\[ A_{100\%-1 1/2''} = \frac{M_l A_l + M_{l+4} A_{l+4} + M_{l+8} A_{l+8} + M_{l+12} A_{l+12}}{M_l + M_{l+4} + M_{l+8} + M_{l+12}}; 1 \leq i \leq 4 \]  
(3)

where: \( i \), is the branch number in the right inside Figure 6.1; \( A \), assay associated to each branch in Figure 6.1.
Table 6.1. Illustration of grade by size data resulting from progressive crushing tests

<table>
<thead>
<tr>
<th>Size Fraction</th>
<th>100%-5&quot;</th>
<th>100%-3&quot;</th>
<th>100%-1 1/2&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>+5&quot;</td>
<td>GbS₁</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+3&quot;</td>
<td></td>
<td>GbS₂</td>
<td>GbS₃</td>
</tr>
<tr>
<td>+1 1/2&quot;</td>
<td></td>
<td></td>
<td>GbS₄</td>
</tr>
<tr>
<td>-1 1/2&quot;</td>
<td></td>
<td></td>
<td>GbS₅</td>
</tr>
</tbody>
</table>

The total number of assays (#Assays₇) required for this analysis is a function of the number of sieves selected (#Sieves₀) as well as the size reduction steps (#SR). In this example 6 sieves (5", 3", 1 ½", 3/4", 3/8" and 1/8") with 3 size reduction steps (100% -5", 100% -3" 100%, 1 ½") generated 32 discrete assay samples (Eq.4). However, settings (sieves sizes, CSS aperture and size reduction steps) can be customised as a function of sample size (ROM, SAG feed, pebble stream) as well as the purpose of the analysis (secondary, tertiary crushing modelling effect).

\[
#\text{Assays}_T = (\#\text{Sieves}_0 - \#SR + 1) \times 2^{\#SR}
\]  \hspace{1cm} (4)

4. Analysis Methodology and Results

An extensive preferential grade by size ROM characterisation program was conducted involving 3 distinctive geological style deposits with preferential grade by size deportment responses (RR) ranging from low (~20 RR units) to high (~100 RR units), whilst increasing feasibility of applying size based preconcentration.

In estimating RR it is desirable to determine the uncertainties in the fitted values. Preferential grade by size characterisation is mainly exposed to two sources of error (once the sampling error associated with splitting has been appropriately addressed; see earlier discussion on progressive crushing, section 2)

1. Error propagation of chemical assays (e.g. Eq.2, Eq.3)

2. Model fitting error (Eq.1).

The former is calculated by inspecting assay measurement error within each of the final branches (32 in total) depicted in Figure 6.1, preferably from assay repeats. The grade of each size fraction in
each size reduction step comprises of the weighted average grade of related branches (e.g. Eqn.2 and Eqn.3). The error propagates from right to left in Figure 6.1. Napier-Munn (2014) describes different methods to examine the extent of error propagation. In this work assay error is assumed to be the only significant source of error and therefore the masses of each of the branches (Figure 6.1) are treated as constants in the partial differential formula governing error propagation (it was assumed that the mass measurement error is small compared with the assay error, which is usually the case). If sample repeats are not available to estimate assay error, there are general expressions that can be utilised instead (Jansen et al., 2007; RSC, 2004; Napier-Munn, 2014, Chapter 12) to determine assays error and its impact on grade by size data analysis.

The RR model fitting error is obtained through the minimisation of least squares (Eq.5) involved in the estimating of RR from grade by size data. Analysis of variance (ANOVA) is employed to assess whether the model proposed (Eq.1) is statistically significant. The model has to reject the null hypothesis: “the model mean square is not significantly different to the error mean square” (Napier-Munn, 2014). If the null hypothesis is not rejected, this would lead to

1. Discarding the grade by size data, since sample preparation might have been compromised, or
2. A different model is required to describe grade by size responses.

The first can be explained by poor sampling procedures or low screen efficiency. This can be assessed by analysing the particle size distribution (PSD) curve smoothness prior to chemical assays. PSD should be approximately described as a straight line in log-log space. The absence of certain mass size fractions, for instance, would clearly reflect sample preparation issues.

In this work the model proposed in Eq.1 is suitable for the samples employed in this analysis. Eq.6 describes how the model fitting standard error (SE) is calculated from the total fitting sum of squares (SST). Model fitting error (uncertainty of the model) is approximately one order of magnitude higher than the error propagation due to mass balancing (Figure 6.2) and therefore needs to be estimated. The model SE is input to a Monte Carlo simulation to determine the confidence intervals for the RR values.

$$\text{SST} = \min \sum_{r=1}^{n} (y_t - \hat{y}_r)^2$$  \hspace{1cm} (5)

where $y_t$ is the experimental value and $\hat{y}_r$ is the value predicted by the fitted model.
where \( n \) is the number of data points and \( p \) the number of model parameters.

\[
SE = \sqrt{\frac{SST}{n - p}} \tag{6}
\]

![Figure 6-2. Comparison between RR error prediction due to model and mass balance propagation of errors for a sample as example. GbS set defines a certain parent size and size reduction step - see Table 6.1 as illustration.](image)

The term “Monte Carlo method” is used to describe a wide range of simulation techniques, all of which are based on the use of random numbers. These methods are used to explore the behaviour of systems which are either too complex or too large to be calculated analytically. Monte Carlo methodology involves the use of a computer program with a random number generator to conduct many iterations. The results are then examined and then conclusion can be drawn in terms of characteristics of the system.

The method employed here is described in detail by Napier-Munn (2012, 2014). It determines the uncertainty/error of the model parameters and predictions. The RR\(_i\) is first fitted from the data and the model predictions determined for each point. These predictions are then perturbed using normally distributed random numbers with a mean of zero and a standard deviation equal to the standard error of the original fit (Eq. 6). This process is repeated a large number (say 1,000) times,
and key information such as model predictions and RR values are recorded. The process is illustrated in Figure 6.3.

![Figure 6.3. Illustration of Monte Carlo analysis to infer confidence interval on model parameters.](image)

Results of preferential grade by size characterisation for the six samples are displayed in Table 6.2. RR$_g$ has been calculated by using assay as inputs of the branch associated with each main size fraction sieved, i.e. +5” (1-16 branch), -5+3” (17-24 branch), -3+1 ½” (25-28 branch), -1 ½ +3/4” (29 branch), -3/4+3/8” (30 branch), -3/8+1/8” (31 branch) and -1/8” (32 branch) in Figure 6.1. Figure 6.4 depicts the “goodness” of fit as well as the 95% confidence interval of RR for sample A1 as an example.

<table>
<thead>
<tr>
<th>Site ID</th>
<th>RR$_g$</th>
<th>RSD %</th>
<th>Deposit Style</th>
</tr>
</thead>
<tbody>
<tr>
<td>A$_1$</td>
<td>53</td>
<td>2.1</td>
<td>Cu-Mo Porphyry Breccia</td>
</tr>
<tr>
<td>A$_2$</td>
<td>76</td>
<td>2.2</td>
<td>Cu-Mo Porphyry Breccia</td>
</tr>
<tr>
<td>A$_3$</td>
<td>47</td>
<td>4.9</td>
<td>Cu-Mo Porphyry Breccia</td>
</tr>
<tr>
<td>B$_1$</td>
<td>22</td>
<td>3.6</td>
<td>Cu-Mo Porphyry Volcanic</td>
</tr>
<tr>
<td>B$_2$</td>
<td>21</td>
<td>4.6</td>
<td>Cu-Mo Porphyry Volcanic</td>
</tr>
<tr>
<td>C$_1$</td>
<td>119</td>
<td>1.2</td>
<td>Cu Stockwork vein hosted</td>
</tr>
</tbody>
</table>
Progressive crushing grade by size test work enables the generation of a coarse liberation matrix (CLM), spanning preferential grade by size responses ($RR_i$) per size fraction/parent size (row, Table 6.3) and the evolution of this response as size reduction increases (column, Table 3). $RR_i$ factor describes the “intra” grade by size response variability in contrast to $RR_g$ which describes grade by size response at “inter” sample scale.

It is noteworthy that to develop the CLM it is necessary to confirm whether the model proposed to describe preferential grade by size responses is still statistically valid. Monte Carlo is again used to determine the confidence intervals of $RR_i$ in the CLM.

A two sample t-test is performed to confirm the statistical significance of the difference among the $RR_i$ values within the CLM. In 5 of the 6 samples the size reduction does not statistically alter the $RR_i$ of each of the size fractions within the ROM samples (row, CLM, Table 6.3). The same pattern was identified by Vizcarra et al. (2010) when investigating mineral liberation in sulphide ores with different breakage mechanisms (conventional grinding versus particle bed-breakage). Although the “liberation” in this sense is defined based on the mineral surface exposure, there is intrinsically a relationship with the amount of valuable phase in a given size class (i.e. grade). Vizacarra et al. (2010) assert that the degree to which the sample is comminuted does not affect size by size liberation properties when samples of ~3 mm top size are tested. In the current data set, however, there was one sample, $B_2$, where the RR significantly increased (with 95% confidence) due to size reduction, in the coarser fraction, +5” (Figure 6.5a).
Furthermore for the totality of samples tested, RR\textsubscript{i} values within each size fraction (i.e. +5”, -5+3, -3+1 ½”, -1 ½”, Table 6.3) are statistically different with 99% significance (row, CLM, Table 6.3). RR\textsubscript{i} increases with a corresponding decrease in particle size, displaying a liberation behaviour also described by Vizcarra et al. (2010).

Table 6.3. RR\textsubscript{i} obtained through non-linear regression.

<table>
<thead>
<tr>
<th>Size Fraction</th>
<th>100%-5”</th>
<th>100%-3”</th>
<th>100%-1 ½”</th>
</tr>
</thead>
<tbody>
<tr>
<td>+5”</td>
<td>28</td>
<td>30</td>
<td>33</td>
</tr>
<tr>
<td>+3”</td>
<td></td>
<td>37</td>
<td>42</td>
</tr>
<tr>
<td>+1 ½”</td>
<td></td>
<td></td>
<td>45</td>
</tr>
<tr>
<td>-1 ½”</td>
<td></td>
<td></td>
<td>49</td>
</tr>
</tbody>
</table>

The information contained in the CLM is plotted as a function of the relative size (\(x_{i,j}\) defined as the ratio of geometric mean of the size class and maximum particle size (Eq.7).

\[
x_{i,j} = \frac{d_{\text{mean},i}}{d_{\text{max},j}} = \sqrt{\frac{d_{i+1} \times d_i}{d_{\text{max},j}^2}}
\]

where i is the size class and j the sample.

Fig. 6.6 shows the relationship between the fitted RR\textsubscript{i} and relative size. From analysing Figure 6.6, two conclusions can be drawn:

1) There is a clear relationship between the particle size (\(x_{i,j}\) Eq.7) and the RR\textsubscript{i} (intra sample grade by size response variability). This means that the propensity of deporting metal into finer fractions increases as relative size decreases.
2) The pattern described in (1) is more noticeable (RR<sub>i</sub> vs x<sub>i</sub> slope increases) at higher RR<sub>g</sub> (Table 6.2).

![Figure 6.6. RR<sub>i</sub> vs relative size (x<sub>i</sub>). 95% confidence intervals calculated according Eqn.6 (1.96*SE). (Appendix C contains raw data associated)](image)

A higher RR<sub>g</sub> suggests significant physical differences in the mineral assemblage (texture) across size classes which is supported by the higher RR<sub>i</sub> internal grade by size intra-sample variability. A homogeneous rock (i.e. low RR<sub>g</sub>), will have a low RR<sub>i</sub> variability across size fractions within a sample.

The rate of change of RR<sub>i</sub> seems to increase as x<sub>i</sub> decreases, which is particularly noticeable for samples A<sub>2</sub> and A<sub>3</sub> (Figure 6.6). Eq.8 defines this relationship, where C is a constant related to the material properties, affecting metal deportment (i.e. texture, mineralisation) and α is the shape of the RR<sub>i</sub> vs x<sub>i</sub> curve. This differential equation is similar to that proposed by Walker et al. (1937) relating specific energy to size reduction in comminution. By integrating Eq.8, a general expression (Eq.9) is obtained describing RR<sub>i</sub> as function of particle size (x<sub>i</sub>). A ϕ function has been included in Eq.9 to account for a possible size reduction effect in conditions different to those encountered in the present work, for example:

1) Material having different textural characteristics, or
2) Different breakage mechanisms under assessment.

Figure 6.5 suggests that the $\varphi$ function depends on the size reduction as well as its parent size.

\[
\frac{dRR_i}{dx} = C \times_i^{p-1} \tag{8}
\]

\[
RR_{i,j} = \varphi_j \left( \mu_j x_i^p + \theta_j \right) \tag{9}
\]

where $i$ is related with the particle size and $j$ sample.

In regression analysis it is often implicitly assumed that each observed value ($y_i$) has the same uncertainty (Eq.5). This is often untrue, but it is a convenient assumption when the uncertainties are unknown. However, where the error model is known, a weighted regression analysis is more appropriate to give improved parameter estimates and uncertainties (Napier-Munn 2014; Kojovic, 2012). The use of measurement variance is typically the numerical weight that is added to Eq.5. (Eq.10). This means that a value with low uncertainty (low variance) has an increased weight. The $RR_i$ variances have been determined through the Monte Carlo analysis. The significance of the model fitting can then be determined by the chi-square test (Eq.10) which provides the $P$-value with $n-p$ degrees of freedom. Press et al. (1992) provides a set of $P$-values that can be used as reference to determine the significance of the model.

\[
\chi^2 = \sum \left( \frac{y_i - \hat{y}_i}{\sigma_i} \right)^2 \tag{10}
\]

The $P$-values from the $\chi^2$-test show that there is no significant difference between the two models, Eqs.10 and 11, implying that $\varphi=1$ (no size reduction effect) and $\alpha=1$ in Eq. 10, enabling the simpler Eq.11 to be used instead.

The $\mu_i$ and $\theta_i$ parameters in Eq.11 are strongly related to $RR_g$, indicating that the $RR_i$ variation across size fractions can be potentially estimated by just knowing the $RR_g$ of the sample. Geometallurgical characterisation of $RR_g$ coupled with Eq.11 (Carrasco et al., 2015; Carrasco et al., 2014, Carrasco 2013) also enables the $RR_i$ variation to be embedded within the resource model, aiding the optimisation of coarse separation circuits. However, precautions need to be taken to
confirm whether the model parameters and their interaction hold regardless of the rock characteristics and breakage mechanism under assessment.

\[ RR_{i,j} = \mu_j x_t + \theta_j \]  \hspace{1cm} (11)

Figure 6.7 depicts the RR\(_i\) observed (y-axis) and RR\(_i\) predicted (x-axis) for the present data, by using just the RR\(_g\) parameter to predict RR\(_i\) (R\(^2\)=0.96). It is noteworthy that the RR\(_i\) difference due to size reduction in sample B\(_2\) (Figure 6.5) is within the 95% range of the model prediction (± ~10 RR units, SE = 5.36).

Figure 6-7. Observed vs predicted RR\(_i\), 95% confidence intervals (CI). (Appendix C contains the statistical analysis associated)

5. Conclusions

An empirical coarse liberation model categorised by the RR parameter has been developed based on six ROM samples from three different geological style copper deposits. This model is able to predict the propensity of each size fraction to preferentially concentrate metal into finer fractions
(RR$_i$) as a function of overall preferential grade by size response (RR$_g$) and the size reduction process. It can therefore be used to determine the potential effectiveness of preconcentrating the ore using a simple sizing process.

For the majority of samples tested, RR$_i$ per size fraction does not statistically change due to changes in size reduction. Nevertheless, RR$_i$ increases when size decreases for all the samples tested.

This methodology relies on an extensive sample characterisation program. However, the development of predictive geological driven models will reduce the need for such physical test work. Further testing will determine whether the model proposed is independent of the geological deposit characteristics. Experience to date indicates that the current methodology can be applied to drill core material, reducing material handling and sample statistic requirements compared to ROM bulk samples. However experience to date indicates that scale up factors are required (Carrasco et al., 2014).

The author’s intent is not to develop a universal model, but rather to provide insights regarding coarse liberation and its interaction with the size reduction process.

This model could be used to understand the potential bias in RR when samples at different particle size distributions are being used to characterise preferential grade by size responses (e.g. ROM, SAG feed material).

The model also supports coarse separation circuit flowsheet design as well as process control optimisation, identifying whether size reduction of coarser particles will still deport to economically recoverable ore.

The present work adds an additional dimension, grade, to the current comminution models focused on understanding how new particles are created within a wider range of size reduction equipment. However, in ore bodies where the mineralogy has a large impact upon processing performance, grade deportment should be characterised in conjunction with mineralogical information.

6. Acknowledgments

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7. References


Chapter 7 Unlocking additional value by optimising comminution strategies to process Grade Engineering® streams


1. Abstract
Grade Engineering® comprises a range of techniques aiming to reject low grade uneconomic material (preconcentration) as early as possible within the mining value chain. It has been identified as an effective and feasible operating strategy whereby mining unit metal productivity can be significantly increased. Two Grade Engineering (G.E) levers have been assessed in a copper porphyry deposit: preferential grade by size response, and differential blasting for grade. Those are exploited through a modified blasting fragmentation coupled with screening based process on run-of-mine material to recover upgraded undersize fractions. Application of G.E inevitably alters comminution circuit typical feed particle size distributions, and consequently impact semi-autogenous (SAG) milling performance. A factorial design approach has been employed to assess the extent of this effect. A wide range of different operating scenarios, representing the possible G.E strategies and dynamic processing rock attributes, were simultaneously assessed using the Integrated Extraction Simulator (IES), a new cloud-based process simulator. This enabled the development of a G.E throughput improvement model as function of blasting fragmentation, impact hardness (Axb) and grindability (BMWi), which can be employed to conduct more detailed process modelling as well as resource optimisation. Improvements up to 14% in throughput due to changes in mill feed particle size distributions were observed under the conditions examined. The impact of this effect upon the proportion of material that is amenable to G.E is also discussed.

2. Introduction Grade Engineering®

Grade Engineering (G.E) refers to a range of integrated strategies aiming to improve feed grades by removing low grade uneconomic material prior to energy intensive and inefficient grinding (Walters 2016). In this work the impact of preferential grade by size deportment and differential blasting for grade on comminution circuit performance (throughput) is investigated. Both strategies are exploited through the intimately interplay between modified blasting fragmentation and G.E circuit (Figure 7.1).

Preferential grade by size refers to a “natural” based rock property whereby a significant metal proportion preferentially deports into specific size fractions after breakage (Carrasco et al., 2016a, Carrasco et al., 2014, Carrasco 2013). This geometallurgical attribute is modelled through a “RR” parameter (dimensionless), which describes the mathematical relationship between metal upgrade ($Upg$) and cumulative undersize weight ($CW$) (Figure 7.2), (Carrasco et al., 2016a, Carrasco et al., 2016b; Carrasco et al., 2014). Samples are screened in defined size fractions to determine the proportion of mass as well as grade within each size class to then determine the $Upg$ and $CW$, both being input variables in Eq.1.
In order to understand variability and the extent of this geological-processing response, an extensive and careful sampling program, at drill core scale, is required. Either intact drill core or coarse assay rejects of bench height composites can be employed. Historical experience has indicated that scale up factors need to be applied to transform drilling preferential grade by size responses (typically determined with a material having a particle top size of ~3 mm) to production scale responses for ROM /SAG feed material (Carrasco et al., 2014, Carrasco 2013).

\[
RR = \frac{\ln(U_{pg})}{\ln(CW)}
\]  

(1)

Figure 7.1. Two size based Grade Engineering levers assessed in this work.

Figure 7-2. Characterisation of preferential grade by size response, Metal upgrade versus cumulative weight undersize (%). RR is ~ 75 in this example.
The second G.E strategy involves the application of differential blasting for grade to exploit spatial grade heterogeneity. This relates the presence of spatial high grade and low grade discrete clusters within a certain production volume originally assigned to a single destination based on its average grade. Figure 7.3 shows an example of Copper (Cu) grade variation by blast hole, at bench scale, in a Cu porphyry deposit which was all assigned to the mill based on its average grade. The green clusters describe leach grade, the blue clusters describe waste grade rock, and the red/orange clusters describe mill grade ore. In “induced” G.E blasting fragmentation, high levels of energy are applied to high grade pockets and low energy is imported to low grade zones, allowing high and low grade cluster fragmented rock to be separated based on their different particle size distributions, via screening. Characterisation of the grade heterogeneity amenability attribute is primarily determined by analysing blast hole data, which provides greater grade resolution than block modelling, where grade heterogeneity is smoothed out through geostatistical interpolation techniques such as kriging.

Currently there is no statistical evidence that reflects a relationship between the preferential, “natural” grade by size response (Figure 7.2) and spatial grade heterogeneity (Figure 7.3). This means that both attributes need to be characterised independently. However, when differential blasting is applied, the ultimate combined grade by size response depends on the preferential grade by size response of each of the discrete grade areas in addition to the fragmentation applied. Eq.2 depicts this dependency, where \( F, x_f, CW, Upg \) define the mass, grade, particle size distribution and metal upgrade of each high and low grade clusters respectively. \( \chi_{t}^{High+Low} \) represents cumulative grade (Cu %) by combining both clusters in a composite. Further, since the relationship between

---

**Figure 7.3. Contoured blast hole Cu grades for ore assigned to mill feed.**
and is controlled by RR parameter (Figure 7.2, Eq.1), it is evident there is an interaction between differential blasting and preferential (natural) grade by size deportment.

\[
X_t^{High+Low} = \frac{E^{High}CW_t^{High}x_f^{High UPg_t^{High}} + E^{Low}CW_t^{Low}x_f^{Low UPg_t^{Low}}}{E^{High}CW_t^{High} + E^{Low}CW_t^{Low}}; t = \text{size class} \quad (2)
\]

Figure 7.4 illustrates simulated high and low intensive blasting and the resulting Cu grade by size response of the composite \((X_t^{High+Low})\) when a conservative preferential grade by size (RR~40) and null response (RR=0) are used. In this example differential blasting to exploit spatial grade heterogeneity is applied to high grade pockets (0.9% Cu, making up 30% of the total volume) and low grade (0.4% Cu, making up 70% of the total volume). Comparison of the Cu grade by size responses in differential blasting, indicate a significant difference when preferential grade by size (RR) is present.

The application of these two G.E strategies will inevitably alter the comminution circuit feed particle size distribution (PSD) due to modified blasting fragmentation and additional screening process.

![Graph showing cumulative grade by size](image)

**Figure 7-4.** Cumulative grade by size obtained through the application of differential blasting conservative preferential grade by size (RR~40) and its absence (RR=0). 30/70 mass ratio between high grade (0.9% Cu) and low grade (0.4 %) clusters.

3. Grade Engineering Circuit
Results from the G.E characterisation program enable the population of G.E attributes (see Figure 7.1) into the long term resource block model and the determination of additional economic benefits associated with grade by size based preconcentration. This analysis is conducted in conjunction with the minimisation of additional CAPEX and OPEX committed by installing a G.E preconcentration circuit (Figure 7.4).

The G.E strategy focuses on “Metal Exchange” (Carrasco et al., 2016a) by upgrading material located close to the mill/leach cut-off boundary, exchanging metal across streams initially assigned to a single process location (Waste, Leach, Mill) maximising annual cash flows. Typically, material with metal grade significantly higher than defined by an economic threshold, such as cut-off grade, is not intervened by a G.E treatment strategy and composes the direct mill feed.

The G.E circuit configuration seeks to produce a defined undersize mass pull, i.e. proportion of material under defined size fraction (see “upgraded undersize”, Figure 7.5), which is combined with the product from the primary crushers to form the feed stream to the comminution circuit, via a coarse ore mill stockpile.

It is noteworthy that the proportion of the ore body treated as direct mill feed and the proportion amenable to G.E, with the resultant proportion of undersize fine G.E material (at the same time defined by the blasting fragmentation and screen aperture selected in G.E circuit) determines the particle size distribution fed to comminution and therefore its metallurgical impacts.

![Grade Engineering circuit under assessment](image)

*Figure 7-5. Grade Engineering circuit under assessment.*
4. Particle size distribution effect on comminution

The installation of autogenous and semi-autogenous milling (SAG) for comminution circuits has undoubtedly lead to several economic advantages in mine site mineral processing over the years. However, SAG milling also has some disadvantages, including; performance sensitivity to mill feed input variations, such as; rock competency, and mill feed particle sizing distribution.

SAG mill feed contains particle fractions of ore which serve as the grinding media. Larger ore particle fractions break the smaller particles, and in the process also breakdown into smaller particles, before exiting the mill as a product. Steel balls are charged to the SAG mill to replicate the breakage action of larger ore particles, and also help break the more competent rock lumps, in order to maintain mill throughput. The Mine to Mill (M2M) strategy is a clear example of manipulating the relationship between plant throughput and feed particle size distribution (Kanchibotla 2000, Morrell and Kojovic 1999, Scott et al., 1999). M2M focuses on enhancing milling throughput by controlling blasting fragmentation. The objective is to increase the proportion of sub-grate size material (~20 mm) and reduce mill residence time, in conjunction with decreasing the material particle content within the called as critical size range (20-70 mm) which is difficult to break, limiting grinding capacity. Geotechnical attributes (e.g.: RQD, UCS, mean in situ block size), blasting pattern modification (Burden and spacing), blasting powder factors and type of explosives are the fundamental tools in M2M implementation. Proper application of M2M typically leads to a reduction in the ROM’s top size, easing the work load of the primary crusher. This provides an opportunity to reduce the primary crusher open side setting (OSS), without compromising capacity, thereby reducing the crusher product particle size distribution and increasing fines in the SAG mill feed. The M2M throughput improvements across different deposit styles have ranged from 10 to 20% (Morrell and Kojovic 1999).

The installation of precrushing (secondary crushing) stages applied to the whole or partial SAG feed stream to improve comminution throughput has also been extensively discussed in the literature (Rose et al., 2015, Siddall and Putland 2007, Putland et al., 2004, Atasoy et al., 2001). This operating strategy is particularly effective in cases where the ore is very competent and/or SAG mill is the throughput process bottleneck, which is very often the case in close grinding circuits (Mainza et al., 2011). Rose et al. (2015), related additional plant capacity benefits by installing an additional crushing unit, namely: a significant decrease of SAG and mill steel ball consumption and improvements in grade and mill liner life. The use of secondary crushing can control the variations in SAG mill throughput which the pebble recycle crushing is not able to address. Putland et al. (2004) proposes the installation of an overflow bin to control the proportion of material sent to...
secondary crushing. This strategy provides a high degree of flexibility by controlling the blend of primary and secondary crushing products. This ensures the presence of enough grinding media while balancing the energy utilisation between primary and secondary milling. The throughput improvements reported by additional crushing can be moderate (~20%) to significant (~60%) (Siddall and Putland 2007)

The G.E approach offers similarities in producing a mill feed stream with a fine particle size distribution compared with the secondary crushing strategies previously outlined. The application of G.E to amenable ores offer further economic advantage by improving mill grades through coarse separation preconcentration Combination of these two optimisation strategies could lead to significant improvements in energy efficiency, and thus unit metal productivity.

5. Methodology and Results

The Integrated Extraction Simulator (IES) has been employed to assess G.E’s impact on comminution circuit performance (associated with Cu-porphyry deposit under assessment) due to changes in mill feed particle size distribution. IES is a new cloud-based simulator that is able to integrate blasting, comminution and flotation in one single simulation package (Stange et al., 2014). IES provides a high level of flexibility whereby customised processing models can be easily embedded which can interact with well-known comminution models, for instance the JKMRC SAG variable rates model (Napier-Munn et al., 1996). The calibrated and validated models originally embedded in the JKSimMet package were transferred to IES for this analysis. The following comminution equipment models were used:

- SAG mill: variable rates model (Napier-Munn et al., 1996).
- Trommel: Single component efficiency curve (Napier-Munn et al., 1996).
- Ball mill: perfect mixing model (Whiten, 1976).

The circuit under assessment, is shown in Figure 7.6, which encompasses a SABC circuit (SAG-ball mill-with pebble crusher). The mass simulation capabilities embedded in IES enable a factorial design analysis to assess multiple operating scenarios. This aims to obtain a better understanding of interactions from changes in G.E operating conditions and metallurgical rock properties, specifically in mill throughput. The scenarios examined encompassed:
- Three different blasting profiles (coarse, medium and fine particle size distribution); used in feed to both the gyratory crusher and G.E circuit.

- Five G.E mass pulls (proportion of upgraded fine material from 20 to 60%, in 10% intervals), producing six different particle size distribution profiles per ROM blasting and therefore eighteen mill feed fragmentation profiles (fifteen related with GE strategy and three corresponding to the base case, fine blasting fragmentation depicted as example Figure 7.7).

- Four different impact hardness values (Axb=25, 38, 45 and 60) and four bond mill work index (BMWi= 14, 16, 18, 20, kWh/t). These values comprise the main production rock types range identified within the Cu-porphyry deposit under assessment.

The proportion of G.E circuit product supplied to mill feed was fixed in a 1/3 ratio of direct mill feed, which was determined as economically optimal by intensive characterisation assessment (Section 1).

![Diagram of SABC circuit G.E evaluated](image-url)
Figure 7.7. Mill feed fine blasting particle size distribution at different Grade Engineering mass pulls (relative proportion in the undersize material).

Results across the multiple processing scenarios tested (Appendix C) indicated that the change in SAG mill energy was strongly influenced primarily by changes in mill feed PSD. Nevertheless, ball mill energy consumption did not appear to be significantly changed (this is mainly controlled by the mill ball load, Napier-Munn et al., 1996), indicating that SAG mill performance controls comminution circuit throughput. The SAG gross energy is drastically reduced when G.E streams are fed to comminution circuit (Figure 7.8). The approach employed was to incrementally increase the comminution mill throughput until reaching the base case SAG gross energy (Figure 7.9). This ascertains the additional throughput achieved by exploiting the mill energy available related with the Grade Engineering modified PSD feed.

As an example, the anticipated SAG feed tonnage increase for a 20% G.E undersize mill pull, at medium blasting fragmentation and a mill feed Axb of 25, is depicted in Figure 7.8. In this example an increase of total mill throughput of 12% is observed.

Throughput improvement changes (as percentage), modified G.E mill feed particle size distribution (measured through size fraction 20% passing, F20) and changes in flotation feed P80 relative to base case (as percentage) are depicted in Fig. 7.10 for the three different blasting profiles under assessment. The data indicates that throughput improvements will increase flotation feed particle size distribution (P80) up to 5%. Operating flotation Cu recovery by size, indicates that this effect could slightly decrease flotation recovery by less than 1%, suggesting that the current circuit is
operating within the optimum grind size limits and therefore is not considered as part of the current analysis.

Figure 7-8. SAG mill gross power feed tonnage rate versus SAG feed tonnage with a 20% Grade Engineering mass pull.

Figure 7-9. Gross energy SAG mill base cases analysed for the range of ore competence and blasting fragmentation profile analysed.
Simulation results (Appendix C) indicates that throughput improvement is strongly related with mill feed F20, and to lesser extent ore competence (Axb). By adding a G.E circuit streams to the primary crusher product there is a significant increase in the fine particle content below the F80 in the SAG mill feed (Figure 7.7 as example). This certainly explains the close relationship of comminution throughput with F20 rather than F80, the parameter commonly employed in mill specific energy predictions (Morrell 2004).

The SAG mill throughput improvement, obtained from G.E impacted mill feed, also suggests that the proportion of critical size is not significantly altered, and that the major change results from the fraction lower than 20 mm, a size fraction typically characterised for having limited residence time in SAG milling (less than grate size, See section 3). Within the range of operating scenarios tested, SAG mill throughput of more competent ores is more sensitive to changes in the mill feed particle size distribution, than soft ores, which has been observed in previous mine to mill studies (Section 3). The maximum throughput improvement, ~14%, was observed for the highly competent ores (Axb=25) with a fine fragmentation (Figure 7.11).
Figure 7-11. Throughput improvement as function of F20 for the different Ax b across the three blasting profiles examined.

An algorithm was developed to automatically analyse the significant amount of data obtained from IES mass simulation to identify mill throughput improvement. From clear and strong relationships evident in Figure 7.11, a throughput model was developed (Eq.3) that can be employed for coarse separation process optimisation (e.g. strategic mine planning and scheduling or real time optimisation in the context of process control when online information is available) by taking into account modified mill feed PSD, ore competence (Ax b) and grindability (BMWi).

\[
\Delta T_{ph}(\%) = 100 \times \left(1 - \exp\left(-\frac{F_{20}'}{\beta}\right)^\alpha\right) \tag{3}
\]

\(F_{20}'\) (Eq. 4), represents the normalised \(F_{20}'\) relative to size fraction 20% passing associated with the base case blasting fragmentation (\(F_{20,0}\)). Throughput improvement (\(\Delta T_{ph}\)) tends to zero as \(F_{20}\) (altered mill feed PSD due to G.E) approximates to PSD base case, \((F_{20}' \to 0)\).
Both fitted parameters, \( \alpha \) and \( \beta \) strongly depend of impact hardness and \( F20_0 \) (in particular \( \alpha \), which determines the shape of the \( \Delta \) Tph and F20 relationship, Figure 7.10).

This model allows prediction of throughput simulation performance with a \( \pm 0.6\% \) with a \( 95\% \) of confidence (Figure 7.12).

![Figure 7-12. Throughput boost observed (simulations) vs predicted. Dotted lines represent 95% confidence interval.](image)

6. Discussion of the impact of throughput improvement upon Grade Engineering

First order economic analysis was conducted to understand the interaction between Grade Engineering and increased throughput. The minimum economic (or breakeven) cut-off grade defines the minimum grade that is economic to treat at a processing destination given a set of parameters, as Eq.5 depicts:

\[
X_c = \frac{P_o}{V \times Rc} \tag{5}
\]

where \( X_c \) is cut-off grade, Po, is unit processing cost, V, is unit metal price and Rc, is global metal recovery. It has been determined that the mill operating costs are 55% fixed (A, Eq.6) and 45% inversely proportional to tonnage processed (B, Eq.6) (Rendu 2008).
Eq. 3 can be used to estimate the throughput improvement at different processing scenarios, (a defined blasting profile, a G.E mass pull which determines the ultimate mill feed F20, coupled with AxB) and how this might impact the proportion of material amenable for G.E. An increase in throughput slightly lowers the minimum economic cut-off grade, since the unit processing cost is reduced due to an increase in throughput. This can be illustrated through the approach proposed by Carrasco et al., 2016a. This means that samples with a lower RR (~10 RR units) at the same grade are amenable to G.E (i.e. fall inside the area defined by the mass pull and cut-off grade illustrated in Figure 7.13). It is also accurate to assert that lower feed grades are now amenable at the same RR when throughput improvements are considered due to changes in particle size distribution. This effect, when considered in conjunction with the grade tonnage distribution of the deposit, could have a significant economic impact, in particular for marginal ores.

![Figure 7.13](image-url)  
*Figure 7.13. Slight decrease in cut-off grade due to increase in throughput.*
Chapter 7

7. Conclusions

Grade Engineering® (G.E) involves a range of operating techniques that seek to increase mill feed grades by removing as early as possible low grade uneconomic material prior intensive and inefficient grinding. The G.E assessment example discussed in this paper involved a Cu-porphyry deposit in which two size based G.E coarse separation techniques were characterised and economically assessed, preferential grade by size deportment and differential blasting for grade. The first refers to the natural propensity of certain rocks to concentrate metal in finer particles. While differential blasting for grade exploits intrinsic grade variability, where changes in blasting fragmentation aim to induce the deportment of metal into the finer particle fractions. The G.E exploitation strategy involves the installation of screen and crusher equipment, which alters both the mill feed “typical” particle size distribution and mill feed grade. A factorial design approach was employed to assess the magnitude of this effect across multiple operating scenarios using the Integrated Extraction Simulator (IES).

Three G.E blasting fragmentation profiles, six under size mass pulls (proportion of fine material upgraded within G.E circuit) coupled with a range of comminution ore properties (four Axb, and four BMWi) representing the characteristics of the ore body; comprised the G.E operating conditions tested. It was observed that variations in feed particle size distribution significantly impacts the SAG mill gross energy whereas ball mill grinding capacity was not greatly affected.

The application of G.E increases the proportion of fine feed material under the SAG mill grate size, (~20 mm), hence strong relationship between F20 (20% passing size) and throughput improvement. The increase in comminution circuit throughput is directly proportional to ore competence and blasting intensity, indicating a maximum of ~14% without a detrimental impact in comminution performance under the operating conditions examined.

A G.E comminution throughput improvement model has been developed that takes into account SAG mill feed F20, defined as a function of blasting fragmentation, the proportion of G.E/direct mill feed, and mass pull, coupled with impact hardness (Axb), by using the mass simulation IES capabilities. This model can be used in further G.E coarse separation circuit value based optimisation analysis (i.e. strategic mine planning and scheduling).

The impact of throughput improvements upon the G.E strategy has been discussed. This could decrease the economic cut-off grade for ore and therefore potentially increases the material amenable to G.E. It is noteworthy that cut-off grades are function of costs, revenue and mine operating mode. Therefore, the cut-off grade estimation is not a mere function of throughput. The
increase in comminution feed grade due to Grade Engineering could also lead to a further increase in global metal recovery and therefore cut-off grade. Furthermore, deleterious elements, such as clay and arsenic can certainly have a baneful impact on mining revenue. This analysis coupled with grade-tonnage information is required to determine the magnitude of the additional value resulted as the combination of both, increase in feed grades and throughput strategies.

During the simulations, the operating mill parameters (i.e. critical velocity, ball load) were kept constant. Therefore there is an opportunity to determine more customised comminution “recipes” to optimum process the different rock type operating scenarios. Nevertheless, in this analysis the inherent modelling limitations need to be considered (Bailey et al., 2009) as well as the correlation between processing variables and rock based attributes (i.e. PSD and hardness) to ensure the reliability of simulation results. An important aspect that need to be consider is the change in viscosity and thus performance within the SAG and ball mill due to an increase in fine material. The current models embedded in IES, i.e. SAG mill variable rates and perfect mix ball mill (Napier-Munn et al., 1996), both perfectly describe breakage (through appearance, breakage classification and selection functions) but to lesser extent viscosity. It seems to be that the most effective approach to understand the fines impact upon viscosity is though piloting, since slurry effects are particularly complex to model and therefore to simulate.

8. Acknowledgments

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9. References


Chapter 8  Value Driven Methodology to Assess Risk and Operating Robustness for Grade Engineering Strategies by means of Stochastic Optimisation

1. Abstract
Grade Engineering® spans a range of operational techniques that exploits intrinsic grade variability to remove low grade uneconomic material prior to energy intensive and inefficient grinding. Grade Engineering provides an additional level of operational flexibility whilst also incurring complexity that needs to be managed for an effective operational deployment. An integrated value driven methodology has been developed to manage this complexity by means of stochastic optimisation. This allows the optimum Grade Engineering processing “recipe” to be determined that maximises value per unit of time that can be drawn from a production volume under a set of user defined constraints. The introduction of uncertainty in the stochastic optimisation problem enables the assessment of the risk and operating robustness, both essential in robust decision-making processes. The case study discussed in the paper comprises a large open cut copper porphyry deposit for which two Grade Engineering strategies are assessed: differential blasting for grade, and preferential grade by size response. These size-based coarse separation levers are subsequently exploited through a Grade Engineering circuit. This comprises a set of screens and crushers, with a configuration and operating settings defined by the Grade Engineering recipe. The methodology developed demonstrated that size-based Grade engineering is a robust operating option that can effectively deliver significant improvements in unit metal productivity.

2. Introduction. Mining Moving Towards a Manufacturing Industry through Flexibility
The global mining industry is currently focused on improving unit metal productivity and energy efficiency in order to fulfil increasing demand for natural resources. These are currently being impacted by increases in processing costs and the trend of reduced ore body grade (Napier-Munn, 2015; Bearman, 2012; Prior et al., 2012; Topp et al., 2008).

Novel operating strategies such as flexible circuits (Powell et al., 2014; Foggiatto et al., 2014; Powell and Bye 2009) and Grade Engineering (Walters, 2016; Carrasco et al., 2016a; Carrasco et al., 2016b; Carrasco et al., 2016c) seek to provide an additional level of operating flexibility to exploit inherent ore body variability, enabling resource as well as process optimisation. Nevertheless, this flexibility presents significant challenges to the current standard operating philosophy which is mainly focused on maximising material quantity, rather than quality.

Industries with a significant level of flexibility such as manufacturing, chemical and oil and gas have coped with the associated complexity through the development of decision support and execution systems (Engell and Harjunkoski, 2012; Frost and Sullivan, 2010; Scholten,
This has been done in conjunction with new approaches to data integration to understand the impact of flexible operation decisions across the entire system value chain (Englell and Harjunkoski, 2012; Harjunkoski, 2009; Wassick, 2009; Smith, 2005; Sakizlis et al., 2004). A clear example of this flexibility successfully implemented in the refining process of the oil and gas industry is discussed in this paper. This process can be divided into three areas: crude operation, production and blending.

A variety of crude oil can be fed to the production plant, characterised by its flexibility to accommodate a range of flow rates, compositions and physical/chemical properties (density, flash point, etc.) to produce a variety of saleable products. These are subsequently blended to meet a dynamic product demand. However, variability in feed characteristics are often difficult to quantify and are therefore uncertain (e.g. inconsistencies in the feed stock, coupled with variations in the performance of upstream processes) (Mesfin and Shuhaimi, 2010; Cao et al., 2009). Hence the problem in this flexible production environment is to make the process economically optimal, but still feasible under uncertain feed conditions. As these decisions are made in close to real time, it is essential to take into account the possible nonlinearities in process operations through detailed process models. This is in opposition to simple linear representations of production processes that are generally adequate for strategic/long term based decisions. (Newman, 2010; Wassick, 2009).

This has been addressed through process optimisation under uncertainty, also referred to as stochastic optimisation (Navia et al., 2011; Birge and Louveaux, 2010; Sahinidis, 2004; Wendt et al., 2002). This aims to deliver robust processing decisions which have been extensively applied across process design, operation and control (Gabrel et al., 2014; Sahinidis, 2004) in the aforementioned industries, and to a lesser extent, in mining. A novel decision support tool referred to as Ore Logic® has been developed to support Grade Engineering deployment at an open cut copper porphyry deposit. Two GE size based separation techniques are extensively analysed; preferential grade by size deportment (Carrasco et al., 2016a; Carrasco et al., 2014; Carrasco 2013) and differential blasting for grade (Carrasco et al., 2016c).

The former refers to a natural based rock property whereby a significant metal proportion preferentially deports into specific size fractions after breakage. Differential blasting aims to change blast product fragmentation to “induce” grade by size deportment through the exploitation of deposit spatial grade heterogeneity characteristic. This relates the presence of spatial high grade and low grade discrete clusters within a certain production volume originally assigned to a single destination (e.g. waste, leach, and mill) based on its average grade. In differential blasting for grade high levels of energy are applied to high grade pockets and low energy is imported to low grade...
zones, allowing high and low grade cluster fragmented rock to be separated based on their different particle size distributions, via screening.

These size based separation responses are exploited through a Grade Engineering circuit, comprising of a set of screens and a crusher, which were modelled with the widely accepted JKMRC performance models (Napier-Munn et al., 1996) to better describe the nonlinear interaction between rock based properties and equipment performance. This tool enables the Grade Engineering (GE) strategy to be assessed not merely for value, but for robustness and flexibility. Ore Logic® comprises 5 modules as shown in Figure 8.1. The first module is associated with uncertainty modelling, where information from an industrial GE screening trial has been employed. The aim is to estimate the probability density distributions of the GE inputs later used in the stochastic optimisation module (Carrasco et al., 2016d). The second module takes into account variations in comminution and flotation performance due to changes in standard mill feed particle size distributions (Carrasco et al., 2016c). The third module predicts changes in grade by size responses due to breakage, using a statistically robust coarse liberation model (Carrasco et al., 2016b). The fourth module employs the aforementioned inputs to perform a chance constraint stochastic optimisation (Mesfin and Shuhaimi, 2010; Li et al., 2008; Sahinidis, 2004) through sample average approximation (Shapiro 2013; Pagnoncelli et al., 2009; Shapiro and Wardi, 1996) and a customised genetic algorithm. This determines the optimum material processing destination, a GE processing recipe comprising of the optimum processing path and GE operating settings (screen apertures and crusher closed side setting). The final Ore Logic component, data analysis, performs comparative statistical tests (e.g. t-test) and a robustness analysis analysing the interaction between the objective function and the feasible region defined by the constraints.

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**Figure 8-1. Ore Logic® structure.**

GE: Grade Engineering
IES: Integrated Extraction Simulator
PGBS: Preferential Grade by Size
3. Optimisation under Uncertainty (Stochastic Optimisation)

Optimisation under uncertainty or stochastic optimisation refers to a collection of methods for minimising or maximising an objective function when uncertainty is present. Each of the uncertain data is described in terms of the probability distribution (e.g. Gaussian, log-normal) while its correlation with other variables is also characterised. These uncertain variables are propagated through the process to the output variables. The aim is to integrate the available stochastic information in the optimisation problem.

Stochastic problems can be essentially divided into two different categories, those which involve a sequence of decisions over several time periods (multistage problems), or those involving a single time period (single stage).

The multistage approaches seek to find an optimal sequence of decisions over a certain period of time. This approach has been extensively used in long term strategic scheduling and planning problems. In mining this approach has received great attention in the last decade (Dimitrakopoulos and Godoy, 2014; Montiel and Dimitrakopoulos, 2013; Godoy, 2003; Dimitrakopoulos et al., 2002). The uncertainty is modelled through geological conditional simulation and therefore accounts for ore body knowledge uncertainty. However, this is beyond the scope of this work.

Single stage aims to find a single optimal decision at a given point in time, such as the best set of operating settings for given feed characteristic. Two strategies are employed to solve single stage problems under uncertainty: a two stage programming approach with recourse formulation and probabilistic, or chance constrained programming (Arellano-Garcia, 2006; Charnes and Cooper, 1959). The first has been appropriately employed to solve planning problems with demand under uncertainty (Petkov and Maranas, 1997), while the second approach has been extensively used in production optimisation and process control (Li et al., 2008; Arellano-Garcia, 2006) and thus is the method employed in this paper. In the chance constrained approach, the system’s ability to meet a feasible solution in an uncertain environment is considered, i.e. the system’s reliability/robustness. Therefore, this technique enables the quantification of the compromise between profitability and robustness. The stochastic optimisation using chance constrained provides comprehensive information on the economic achievement as a function of the desired confidence level of satisfying process constraints, particularly important in robust decision making process.
4. Uncertainty Modelling

Li et al., (2008) divides uncertainties into external and internal categories. The former is related to process inputs (i.e. variability in streams properties such as: composition, flowrate), equipment operating settings (e.g. pressure, temperature, ball load) as well as market conditions (e.g. price, demand). Internal uncertainties represent the lack of process knowledge, such as process model parameters. This work focuses on external uncertainties.

In Ore Logic® there are three externalities where GE uncertainty is analysed: input grade, blast fragmentation and preferential grade by size deportment, characterised through a ranking response (RR) parameter (Carrasco et al., 2016a; Carrasco et al., 2016b; Carrasco et al., 2016c). Since differential blasting for grade involves the assessment of two (or more) discrete zones within a blast (i.e. high and low grade), each zone requires identical information (i.e. grade, RR and particle size distribution).

The uncertainty of achieving a blast fragmentation target is a function of poor blasting practices or limited ore body knowledge which is not able to capture its variability (Onederra el al., 2010). The use of online information such as measurement while drilling, and image bucket analysis (La Rosa et al., 2001; Palangio and Maerz, 1999) can be employed to understand the extent of blasting uncertainty and its relationship with geotechnical variability.

In this work a different approach has been used, which consists of fitting a fragmentation model (Eq.1) to define the confidence limits of the model parameters to gauge blasting uncertainty. The application of Eq.1 provides some degree of error fitting, but with less parameters compared to the Swebrec and Kuz-Ram expression in the data analysed. Two data sets were utilised to determine uncertainties associated with the application of differential blasting for grade. First, particle size distributions obtained through muckpile image analysis of a blasting domain in which a set of identical blasting parameters was applied (Production blasting in Figure 8.2). The second source of information (Grade by size trial in Figure 8.2) was taken from daily data gathered during a preferential grade by size trial where the same ore type was periodically screened (Carrasco et al., 2016d).

Eq.1 was applied in both cases. Variability in blasting fragmentation measured through the variability (dotted lines, Figure 8.2) of the blasting fitting parameters ($d^*, n$, Eq.1) was significantly higher than the confidence limits of the fitting parameters applied. Therefore, this range is employed as input in the Eq.1 which defines the uncertainty in the blasting fragmentation. The
parameter’s correlation (i.e. $d^*$, $n$) is not statistically significant, and thus the covariance does not need to be considered when both parameters are simulated using a Monte Carlo analysis in the stochastic optimisation (See Section 4).

$$CW = 1 - \exp\left(-\left(\frac{d}{d^*}\right)^n\right)$$

(1)

$CW$, Cumulative passing weight, $d_p$, size (mm) and $d^*$ and $n$, are fitted parameters

Figure 8-2. Variability of the coefficients in Eq. I determine uncertainty in blasting. 95% confidence interval in dotted lines.

Feed grade to the Grade Engineering (GE) circuit is a crucial parameter for processing material optimisation. Although the variability in blast hole grade information can be employed to gauge feed grade uncertainty (i.e. geostatistical simulation), mining methods, ore handling, crushing, blending and particle size segregation have a significant impact within grade uncertainty, however complex to take into account (Dowd and Dare-Bryan, 2004). Information gathered during the preferential grade by size characterisation trial (Carrasco et al., 2016d) has been employed to model preferential grade by size deportment uncertainty (Carrasco et al., 2016a; Carrasco et al., 2016b; Carrasco et al., 2014; Carrasco, 2013). Carrasco et al (2016d) focused on assessing the influence of different sources of uncertainty upon industrial pilot preferential grade by size validation. Figure 8.3 depicts the modelled probability distributions of the inputs employed in the Grade Engineering assessment. The Anderson-Darling test was employed to determine the significance of each
probability distribution input’s departure from normality (Napier-Munn, 2014). It can be observed that grade is slightly skewed, but normally distributed in log space.

![Figure 8-3. Probability distribution of the inputs parameters employed in Ore Logic®.](image)

5. Grade Engineering Circuit

The optimisation aims to maximise the net value of material treated through the GE circuit (Figure 8.4) relative to operational constraints of production to exploit grade heterogeneity and/or preferential grade by size deportment. In this analysis, three possible processing destinations are considered: waste (Figure 8.4, W), leaching (Figure 8.4, L), and mill (Figure 8.4, M).

The GE circuit comprises of a grizzly screen (G, Figure 8.4) with a fixed aperture (200 mm) to protect downstream processes from potential blasting inefficiencies. A double deck screen, modelled as two independent screen devices (S1 and S2), and a crusher (C, Figure 8.4). Additional flexibility regarding material treatment pathways within the GE circuit is also included and evaluated (each of the branches, \( t_n \) in Figure 8.4). The optimisation routine aims to determine the optimum GE “processing recipe”, consisting of the material treatment pathway, product processing destinations and equipment operating settings (\( O_n \), Figure 8.4), including screen apertures and crusher closed side setting. Tables 8.1 and 8.2 depict integer and mass balance variable constraints respectively.
The screen performance model is based on an efficiency partition model (Napier-Munn et al., 1996) which relates a defined screen efficiency with a real separation point (real size aperture). In the operation of industrial screening, it has been observed that screen efficiency changes with capacity. The maximum efficiency is 95% which very often occurs at 80% of rated capacity. Below 70% of rated capacity, the screening efficiency decreases dramatically because the limited load allows particles to bounce away from apertures (Bothwell and Mular, 2002). This effect has also been incorporated in the screen process model.

The crusher model is based on a classification function coupled with a breakage function (Whiten, 1974). This has been integrated with the coarse liberation model based on preferential grade by size responses deportment responses developed in Carrasco et al., (2016b) to account for changes in grade distribution due to the size reduction process (Figure 8.5). Thus, the material fed to a Grade Engineering circuit is modelled using a population balance approach (Powell and Morrison, 2006; Napier-Munn et al., 1996) that takes into account the non-linear performance responses due to the interaction of equipment models and rock based processing properties.

Figure 8-4. Grade engineering circuit illustration. G, grizzly; S1, screen1; S2, screen2; C, crusher; W, waste; L, leach; M, mill.
Table 8.1. Binary streams constraints.

| \( i_1 \) = 1 | Oversize (O) Grizzly (G) |
| \( i_2 + i_3 = 1 \) | Undersize(U) Grizzly (G) |
| \( i_4 + i_5 = 1 \) | Oversize (O) Screen 1 (S1) |
| \( i_6 + i_7 = 1 \) | Undersize(U) Screen 1 (S1) |
| \( i_8 + i_9 + i_{10} + i_{11} = 1 \) | Crusher (C) product (P) |
| \( i_{12} + i_{13} = 1 \) | Oversize (O) Screen 2 (S2) |
| \( i_{14} + i_{15} = 1 \) | Undersize (U) Screen 2 (S2) |

Table 8.2. Mass balance constraints. U, undersize; O, oversize; P, product. See Table 8.1

| \( M^U_{i3} + M^U_{i7} + M^P_{i9} + M^U_{i15} \) | Mill |
| \( M^U_{i5} + M^U_{i13} + M^P_{i14} + M^P_{i9} \) | Leach |
| \( M^P_{i8} + M^O_{i4} + M^O_{i12} \) | Waste |

Figure 8-5. Illustration interaction of crusher model and coarse liberation model (CLiM) developed using preferential grade by size responses (Carrasco et al., 2016b).

6. Objective Function.

The objective function employed in the optimisation routine aims to maximise the net value of the Grade Engineering circuit with respect to the maximum time required to mine, separate and process the material (expressed as dollars per hour in Eq.2). This approach rewards solutions that increase
grade and throughput when the mill is the bottleneck for production, and solutions that maximise the quantity of metal treated at the mill and recovered in flotation when mining is the bottleneck for production. If the Grade Engineering circuit becomes the bottleneck for production, solutions that balance the internal constraint within the Grade Engineering circuit (opening crusher closed side setting or screen apertures) with the external constraints of the mine or mill will be favoured. As a single stage solution in the short term, the operation of the Grade Engineering circuit to maximise net value relative to active constraints of production will improve the present value of the operation.

The first term of the objective function in Eq.2 reflects the total revenue (value) obtained by sending the GE circuit product to the mill and leach processing destinations (Figure 8.4). The second term refers to the total variable costs (dollars per ton) incurred by processing material through the GE circuit, including material handling, energy and consumable costs related to the processing path and equipment chosen (branch in Figure 8.4). The variable costs have been estimated with reference to the size of the equipment to be installed and the preferred location of the GE plant, which defines the material handling costs to processing destinations. The last term captures the fixed processing costs of the GE circuit and mill which are incurred per unit of time ($/h), where the factor $\theta$ is the maximum time required to treat the material in the chosen pathway and is governed by the bottleneck of the system (Eq.3).

It should be noted that the impact in comminution throughput ($K_{mill}$) due to changes in particle size distribution from the application of Grade Engineering is captured by employing the model developed in Carrasco et al., (2016c). This was obtained by exploiting mass simulation capabilities embedded in the Integrated Extraction Simulator (IES).

Comminution throughput variations ($\gamma$ factor in Eq.3) are predicted as a function of relative changes in mill feed particle size distribution, ore competence (Ax$b$) and grindability (BMWi). However, these parameters are assumed constant in the stochastic assessment ($ Ax_b = 25 $ and $ BMWi = 13 $ kWh/t).

$$ F = \left( \left( \sum_{p=1}^{2} X_p F_p R_p \gamma - \left( \sum_{i=1}^{n} F_{i} (\Omega_i) P C_{li} + \sum_{p=1}^{2} F_p P c_p \right) - (\lambda_{GE} + \lambda_{Milt}) \times \theta \right) / \theta \right) $$

(2)
\[ \theta = \max \left( \frac{F_{\text{Mine}}}{K_{\text{Mine}}}, \frac{F_G}{K_G}, \frac{F_C}{K_C}, \frac{F_{S1}}{K_{S1}}, \frac{F_{S2}}{K_{S2}} \times \gamma \times K_{\text{Mill}} \right) \] (3)

Table 8.3. Detailed description of the variables employed in Eq.2

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F )</td>
<td>Value per unit of time</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Maximum time required to process the material</td>
</tr>
<tr>
<td>( X_p )</td>
<td>Feed grade to processing destination (i.e. leach and mill)</td>
</tr>
<tr>
<td>( F_p )</td>
<td>Tons of material sent to processing destination (i.e. leach and mill), see Table 2</td>
</tr>
<tr>
<td>( R_p )</td>
<td>Recovery at each processing destination (i.e. leach and mill)</td>
</tr>
<tr>
<td>( v )</td>
<td>Cu price ($/Cu ton)</td>
</tr>
<tr>
<td>( F_l )</td>
<td>Tons of material processing through G.E circuit (e.g. crusher, screen 1, screen 2), which is a function of ( O_l )</td>
</tr>
<tr>
<td>( O_l )</td>
<td>Operating setting within G.E circuit (e.g. crusher side setting, Screen Apertures 1 and 2)</td>
</tr>
<tr>
<td>( P_{C_l} )</td>
<td>G.E processing costs per ton ($/t)</td>
</tr>
<tr>
<td>( P_{C_p} )</td>
<td>Processing destination costs ($/t)</td>
</tr>
<tr>
<td>( l )</td>
<td>Integer variable associated with the processing branch, Figure 4</td>
</tr>
<tr>
<td>( \lambda_{G_E} )</td>
<td>G.E processing costs per hour ($/h)</td>
</tr>
<tr>
<td>( F_{G_E} )</td>
<td>Tons of material processed by G.E circuit</td>
</tr>
<tr>
<td>( F_{\text{Mine}} )</td>
<td>Material available</td>
</tr>
<tr>
<td>( F_{\text{Mine}} )</td>
<td>Mine rate</td>
</tr>
<tr>
<td>( K_G )</td>
<td>Grizzly capacity tph.</td>
</tr>
<tr>
<td>( F_G )</td>
<td>Tons of material sent to the Grizzly screen</td>
</tr>
<tr>
<td>( K_C )</td>
<td>Grizzly capacity tph.</td>
</tr>
<tr>
<td>( F_C )</td>
<td>Tons of material sent to the crusher</td>
</tr>
<tr>
<td>( K_C )</td>
<td>Crusher capacity tph.</td>
</tr>
<tr>
<td>( F_{S1} )</td>
<td>Tons of material sent to Screen 1</td>
</tr>
<tr>
<td>( K_{S1} )</td>
<td>Screen 1 capacity tph.</td>
</tr>
<tr>
<td>( F_{S2} )</td>
<td>Tons of material sent to Screen 2</td>
</tr>
<tr>
<td>( K_{S2} )</td>
<td>Screen 2 capacity tph.</td>
</tr>
<tr>
<td>( \lambda_{\text{Mill}} )</td>
<td>Mill processing costs per unit of time ($/h)</td>
</tr>
</tbody>
</table>
\( F_{p=mill} \) Tons of material sent to the mill via G.E circuit

\( K_{mill} \) Mill rate, tph

\( \gamma \) Estimated using model developed by Carrasco et al., 2016c; \( \gamma = \frac{K^{GE}_{mill}}{K_{mill}} \)

The following upper and lower variables constraints were applied, which represent the operating setting limits of the equipment (i.e. screen aperture and crusher side setting).

\[
SA_{2,1} \leq O_1 \leq SA_{2,2}
\]

\[
CSS_1 \leq O_2 \leq CSS_2
\]

\[
SA_{3,1} \leq O_3 \leq SA_{3,2}
\]

7. Method to Solve Optimisation under Uncertainty

Sample Average Approximation (SAA) has been employed given its relatively easy numerical implementation and good convergence properties (Shapiro, 2013; Shapiro and Wardi, 1996; Robinson, 1996). SAA is a two part method that uses Monte Carlo sampling and deterministic optimisation to solve Eq.2. Essentially the profit function is approximated by the expected value (Eq.7) of the independent realizations \( (\xi_i, \text{Eq.7}) \) defined by the probability density distribution utilised to characterise the uncertainty of the optimisation inputs. The right hand side of Eq.8 (see Eq.2) is deterministic, so deterministic optimisation methods can be used to solve the approximate problem.

\[
f(x) = \max \{E(F(x, \xi_i))\}
\]

\[
f(x) = \max \left\{ F(x, \xi) \approx \frac{1}{n} \sum_{i=1}^{n} F(x, \xi_i) \right\}
\]
Heuristic based methods are extensively used to solve non-linear problems. Heuristics search methods start with an initial solution then explore all solutions in the neighbourhood of the point to look for the best one. This is followed by repeats if an improved point is found. Heuristics algorithms such as Tabu Search (TS), Scatter Search (SS), Simulated Annealing (SA) and Genetic Algorithm (GA) guide and improve the heuristic algorithm. These perform the searching more efficiently while avoiding becoming trapped in a local optimum (Edgar et al., 2001). GA have been successfully applied in the area of mineral processing, due to the ease of implementation and robustness in solving non-linear and non-convex optimisation problems (Mhlanga et al., 2011; Bengtsson et al., 2009; Svedensten 2007; Husband et al., 2006; Svedensten and Evertsson, 2005; Contoni et al., 2000). GA is based on Darwin’s principle of natural selection. This comprises three steps: selection, crossover and mutation. GA requires an initial population of candidates, from which some are selected (based on the fitness value) as “parents” and are used to create the next generation of individuals, called “children”. An additional population of children is also generated by combining different pairs of parents in a process called crossover. A third children’s generation is obtained by applying random changes to individual parents in a step named mutation. The children now become the candidates from which the next generation of parents is selected for the subsequent iteration. The iterations finish when a set of user defined conditions is met (Goldberg, 1989).

A customised GA algorithm was developed to cope with time inefficiencies observed during the optimisation under uncertainty. This consisted of determining the initial population ($S_0$, Figure 8.6) through an iterative process while determining the optimum processing path for the candidate (integer variables in Eq.2 and $i^0_k$ in Figure 8.6). The integer together with continuous variables (operating settings, $O^0_k$) are then fed to a tuned GA engine. Extensive analysis was conducted to determine the optimum trade-off between time convergence and the global optimum solution. Considering: the initial population’s range of candidates, mutation rate, fitness tolerance (difference between the optimum values, i.e. Eq.2, in each iteration) and the number of generations required. The integer component of the final solution ($i^f_k$) is perturbed and compared with the GA solution ($S_f$). If the perturbed value is higher, the algorithm repeats the process assuming $S_0 = S_f$ otherwise a solution is found.
8. Results and Analysis

8.1. Grade Engineering scenarios tested

The differential blasting for grade fragmentation was determined by taking into account geotechnical characteristics of the distinctive high and low grade areas (UCS, RQD, mean block size) and production and safety impacts. The last comprised blasting induced vibrations on wall stability (in high intensive blasting) as well as shovel productivity, (i.e. diggability in low intensive blast). Burden and spacing was assumed constant, and therefore the difference in blasting fragmentation is mainly driven by changes in powder factor (i.e. steaming high) and explosive type. Figure 8.7 depicts the high and low intensive blasting envelope employed in this analysis. Eight Grade Engineering operating scenarios are investigated (Table 8.4). Preferential grade by size responses have been assigned based on the range encountered during the geometallurgical characterisation. For the first four scenarios (Scenarios 1 to 4, Table 8.4) differential blasting (DB, Table 8.4) for grade is applied, whereas for scenarios 5 to 8, identical blasting fragmentation was employed (assuming a low intensive blast, see Figure 8.8) and therefore just preferential grade by size is evaluated.

Since the stochastic optimisation assessment requires significant computer power, the tool developed by Carrasco et al. (2016a) can be used to filter out scenarios that do not require a further size based separation assessment, thus easing computing requirements while improving time efficiency. However, since the Grade Engineering circuit can be also utilised as a pre-crushing circuit, the effect on changes in mill feed particle size distribution is still performed.
The stochastic assessment enables the use of statistical comparative tools, such as t-test to determine the significance of the optimised Grade Engineering strategy relative to the base case as well as across different scenarios under assessment. However, the application of the aforementioned statistical techniques require that population distribution under assessment is normally distributed. Alternatively two options can be considered: the use of a transformation function (e.g. Box-Cox) for converting non – normal data to normal, or the application of non-parametric tests (e.g. Mann-Whitney test, an alternative to the 2-sample t-test and Siegel-Tukey to F-test) (Napier-Munn, 2014).

Assuming that the mill capacity is able to process the entire volume of production, the optimum Grade Engineering processing recipe across the scenarios tested comprises essentially a pre-crushing circuit seeking to maximise throughput by increasing the proportion of fine material (typically below SAG grate size). (Figure 8.8, Table 5). This is achieved by minimising the

**Figure 8.7.** Particle size distribution envelope employed in differential blasting assessment.

**Table 8.4.** Scenarios Tested (DB= differential blasting for grade, HG= high grade, LG= low grade).

<table>
<thead>
<tr>
<th>Scenario</th>
<th>DB</th>
<th>Grade_HG</th>
<th>Grade_LG</th>
<th>Area_HG%</th>
<th>Area_LG%</th>
<th>RR_HG</th>
<th>RR_LG</th>
</tr>
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<tbody>
<tr>
<td>1</td>
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<td>0.41</td>
<td>0.21</td>
<td>80</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
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<td>20</td>
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</tr>
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<td>80</td>
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<td>20</td>
<td>80</td>
</tr>
<tr>
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<td>No</td>
<td>0.41</td>
<td>0.21</td>
<td>80</td>
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<td>0.41</td>
<td>0.21</td>
<td>80</td>
<td>20</td>
<td>20</td>
<td>80</td>
</tr>
</tbody>
</table>
crusher’s CSS in the GE circuit (Table 8.5). Nevertheless, scenarios representing the differential blasting for grade strategy (scenarios 1-4, Table 8.4) add more value than preferential grade by size scenarios (scenarios 5-8, Table 8.4), even though the Grade Engineering circuit has the same degree of operational flexibility (Figure 8.9). This indicates that despite the ability of the circuit to produce fines (lower than SAG grate aperture), the fines produced by blasting to a large extent influence the comminution throughput performance. However, it is well known from Mine to Mill studies that the reduction of material oversize coupled with crusher choke feed conditions promote the creation of fine material, which has not been incorporated in the current crusher model and is therefore not being captured in the Grade Engineering optimisation.

![Figure 8-8. GE processing recipe, pre-crushing circuit.](image)

**Table 8.5. GE recipe, processing path and operating settings after optimisation under uncertainty.**

<table>
<thead>
<tr>
<th></th>
<th>i1</th>
<th>i2</th>
<th>i3</th>
<th>i4</th>
<th>i5</th>
<th>i6</th>
<th>i7</th>
<th>i8</th>
<th>i9</th>
<th>i10</th>
<th>i11</th>
<th>i12</th>
<th>i13</th>
<th>i14</th>
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<th>CSS</th>
<th>S1</th>
<th>S2</th>
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<td>NA</td>
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</tr>
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</table>
Table 8.6 depicts the P-value results of conducting t-test comparing the objective function mean (i.e. value per unit of time) values for the different GE scenarios, assuming unequal variances. The lower the P-value the higher the statistical difference of the means. Shaded cells present P-values lower than 0.05, representing a significant difference in the means with a 95 % confidence level. This analysis indicates all the Grade Engineering scenarios provide a significant value (first row, Table 8.6) relative to the base case (send material to the mill). Within the scenarios, however, two groups are statistically different, scenarios 1 to 4, comprise one group (unshaded cells, no statistical difference between them) and scenarios 5 to 8. It is noteworthy that these comparisons (t-test) depend strongly of the number of realizations (n, Eq.8) defined by the convergence rate of the sampling method. Shapiro (2003) provides a guide to determine the minimum sampling size essentially as function of the expected variance, degree of statistical confidence and precision. In this analysis 250 realizations were employed.
Table 8.6. P-values obtained by comparing the objective value means of each of scenarios under assessment, 
(Base Case, BC; Scenario, Sc)

<table>
<thead>
<tr>
<th>ttest</th>
<th>BC</th>
<th>Sc1</th>
<th>Sc2</th>
<th>Sc3</th>
<th>Sc4</th>
<th>Sc5</th>
<th>Sc6</th>
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</thead>
<tbody>
<tr>
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</tr>
<tr>
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<td>0.6</td>
<td>0.4</td>
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<td>0.0</td>
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</tr>
<tr>
<td>Sc2</td>
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<td>0.7</td>
<td>0.0</td>
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<td>0.0</td>
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</tr>
<tr>
<td>Sc3</td>
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<td>0.1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
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<tr>
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<td></td>
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</tr>
</tbody>
</table>

8.2. Sensibility analysis of mill treatment, Changes in Operating Mode

A sensitivity analysis of the Grade Engineering strategy is conducted by changing the capacity of what is very often identified as the bottleneck within the current mineral processing circuit as the available comminution capacity \( K_{mill} \) (Eq.3). This enables a determination to be made of the impact of a defined operating mode upon the value that GE is able to deliver in conjunction with an associated GE recipe. Four relative milling capacities, \( K' \), were employed (1, 0.7, 0.5 and 0.3) \( (K' = K_{l}/K_{mill}) \). This is defined as the ratio of the available to nominal mill capacity. Figure 8.10 and 8.11 depict respectively the optimum as well as the standard deviation (error bars) associated with the differential blasting for grade scenario (1 to 4) and preferential grade by size only (5 to 8) in addition to the base case respectively, for difference mill capacities. Scenarios 2 and 3 add statistically significant value to base case (95% of confidence) of up to 0.5 of nominal mill capacity. Scenarios 6 and 7 at a mill rate of 0.5 to nominal mill capacity do not provide statistical meaningful differences.

Figure 8.12 illustrates the different Grade Engineering circuit configurations as a response to changes in mill capacity \( (K') \). As the mill available capacity decreases the circuit focuses on intensively extracting and sending the best material to each processing destination (i.e. Leach and Mill). The GE circuit flexibility achieves this essentially by increasing the selectivity through increasing the probability of material separation events (from 1 to 3 in Figure 8.12, Appendix C).
Figure 8-10. Value per hour for base case and GE scenarios from 1 to 4 against milling capacity available ($K'$).

Figure 8-11. Value per hour for base case and GE scenarios from 1 to 4 against milling capacity available ($K'$).
8.3. **Robustness Analysis – A Compromise between Objective Function and Constraints**

The chance constrained approaches allow an assessment of the relationship between user defined operating constraints and the objective function. Those constraints are often associated with safety and product quality, which sometimes are equally important as the objective functions (Eq.2). This interaction (optimal-constraints) can be obtained by determining the optimal solution for different confidence levels (Eq.9), where $Pr\{\}$ represents the probability (reliability) of complying the inequality constraints, while $\alpha$ (Eq.9) is a user-predefined confidence interval. Single or Joint constraints can be employed as a function of the problem/system characteristics under assessment. For the former, individual confidence levels are assigned to each constraint equation, while in joint constraints identical confidence levels are applied regardless. In this analysis, single is used and the constraints employed are related to product quality (i.e. mill feed grade and mill particle size distribution). Five confidence intervals are employed: 0.99 (i.e. requiring a 99% probability of realization over the used defined constraint), 0.95, 0.9, 0.8 and 0.5. The shape of the curve describes
the robustness of the solution, which is crucial for decision making. The steeper the curve the less robust is the optimum value drawn from the optimisation.¹

\[ Pr\{g_i(u,x) \geq 0\} \geq \alpha_i; 0 \leq \alpha_i \leq 1; \text{where } \alpha \text{ is used defined confidence level} \]  \hspace{1cm} (9)

As Scenarios 2, 3, 6 and 7 provided additional value, these were further analysed. Figure 8.13 depicts the interplay between the objective function and the degree of confidence when the size fraction corresponding to 20% passing the mill feed (i.e. F20) is included in the constraints. The degree of confidence in sending a F20 greater or equal to 10 mm is employed as reference. The scenarios associated with preferential grade by size are more robust compared with differential blasting for grade. Identical analysis can be performed for grade sent to the mill (Figure 8.14). By using 0.45% Cu as a reference, only Scenarios 2 and 3 were capable of providing feasible solutions. Although Scenario 2 delivers more value, both present similar behaviour in terms of robustness (Appendix C).

![Figure 8.13. Objective function against degree of confidence for F20 fed to the mill, at 0.7 nominal mill capacity.](image)

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¹ The robustness of the solution, which is crucial for decision making. The steeper the curve the less robust is the optimum value drawn from the optimisation.
Figure 8.14. Objective function against degree of confidence for Cu grade fed to the mill, at 0.7 nominal mill capacity.

Eq.10 can be employed to mathematically rank the operating robustness across different operating scenarios. This function mimics the curve shape observed in Figure 8.13 and Figure 8.14. The factor represents the maximum value of the objective function, encountered at the minimum confidence interval ($\alpha=0.5$) whereas the parameter refers to the objective’s minimum value ($\alpha=0.99=\alpha_{\text{max}}$). The robustness factor, $\xi$, is then obtained by through minimisation of least squares. The higher $\xi$ is the lower the robustness of the optimum relative to the constraint analysed.

$$\varphi + (\omega - \varphi)(1 - \exp(-(\alpha - \alpha_{\text{max}})\xi)) \quad (10)$$

The objective function, its associated uncertainty (standard deviation, shown as error bars) and operating robustness can be plotted in the risk-value diagram (Figure 8.15). Mill feed particle size distribution (F20) has been employed to determine $\xi$ (see Figure 8.13).
9. Conclusions

An optimisation under uncertainty approach was employed to determine the optimum Grade Engineering recipe that maximises value per unit of time of a volume of material. This “recipe” comprises the optimum processing path sequence as well as the crusher closed side setting and screen apertures and was conducted by integrating the Grade Engineering outcomes from previous studies. Uncertainties have been modelled primarily using the information from an Industrial Grade Engineering validation trial (Carrasco et al., 2016c). A Coarse liberation model developed in Carrasco et al., (2016b) has been integrated with the Whiten crusher model (Whiten, 1974) to incorporate changes in grade distribution due to size reduction processes. Changes in comminution performance (i.e. throughput) have been accounted for using a throughput model which spans the dynamic range of ore type comminution attributes and likely blasting fragmentations (Carrasco et al., 2016c). The integration of the aforementioned sources of information and the subsequent data
analysis has been named as Ore Logic®, a value driven decision support tool under uncertainty to aid the operating deployment of the two size-based coarse separation techniques, pertaining to Grade Engineering. The introduction of uncertainty in the optimisation enables the assessment of the risk and operating robustness, both essential in robust decision-making process.

Changes in mill capacity were assessed across the Grade Engineering scenarios defined. This clearly shows, unsurprisingly, that the Grade Engineering processing recipe is sensitive to changes in mill available capacity.

Differential blasting for grade coupled with appropriate preferential grade by size deportment add statistically significant value per ton to the base case up to 50% of available nominal mill capacity. The use of only preferential grade by size adds value, but to a lesser extent than the combined effect and up to 70% nominal mill capacity. Thus, differential blasting increases flexibility while increasing value when changes are made to mill rate capacities.

Chance constrained stochastic optimisation enables the interplay between user-defined operating constraints and the objective function (value per unit of time) to be examined. Two constraints were employed, the F20 (size fraction at 20% mass passing) and grade fed to the mill. In terms of associated mill feed particle size distribution to the optimisation constraints, preferential grade by size is a more robust operating strategy than differential blasting for grade, due to the fact that the objective function does not change significantly when different confidence levels in fulfilling this constraint are analysed. However, differential blasting for grade was the only alternative available to produce a feasible solution when mill feed grade is considered as the operating constraint.

The relationship between the objective function and confidence level in achieving a defined operating constraint has been mathematically modelled through an operating robustness parameter (\(\xi\)). The risk (standard deviation obtained during stochastic optimisation), value (objective function), operating robustness (measured through \(\xi\)) diagram is an effective tool to compare several Grade Engineering Scenarios.

10. Acknowledgments

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11. References


Chapter 9  Research Conclusions

This chapter outlines the conclusions pertaining to the value driven assessment of size based Grade Engineering techniques.

1. Overview

Four research hypothesis were presented for the current research aiming to develop a methodology to assess size based Grade Engineering operating strategy from a value, risk and operating robustness perspective in the context of production control (Figure 9.1).

Research Hypothesis: Differential blasting for grade and preferential grade by size deportment can be effectively described by a mathematical function which can be effectively embedded within current available equipment performance models to conduct process optimisation. The associated Research Aim was to integrate the size based Grade Engineering attributes studied in this thesis with equipment performance models to enable its interaction with operating parameters to render process optimisation (Chapter 3, Chapter 6).

Research Hypothesis: Cut-off grades, proportion of material upgraded through screening need to be taken into account in addition to preferential grade by size response to conduct an economic as well as operating appraisal. The associated Research Aim was to understand key variables associated within the exploitation of preferential grade by size signatures and its operating impact (Chapter 4, first order assessment).

Research Hypothesis: Process simulation allows to assess the impact on comminution performance due to changes in mill feed particle size distribution due to the application of size based Grade Engineering. The associated Research Aim was to assess impact of modified mill feed particle size distribution upon comminution performance due to the application of size based Grade Engineering techniques (Chapter 7).

Research Hypothesis: The introduction of uncertainty/variability enables the assessment of size based Grade Engineering operating strategies from a risk and operating robustness perspective in
addition to value. The associated Research Aim was to characterise and integrate the likely size based Grade Engineering process uncertainty within the economic as well as operating assessment to determine entailed operating robustness and risk in addition of value (Chapter 5, Chapter 8).

It is considered that these research aims have been achieved on the basis of outcomes presented in Chapters 3 to 8.

![Figure 9-1. Relationship between Research Aims/Hypothesis and PhD Chapters.](image)

### 2. Conclusions

A value driven methodology by means of stochastic optimisation to assess size based Grade Engineering (i.e. preferential grade by size, differential blasting for grade) was developed. This determines the optimum size based Grade Engineering circuit operating set points (“recipe”) and configuration that maximises the net value per unit of time that a production volume can be drawn by exploiting the associated size based Grade Engineering attributes. This was achieved by integrating multiple and diverse methodologies (i.e. characterisation, process modelling and simulation, optimisation) under a probabilistic decision support system for production control.

This spanned:

- The development and application of a mathematical model to describe preferential grade by size responses through a Response Ranking (RR) attribute. This parameter has been extensively employed at different sample sizes (ROM production, drill core, blast hole) across different geological style deposits (stock work vein hosted, Cu-Mo breccia porphyry, and Cu-Mo volcanic porphyry, see Chapter 6) in this PhD thesis. Response Rankings can
characterise preferential grade by size responses across a deposit and in process optimisation by integration into equipment performance models.

- First order ore body size based Grade Engineering amenability analysis through a novel visualisation method. This clearly depict that the exploitation opportunity of sized based separation techniques in the minerals industry is not merely a function of the preferential grade by size response measured through a RR parameter but also relies on metal head grade, proportion of material upgraded (i.e. mass pull) and the optimal economic material destination (i.e. defined through grade cut-off). However, the understanding of the interaction among of the aforementioned variables and their subsequent operating and economic impact is essential from a production control perspective. This has been addressed through the size based Grade Engineering characterisation (Chapter 5), process modelling (Chapter 6) and simulation (Chapter 7).

- The derived attributes pertaining to preferential grade by size validation at industrial scale were employed to characterise the likely production uncertainties associated with an eventual size based Grade Engineering application. This comprised screening of 40.000 tons of ROM material during approximately one month. A front end loader sampled the four size fractions produced by a screen plant with a capacity of ~1.500 tons per day. Samples were further prepared for assaying in order to ascertain a RR value per day. The RR values at production scale, were extensively analysed by taking into account three possible sources of uncertainty: fitting error related to RR mathematical estimation, sampling error (Gy’s sampling theory through fundamental sampling error) and impact of screening production inefficiencies. The RR fitting error was significantly lower compared with the uncertainties entailed with the production trial, reflecting the robustness of the method to describe preferential grade by size deportment responses applied at production scale. This novel methodology enables decoupling the different sources of uncertainty to distinguish the RR dynamic variation underpinned with inherent geological variability. This novel methodology can be employed to understand the nature of scale up factors observed to transform responses at drilling to production scale (Carrasco, 2013).

- A statistical robust coarse liberation model based on preferential grade by size deportment responses (RR) was developed. This accounts for the impact on RR values due to changes in particle size distribution. This interaction is essential in the integration between preferential grade by size deportment and process optimisation. The model developed comprised
extensive ROM sample characterisation through a novel preferential grade by size characterisation test, the “progressive crushing test” (PCT) and novel data analysis techniques (i.e. ANOVA, Monte Carlo Simulation and t-test). The PCT enables the generation of a coarse liberation matrix (CLM), spanning preferential grade by size responses (RR) per size fraction/parent size and the evolution of this response as size reduction increases. The uncertainty of the RR values within the CLM is investigated by analysing two sources of error: error propagation of chemical assay and mass balance associated and model fitting error. For the majority of the samples tested the RR per sample per size fraction does not statistically change due to changes in size reduction. Nevertheless, the RR values increases when size decreases. This model was integrated with the Whiten crusher model based on a classification and breakage function to track metal grade in addition of mass per size fraction.

- The impact of modified mill feed particle size distributions upon comminution throughput was estimated employing a factorial design approach coupled with mass simulation capabilities embedded in the Integrated Extraction Simulator (IES) a new cloud based process simulator being developed within CRC ORE. The comminution circuit pertaining to the Cu porphyry deposit under assessment comprised of a Semi Autogenous mill (SAG), pebble crusher that divert crushed product to two ball mills.

Several operating strategies were simultaneously evaluated to obtain a more thorough understanding of interactions from changes in size based Grade Engineering operating conditions and metallurgical rock properties. This encompassed three different blasting fragmentation distributions, six mass pulls (proportion of upgraded fine material dispatched to the mill), four impact hardness (Axb), and four bond mill work indices (BMWi). Results across the multiple simulated processing scenarios indicated that the change in SAG mill energy was strongly influenced primarily by changes in mill feed particle size distribution. Ball mill energy consumption did not appear to significantly change, indicating that SAG mill performance controls comminution circuit throughput evaluated. The approach employed was to incrementally increase the comminution throughput until reaching the SAG gross energy base case. The throughput improvement due to Grade Engineering is directly proportional to the F20 (i.e. size fraction where twenty percent mass passing) and ore competence (i.e. Axb), indicating a maximum of ~14% improvement in throughput under the conditions examined.
The aforementioned methodologies were integrated in a size based Grade Engineering circuit simulation platform developed in Matlab® software. The circuit performance was modelled through the widely accepted semi empirical JKMRC models to better describe the nonlinear interaction between rock based properties and equipment performance crucial in the production control context.

The objective function employed in the optimisation routine maximised the net value that can be drawn from a production volume with respect to the maximum time required to mine, separate and process the material through the size based Grade Engineering circuit. To support this objective realistic process bottleneck scenarios were incorporated in the objective function.

Two techniques were employed to perform the optimisation routine. The use of sample average approximation (SAA) and genetic algorithms (GA). SAA enables transformation of a probabilistic outcome into a deterministic one, in which the expected value of the discrete scenarios defined by Monte Carlo simulation is maximised. GA is widely used in similar optimisation problems (i.e. mineral processing circuit design as well as crushing circuit process optimisation) and therefore the method selected. A customised GA algorithm was developed to cope with time inefficiencies observed during the optimisation under uncertainty. This consisted of determining the initial population through an iterative process while determining the optimum processing path for the candidate (i.e. size based Grade Engineering configuration). The integer together with continuous variables (size based Grade Engineering operating settings) are then fed to a tuned GA engine. Extensive analysis was conducted to determine the optimum trade-off between time convergence and the global optimum solution.

The integrated valued driven methodology developed in this PhD thesis is able to account for the synergies between mine to mill strategies (increase throughput due to modified particle size distribution) and metal upgrade through screening. This has been demonstrated assuming that the mill capacity was able to process the entire volume of production (although a different operating constraint could be employed). The optimum size based Grade Engineering processing recipe across all scenarios tested indicates that a pre-crushing circuit renders the maximum net value per unit of time under this deliberately imposed operating constraint (i.e. fill mill capacity available to process Grade Engineering streams).

However, differential blasting for grade strategy added more value than preferential grade by size. This indicates that despite the ability of the circuit to produce fines (i.e. lower than SAG grate aperture ~20 mm), the fine produced by blasting to a large extent influence the comminution
throughput performance. Therefore, this suggests that differential blasting for grade is able to provide an additional level of system flexibility to the size based Grade Engineering strategy purely from a size-throughput perspective (i.e. Mine to Mill philosophy).

Changes in available mill capacity were assessed across the size based Grade Engineering scenarios defined. This enables a determination of the impact of a defined operating mode upon the value that size based separation is able to deliver in conjunction with an associated size based Grade Engineering recipe. This clearly shows that the Grade Engineering processing recipe is sensitive to changes in mill available capacity.

Within the operating scenarios examined the decision support system developed indicates that differential blasting for grade coupled with an appropriate preferential grade by size response (RR) add statistically significant value per ton to the base case until 50% of available nominal mill capacity. The use of only preferential grade by size adds value, but to a lesser extent than the combined effect and up to 70% nominal mill capacity. Thus, differential blasting for grade increases flexibility while increasing value when changes are made to mill rate capacities.

The chance constrained method in single stage stochastic optimisation techniques enable the interplay between user-defined operating constraints and the objective function (value per unit of time) to be examined. Those operating constraints are often associated with safety and product quality, which can be equally as important as the objective function particularly in production control. Two constraints were employed to demonstrate how this technique can be used to aid operating decisions. F20 (size fraction at 20% mass passing) and feed grade to the mill. In terms of associated mill feed particle size distribution to the optimisation constraints, preferential grade by size is a more robust operating strategy than differential blasting for grade, due to the fact that the objective function does not change significantly with different confidence levels (when fulfilling this constraints are analysed). Differential blasting for grade was the only alternative available to produce a feasible solution when mill feed grade is considered as the operating constraint which highlights the importance of spatial grade heterogeneity exploitation in a size based Grade Engineering strategy. The relationship between the objective function and confidence level in achieving a defined operating constraint was mathematically modelled through an operating robustness parameter (ξ). This parameter can be employed to rapidly rank the robustness across different operating scenarios. The higher ξ is the lower the robustness of the optimum relative to the constraint analysed.
The novel size based Grade Engineering decision support system developed effectively compresses the results of the stochastic value driven assessment in a scatter plot. The y-axis comprises the value (maximum value of objective function in the optimisation routine) with errors bars representing the risk (uncertainty of the inputs propagated in the optimisation) whereas in the x-axis, the operating robustness (measured through $\xi$) associated. This diagram is an effective tool to compare several size based Grade Engineering Scenarios.

The current approach can be employed to aid coarse separation circuit design by testing different circuit layouts and equipment capacities by representing the input’s uncertainties through geometallurgical test work of the relevant ore body properties. This will avoid any equipment oversizing and therefore better capex estimation.

Economic as well as production targets from the upper decision layers (strategic planning) are deterministic due to the scarce level of resolution of the inputs employed. This tool enables determination of the statistical significance of achieving those by comparing the economic and entailed production planning goals with the probability distribution generated by stochastic optimisation. Chance constrained methodology quantifies the interaction with user defined constraints.

The set of novel methodologies presented across this PhD thesis enables the assessment of coarse size based separation techniques in the context of production control irrespective of the geological deposit style. Rigorous and sophisticated mathematical and statistical techniques have been applied across this PhD thesis, appraising the distinctive sources of uncertainty/variability associated in rock based processing characterisation, process modelling and simulation, production scale process uncertainty characterisation and process optimisation. This is the first study that supports the emerging Grade Engineering concept by developing a novel decision support system to aid deploying size based Grade Engineering techniques. Although size based separation concept underpinned the methodologies developed and their subsequent integration, the framework outlined in this work is applicable to any separation technique. Furthermore, the characterisation of process uncertainty/variability and its utilisation in a process optimisation scheme enables the rapid assessment of several operating strategies which could not be necessarily associated with size based separation (e.g. sensor based sorting, dense media separation). This represents a breakthrough in how highly flexible operating options can be simultaneously assessed from a value, risk and operating robustness perspective.
Chapter 10  Recommendations for Further Work

This study has provided major advances in the methodologies required to deploy size based Grade Engineering. This has been achieved by considering additional operating flexibility, likely size based Grade Engineering uncertainty and processing responses as result of the non-linear interaction between operating machine parameters and rock attributes. However, there are areas requiring additional development for operational aspects of size based Grade Engineering.

1. Introduction

This chapter provides recommendations for future work in the areas outlined in Figure 10.1 focussing on refining the current modelling methodologies and evaluations developed in this PhD thesis. Suggestions to improve the integration across the associated decision layers are also provided.

![Figure 10-1. Recommendations for further work associated with current PhD thesis.](image-url)
2. **Coarse Liberation Modelling**

The current available empirical model was developed based on an intensive characterisation program applied to a limited number of Run of Mine (ROM) samples pertaining to three geological style deposits (stock work vein hosted Au, Cu-Mo volcanic porphyry, Cu-Mo breccia porphyry). The methodology enables the understanding of variations on RR values at different particle size distributions. This methodology was exclusively applied to pay elements (e.g. Cu, Au). Therefore, further work is required to understand the deportment by size behaviour of deleterious elements (e.g. Arsenic) and rock forming elements (e.g. Al) which could significantly impact downstream process efficiencies (i.e. comminution throughput, flotation recovery).

Compressive breakage (i.e. jaw crushing) has been solely studied in this regards. The assessment of further breakage mechanisms (e.g. bed breakage) in conjunction with additional test work comprising different geological deposit styles will determine whether the proposed model is independent of the breakage process and geological characteristics. The application of a coarse liberation methodology developed in this PhD thesis to drilling products is suggested. This will ease material handling and sampling statistics requirement compared to ROM bulk samples. However, scale up factors will be required to transform RR values obtained at drilling (i.e. crushed drill to production scale. Although the intra sample RR variability (i.e. RR per size fraction relative to global RR, see coarse liberation matrix, Chapter 6) could remain constant when spatially related samples are compared.

This novel approach can be combined with mineralogical and textural information at defined breakage mechanism to develop predictive geological driven models to reduce the considerable reliance of physical test work. Further investigation is required to integrate this outcomes in blasting design where several breakage mechanism are involved (e.g. attrition, abrasion, impact breakage). While the current work has assumed that the application of differential blasting for grade does not have an impact on the magnitude of the associated RR values, this comprehensive study will confirm this hypothesis.
3. Industrial Pilot Trial Grade Engineering Validation

In this work Gy’s sampling theory was adequate to quantify the uncertainty associated with sampling (i.e. through fundamental sampling error, FSE) strategies pertaining to production scale preferential grade by size trial (Chapter 5). In situations where there is a reason to consider the presence of a significant correlation (i.e. time, space) across individual increments (i.e. lots, sampling volumes) a variogram error analysis and its auxiliary functions is recommended (Petersen et al., 2005). This enables a robust sampling strategy to be designed to ensure that increment correlations and hidden process variations are taken into account. This will improve the confidence RR estimation as well as the linkage with geological variability. RR values obtained from the characterisation of spatially related drill core samples can then be compared with production RR values to understand the nature of scale up factors.

4. Uncertainty Process Modelling

Uncertainty modelling is essential in the application of stochastic optimisation. This thesis has thoroughly examined the information gathered during the preferential grade by size industrial trial to characterise the likely size based Grade Engineering process uncertainty entailed at production scale.

Less emphasis was provided to metallurgical parameters, such as impact hardness (Axb) and grindability (BMWi). During the size based Grade Engineering scenarios examined, a constant Axb=25 and BMWi=13 kWh/t values was employed. Additional investigation should aim to integrate ore body metallurgical variability (i.e. uncertainty in stochastic optimisation) within the process optimisation framework developed. This analysis will render more detailed insights regarding the extent of synergies between throughput based and size based separation strategies. It is encouraged the application of class-based analysis and modelling (Keeney, 2010). This methodology provides more effective predictive capabilities, compared to universal models, enabling robust predictions of processing performance attributes to be made. This is critical for providing data support necessary for geostatistical modelling of geometallurgical attributes.

In this PhD thesis process uncertainty was modelled as constant and independent of the operating scenarios analysed (see size based Grade Engineering scenarios, Chapter 8). The further
appraisal of online production information (and certainly the mentioned geometallurgical analysis) will improve the uncertainty modelling particularly in cases where the process models predictability is limited and/or geological information cannot be employed fittingly by itself. A clear example is the blasting fragmentation, a function of geotechnical information (e.g. UCS, block size) and blasting configuration (e.g. stemming high, burden spacing). The application of time series modelling through ARIMA (auto-regressive integrated moving average) models enables the dynamic characterisation of uncertainty via scenarios generation. Napier-Munn (2014) provides a thorough description of the ARIMA methods available. This approach has been extensively employed in electricity markets when decisions under uncertainty problems are studied (Conejo et al., 2010). Therefore the integration of this approach to current stochastic optimisation framework is suggested.

5. Integrated Process Simulation

The Integrated Extraction Simulator was employed to determine the impact of modified mill feed particle size distribution due to the application of size based Grade Engineering within several operating scenarios (Chapter 7). Likely changes in rock based processing attributes (i.e. Axb, BMWi) per size fraction associated with post screening streams were not considered within the scenarios examined. The impact of the new hardness by size profile upon the available comminution process models (i.e. Napier-Munn et al., 1996) outputs from a reliability perspective need to be thoroughly understood. During simulations, the operating mill parameters (e.g. critical velocity, ball load) were kept constant. Furthermore, flotation response was modelled using a generic P80 and metal recovery relationship without talking into account the potential increase in flotation feed liberation due to size based Grade Engineering. Therefore there is a significant opportunity to determine more customised comminution/flotation recipes for optimum processing of different rock type scenarios.

It is noteworthy that to take full advantage of the mass simulation capabilities embedded in IES and system modelling integration a thorough analysis of inherent semi-empirical/mechanistic modelling limitations and uncertainty of the models predictions will be required. A clear example of the former, is the SAG mill model limitation. The variable rates employed to predict SAG mill performance was ascertained with operating data from a series of mill with a fixed average total load volume of 25 percent. Therefore, simulations need to be conducted at 25 percent of load to
ensure the reliability of simulation results. Since simulation is merely a representation of reality, there is a degree of error attached within the model’s prediction. The Mass simulation capabilities embedded in IES provide an excellent opportunity to understand modelling error propagation when a full integrated processing system is evaluated (i.e. referred to as internal uncertainty in stochastic optimisation). This could be achieved by applying unsupervised machine learning algorithms (e.g. k-means, principal component analysis) to online production information (i.e. stored in Data Historian module in process control layer) to decouple geological variability from stochastic equipment response. Online information provides larger system visibility compared with the typically conducted cost intensive sampling programs which capture idealised operating scenarios that rarely reflect the expected ore body metallurgical variability. This research study will determine whether the response of the distinctive operating scenarios examined truly mirror the interaction of operating parameters and rock variability rather than uncertainty associated with the model predictions.

6. Application of Discrete Event Simulation (DES)

The current work has sought to determine robust Grade Engineering recipes by introducing rock based uncertainty (i.e. grade, blasting fragmentation, RR) into the optimisation assessment. Nevertheless the use of Discrete Event Simulation (DES) to embed equipment stochastic behaviour needs to be investigated. The use of optimisation coupled with DES provides a more realistic evaluation of system capability and equipment interactions. This can be conducted by extensively analysing maintenance and operating data where the mean time between failure (MTBF) and mean time to repair (MTTR) are employed. MTBF is the elapsed time between failures of a system during operation. The definition of MTTR is the elapsed time starting from equipment breakdown until it is fixed and back to operational status. DES can be employed in conjunction with the value driven optimisation scheme developed in this work to determine:

- The impact of Grade Engineering upon material movement to a beneficiation plant as well as assets productivity.
- Bottlenecks within Grade Engineering and the most sensitive control points.
- The reliance of Grade Engineering equipment breakdowns.
- Size of the inventories to ensure Grade Engineering application robustness.
7. Optimisation Scheme

Significant mathematical effort was made to derive a parametrised objective function that could be employed in the stochastic optimisation engine (Chapter 8). This represents the value per unit of time driven by the system bottleneck using processing streams obtained by the population balance approach (Napier-Munn et al., 1996). Blasting design parameters were not considered directly (i.e. blasting fragmentation) in the optimisation scheme. Additional work is required to integrate more detailed blasting models and associated costs within the stochastic optimisation framework developed.

Further objective functions relevant within production context can be also employed (e.g. energy consumed per unit of metal in conjunction to value per unit of time, see Kallrath, 2002). The use of multi-objective optimisation in conjunction with Pareto techniques (Pareto profile or Pareto front) to find the optimal trade-off of feasible solutions is recommended.

8. Integration with Planning and Production Scheduling

This work has employed Genetic optimisation algorithms coupled with simple average approximation to determine the optimum Grade Engineering recipe (i.e. operating set points) to solve an optimisation problem where the likely process uncertainty at production scale has been included. Further work needs to be pursued to integrate the current methodology developed across planning and production scheduling. This will shift the problem’s mathematical nature from single stage, time independent towards a dynamic, multistage optimisation. The evaluation of different optimisation algorithms should be performed, particularly the use of Simulated Annealing algorithms. These are extensively used in multistage long term planning and scheduling problems in mining when essentially grade uncertainty is modelled through geological conditional simulation. The use of IES as dynamic simulator by exploiting the mass simulation capabilities is also recommended.
9. Integration with Process Control Strategies

The offline optimisation methodology developed in this thesis is able to render a basis for a real
time optimisation approach, whereby optimum crushing circuit (pertaining to Grade Engineering
strategy) operating points are dynamically rendered. Further work is required to determine whether
the current processing models and optimisation algorithms (i.e. Genetics optimisation algorithm
coupled simple average approximation) are suitable for on-line optimisation. Control philosophies
to address real time process disturbances and transition of the process towards an optimum regime
determined by the optimisation scheme need to be investigated (i.e. fuzzy logic, model based control).

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Appendix A. Background to Gy’s Sampling Theory

Overview of Gy’s sampling theory to estimate fundamental sampling error is provided. This framework was employed in Chapter 5.

1. Introduction

Sampling particulate systems is not an easy task given the several sources of error involved, which could potentially compromise results and ultimately disguising the real Grade Engineering application potential. Gy’s sampling theory (Gy, 1982) has been proven over time to be a powerful tool to control and manage measurement’s bias. Gy’s divided the sampling errors in several independent components (Figure 1). Each of them can contribute significantly to the uncertainty of the parameter of interest (Eq.1). Preparation error is relevant when the sample has not been process properly. Typical poor sampling practices span: sample contamination, sample mixing, wrong identification/labelling and so forth. Delimitation and extraction errors significantly occur when the volume of the increment (sample) to be taken has not been well defined. For example, not taking a whole stream cut and just a portion. Weighting and periodic quality fluctuation errors are a result of the natural variability of the properties of a stream. These strictly apply when increments from moving streams are being analysed. Nevertheless, Fundamental sampling error ($\sigma_{FE}^2$) is a critical consideration in designing any sampling scheme. Fundamental sampling error can be defined as the minimum error of an ideal sampling procedure. This defines the minimum amount of sample required from obtaining a reliable and representative portion of the lot.
Appendix A

2. Fundamental sampling error

Gy’s formula (Gy, 1982) relates the fundamental sampling error through a constant (C), maximum particle size within the lot and the mass of the lot ($M_L$) to be sampled ($M_S$) (Eq.2).

$$\sigma_{FE}^2 = C d^3 \left( \frac{1}{M_S} - \frac{1}{M_L} \right)$$  \hspace{1cm} (2)

Where:

- $\sigma_{FE}^2$ Fundamental error variance expressed as relative proportion
- C Sampling constant
- D Largest size of material to be sampled (cm)
- $M_S$ Mass of the sample (g)
- $M_L$ Mass of the Lot (g)
Gy’s sampling constant $C$ as a function of the ore properties, is not quite constant, as it more specifically relates to how the phase of interest is liberated and distributed (which depends on $d$). In all cases, $C$ can be equated to the following expression: (Eq.3).

$$C = f \, g \, l \, m$$  \hspace{1cm} (3)

Where

- $f$: Particle shape factor, which varies between 0 and 1.
- $g$: Granulometric factor which describes the size range of particles within the lot.
- $l$: Liberation factor, where $d_l$ is the comminution size at which the mineral interest is fully liberated, and $d$ is the largest fragment size within the lot (i.e. nominal comminution size) (Eq.1). Poor, over simplistic models have been proposed by Gy and more performant models were later established by Francois-Bongarcon (see below).
- $m$: is essentially the expected volumetric fraction of the mineral of interest the mineral composition factor, estimated according (Eq.5):

$$m = \left(1 - \frac{a_L}{a_L}ight)[(1 - a_L)\rho_m + a_L\rho_0]$$ \hspace{1cm} (5)

Where $a_L$ is the expected volume fraction content of the mineral of interest within the lot and $\rho_m$, $\rho_0$ the density of the mineral interest and the gangue.

Sampling constant ($C$) as well as the sample mass ($M_S$) are very sensitive to aforementioned parameters, therefore they have to be carefully estimated (Minnitt et al, 2007; Minkkien, 2004; Allen, 1981).

Francois-Bongarcon (1993) treats Eq.3 in a more general form, and can be written in the following form (Eq.6) when the ML is significant higher than $M_s$.

$$K = cf \, g \, d \, lm^{3-a}$$ \hspace{1cm} (6)
Where $\alpha$ is a parameter that depends on the deposit’s mineralization characteristics. The parameters $dl$ (in K) and $\alpha$ of the Eq.6 can be then calibrated by taking the logarithms and by means of heterogeneity tests (Francois-Bongarcon, 1993; Minnitt et al., 2007).

The aforementioned information can be translated into a sampling nomogram (Figure A.2). This diagram depicts the relationship between sample volume, top size (95 % passing) and fundamental sampling error (Eq.2). This plot is a risk sampling diagram whereby the optimum combination between mass volume and top size are defined by the area above the line defined by the sampling constant (Eq.3) and a defined error threshold (defined by the fundamental sampling error).

![Sampling Nomogram](image)

*Figure A.2. Sampling Nomogram.*

Figure A.3 depicts different sampling constants and different sampling options. The 3 sampling options are safe, i.e. above the line defined by the sampling constant equal to 1 and a 3 % fundamental sampling error defined as threshold. Nevertheless, as the sampling constant increases (maintaining 3 % as constant), option 2 presents the most robust sampling strategy, given that sampling route is above the 3 sampling constants considered in this example. This involves top size reduction from 100 cm (Run of Mine material) to a 1 cm, where the sampling volume required to obtain a representative sample can be significantly lower compared with the original volume.
Figure A-3. Sampling nomogram depicting different options across different sampling constants.

3. References


Appendix B. Overview Sampling Strategies Industrial Grade Engineering Pilot Trial

Different sampling ROM strategies to validate size based Grade Engineering levers are examined.

1. Introduction
Grade Engineering validation consists of determining whether there is a noticeable difference in grade across size fractions. The resources available as well as the site logistics will define the sampling strategy, divided as follows:

1) Screen plant sampling
2) Crusher sampling
3) Screen plant in line with crusher sampling

2. Screen Plant Sampling
Samples are collected at the discharge of each screen plant’s conveyor belt usually using a front end loader (Figure B.1). Each conveyor belt is related to a well-defined size fraction, which need to be kept separately for obtaining metal grade per size and therefore preferential grade by size responses (RR).

Advantages:
1.-The results of either preferential grade by size or differential blasting for grade characterisation can be directly be used to predict production responses based on the comparison of spatially related drill core/blast hole samples. Results truly represent preferential grade by size responses of production streams.

2.-Segregation error are decreased since wide size range of particles has been reduced by screening.

Disadvantages.
1.-Material handling becomes extremely difficult with the presence of +100 mm streams, in particular when ROM material is being screening to assess preferential grade by size responses.
2.- The production of significant amount of boulders can potentially jeopardise plant screen efficiencies. Screened streams will contain particles that should not be recovered on those, compromising the further grade by size analysis. It is highly recommended to address this operational contingency on site. The significant volumes produced while trial is being conducted, could make almost impossible to further screen those streams at a commercial laboratory. A second screen unit can effectively address this issue by feeding any coarse stream that was not efficiently processed in the originally allocated screen plant. These two stations can be operated in parallel if enough space is available and safety associated risks due to equipment congestion is managed. Figure 4 illustrates the utilisation of a second screen unit (green) focused on screen the +6” stream from the static production’s grizzly (red). In this example, approximately 30% of fines (-6”) were misreported in the original +6”stream by visually inspecting the products of the second installed screen (green). It is noteworthy that this problem can be exacerbated when differential blasting for grade is being assessed, since the screen plant will be periodically fed with very coarse ROM particle size distribution.

![Figure B-1. Second screening unit (green) to address low production screen efficiencies (red).](image)

The problem that needs to be addressed then, it is how to get a representative sample from a coarse fraction (in this example +6”). Large masses required for acceptable sampling precision are a result of large particle sizes. Large samples can be avoided by reducing the top size by crushing. The material can be set it in motion by using a crusher while producing smaller sizes. Therefore, lower
amount of sample is required without compromising precision. Cone and quartering is an option, but it should be carefully assessed, and can be shown theoretically to be quite insufficient to reduce the effects of segregation.

3. Crusher Discharge Sampling

Samples are collected at the crusher’s discharge (Figure B.2) and therefore these represents a sample that needs to be further screened to obtain grade by size information.

Advantages

1.-Reduction of size fractions significant aid material handling and sample volumes to obtain a representative sample.

2.-Less equipment is required on site (crusher, shovel and front end loader).

Disadvantages

1.-Changes in particle size distribution can affect either preferential grade by size and more importantly differential blasting for grade. For preferential grade by size, a progressive crushing test can explain the extent of particle size distribution impact. However, the distinctive fragmentation resulted to exploit grade heterogeneity in differential blasting will be greatly smoothed, likely compromising (diluting) grade by size responses. Therefore this sampling strategy is not suitable for characterising differential blasting grade by size responses.

2.-Screening needs to be conducted in a laboratory. This can lead to extensive amount of work given the magnitude (volume) of the streams produced on site, which can impact negatively in costs and results turnaround.
4. Screening in-line with Crushing Sampling

Samples are collected at each screening discharge while the ROM sample is crushed aiding material sampling (B.3) but likely affecting preferential grade by size responses due to additional size reduction process.

Advantages

1.- Segregation error are reduced since the wide particle size variation within the stream is decreased by means of screening process.

2. - Coarse particles are crushed aiding the screening efficiencies and material handling.

Disadvantages

1.- Changes in particle size distribution can affect either preferential grade by size and more importantly differential blasting for grade response.

2. - Characterisation results are not directly applicable for an eventual ROM metal preconcentration by size, since particle size distributions under assessment will be largely different from ROM production blast. Progressive crushing test is entailed to understand the relationship between size fragmentation and preferential grade by size.
Figure B-3. Sampling procedure crusher with in line screening plant.
Appendix C. Digital Information

This appendix (Appendix_C.rar) outlines the information provided in digital form and its relationship with the relevant Chapter in this work.

1. Raw data and statistical analysis pertaining to the development of a coarse liberating model, (Chapter 6). (File: Appendix_C.xlsx)

2. IES Simulation results (288) employed in the development of size based Grade Engineering throughput model (Chapter 7). (File: IES_MassSimulations.rar)

3. Results of the stochastic optimisation and the flexibility analysis (Chapter 8). (File: Appendix_C.xlsx)