New Approach for Characterising a Breakage Event as a Multi-stage Process

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Abstract

In any metallurgical plant, optimum comminution can be assured when a predictive process model is established. However, the current ore breakage characterisation practice, in addition to several other factors, is determined as one of the barriers for the development of predictive comminution models. In this project, a comprehensive review of the literature was conducted to investigate this issue. Hence, the current comminution models were discussed from the ore breakage characterisation perspective and drawbacks were identified and addressed. It was found that the existing practice does not represent the true properties of rock; what it reflects as ore hardness is a combination of ore properties and the effect of breakage environment, such as geometry, stressing velocity and etc. In other words, the characteristics of rocks and the breakage system are combined together in the outcome of a breakage test, which contradicts the original purpose of characterisation. This necessitated the understanding of the most fundamental element of breakage; the breakage of a single particle.

Based on the literature, it was hypothesised that a single particle breakage test can be seen as a sequential process of several other sub-processes, referred to as primary breakage, classification, and selection. Each sub-process was determined to depend on either the properties of rock or the breakage environment or both. For instance, the “primary breakage function”, also known as the appearance function; the size distribution resulted from initial fragmentation of particles was related solely to the properties of materials. The “selection function”; referred to as the probability of particle selection was associated with the geometry of the comminution environment and other factors, such as stressing velocity. The classification function was linked to the geometry and the spatial distribution of fragments during breakage. Also, a model was developed for this process based on the three sub-processes. Each sub-process was then modelled individually. The characteristics of primary breakage were considered as the main component of the model that repeated itself in each sequence of breakage. Then, the effect of other selection and classification functions was incorporated into the model.

The model was examined in its response to the three elements by conducting a sensitivity analysis that indicated promising results. Also, the model was validated for a single particle breakage characterisation approach known as drop weight testing mechanism for three types of materials, such as quartz, apatite, and silicate. In the case of quartz, the classification function
was altered to reflect the brittle behaviour of quartz and the formation of wide spatial distribution which results from an extensive ejection of the fragments around the breakage environment. The results indicated a good agreement with the experiments. For the apatite particles, the selection functions were changed to reproduce the size distribution resulting from using three different geometries in the drop weight tester. In the case of silicate, a good agreement was found between the model and experiment. However, a substantial mismatch was found between the model and experiment at high energy levels and it increased as the applied energy increased. This error was related to the limitations of the model, i.e. the model does not account for the formation of a bed that occurs at high energy levels as well as the non-normalised breakage function. Also, an error propagation analysis was conducted to investigate how the error in each stage is propagated into the next levels. Later, the areas of improvement were identified.

The concept of “a single breakage event as a process” was applied to study the effect of stressing velocity; a parameter that changes with the system of breakage (breakage mechanism). This means each sub-process such as primary breakage, classification and selection elements of the model were tested in their response to the stressing velocity. It was hypothesised that primary breakage characteristics such as primary breakage appearance function and associated fracture energy are not affected by the stressing velocity. An experimental procedure that involved two different mechanisms of breakage; compression and impact – the two mechanisms provide widely different stressing velocities - was developed for this purpose. The experiments were conducted on two different types of ore; magnetite and silicate. The results demonstrated that primary breakage characteristics such as appearance function and fracture energy were insensitive to the effect of stressing velocity, but sensitive to the properties of the rock. It was perceived that although the stressing velocity does not influence the primary breakage characteristics, it can possibly affect the selection and classification of the fragments in the breakage environment. Hence, the next hypothesis was set to examine this issue. Testing this hypothesis required that experimental procedure to be extended for compression and impact mechanisms beyond the first fracture characteristics. Hence, other experiments were conducted at higher levels of energy, allowing for progression of breakage post the initial fragmentation. The results indicated that applied strain rate can impact the classification of the fragments, resulting from the difference of the brittle behaviour of materials.
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NOMENCLATURE

A : cross sectional area ($m^2$)
A×b : Hardness index
α : local deformation (m)
B_{ij} : breakage progeny distribution
b_{ij} : primary breakage function
β : the slope of the breakage distribution at coarser sizes
C_α : strain wave velocity (m/s)
C_i : classification function of size class i
C : Ratio between actual and theoretical velocity
d_{p,o} : parameter of the effect of size on particle fracture energy (mm)
d_p : particle size (mm)
d_{p,o} : characteristic size of the material
Δx : displacement (m)
E_k : kinetic energy (J)
E_l : energy losses during loading of particles (J)
E_{m,o} : parameter of Weibull distribution of fracture energy
E_{m,0} : parameter of the Weibull distribution of fracture energies (J/kg).
E_{m,∞} : residual fracture energy
E_{m50,i} : median mass-specific fracture energy
E_p : energy absorbed by the particle (J)
E_{cs} : Mass-specific comminution energy (J/kg, kWh/t)
E_r : energy absorbed by impact load cell bar (J)
E_{res} : energy absorbed for restitution of impactor (J)
ε : strain
F : local force (N)
Ø : material constants
g : gravitational acceleration (m/s$^2$)
h : drop effective height (m)
k_b : ball stiffness (GPa)
K_e : elastic constant of Hertzian constant (Pa)
$k_p$: particle stiffness (GPa)

$K$: Hooke’s stiffness (N/m)

$\kappa$: selection function model parameter

$m_i$: mass fraction of particle in size class i

$m_{i}^{*n}$: mass fraction of particle in size class i and stage n of the breakage

$m_p$: Mass of the particle (kg)

$N_r$: Number of revolution

$n$: number of stages in model

$P(S)$: probability of strength distribution

$P$: particle compressive strength

$\rho$: specific density

$r$: Radius of rotor (m)

$S_i$: selection function of size class i

$S_o$: a constant

$S$: tensile stress of the particle

$t_{10}$: Cumulative percentage passing one tenth of the particle original size (%)

$t$: time (s)

$t_n$: Cumulative percentage passing one nth of the particle original size (%)

$u$: displacement (m)

$u_f$: coefficient of uniformity

$V_i$: Impact velocity (m/s)

$V_{it}$: Tangential impact velocity (m/s)

$v_o$: initial impact velocity (m/s)

$\sigma_E^2$: variance

$Y$: modulus of elasticity (GPa)

$\gamma$: the slope of the breakage distribution at fine sizes

$\delta$: selection function model parameter

$\phi$: percentage of the breakage distribution at fine end
1. INTRODUCTION

The mineral processing plants, particularly comminution circuits are significantly energy intensive (Musa and Morrison, 2009). An energy efficient process requires an optimum comminution circuit to maintain its performance with variation in ore characteristics as well as the operation parameters. This can be achieved by implementing models which are robust in their prediction over a wide range of conditions. However, the predictions of current models are generally only good when the design operation is in the same regime as the database. As parameters of such models are fitted to a particular operating condition, it is dangerous to use these methods to extrapolate to new conditions or ore types, let alone new types of applications (Powell and Morrison, 2007). Among different shortcomings of current models, ore characterisation is recognised as one of the significant concerns. The weaknesses of ore characterisation within such models can be summarised in the following statements:

1- Characterisation experiments measure the response of the rock to a certain mechanism of breakage but the resulting parameters are treated as the properties of material rather than a response to the system that applies mechanism.

2- The combination of breakage stressing modes implemented in characterisation experiments is different from the combination of modes present in industrial scale comminution devices.

3- Ore characterisation experiments always test a fixed combination of stressing modes, while in comminution devices the combination of breakage mechanisms changes with variation in the operating condition.

To address the drawbacks of current characterisation techniques and how they are incorporated into models, a new attempt has been made toward developing mechanistic methods which can provide a phenomenological understanding of breakage processes. Despite significant improvement in this field, a comprehensive study on appropriate rock breakage characterisation experiments which provides required parameters for mechanistic models is still scarce. This is a critical step which requires an understanding of the most fundamental element of any breakage mechanism: the breakage of a single particle.
One particularly interesting approach to characterisation is to consider breakage of a particle, for instance in a drop weight tester, as a sequence of primary and secondary breakage components. Secondary breakage is defined as subsequent breakage of progeny that takes place after the primary fracture of the parent particle due to surplus energy that is provided by the comminution or breakage testing device. Due to the complex nature of secondary breakage, most of the researches in the past have either been dedicated to understanding primary fracture as a basic element of every comminution process or to describe the breakage process as a whole. However, the progression of breakage may be characterised and modelled properly by incorporating the sequential breakage of fragments into a breakage process. In this present work, a novel approach is proposed to describe a single breakage event by modelling it as a process through several stages of primary fracture while appreciating the effect of sequential fragmentation. This approach also can be used to compare different mechanism of breakage.

This dissertation is divided into six chapters. After the introduction, Chapter Two is allocated to review of the literature and distributed into four sections. In section one of this chapter, the existing comminution models are studied from the ore characterisation perspective. Empirical models of ball mill, semi-autogenous (SAG) mill and high pressure grinding roll (HPGR) are presented as examples. In section two, the mechanism of breakage inside common single-particle breakage characterisation devices is discussed and compared with one another in detail. In section three, the working principle of impact load cell (ILC), used to study the fracture of a single particle is reviewed. In section four, the concept of secondary breakage is presented. Following the literature review, in Chapter Three, hypotheses and objectives are stated based on the precedent studies to identify the research gap. In Chapter Four, the materials and the experimental procedures to address the stated hypotheses are explained in detail. In Chapter Five, the model development and its validation are presented. In Chapter Six, the experimental results on the effect of stressing velocity are elaborated and discussed. Finally, the results are summarised and a conclusion is drawn.
2. LITERATURE REVIEW

2.1. IMPLEMENTATION OF ORE CHARACTERISATION IN EXISTING COMMINUTION MODELS

The parameters obtained by breakage characterisation techniques are used directly in comminution models; hence, it is worthwhile to elaborate on such methods. The following section provides a discussion of the use of ore characterisation in a few selected comminution models, focussing on the strengths and limitations of the approaches. Three major equipment types are reviewed: the SAG mill, ball mill, and HPGR; with a brief description of major accepted process models upon which to base the analysis of how ore characterisation is used in modelling.

This section is not intended to be a comprehensive review of all equipment and associated models, the focus is on the needs of ore and limitations that are still to be addressed.

2.1.1. SAG Mill Model

One of the failures of the current models according to Powell and Morrison (2007) is the difference between the major modes of breakage that the model developed based on the perceived conditions in the comminution equipment versus the actual breakage conditions. The SAG mill model developed by Leung et al. (1987) uses the appearance function with two components: high energy impact breakage and abrasion. Ore response to high energy impact is characterised by breakage of the rocks in drop weight tester (DWT) or rotary breakage tester (RBT). Abrasion parameters of the sample are determined from tumbling rocks in the abrasion laboratory mill (Napier-Munn et al., 1996). The overall appearance function generated for each size fraction is the combination of high and low energy appearance functions proportionally. However, the studies of rock breakage inside a SAG mill using discrete element model (DEM) demonstrates that a large portion of energy contributed to breakage comes from low energy impact, causing cumulative damage over time, instead of high energy impact which breaks the particles in a single event (Cleary and Morrison, 2004; Morrison et al., 2002). Based on this study, the large AG and SAG mills which are commonly used are unable to supply enough energy to break particles larger than around 10 millimetres in a single event. Even the breakage of the smaller particles which are
expected to experience more severe breakage than the larger ones seems likely to be the result of repeated small to moderate collision energy. There are also other concerns about the appropriate use of DWT for SAG milling environments. The breakage mechanism of the DWT is two-point contact, in which a flat impactor is dropped from a certain height onto a single particle sitting on a flat surface. Also, the maximum achievable fragmentation and the rock hardness index obtained by this experiment are interpreted as the properties of material. However, looking into a more sophisticated ball milling environment, it is evident that most of the impacts that take place involve a ball or two (curved-shaped) as an impactor, which is different from the geometry that is applied in DWT (Flat surface). This casts doubt on using drop weight tester as the characterisation device for any milling environment with ball media. It is also expected that the size distribution and the maximum fragmentation that can be achieved in the milling environment are different from what is obtained in DWT. In other words, the resulted size distribution is also geometry dependent and shouldn’t be interpreted only as the material properties.

Leung’s approach for the characterisation of the abrasion mechanism in a SAG mill utilises the outcomes of the JK abrasion tumbling mill test, using -55+38mm particles ground for 10 min in a 300mm diameter by 300mm long tumbling mill running at 70% of critical speed. Rather than the mechanism of sliding or rubbing of the particles due to particle to particle and particle to shell interactions which is expected with this test, the actual rock breakage inside this testing device results from (1) chipping off the particle corners due to low energy impact, (2) surface breakage of smoothed particles due to low energy impact, (3) body breakage due to incremental low energy impacts (Khanal and Morrison, 2008). The total number of particles in the mill and the speed of mill determine the number of times each particle is lifted and dropped in the mill (Austin et al., 1986.; Hennart et al., 2009). Considering the abovementioned mechanisms of breakage associated with the JK abrasion test, realistic application of this test for abrasion characterisation to the SAG mill model is doubtful. The so-called ‘Abrasion’ tests considering the speed of the test mill cannot be a representative test for abrasion condition in industrial mills over a wider distribution of energy.

The variation of operating conditions changes the breakage mechanisms, energy spectra and stressing condition within the mill. Despite this, the combination of breakage
mechanisms inside the breakage characterisation device is fixed and does not vary along with it. The study of SAG mills by Cleary and his colleagues (Cleary et al., 2003) reveals even changing the particles shape from sphere to angular can lift the charge shoulder position slightly and put the cataracting stream at a moderately higher position. They also showed the transition from cascading to cataracting happened when the mill speed changed to a higher value. In their work, it is discussed how the positions of toe and shoulder are affected by variation in mill load as well as the mill speed. The complex nature of breakage process in SAG and AG mills can be seen as the root cause for an overwhelming number of tests, developed to characterise the ore properties within this environment. According to Figure 2-1 by Yahyaei et al. (2015), there are at least 9 characterisation tests which are commonly used in the design and modelling of autogenous and semi-autogenous grinding mills.
Leung et al. (1987) model was later modified by Valery Jnr and Morrell (1995) to account for mill variables such as feed size and ore hardness. Despite great improvements in this area, the model in its nature is still empirical, relying on impact test or tumbling mill tests to obtain the appearance functions.

2.1.2. Ball Mill Model

The power-based Bond model (Bond, 1961) successfully correlated a well-controlled laboratory test to the traditional standard rod and ball mills. Bond third law of comminution established a relationship between energy input and particle size made from a given feed size: Bond work index ($W_i$), referred to as hardness index should be obtained from actual plant data. It is alternatively can be achieved by conducting laboratory test in which energy,
product size and feed size are measured. One of the assumptions in Bond’s equation is that the net energy consumption per revolution of the test mills is constant. He also suggested that laboratory tests were carried out so they can generate similar size distribution. However, when the full-scale conditions were applied, the results significantly deviated from those of the laboratory tests. A number of correction factors were applied by Bond to address such variations. According to the Bond’s equation, work index should remain constant for a given comminution step, irrespective to the feed and product size. This can be true if the resistance of the rock remains constant as well the efficiency of the comminution step. However, according to Morrell (2004) and based on the industrial mills, the Bond’s work index changed with different ball charges, closing screen sizes and feed sizes. Modifications were made since the Bond’s time to address the drawbacks of the original model to cover larger mills, wider ball size ranges, changes to mill speed, etc.

A common approach to ball mill modelling is the population balance method (PBM) which utilises a breakage and selection function (Herbst and Fuerstenau, 1968). The breakage distribution function – often called only breakage function – may be estimated by conducting controlled experiments with narrow size fractions, running the batch ball mill for a very short grinding time in order to prevent particles suffering more than one breakage event (Austin et al., 1984). The breakage function is regarded as relatively insensitive to most of the system variables. Indeed, it is common to assume that the breakage function is predominantly a material-specific function (Kelly and Spottiswood, 1990). The selection function, known as the selection probability of particle, on the other hand is sensitive to the system variables such as mill speed and ball load. It is obtained by back-fitting to mill performance.

A simplified version of the PBM was developed by Whiten (1976) in the form of a perfect mixing model that converts rates to \( R/d \) (rate over discharge). This assumes that the discharge function is constant, and thereby enables fitting of the model to site survey data. It is generally used with a fixed average appearance (or breakage distribution) function that is not energy or environment dependent. Allowance for changes in milling conditions are generally applied through empirical adjustment of the breakage rates (or selection function), with fitting of the breakage rates accommodating both the difference in the ore characteristics and the machine characteristics.
There is a range of work (Cleary, 2001; Kiangi et al., 2013; Loveday and Dong, 2000; Radziszewski, 1999; Usman, 2015) that shows the change in breakage conditions as the mill operating conditions change, and even within different zones within the mill. The initial DEM studies of charge motion by Radziszewski (1999) shows a typical charge profile in a ball mill comprises four zones, referred to as flight, crushing, grinding, and tumbling. However, the boundaries of those zones are not stationary and change by changing the operating conditions such as mill filling, throughput and feed size distribution. According to Loveday and Dong (2000), increasing the mill load also can accelerate the wear rate of particles inside the mill. In addition, a study by Banini (2000) shows the overcrowding at the toe of the mill which results in a more rock-rock collision can increase the generation of ultra-fines particles inside the mill. Loveday and Dong (2000) also found the rate of rock abrasion can be reduced by the presence of fine material in the feed to the mill. Hence, an in-depth understanding should be developed about the combination of different breakage mechanisms as the result of varying conditions for the purpose of modelling changing operational environments in ball mills.

In addition, the current empirical comminution models of ball mills are not capable enough of modelling the breakage product of a more complex process such as the one which involves a bed of particles. The impact fracture of particle beds was studied initially by Cho (1987) and Hofler (1990) who attempted to measure the fragment size distribution at different heights of bed and stressing geometries. He showed that the broken mass of fragments increased with the increasing input energy. However, further experimentation indicated a slower increase of fragmentation with increasing the input energy, suggesting a limit to the maximum achievable fragmentation. According to Hofler (1990), the size distribution of fragments is not impacted by the height of the beds with 2 to 10 layers of particles. However, the number of layers itself, is affected by a combination of parameters such as particle shape, size, and impact conditions such as impact velocity and stressing geometry.

Later, Bourgeois (1993) attempted to relate the particle bed fracture in an unconstraint condition to the fracture of the individual particles inside it. It is stated that in an unconstraint bed of particles, the movement or rearrangement of the particles and their fracture are
separate processes with almost no overlap. Therefore, it is of high importance to know the configuration of the particles inside in the breakage zone when the breakage phase begins.

One of the most recent mechanistic models for breakage of particles contained in a bed was developed by Barrios et al. (2011). It investigates breakage of a bed of particles impacted by a falling steel ball in unconstrained conditions, such as those that are likely to be found in tumbling mills. For each of the variables affected by milling environments such as particle size, impact energy, ball size and bed configuration, a mathematical model was developed. From the characterisation perspective, this model uses the single particle breakage data such as distribution of particle fracture energy.

2.1.3. HPGR Model

The breakage mechanism inside HPGR is modelled using the basics of the well-known population balance model (Morrell et al., 1997). It assumes two breakage modes for different regions of the HPGR. If particles are bigger than a certain critical size they may be broken directly by the roll faces, similar to a conventional rolls crusher. The breakage in this zone is referred to as a ‘pre-crushing’ mode and the product may subsequently pass to a region where a bed under compression is formed. The boundary between the pre-crushing and bed compression regions in this model is defined by a critical gap (\(x_c\)). Particles larger than the critical gap are crushed in a single particle breakage mode.

Breakage at the edge of the rolls is different to that at the centre and conforms more to that experienced in a conventional rolls crusher. The proportion of relatively coarse particles usually seen in HPGR product results from this area and is referred to as “edge effect” (Knorr et al., 2016). This occurs because of the gradual pressure drop that is extended towards the edge of the rolls. This extension is assumed to be a function of the working gap. It is assumed that in both pre-crushing and edge zones, rock breakage takes place in single particle breakage mode. For this part, the DWT is used to generate the breakage function. However, application of this testing method for ore characterisation can be questioned since the breakage mechanism utilised by DWT is impact with high energy intensity which is different from compression; the dominant mechanism in HPGRs. On the other hand, in the compressed bed crushing zone of the model, size reduction is assumed to be similar to that experienced by a bed of particles in a piston press. The parameters used to describe size
reduction are determined from experiments with a laboratory or pilot scale HPGR along with breakage tests in a piston press. The test uses several fractions of particles within a narrow size distribution and applies different pressure which is related to various input energy levels applied to the bed. However, the size distribution of the feed and angularity of particles affect their interaction within a bed with their neighbour particles. In addition, the arrangement of the bed which differs in monolayer compared to multiple layers and confined or unconfined condition influences the stress distribution within the bed and eventually affects the product size distribution. In stressing beds under the unconfined condition, a fraction of material leaves the bed and escapes from being stressed, whereas under confined conditions only a rearrangement of particles can occur. In addition, stressing single particles produces a steeper size distribution than stressing a bed of particles (Ozcan et al., 2015; Schöenert, 1996). The variation in breakage response could increase when rolls with different surface profile (Knorr et al., 2016) and roughness are used (Ozcan et al., 2015; Schoenert and Lubjuhn, 1990; Schonert, 1991).

One of the most recent models for breakage of particles inside HPGR has been developed by Dundar et al. (2013). The model uses the concept of population balance model. The breakage function is obtained by conducting experiment in a bed compression configuration at fixed range of different pressures. The relationship between energy and size reduction is expressed by \( t^{10}-E_{cs} \) approach; in the same way as described for breakage of a single particle. The \( t^{10}-E_{cs} \) method is presented in this chapter in Section 2.2.1. The breakage rate which is a function of pressure and particle size is back-calculated. Table 2-1 summarises an overview of the previously described comminution models.
Table 2-1: Summary of characterisation techniques, corresponding to different industrial equipment

<table>
<thead>
<tr>
<th>Equipment</th>
<th>Model</th>
<th>Characterisation tests</th>
<th>issues</th>
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</table>
| SAG mill           | Leung              | high energy impact (DWT or RBT)                    | • The majority of breakage caused by low to moderate impacts, instead of high energy, resulting in cumulative damage  \  
                    |                    |                                                    | • Changes in the mode of breakage by factors such as transition from cascading to cataracting, changes in the position of toe and shoulder, particle shapes and mill speed |
| Abrasion laboratory mill |                |                                                    | • Abrasion takes place at a wide range of energy, not only at low energies                                                          |
| Ball Mill          | Power-based        | Bond test                                          | • Do not respond well to new generation of ball mills that include larger mills, wider ball size ranges, changes to mill speed        |
| HPGGR              | Population balance | DWT, laboratory or pilot scale HPGR and piston press | • Do not account for the variation in the shape of the particles, the configuration of bed from monolayer to multiple layers in confined and unconfined |

2.1.4. Conclusion

In existing models of comminution, a correlation is established between the industrial equipment and the breakage characterisation device. However, the differences of the stressing condition between the testing device and the actual industrial scale equipment cast doubt on the reliability of the current testing techniques for the corresponding equipment modelling. The current testing techniques are seen as a major limitation to the modelling progress, as they do not correctly reproduce the modes of breakage found in comminution equipment and do not have the flexibility to do so. Hence, any variation in system parameters (operating conditions) such as mill speed, filling, throughput can change the breakage characterisation parameters which were originally interpreted as material properties, not the system. To overcome this problem, the existing empirical models should apply data fitting over a broad range of industrial data to accommodate this variation. This shortcoming of the current models highlights the importance of movement toward a novel and mechanistic approach in ore characterisation techniques. Thereupon, it is crucial to have a fundamental understanding of the underlying breakage processes. This research attempts to understand mechanics of breakage under different mechanisms and identify key factors affecting ore
response to breakage. This will enable the author to propose a novel method of ore characterisation by investigating breakage principles inside the available testing devices. For this purpose, the following chapter is allocated to the study of current breakage characterisation techniques and the mechanisms applied by them in greater details.

2.2. SINGLE PARTICLE BREAKAGE CHARACTERISATION

In industrial comminution processes, particles are mainly fractured by compressive loading and the fundamental properties of the fracture process can be studied most effectively by conducting experiments on single particles. The single particle testing offers a simple practical approach in which a well-defined and controlled loading can be applied to a single particle (Zhang and Ghadiri, 2002). Although the particles break differently in the vicinity of other particles, it is believed that the behaviour of particles in any breakage event can be studied from the breakage of each individual particle (Tavares, 2007).

Narayanan (1986) used the single particle breakage data, obtained by conducting experiments in a twin pendulum tester, for modelling the performance of industrial ball mills. Also, Sahoo (2006) reviewed a number of single particle characterisation techniques such as drop weight tester (DWT), pendulum tester and slow compression for coal handling systems. Bearman (Bearman, 1996) attempted to relate the mechanical properties of rock particles, such as mode I fracture toughness to breakage comminution parameters such as $A\times b$ obtained from DWT or pendulum testing.

A more mechanistic insight into characterisation techniques is provided in the work of King and Bourgeois (1993), Bourgeois and Banini (2002) and Tavares (2007) who used the impact load cell for measurements of fracture energy distribution to implement into tumbling mill models.

Ghadiri and Zhang (2002) also, studied other mode of failure such as attrition in the case of single particle. As many solids fail by semi-brittle mode, the study was exclusively conducted on the semi-brittle material such as single crystals of KCl, NaCl and MgO. This is relevant to some comminution processes such as the jet-and attrition-milling, where the localised surface damage is responsible for particle breakage instead of the large-scale fracture which is the common mechanism for comminution.
In the following sections, a review on single particle breakage characterisation methods using common testing devices such as drop weight tester (DWT), short impact load cell (SILC), rotary breakage tester (RBT) and single particle compression is presented. In this section, the stressing mechanism of each device and the significance of a number of factors such as the stressing velocity and the number of contact points that influences the product of breakage in these devices are discussed in greater detail.

2.2.1. JK Drop Weight Tester (JKDWT)

The drop weight test principle was first used by Gross (Gross, 1938), in order to establish a relationship between the net breakage energy and new surface area generated. Schönert (1972) also employed a dropped weight to determine energy utilisation in single particle crushing and used it as a basis to evaluate the performance of an actual grinding mill. Despite the use of a dropped weight before in the area of ore comminution, it was not until 1992 that this technique was employed by JKMRC to replace the twin pendulum as a more convenient way for breakage characterisation (Brown, 1992). Figure 2-2 depicts a schematic picture of this apparatus. This standardised and controlled the application of a dropped weight for testing, being named the drop weight tester (DWT).

![Figure 2-2: A schematic picture of DWT used in JKMRC (Napier-Munn et al., 1996)](image)

The DWT which is used for single particle breakage testing utilises a combination of drop mass and height to break rock samples with a given energy. The stressing mechanism in this device is generally known as two-point contact in the field of comminution, which
means that particles are compacted from the two opposite sides. The weights in a typical
drop weight tester cover a broad range from 2.8 kg to 49.8 kg that allows rock
characterisation with specific energies between 0.1 kWh/t to 2.5 kWh/t. The velocity at the
time of impact is calculated by knowing the drop effective height (i.e. initial height of the
drop weight (h₀) minus the final height of the weight resting on top of broken particles) using
the following equation.

\[ V_i = \sqrt{2gh} \]  
Equation (2.1)

Where:

\( V_i \): impact velocity (m/s)

\( g \): gravitational acceleration (m/s²)

\( h \): drop effective height (m)

The DWT takes advantage of a guiding system to control the drop of the falling mass.
In this case, the free falling condition is no longer of sufficient accuracy as it needs to account
for the frictional losses that reduce speed. Data obtained by DWT is used to establish a
function for ore breakage based on the supplied energy. (Narayanan and Whiten, 1983) of
the JKMRC developed a convenient method for presentation of breakage data resulted from
breakage of different ore types in DWT. This method, which is referred to as the \( t_n \) family
of curves is used to predict the size distribution of any given type of ore with any known
degree of breakage. The parameters \( t_2, t_4, t_{10}, t_{25}, t_{50}, t_{75} \) are defined as cumulative
percentage passing of the \( \frac{1}{2}, \frac{1}{4}, \frac{1}{10}, \frac{1}{25}, \frac{1}{50}, \frac{1}{75} \) of the initial size of the particle. The size of the
particle in mineral processing context refers to the aperture size of a screen in the root two
sieve series that particles will remain on top of that. Knowing the curves for a particular
material at a given \( t_{10} \), it is possible to reconstruct a full size distribution. \( t_{10} \) is also referred
to as breakage index and is related to specific breakage energy (Ecs) by the following
expression:

\[ t_{10} = A[1 - e^{-b \times Ecs}] \]  
Equation (2.2)

Ecs is in kWh/t and can be converted to SI unit, J/kg.
A and b are regarded as ore breakage parameters and represent the properties of the ore. Parameter A indicates the maximum fragmentation that can be achieved for a certain type of ore. A higher value of b represents softer ore (or lower hardness) and the curve becomes steeper. The product of the two parameters (A×b), which is the slope at the origin, has been found to be a good measure of rock competence and is used to compare different types of ore. Figure 2-3 illustrated the tn-t10 family curves for a typical copper ore.

![Figure 2-3: tn-t10 family curves](image)

### 2.2.2. Ultra Fast Load Cell (Instrumented DWT)

The energy that is applied by the DWT to break particles is usually more than that required energy by particles for their first fracture. Therefore, to study the primary fracture, an instrumented drop weight tester, referred to as impact load cell (ILC) with a number of variations such as short impact load cell (SILC) and ultra-fast load cell (UFLC) were developed at different stages. UFLC was originally designed by (Weichert and Herbst, 1986) at University of Utah for the study of breakage mechanics common in comminution devices such as tumbling mills. The specific design features of this device allow researchers to study fundamental ore breakage properties. The impactor in this device is usually a steel ball. When the ball strikes the particle sitting on top of an instrumented rod, the stress waves, generated due to interactions between the ball, rock and anvil travel into the rod and cause elastic deformation. The deformation of the rod caused by the strain waves can be measured by fast strain gauges that are attached to the rod. The strain gauge is wired in a certain
configuration using a Wheatstone bridge that measures changes in voltage due to the travelling of strain waves through the rod. Later, the voltage measurements at various stages of breakage are translated to force measurements versus time for the calculation of energy consumed by particles. The load applied to the top of the rod by the particle during an impact is calculated from the law of proportionality of strain gauges and Hooke’s law. Details of this system and the related measurements and calculations are presented in other works (King and Bourgeois, 1993; Weichert and Herbst, 1986) and will be discussed in greater details in Section 3. A schematic picture of this device is illustrated in Figure 2-4. Impact load cell can be used for measurements of three fundamental fracture characteristics of brittle materials such as particle fracture energy, particle strength, and stiffness (Tavares and King, 1998). The measurements associated with this device will also be discussed in Section 3.

Figure 2-4: A schematic view of UFLC from Tavares (2007)

2.2.3. Rotary Breakage Tester (RBT)

The rotary breakage tester (RBT) uses the same principle for rock breakage as vertical shaft impact (VSI) crushers and laboratory pulverisers. Since the exact amount of energy applied in the process is not well controlled for the latter two, their application is limited to size reduction processes rather than breakage characterisation device, which is the case for RBT. Shi et al. (2009) used the RBT to control applied energy and characterise rock
breakage. In fact, this device is used in the JKMCR interchangeably with the DWT to characterise breakage of different types of ore and compare them with each other. The stressing mechanism in the RBT is in the form of one-point contact, which means the particles are impacted from one side, unlike the DWT. The RBT uses the kinetic energy gained from a spinning rotor to project rocks onto a circumferential set of anvils specially shaped to ensure perpendicular impact. Figure 2-5 depicts this type of device.

The stressing velocity of rock particles in the RBT is larger compared to the DWT at identical energy levels. It is estimated that for the same input energy, the stressing velocity (or applied strain rate) that particles undergo in the RBT is 22 times higher than the DWT. The energy that is imparted to particle breakage in the RBT is calculated from the speed of the rotor and the mass of the particle. However, the mass of particles is no longer effective in this calculation, given the interest of energy per unit mass of particles. The following equation describes this relationship:

\[ \frac{E_{cs}}{m} = \frac{0.5 \times m \times V_{i}^2}{m} = 0.5 \times V_{i}^2 \]

Equation (2.3)
The calculation of speed is based on the rotor tangential and radial velocity. The two components of speed are considered to be equal in theory, which results in the following equation:

\[ V_{l(\text{thory})} = \sqrt{2} \times V_t \]  

Equation (2.4)

Taking into the effect of frictional losses in the guide channels, the actual velocity at the time of impact is lower than the theoretical velocity as it can be described by the below equation.

\[ C = \frac{V_{\text{actual}}}{V_{t \text{theory}}} \]  

Equation (2.5)

And the energy can be written as:

\[ E_{cs} = \frac{0.5 \times C \times \sqrt{2} \times \left(\frac{2 \times \pi \times N_r \times r_i}{60}\right)^2}{3600} = 3.046 \times 10^{-6} C^2 N_r^2 r^2 \]  

Equation (2.6)

Furthermore, high-speed video filming was used for the measurement of actual speed and the corrections were made to account for the losses. Constant C was determined to account for various factors. The most influential factors that contributed to the measurement of C were listed as particle size, rotor speed and the machine itself.

Vogel and Peukert (2003) used an impact testing device similar to RBT to determine the material parameters in their developed model to quantify the impact grinding performance of different materials. In this work, a mastercurve was developed for the breakage probability of various materials. The breakage probability was discussed to depend on the two material parameters; \( f_{\text{Mat}} \), characterises the resistance of particulate material against fracture in impact comminution and \( W_{m,\text{min}} \) defines the mass specific energy which a particle can absorb without fracture. These parameters of material then were applied to a simple model for the simulation of grinding of three different products in two different impact mills. It is also discussed that the derived material properties can be utilised in similar applications for other types of impact mills as the two parameters were determined independent from the mill.

2.2.4. Single Particle Slow Compression

In slow compression of single particle testing, the rock is situated between two flat plates and pressed until the first fracture occurs. Thus, the mechanism of breakage in this
apparatus is two-point contact, similar to what takes place in DWT. However, it is differentiated from the mechanism of breakage in DWT in the sense that the stressing velocity in slow compression is far lower than impact. The compression test requires a longer time to break the rock as opposed to the impact breakage. Therefore, it can be concluded that mechanism of breakage in the compression tests is strain-driven (Unland and Szczelina, 2004). Figure 2-6 illustrates a compression machine with upper and lower platens and the rock specimen in between. Once the contact between compression platens and rock particles are provided, the force increases until the first fracture take place, then the force drops rapidly.

![Compression test using Instron machine set up](image)

Figure 2-6: Compression test using Instron machine set up

The energy absorbed by the samples is calculated from the area underneath the force-deformation curve. The time required to cause fracture depends on the deformation rate, the rate of loading and the materials properties. Under a constant loading rate, the energy transmitted to the rock depends on materials characteristics. Prior to the first fracture, the force increases linearly with the deformation. However, once the first crack is generated, the force levels off and the crack starts to propagate while the deformation still continues to disintegrate the rock into more fragments. According to a study by Antonyuk (Antonyuk et al., 2005), the pattern of fragmentation in compression testing is dominated by the elastic properties of materials under compression loading. They studied three types of materials with different elasticity and observed that for plastic materials, unlike geological materials, the crack velocity is small and stable and follows the compression velocity. Slow
compression of several types of materials including quartz glass, borosilicate glass, Feldspar, marble, limestone, gypsum was studied broadly by Yashima et al. (1979). In order to prevent scattering of the fractured fragments and secondary breakage, the specimen was enclosed in gelatine. According to this study, different materials exhibit different fracture pattern under slow compression. In the case of borosilicate glass, the core of specimen was fractured into very fine particles, whereas the surrounding pieces were broken into several crescent-shaped fragments. Quartz glass behaved similarly to borosilicate glass. Feldspar and quartz behaved similarly as they both created 4 to 5 crystal grains. Marble, limestone and gypsum all were broken into two semi-spheres. Unland and Szczelina (2004) used a compression device to enforce a determined displacement to rock samples in order to simulate the conditions prevailing in crushing equipment; it is believed that particles break individually and in a single-mode in jaw and gyratory crushers. Taking into account the type of breakage that is common in comminution devices such as crushers, compression devices are used to study single-particle and inter-particle breakage characteristics and fragmentation for crushers (Dowding and Lytwynshyn, 1982). The machine that is often used to perform compression mode of breakage and measure fundamental properties of materials such as compression strength can also be implemented to measure the other characteristics of brittle and non-brittle materials such as tensile strength and Youngs modulus of elasticity.

2.2.5. Differentiation of Stressing Mechanism

Breakage mechanisms are differentiated based on the mode of stress that they apply to disintegrate rock particles and cause fragmentation. Notably, two main factors, such as the number of contact points and the stressing velocity are recognised to be responsible for this differentiation. In the following section, the previous studies on the effect of these two factors are presented. For this reason, crack propagation, stress distribution, fracture energy and type of fragmentation resulting from various mechanisms are discussed and compared with each other.

Number of Contact Point

According to Brown (1996), one interface of breakage which is referred to as “one-point of contact” results in higher fragmentation of rock compared with two-points of contact. In two-point loading, a test specimen is crushed between two hard surfaces at a
certain deformation rate. Using the Hertz contact theory, Weichert (1992) showed that to reach the same level of tensile stress as one-point contact, twice the amount of energy would be required in two-point contact. The former is the dominant mechanism of breakage in a RBT. The crack propagation pattern in one-point contact was also studied by Arbiter et al. (1968) who concluded in one-point contact, cracks initiate along meridian planes of fracture and the number of cracks is proportional to the breakage energy. However, multiple oblique planes are also formed and developed at higher energy levels. The study of stress distribution by Khanal and Morrison (2008) shows that in one-point contact breakage when the particles are compressed from one face, a wedge-shaped region is formed. Besides, the other side of the contact surface is disturbed in a similar manner which generates another wedge shape region. This is explained by the travelling stress wave which reflects to and from both directions.

The condition of stress in the vicinity of contact points were studied broadly by Hertz (1881) and Antonyuk et al. (2005). Chau et al. (2000) also, simulated the stress distribution of a sphere in one-point and two-point contact. Analysing the stress of two-point impact of a complete sphere showed the compressive stresses are strong at the point of two contacts surfaces. At the same time, the shear stresses also exist but appear in a smaller area compared to compressive stresses. However, the tensile stress with strong intensity in the very small area appears at the points of contacts trying to disintegrate the material in the contact point. In the middle of the complete sphere, where the opposite stress counters join together, there is a small area of compressive stresses as well. When the stresses propagate from one side, the effects are counteracted with the opposite side. Compared to the results of stress propagation in one-point contact, the effect of stresses in two-point contact is damped and the progeny appears with coarser fragmentation. The cone of fines also appears in the contact point and becomes wider; the extension depends on the properties of materials.

**Stressing Velocity (Applied Strain Rate)**

Rock fragmentation has a wide range of physical relevance including, but not limited to, rockburst, earthquake, blasting, crushing and milling. In these cases, fragments are produced under various ranges of applied strain rate (Lankford and Blanchard, 1991). The applied strain rate is defined as the rate of deformation that is applied by the commination
device. Generally, it is categorised into static, quasi-static and dynamic with the lowest rate belonging to static loading. Blasting operates a high range of loading from $10^2 \text{s}^{-1}$ to $10^4 \text{s}^{-1}$ and is considered a dynamic mechanism of breakage (Figure 2-7). However, the mechanism of breakage inside common ore comminution devices takes place in far lower ranges. The loading in any comminution device is quasi-static if the velocity of elastic wave is significantly faster than the crack propagation velocity (Vogel and Peukert, 2003). Common comminution devices deal with loading at rates such that the duration of the contact is sufficient to allow the stress to propagate and equilibrate throughout the particle (Tavares, 2007). As a result, these techniques are called ‘quasi-static’. High pressure grinding rolls (HPGR) and crushers use compression force to break down particles rather than the typical impact forces employed in tumbling mills. The compression velocity in HPGR ranges from 0.05 m/s to 0.1 m/s. On the other hand, the applied rate of loading in tumbling mills ranges from half a meter per second to a maximum 10 m/s. Figure 2-7 illustrates the applied strain rates associated with different types of loading and equipment.

![Figure 2-7: Strain rates associated with different types of loading](image)

The effect of stressing velocity was first studied in the field of rock mechanics and the study of rock failure mechanisms. As it was presented in the work of others (Wu and Pollardy, 1993), the fracture geometry or the shape of the fragments depends strongly on strain rate due to the lengths of the fracture that are affected by their propagation velocity. For instance, at high strain rates, the number of fracture decreases while rapid propagation of some cracks inhibits some others from opening. It is known from the study that Yashima et al. (1973) conducted on silica glass, Borosilicate glass, quartz, limestone and marble that applied strain rate can impact the mechanical properties of brittle materials such as their Youngs modulus of elasticity and compressive strength. However, its effect on fracture...
energy is negligible. According to Schönert (1988), the strain rate in elastically deforming particles affects the state of stress only when shock waves are created, and this doesn’t occur with deformation velocities below 200 m/s. Therefore, the strain rate doesn’t influence the breakage strength under conditions that prevail in usual comminution devices. A range of other studies that were conducted by Tavares and King (1998) indicates that breakage characteristics such as fracture force and energy in drop weight tester are not affected significantly by the speed of applied force covered in this type of device. However, the differences between slow compression and drop weight tester were more substantial (Tavares, 2007).

The effect of strain rate varies with the type of material and their elastic-plastic behaviours. Hence, to investigate the effect of strain rate on the breakage behaviour of rock particles, one must understand the type of fracture and shapes of fragments in various ranges of loading and on different materials. Tavares and King (2002) suggested that some materials show inelastic response due to gradual damage accumulation – growth of a network of cracks – before fracture, exhibiting lower (subcritical) net crack growth velocity. The result is that stressing rate effects may be present for this type of material. A comparative study of dynamic and static loading by Chau et al. (2000) using plaster samples with different water content showed that the required maximum force for fracture in impact loading (dynamic) is less than for compression loading (static). However, the deformations with the impacting process are of higher values which cause more energy consumption. They also found the stress distribution within the sphere varies in the two different cases; the failure pattern is orange-slice/lunar fragments for compression and lateral separation fracture for impact.

The effect of strain rate was attributed to manifest itself on the efficiency of transition from compression to tensile stress which is high for high strain rate processes (Sadrai et al., 2011). Despite the insignificant effect of applied strain rate on the fracture characteristics of many geological materials, it may manifest itself on the characteristics of these materials after their first failure (Schöenert, 1991).

The static, quasi-static and impact stressing mechanisms were studied in detail in the work of Baumgardt (1976), Baumgardt (1973), Baumgardt (1975) and (May, 1974). Shi and Kojovic (2005) linked the differences of size distribution resulting from DWT and RBT to
the applied strain rates and change from two- to one-point contact that the particles undergo in these devices. The strain rate that particles experience in the RBT is higher than in the DWT since the impact velocities are almost 22 times that in the DWT. They compared the results of the two testing methods at nominal equivalent energy levels for two rock samples with significantly different hardness indices. According to their study, the RBT is more efficient than the DWT for producing fine particles at both low and high energy levels. The difference was more significant for the soft rock.

2.3. IMPACT LOAD CELL AND ASSOCIATED MEASUREMENTS

2.3.1. Working Principle

Measurement of the force and displacement is enabled using the ILC. The strain gauge which is attached to the surface of the rod measures the force that acts on the contact area. The force can be calculated based on the propagation of the wave and the Hooke’s law. According to the longitudinal wave propagation theory, compressive strain wave propagates through a homogeneous rod, according to the following equation (Tavares, 1997):

\[ \frac{\partial^2 u}{\partial t^2} = \frac{Y}{\rho} \frac{\partial^2 u}{\partial x^2} \]  

Equation (2.7)

\( u \), is the local deformation of rod. \( Y \), is the Young modulus of elasticity of rod. \( \rho \) is the rod density. Based on this equation, strain wave propagates in a cylindrical rod with the velocity \( C_o \), given by:

\[ C_o = \sqrt{\frac{Y}{\rho}} \]  

Equation (2.8)

According to the law of Hooke’s, the force that acts on the surface of the rod is:

\[ F = k \cdot \Delta x \]  

Equation (2.9)

\( \Delta x \) is the rod deformation from the surface of the rod.

And

\[ K = \frac{Y \cdot A}{L} \]  

Equation (2.10)

Where \( K \), \( A \) and \( L \) are rod’s stiffness, surface area and length

Therefore:

\[ F = \frac{Y \cdot A}{L} \cdot \Delta x \]  

Equation (2.11)

On the other hand, \( \varepsilon \); the axial strain of the rod is written as:
\[ \varepsilon = \frac{\Delta x}{L} \]  
Equation (2.12)

By substituting Equation (2.12) in Equation (2.11):

\[ F = AY\varepsilon \]  
Equation (2.13)

The axial strain of the rod; \( \varepsilon \) is defined as the derivative of the displacement along the x-axis:

\[ \varepsilon = \frac{\partial u}{\partial x} \]  
Equation (2.14)

The transmission of the strain wave can be written as:

\[ \left( \frac{\partial u}{\partial x} \right)_t \left( \frac{\partial x}{\partial t} \right)_u \left( \frac{\partial t}{\partial u} \right)_x = -1 \]  
Equation (2.15)

Also, \[ C_o = \left( \frac{\partial x}{\partial t} \right)_u \]  
Equation (2.16)

By substituting Equation (2.16) and Equation (2.14) in Equation (2.15):

\[ \left( \frac{\partial t}{\partial u} \right)_x = \frac{-1}{\varepsilon C_o} \]  
Equation (2.17)

Therefore:

\[ \varepsilon = \frac{-1}{C_o} \left( \frac{\partial u}{\partial t} \right)_x \]  
Equation (2.18)

By re-writing Equation (2.8), Young modulus of elasticity can be obtained as a function of \( C_o \).

Therefore:

\[ Y = \rho C_o^2 \]  
Equation (2.19)

By substituting Equation (2.18) and Equation (2.19) in Equation (2.13), force is calculated as:

\[ F = \rho A C_o \frac{du}{dt} \]  
Equation (2.20)

However, there are a number of assumptions for this calculation:

1- no dispersion or attenuation of the wave takes place from the point of contact to the measuring station (strain gauges) and

2- the bulk deformations inside the rod are predominantly elastic (which is commonly valid, The strain gauge measurement is based on the Wheatstone bridge circuit, with a certain configuration of four resistances. This will not be discussed in detail as it is discussed in detail in the work of Bourgeois (1993).

Unlike force, the direct measurement of particles compression during impact is not possible. As illustrated in Figure 2-8, the deformation that particle situated on the top of anvil experience can be written as (Tavares and King, 1998):
\[ \alpha(t) = u_b - u_r \]  
Equation (2.21)

This means the deformation of each body, \( u_b \) (for ball) and \( u_r \) (for rod) should be calculated separately. The motion of ball can be calculated from its momentum as:

\[ m_b \frac{d^2 u_b}{dt^2} = -F + m_b g \]  
Equation (2.22)

\( m_b \) is the mass of ball, \( F \) is the force that acts on the contact surface. The initial condition is considered at the moment that contact is established between the striker and particle. Therefore, when \( t = 0 \), No force is applied (\( F = 0 \)). Also, at the time of contact, velocity \( \frac{du_b}{dt} \) is equal to \( v_o \) (calculated from the equation of free falling of an object from a known height). By re-writing Equation (2.21) and integrating it twice and using initial condition, \( u_b \) can be calculated.

\[ u_b(t) = v_o t + \frac{gt^2}{2} - \frac{1}{m_b} \int_0^t \int_0^\tau F(\tau) d\tau dt \]  
Equation (2.23)

The Equation (2.20) for rod deformation can be rewritten as:

\[ \frac{du_r}{dt} = \frac{1}{\rho A_r C_r} F_r(t) \]  
Equation (2.24)

By integration,

\[ u_r(t) = \frac{1}{\rho A_r C_r} \int_0^t F(\tau) d\tau \]  
Equation (2.25)

Hence, it is possible to calculate the displacement of each separate body, leading to the calculation of particle displacement. By substituting Equation (2.23) and Equation (2.25) in Equation (2.21), the displacement of particle can be calculated as:

\[ \alpha(t) = v_o(t) + \frac{gt^2}{2} - \frac{1}{m_b} \int_0^t \int_0^\tau F(\tau) d\tau dt - \frac{1}{\rho A_r C_r} \int_0^t F(\tau) d\tau \]  
Equation (2.26)

As discussed in the previous chapter, by conducting an experiment in impact load cell, the force-time signal is recorded. However, the measured signal is a convoluted version of the actual force–time history experienced by a particle sitting on the top of the rod. This is particularly because the measurement takes place at the strain gauges which is located with distance from the contact surface. Therefore, the measurement is a time-delayed signal which needs deconvolution to reflect the actual force-time history. The details of the deconvolution procedure and calculations can be found in the work of Bourgeois and Banini (2002). Knowing the force-time signal and the mass of striker, the abovementioned equation can be integrated using numerical approach.
Knowing the deformation experienced by particle a well as the force, the energy absorbed by the particle is given by:

\[ E_p = \int_{t_0}^{t_f} F(t) \alpha(t) \, dt \quad \text{Equation (2.27)} \]

\( t_f \) is the time when the particle fractures.

By substituting \( \alpha(t) \) from Equation (2.26) into Equation (2.27), calculation of energy is possible:

\[ E_p = v_o \int_{t_0}^{t_f} F(\tau) \, d\tau + g \int_{t_0}^{t_f} F(\tau) \, d\tau - \frac{1}{2m_p} \left( \int_{t_0}^{t_f} F(\tau) \, d\tau \right)^2 - \frac{1}{\rho A_r C} \int_{t_0}^{t_f} \rho A_r C \frac{d^2 \alpha}{dt^2} \, d\tau \quad \text{Equation (2.28)} \]

\[ \text{Figure 2-8: deformation experienced by particles and bodies of contact (Tavares and King, 1998).} \]

2.3.2. Energy Balance

The energy supplied by the falling ball striker (\( E_{\text{input}} \)) is converted to the other forms of energy during impact. Therefore:

\[ E_{\text{input}} = E_p + E_r + E_{\text{res}} + E_l \quad \text{Equation (2.29)} \]

The input energy is known from the height and the mass of striker when the impact is in the form of free fall. The energy used for particle breakage \( E_p \) is given in Equation (2.28). Similarly, \( E_r \) is the energy absorbed by the rod during its deformation, obtained by Equation (2.29).

\[ E_r = \int_{t_0}^{t_f} F(t) \, du_r(t) \quad \text{Equation (2.30)} \]

Using Equation (2.24) for rod deformation, as well as the force-time signal, \( E_r \) is calculated as:
E_r = 1/\rho_r A_r C_r \int_0^{t_f} F^2(t)\,dt \quad \text{Equation (2.31)}

By assuming that the losses due to friction, heat, etc. are negligible (E_l = 0), it is possible to calculate the residual energy of the striker. Knowing the residual energy of the striker and the input energy, the coefficient of the restitution of ball can be calculated.

2.3.3. Measurements of Fracture Characteristics Using ILC

**Particle Strength**

Particle strength is one of the measurements, associated with the application of the impact load cell. The strength of rock particles, impacted by a striker in an ILC can be obtained by the resemblance of this circumstance with compression of a particle using point loading. In fact, Hiramatsu and his colleagues (Hiramatsu, 1966) conducted stress analysis of point load compression on spherical specimens and presented an equation for the tensile strength of specimen.

\[ S = \frac{2.8P}{\pi d^2} \quad \text{Equation (2.32)} \]

S is the particle compressive strength. d is the distance between the loading points.

They calculated the strengths of irregular-shaped particles using Equation (2.31) and compared them with the measurement of tensile strengths of the same materials in a Brazilian test and concluded that they are in agreement with each other.

The variability of strength is linked to the variability of cracks and flaws present in particles. Reduction of particle size also leads to the increase of the strength, which can be explained by Griffith’s theory of fracture. According to Griffith, the presence of flaws and cracks in particles decreases as the particles become smaller. Weibull (Weibull, 1939) related the particles strength to their volume:

\[ S = S_0 V^{-\frac{1}{u_f}} \quad \text{Equation (2.33)} \]

uf is the Weibull’s coefficient of uniformity and is thus a constant. So is a constant. Both So and uf are obtained by fitting the measurements of particles strength. Weibull also defined the probability distribution of strength as:

\[ P(S) = 1 - \exp[-\left(\frac{S}{S_0}\right)^{u_f}V] \quad \text{Equation (2.34)} \]
Yashima et al. (1973) obtained the value of $u_f$ for some materials of sphere shape using a slow compression testing device. However, (Tavares, 1997) obtained this value for irregular-shaped particles of some other types of materials using the ILC. By fitting strengths obtained by experiment in ILC and relating that to mean strength, he found strong correlations for most of the tested materials that have a high brittleness. However, poor correlation was obtained for marble and iron ore, which are less brittle. This demonstrates that strengths of irregular-shaped particles cannot be linked only to their size, as the shape of particles, besides the internal flaw structure, plays a crucial role in determining the stress state of the particles.

**Fracture Energy**

As indicated before, by obtaining the force-time signals resulting from conducting experiments in the ILC, and conducting an energy balance around the contact area, calculation of the energy absorbed by particles is possible. When the energy is calculated up to the first fracture point, it is referred to as particle fracture energy. By conducting fracture experiments in characterisation devices such as impact load cell or compression testing device for a certain type of rock, a range of data are obtained that should be explained statistically for that particular material, in that particular size. A number of distributions have been used to describe the variability of the fracture energy. The fracture energy distribution can be described using Weibull distribution of strengths (Weibull, 1939), as follows:

$$P(E_m) = 1 - \exp\left[-\frac{E_m}{(E_m, o)^{u_f}}\right]$$

Equation (2.35)

$E_m, o$ is the parameter of the Weibull distribution of fracture energy. In fact, the distribution of fracture energy is linked to the distribution of strength, as the result of the distribution of cracks and flaws in particles. This type of distribution is best fitted to data obtained from fracture of spherical or semi-spherical particles (Weichert, 1992). Tavares (1996) successfully applied the Weibull distribution to describe the variability of strengths of cement clinker obtained by conducting experiments in UFLC. However, for irregularly-shaped particles, the Weibull distribution does not provide an appropriate fit to experimental data. This is particularly because assumptions in Weibull theory are based on the variability of strengths due to the presence of flaws and cracks in particles, not due to their shapes and other complexities of material structure. (Tavares, 1997) used various forms of normal distribution to describe the distribution of fracture energies of quartz particles in the different size range.
The log-normal distribution was found to provide a better fit to experimental data.

\[ P(E_m) = \frac{1}{2} \left[ 1 + \text{erf}\left( \frac{\ln E_m - \ln E_{m,50}}{\sqrt{2}\sigma_E} \right) \right] \quad \text{Equation (2.36)} \]

\( E_{m,50} \) is the median of the distribution and \( \sigma_E^2 \) is its variance.

Using Weibull fracture theory, failure rate function and a number of concepts in fracture mechanics, (Tavares, 1997) proposed a model of fracture energy as a function of particle size.

\[ E_{m,50} = E_{m,\infty} \left[ 1 + \left( \frac{d_{p,o}}{d_p} \right)^\phi \right] \quad \text{Equation (2.37)} \]

\( E_{m,50} \) is referred to as median specific fracture energy. In this model, \( E_{m,\infty}, d_{p,o}, \) and \( \phi \) are material constants, where \( E_{m,\infty} \) represents the residual fracture energy of the material at a coarse size. This means the specific fracture energy of particles does not change significantly for fragments larger than this size. The \( d_{p,o} \) is a characteristic size of the material microstructure.

This equation was also fitted to a variety of data from the fracture distribution of a variety of different materials and over a range of particle sizes (Tavares and King, 1998). As found from fitting data of minerals, ores and rocks to the model, \( E_{m,\infty} \) of minerals is lower than ores and rocks. However, the transition size, \( d_{p,o} \) of minerals is higher than rocks and ores.

**Particle Stiffness**

Hertzian contact theory was used by Tavares and King (1998) to derive an equation for stiffness of particles. Force is related to deformation by the following equation,

\[ F = \frac{d_p^{1/2}}{3} K_e \alpha^{3/2} \quad \text{Equation (2.38)} \]

\( K_e \) is the local deformation coefficient of the Hertzian contact and can be obtained by the following equation:

\[ K_e = \frac{k_p k_b}{k_p + k_b} \quad \text{Equation (2.39)} \]

\( k_p \) and \( k_b \) are stiffness of the particle and the ball, respectively. Each is obtained as:

\[ k_p = \frac{Y_p}{1-\mu_p} \quad \text{and} \quad k_b = \frac{Y_b}{1-\mu_b} \quad \text{Equation (2.40)} \]

\( Y \) is the Young’s modulus and \( \mu \) is the Poisson’s ratio.

By substituting Equation (2.38) into Equation (2.27), particle fracture energy is calculated as:

\[ E = \frac{2d_p^{1/2}}{15} K_e \alpha^{5/2} \quad \text{Equation (2.41)} \]
Using Equation (2.41) combined with Equation (2.38), $K_e$ can be related to critical load and the particle fracture energy:

$$K_e^2 = 0.576 \frac{F_c^3}{d_p E_c^2}$$  \hspace{1cm} \text{Equation (2.42)}

2.4. MULTI-STAGE BREAKAGE

2.4.1. Secondary Breakage

A size reduction process, based on the size-mass balance model, may be described in its simplest form by a combination of two basic components: the breakage function and the breakage rate function, this latter also known as selection function. For instance, in the case of the ball mill, the breakage function may be estimated by conducting controlled experiments with narrow size fractions, allowing a very short grinding time in a batch ball mill for particles to only suffer primary fragmentation (Austin et al., 1984). Unlike the selection function, this function is regarded as relatively insensitive to most of the system variables. Indeed, it is common to assume that the breakage function is predominantly a material-specific function (Kelly and Spottiswood, 1990).

An alternative size-mass balance model and breakage characterisation methods moved away from the concept of constant material specific breakage function towards a breakage function that depends on the average specific energy in a mill (Narayanan and Whiten, 1983). This energy-size reduction relationship, proposed by Narayanan and Whiten (1983), has been described on the basis of single-particle breakage tests conducted in a DWT. It is a method that allows the estimation of the size distribution from a given input energy resulting from an impact event in the milling device. It is also a method that gained great popularity in industry to characterize the response of ores to impact breakage, resulting in the well-known “A×b” breakage index (Napier-Munn et al., 1996) which is estimated on the basis of the so-called t10 curves. However, recent work demonstrated a number of distinct limitations for this method that need to be taken into consideration before being used by models (Powell et al., 2014). Most importantly, the shortcomings of t10 method even come to close attention, where it is used to predict stressing conditions that differ from the flat-flat geometry used in the DWT. This is extensively described in the work of Tavares (2007) that the degree of fragmentation is potentially affected by the test geometry, casting doubt on
suitability of the DWT for the modelling of tumbling mill environment in which various geometries of impact are used for ore breakage. In the mentioned study, three different geometries including ball-ball, ball-flat and flat-flat were used on the breakage of fluorapatite samples and the results indicated that at low energy impact, the three geometries resulted in almost the same size distribution of the progeny fragments. In this case, a large fraction of input energy is consumed by the particles to create the first fracture and only a small amount of energy was left available to cause further breakage. However, when the input energy increased, the effect of geometry became more evident. This was referred to the amount of available remaining energy after the initial fracture which was consumed for progressive breakage. Flat-flat geometry involves larger active zone of breakage that captures more material for breakage as the fragmentation progresses. Figure 2-9 illustrates this difference at low and high energies. Following this, it is also suspected that the “A” value or the maximum achievable fragmentation in this characterisation test and “A×b”, cannot be understood as solely rock properties, but the response of the rock to certain well-defined stressing conditions, which involves two flat metal surfaces, impacting against each other, with the particle in between.

![Figure 2-9](image)

**Figure 2-9: Breakage of 2.0-2.8mm fluorapatite at different impact energy levels and with three different geometries, redrawn from Tavares (1997)**

Progressive breakage of particles is an active phenomenon in many ore characterisation tests. The breakage process usually continues after the first failure due to the excessive energy of the falling ball or the comminution device. This energy is consumed
during the next stage of the process after the initial fracture of the particle and leads to secondary fracture or several subsequent fractures depending on the kinetic energy of the rock fragments (Tavares and King, 1998). The effect of kinetic energy is recognised in the work of Bergstrom et al. (1962), who used a steel retaining ring to surround each particle while it was being crushed. When the ring was not used, the fragments flew in all directions. They found that the fragments were coarser than those collected in the steel chamber.

According to Schönert (1988), the subsequent breakage is determined strongly by the spatial arrangement of the fragments after their first fracture when they form a pile. Based on this study, the expansion of the pile depends on the materials hardness, the arrangement of the initial particles and the loading velocity. By conducting some experiments on quartz and gypsum particles, he found that the quartz fragments are distributed widely but the gypsum fragments form a narrow pile. The work of Chandramohan (2013) in addition to Schönert (1988) emphasised the impact of shape on the re-breakage of a single-particle. He related the survival of the broken fragments to their projected velocities in comparison with the velocity of the anvil. If the velocity of falling anvil surpasses the velocity of the fragments; driven by their released strain energies, then the fragments are subjected to repeated loading and re-break. The release of strain energy is specific to the type of materials. The secondary breakage is then related to the velocity of the impactor and the strain energy of materials. In respect to the work he conducted, he concluded that during impact, due to the similar contact area of the vertically orientated flakes and the non-flakes, the new surfaces are created with similar strain energies. However, due to the larger contact area of horizontally oriented flakes, the induced stress field during contact is different that lead to significantly different released strain energy in comparison to the previous arrangements. As the previous work demonstrated, the degree of fragmentation also is affected by the system properties, e.g. geometry of impactor. The dependency of breakage characterisation parameters on the system parameters and spatial distribution of fragments emphasises that a single breakage test is actually a sequence of events in which the degree of fragmentation is affected by the way that rock fragments are captured after primary fracture.

2.4.2. Summary

Different approaches have been used to quantitatively describe the outcome of a single particle breakage test. The simplest have been the empirical approaches that merely describe
the experimental data under standardized conditions, of which the t10 method is the most well-known (Napier-Munn et al., 1996). On the opposite side of the complexity spectrum are advanced computational techniques such as the finite element or the discrete element method (Potapov and Campbell, 1994; Potyondy and Cundall, 2004; Schubert et al., 2005) that have been successfully used to gain insights into the response of single particles to stressing, but that cannot be easily scaled to describe a full scale process. However, by regarding a single breakage event as a multi-stage process, the present work proposes the application of a different modelling approach to describe a single breakage event. Such a model would provide a framework to describe a single breakage event as a process, so that the size distribution as a function of stressing energy is an outcome of the model. The most recent work on the breakage of a single particle as a sequence of subsequent breakage events was described by Tavares (2004). In summary, it models the breakage of a single particle as a multi-stage process of repeated crushing stage while the classification is ideal. The size distribution is calculated after several loading stages, each treats the generated fragmentation as fresh mono-size that is broken out. In terms of energy, relative energy in each impact is kept constant, so the total energy, consumed for the whole process is the summation of the impacting events.

By using the approach proposed in this current project, the most elementary processes governing particle breakage can then be identified, being of key importance when modelling comminution in crushers and mills. This would allow describing not only the influence of stressing energy in breakage, but also of geometry and velocity which consequently will allow investigation of the response of the material in primary and subsequent breakage events. The proposed method also can benefit the area of ore breakage characterisation. It is believed that a simplified and streamlined characterisation is only possible when the relation between the characterisation of first breakage and subsequent breakages is well determined. In the present work, this approach is used at a preliminary stage to explain the combined roles of stressing energy and shape of contacting surfaces on the fineness and shape of the progeny size distribution.
3. HYPOTHESES, OBJECTIVES AND RESEARCH METHODOLOGY

3.1. MULTI-STAGE PROCESS

As seen in the previous chapter, a single breakage event may be explained well by considering it to be a sequence of events that initiates with the first fragmentation of a fresh rock. It occurs when a particle fractures by applying a magnitude of force that exceeds its strength, as portrayed in Figure 3-1(a). Following the initial fragmentation, some fragments may leave the breakage environment, depending on the extent of their spatial distribution. In this work, this stage is referred to as “classification”, depicted in Figure 3-1(b). As the breakage continues due to the excess energy, those fragments that remained in the breakage zone may become selected for further fragmentation (Figure 3-1(c)). The probability of selection may vary for particles of different sizes and environments with different geometries. When the fragments are selected, they may break for the second time, depending on their strength. The progression of this process, relies on further classification and selection of the generated progenies.

Figure 3-1: Breakage sub-processes (a) First fracture (b) Classification (c) Secondary breakage

3.2. HYPOTHESES

Based on the multi-stage nature of a single breakage event, discussed in the previous section, three main hypotheses are stated as followed:

HYPOTHESES 1: A characterisation test is a sequence of breakage events that can be modelled in a way to decouple the fundamental properties of rock from the geometry of the testing system.
This hypothesis examines whether breakage of a single particle, such as those that occur in a characterisation device, is a sequential event. By considering this single event as a sequence of other events, it is also possible to model it as a process by incorporating the contribution of its sub-events. Each individual sub-event then can be modelled independently and that model is responsive to the factors that affect it. As explained in section 3.1, the first sequence starts with the first fracture and produces progeny that is referred to as primary fragmentation. Hence, the first fracture is a sub-event and may be explained by a model. Instantly after the first breakage, another sub-event, referred to as “classification” contributes to ongoing breakage. Hence, it is regarded as a sub-event and modelled based on the parameters that impact its extent. Once classified, the selection probability of fragments as the next sub-event should be incorporated into the structure of the model as a separate function that responds to the variation of factors that influence it.

As stated in the second part of the first hypothesis, the model should be structured in a manner that allows isolation of the fundamental properties of rock from the impacts of the testing system. For this purpose, each sub-model that constitutes the overall model should respond to either property of materials or the breakage system and changes as they change.

It is also perceived that after primary fragmentation, each individual generated fragment breaks in the same way as it broke initially, regardless of the breakage environment and parameters that define a system of breakage such as geometry and stressing velocity. In other word, the primary breakage of a particle results in a size distribution that is solely the properties of the material not the breakage mechanism. Hence, the next hypothesis was introduced as:

**HYPOTHESES 2**: Primary breakage characteristics (i.e. appearance function and fracture energy) are not affected by the applied strain rate.

As explained, the first sub-event of breakage starts with the first fracture that has characteristics and properties. When particles break, they produce a progeny, and to form this progeny, they need some amount of energy. Hence, the size distribution of progeny and the energy consumed are considered to be the characteristics of primary breakage function. They are referred to as primary breakage appearance function and fracture energy, respectively.
The second hypothesis states that the characteristics of primary breakage function which is its appearance function and fracture energy are only affected by the material properties not the properties of the breakage system such as geometry and the stressing velocity. To prove this hypothesis in this project, only the characteristics of primary breakage are examined in response to the stressing velocity. The other parameters that are the properties of breakage environment such as the geometry will not be investigated.

The separation of primary breakage function from the other sub-processes of breakage is particularly important from the characterisation and modelling perspective if proven to be solely the properties of materials, not affected by the environment of breakage. However, the other sub-processes that determine the extent of the breakage and the final shape of the size distribution can be system dependent. Hence, the last hypothesis is stated as:

**HYPOTHESES 3: Applied Strain rate changes the classification and selection functions**

This hypothesis states that although the properties of a breakage system, such as its stressing velocity does not change the way that each individual fragment breaks (primary fragmentation), it can influence the way that particles are selected and classified. Hence, the shape of the size distribution curve is affected as a result of this factor.

The next section sets objectives and defines the methodologies to address each individual hypothesis.

### 3.3. OBJECTIVES AND METHODOLOGIES

**OBJECTIVE 1:** To address the first hypothesis the objective is to “Develop a model for a single particle breakage event, such as those occurring in a drop weight test”. Because the model has several components, the following methodology can be applied:

**METHODOLOGY 1:**

1. Model each individual sub-event as a function, i.e. primary breakage, classification and selection functions.
2. Build a sequence (stage) by combining the model of different sub-events to form the final size distribution. Once the sequence is built, it should be repeated.
3. Calculate the energy consumption by the fragments in a sequence.
4. Calculate the total energy consumption after constructing the model from the combination of different sequences.
5. Validate the model with experimental data.

To validate the model, the experimental data are obtained by conducting breakage tests in a drop weight tester. This experiment should provide the following information:

1. Measurement or calculation of primary breakage characteristics such as fracture energy and resulting primary breakage function.
2. Size distribution of breakage progeny at energy levels beyond the first fracture.
3. The consumption of energy by particles.

Once the characteristics of the first fracture event are determined, modelling the total breakage process based on this function and the other subsequent processes is possible. Hence, the first part of the experimentation is dedicated to the measurement of these characteristics. The model validation should meet the following criteria:

1. The shape of the size distribution by the model should match the experimental results. This means that the overall shape of the fragmentation curve (cumulative passing) in log-log scale should match the experimental curve.
2. The energy consumption by the particles, calculated in the model also should match the energy consumption calculated from the experimental data.

Figure 3-2 illustrates a diagram depicting different components of the single-particle model validation.
DATA FOR MODEL VALIDATION

The data used for the purpose of model validation was obtained from different sources. All the experiments are conducted using an impact load cell. In summary, there are three different types of geological materials including rocks and minerals. Table 3-1 provides a summary of the data used for the model validation.

Table 3-1: Summary of data used for model validation

<table>
<thead>
<tr>
<th>Samples</th>
<th>Source of data</th>
<th>Size (mm)</th>
<th>Type of information</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quartz</td>
<td>Bourgeois (1993)</td>
<td>3.35-4.75</td>
<td>Size distribution, Primary breakage function</td>
</tr>
<tr>
<td></td>
<td>Tavares (1997)</td>
<td>0.25-4.75</td>
<td>Energy transfer efficiency, Fracture energy distribution for various sizes</td>
</tr>
<tr>
<td>Apatite</td>
<td>Tavares (1997)</td>
<td>2.0-2.8</td>
<td>Different types of geometry, Size distribution, Breakage at a low energy (Primary breakage function), Fracture energy distribution for various sizes</td>
</tr>
<tr>
<td>Silicate</td>
<td>Conducted in this study</td>
<td>3.35-4.75</td>
<td>Size distribution, Breakage at a low energy (Primary breakage function), Fracture energy distribution for various sizes, Energy transfer efficiency</td>
</tr>
</tbody>
</table>

OBJECTIVE 2: To address the second hypothesis, the objective is to “Investigate the dependency of primary breakage characteristics on the stressing velocity”.

METHODOLOGY 2:
The experimentation should result in measurements or calculation of primary breakage characteristics i.e. primary breakage appearance function and fracture energy. Also, the test set-up should allow the examination of such characteristics under a range of stressing velocity. For this purpose, it should be conducted using devices that allow manipulation of this parameter. This means that a part of experiments is required to be carried out under a low rate of stressing velocity, using techniques such as compression. However, the other part should be carried out at higher rates, such as the one that involves an impact load cell. The product resulting from each setting should be collected and the size distributions measured and compared to each other. Also, the fracture energy is measured or calculated with the two different stressing velocities and compared to one another. If the primary breakage functions obtained by two techniques are identical, it suggests that stressing velocity does not affect the primary breakage characteristics. Hence, this hypothesis will be proven.

OBJECTIVE 3: To address the third hypothesis, the third objective is set as “Develop a method for measuring the effect of stressing velocity on the other sub-events such as selection or classification.

METHODOLOGY 3:

As discussed in the previous sections, with the available energy and the breakage that continues beyond the first fracture point, the other sub-processes such as classification and selection of the fragments also contribute to the final size distribution of the breakage product. Therefore, the experiments should be conducted beyond the first fracture point at various levels of energy to allow measurement or calculation of other factors. Also, similar to the previous methodology, experiments should be conducted at different rates of stressing velocity. To extend the experiments beyond the first fracture point and apply additional energy for breakage, samples in compression experiments should undergo increasing displacements, whereas in impact tests, this can be done by changing the combination of drop weight mass and height.

The experimentation should be conducted under contained and uncontained conditions to account for the effect of spatial distribution. Containment of the fragments inside the breakage zone does not allow broken fragments to leave the breakage environment. Therefore, the effect of spatial distribution is eliminated and the other sub-
processes can be understood. Once this objective is achieved, the experiments can be conducted in uncontained conditions to account for the influence of spatial distribution. For the purpose of experimentation, at first, the spatial distribution of fragments was minimised by containing the rock particles between compression device and the anvil (Classification of 1). A light plastic wrapping was used to keep particles in the breakage zone without any risk of imposing bed breakage.

All the experiments should be performed at different energy levels. Using this approach, it is possible to quantify the influence of stressing velocity on the fragmentation of particles and the resulting size distribution, without being affected by the spatial distribution. Later, experiments have to be conducted without any containment. If the size distribution is affected by the containment of fragments versus the uncontained condition, this proves that classification of fragments is affected by the stressing velocity.

Figure 3-3 illustrates a schematic view of the different elements of testing methodology; each is discussed in detail in the following sections.
4. EXPERIMENTAL METHODS

The experimental procedures in this project are distributed into three sections. The first part is dedicated to experimentation designed for validation of the single-particle breakage model that will be presented in the following chapter. It is worth mentioning that the model validation also takes advantage of the experimental data provided in the work of others (Bourgeois, 1993; Tavares, 1997). The next section is designed to examine the effect of stressing velocity on breakage characteristics of rocks. In order to investigate the effect of stressing velocity on breakage characteristics of rocks, each sub-process is examined separately in response to this parameter. For this purpose, an experimental method for isolating the effect of each sub-process, including first fracture, selection and the spatial distribution of progeny is suggested. In the following section, high-speed video filming was conducted to gain further insight into fundamental aspects of a breakage process, such as capture and spatial distribution of fragments for different types of geological materials. The video filming provides an insight into the detail of single particle breakage using geological materials with different hardness.

4.1. MATERIALS

The main objective of conducting experiments in this study was to identify those characteristics of rock particles whose response to a breakage event is independent of the breakage mechanism. Suitable materials were selected and a proper experimental design implemented for achieving the objective. The experiments were mainly conducted using rock samples of magnetite and silicate in different size fractions.

For the purpose of model validation, silicate samples, collected from Beaudesert Quarry Mine in Australia were tested. The experiments were carried out in six different size fractions: 4.75-5.6 mm, 3.35-4.75 mm, 2.8-3.35 mm, 2.35-2.8 mm, 2.35-2.00 mm, 1.7-2.00 mm. The samples were smoothed to remove excessive angularity. For this purpose, the crushed rock samples with dimensions smaller than 20 mm were smoothed in the JK abrasion mill (Devasahayam and Kojovic, 1995). To facilitate this process, a modified abrasion test was implemented (Yahyaei et al., 2015). The main difference of the procedure used in this study is using a small mill instead of the conventional 1.8m diameter abrasion mill. The smaller mill allows preparation of smaller amount of samples. When the materials
were smoothed, they were washed, dried and sieved. Particles smaller than 1.7mm were not tested due to the limitations of the testing device and the implementation of an appropriate method concerning the small particles. These conditions will be discussed later in the following sections. For each size fraction, a number of 100 particles were selected randomly as a representative collection of the entire population.

To investigate the effect of stressing velocity, a two-stage experimental method was implemented. In the preliminary stage of the work, samples of soft (magnetite, A*b>100) (Napier-Munn et al., 1996) and tough (silicate, A*b<40) components of the LKAB Mine in Sweden were tested. 20 irregularly-shaped particles contained in the size range of 26.5-31.5 mm from each soft and tough component were tested.

The main stage of experimentation on the effect of stressing velocity was conducted using magnetite samples that were also used in the preliminary stage. The silicate samples were from the same type used for model validation. The hardness indices of the two types of rocks are significantly different from each other. Using the two types of rock with a significant difference in their hardness allows investigation of ore breakage characteristics for soft and hard components. Materials in size fraction 6.7-8.0 mm were considered to be appropriate choice for the aim of experimentation, considering the limitations of the testing devices used for the purpose of experimentation. That will be explained later in the experimental procedure.

Other material such as quartz in addition to Beaudesert silicate was selected for the purpose of high-speed video filming to gain further insight into the process of breakage. Thus, the study of fracture properties and fragmentation such as fracture energy, its probability and the spatial distribution of fragments is possible. A summary of the material properties is shown in Table 4-1.
Table 4-1 - Summary of material property tested in this study

<table>
<thead>
<tr>
<th>Material</th>
<th>Source</th>
<th>Size (mm)</th>
<th>Density (g/cm³)</th>
<th>Composition</th>
<th>Index of impact strength (A×b*)</th>
<th>Preparation</th>
<th>Purpose</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetite</td>
<td>LKAB mine</td>
<td>6.7-8.0</td>
<td>5.19</td>
<td>Mineral, Pure magnetite</td>
<td>72</td>
<td>Smoothing</td>
<td>Strain rate effect</td>
</tr>
<tr>
<td>Silicate</td>
<td>Beaudesert Quarry Mine</td>
<td>6.7-8.0</td>
<td>2.85</td>
<td>Rock, Feldspars</td>
<td>NA</td>
<td>Smoothing</td>
<td>Strain rate effect</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.75-5.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Model Validation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.35-4.75</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.8-3.35</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.35-2.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.35-2.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.7-2.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.7-8.0</td>
<td></td>
<td></td>
<td></td>
<td>NA</td>
<td>None</td>
</tr>
<tr>
<td>Quartz</td>
<td>Bundaberg Quarry</td>
<td>6.7-8.0</td>
<td>2.65</td>
<td>Mineral, Pure quartz</td>
<td>58</td>
<td>None</td>
<td>Study details of breakage mechanism</td>
</tr>
</tbody>
</table>

* A×b values are given for the specified size range mentioned in the same row

Preparation of magnetite and silicate for the effect of stressing velocity involved smoothing the irregular rock particles to remove excessive angularity. Additionally, for each set of experiments, a population of 50 particles with a spherical aspect ratio (aspect ratio of ~ 1) was selected. This practice is to ensure that statistically consistent results can be achieved with a smaller number of particles. For this purpose, the flaky particles were rejected from a batch of prepared samples using Australian flakiness standard (1999). According to this standard, flakes are particles with a minimum dimension (thickness) less than 0.6 of its mean dimension. In addition to flakiness, the elongation also causes inconsistency in results. Thus, elongated particles which have one dimension more than 1/0.6 of the mean dimension were removed.

4.2. TEST SET-UP FOR MODEL VALIDATION

For the purpose of model validation using silicate samples, all the experiments were conducted in a short impact load cell (SILC); a shortened version of Impact Load Cell (ILC) (Bourgeois and Banini, 2002). Figure 4-1 depicts a front view of the SILC.
For single particle model validation, particles of all shapes were tested. This set of experimentation consists of two parts; first part deals with the measurement of first fracture energy for a number of size fractions. The second part relates to the measurement of the size distribution at various energy levels.

As mentioned earlier in the previous chapter, the test set-up should allow measurements of primary and secondary breakage characteristics. For this purpose, the particles were individually weighed using a high precision scale. The height of each separate particle was measured using a calliper. After these measurements, each sample was placed on top of the anvil, individually, in the centre. Usually, their smallest dimension is along the axis of falling weight. An appropriate drop height was chosen in order to avoid excessive secondary breakage when measurement of first breakage characteristics was targeted. Often, excessive energy results in the unclear signal record, in which the distinction of primary fragmentation from many other subsequent fractures becomes problematic (Tavares, 2007). All the experiments were conducted using the small SILC with a 20 mm diameter. Steel balls with two different dimensions, 25.4 mm (1 inch) and 50.8 mm (2 inches) were used as impactors. During each impact event the voltage versus time signal was recorded, then collected for processing and analysed for measurement of fracture energy. It is worth mentioning that some of the signals were excluded from the calculation of fracture energy due to their low quality. Once the samples were broken, the debris was brushed from the anvil and collected in a plastic bag and labelled. The samples then were taken for manual sieving and the size distribution were determined.
The primary breakage characteristics were measured for all the six size fractions. Table 4-2 shows the combination of drop height and weight for measurement of first fracture energy related to each size fraction.

Table 4-2: combination of drop height and mass for fracture energy measurement

<table>
<thead>
<tr>
<th>Size (mm)</th>
<th>Drop Height (m)</th>
<th>Drop Weight (kg)</th>
<th>Ave. Particle Height (mm) ± (mm) 95% conf. Intv.</th>
<th>Ave. Particle Weight (gr) ± (gr) 95% conf. Intv</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.35-4.75</td>
<td>0.040</td>
<td>0.535</td>
<td>3.31 ± 0.51</td>
<td>0.130 ± 0.027</td>
</tr>
<tr>
<td>2.8-3.35</td>
<td>0.025</td>
<td>0.067</td>
<td>2.07 ± 0.38</td>
<td>0.051 ± 0.010</td>
</tr>
<tr>
<td>2.35-2.8</td>
<td>0.030</td>
<td>0.067</td>
<td>1.77 ± 0.32</td>
<td>0.032 ± 0.006</td>
</tr>
<tr>
<td>2.0-2.35</td>
<td>0.030</td>
<td>0.067</td>
<td>1.54 ± 0.27</td>
<td>0.021 ± 0.003</td>
</tr>
<tr>
<td>1.7-2.0</td>
<td>0.015</td>
<td>0.067</td>
<td>1.29 ± 0.24</td>
<td>0.013 ±0.002</td>
</tr>
</tbody>
</table>

Obtaining higher impact energies is possible by changing the combination of the mass of strikers and the drop heights. The experiment was conducted at high energy levels for the size fraction 3.35-4.75 mm. The mass and the height of each individual sample were measured separately as shown in Table 4-3 for the calculation of specific input energy (refer to Section 2.2.1). Samples were situated on top of the anvil. After the test, the fragments were collected in a plastic vessel and sieved.

Table 4-3: Combination of drop height and mass at different energy levels

<table>
<thead>
<tr>
<th>Specific Input Energy (kWh/t, J/kg)</th>
<th>Drop Height (mm)</th>
<th>Drop Weight (kg)</th>
<th>Ave. Particle Height (mm) ± (mm) 95% conf. Intv.</th>
<th>Ave. Particle Weight (gr) ± (gr) 95% conf. Intv</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10, 377</td>
<td>13</td>
<td>0.535</td>
<td>3.31 ± 0.51</td>
<td>0.134 ± 0.027</td>
</tr>
<tr>
<td>0.51, 1854</td>
<td>20</td>
<td>0.535</td>
<td>3.24 ± 0.55</td>
<td>0.136 ± 0.032</td>
</tr>
<tr>
<td>0.81, 2910</td>
<td>30</td>
<td>1.535</td>
<td>3.27 ± 0.53</td>
<td>0.138 ± 0.034</td>
</tr>
<tr>
<td>1.07, 3852</td>
<td>37</td>
<td>1.535</td>
<td>3.24 ± 0.49</td>
<td>0.131 ± 0.028</td>
</tr>
<tr>
<td>2.08, 7477</td>
<td>65</td>
<td>1.535</td>
<td>3.36 ± 0.55</td>
<td>0.124 ± 0.029</td>
</tr>
</tbody>
</table>

The error in the measurement of size distribution was determined for a number of experiment. This was conducted by repeat of certain experiments twice. Hence, the standard deviation was measured for each size fraction separately.

4.3. TEST SET-UP FOR THE EFFECT OF STRESSING VELOCITY

To examine the effect of stressing velocity, samples were tested using low and high-rate breakage mechanisms. For the high rate, an impact device and for the low rate, a
A compression device was employed. Table 4-4 presents a summary of testing conditions used in SILC and Instron compression device for this study.

**Table 4-4: Summary of experimental condition**

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Test apparatus</th>
<th>Applied strain rate (m/s)</th>
<th>Input energy (kWh/t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impact</td>
<td>Short impact load cell (SILC)</td>
<td>0.5-2</td>
<td>0.06-1.61</td>
</tr>
<tr>
<td>Compression</td>
<td>Instron compression device</td>
<td>8.3×10^{-6}</td>
<td>0.03-0.5</td>
</tr>
</tbody>
</table>

To eliminate the effect of test geometry, identical testing environments; flat-flat type of geometries (flat impactor and a flat anvil) were set-up in both Instron and SILC. Anvils with 50mm diameter were used in both devices. Flat-flat geometry also maximises material capture during experiments. To ensure classification of 1, samples in both SILC and Instron were contained inside a transparent plastic wrap that surrounded the compression anvils. To examine the effect of spatial distribution in both compression and impact mechanisms, the experiments were repeated without any containment and the same procedure was followed. After experiments, samples were brushed from the compression anvil and sized on a root 2 series of sieves down to 38 microns.

The compression experiments were performed in a compression machine, Instron 4505, equipped with a 5 kN load cell. This device was used to apply a certain amount of compression displacement to break rock particles. The signature of force versus displacement was recorded when the contact of compression platen and the particles was established. Once the fracture takes place, the force drops rapidly, indicating the immediate contact of compression platen and rock particle is lost. Instron’s Bluehill 3 software was used to run the compression load cell. This type of systems can perform a variety of tests such as tensile (pull), compression (push), flex (bend), cyclic, creep, and relaxation, depending on the purpose of the test. Each system requires a computer and uses the software to control and monitor tests, collect data, analyse and calculate results, produce graphs, and generate reports required by the user (Corporation, 2004). The compression speed was set to as low as 0.5mm/min and the data were recorded at 0.0001s intervals. First fracture was controlled by a drop in force. Also, the maximum allowable force for experiment to reach was set as 4500 N for safety purposes.
Each individual sample was placed on a 50mm diameter, hardened steel anvil in the Instron. To measure the first fracture characteristics, including the primary breakage function, fracture energy and its distribution in compression tests, a reasonable number of samples were compressed individually between the platen and the anvil until the force dropped by 65% from the maximum value. A low drop of force is normally due to the chipping or fracture of small pieces from the corner of the sample. It also can be due to the relocation of particles during the test. In Figure 4-2, forces vs. extension for two samples of Beaudesert Quarry rock in size fraction 6.7-8.0 mm are shown. In this figure, chipping is differentiated from the main breakage.

![Figure 4-2: Chipping and main breakage of silicate rock samples in size fraction 6.7-8 mm](image)

The force drop of 65% from the maximum value also occurs when a particle is chipped or dislocated (detached) and loses contact with the compression platen. Due to this reason, a conventional criterion for breakage such as 10% loss of the original mass is also required to ensure that primary body breakage has taken place. Therefore, particles were collected after breakage and checked for this criterion. To obtain the first fracture energy, the area under the force versus extension up to the fracture point for those particles with body breakage is calculated. This procedure was repeated for each individual particle and the first fracture energy of each individual particle was calculated. In addition, the products of main breakage were collected all together in a plastic bag and sieved for the measurement of the primary breakage function. The chipped particles were excluded. To extend the energy measurement beyond the fracture point in the Instron, samples were compressed to certain displacements.
of 0.4 mm, 0.7 mm, 1 mm, 1.5 mm and 2.5 mm by changing the setting in the Instron software. The energy was measured from the area under the force-displacement curve. Table 4-5 illustrates the measured energy levels for this experiment. With the same compression displacement, particles with energy consumption values close to each other were collected in the same bag for sieving. It is worth mentioning that using the compression device to conduct experiments on small particles is accompanied by the risk of machine and test failure. When small rock samples are used, the compression platens should be adjusted in a small distance from each other before fracture takes place. Progression of breakage at higher energy levels requires more displacement; hence the risk of the contact of two metal surfaces against each other increases. Hence, failure of the machine and test is high.

<table>
<thead>
<tr>
<th>Displacement (mm)</th>
<th>Specific input energy (kWh/t)</th>
<th>Displacement (mm)</th>
<th>Specific input energy (kWh/t)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Contained</td>
<td>Uncontained</td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>0.07</td>
<td>0.12</td>
<td>0.4</td>
</tr>
<tr>
<td>0.7</td>
<td>0.13</td>
<td>0.12</td>
<td>0.7</td>
</tr>
<tr>
<td>1.0</td>
<td>0.16</td>
<td></td>
<td>1.0</td>
</tr>
<tr>
<td>1.5</td>
<td>0.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.5</td>
<td>0.49</td>
<td>0.50</td>
<td></td>
</tr>
</tbody>
</table>

All the impact tests were conducted in the SILC. Table 4-6 shows this combination, used for impact load cell experiments. The height of particles was considered in the calculation of actual drop height. Therefore, the calculation of the specific input energy can be completed with more accuracy.

<table>
<thead>
<tr>
<th>Drop weight mass (kg)</th>
<th>Drop height (mm)</th>
<th>Input energy (kWh/t)</th>
<th>Stressing Velocity (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.683</td>
<td>30</td>
<td>0.06</td>
<td>0.77</td>
</tr>
<tr>
<td>0.683</td>
<td>40</td>
<td>0.09</td>
<td>0.88</td>
</tr>
<tr>
<td>1.535</td>
<td>45</td>
<td>0.22</td>
<td>0.94</td>
</tr>
<tr>
<td>1.535</td>
<td>70</td>
<td>0.35</td>
<td>1.17</td>
</tr>
<tr>
<td>1.153</td>
<td>150</td>
<td>0.50</td>
<td>1.54</td>
</tr>
<tr>
<td>3.200</td>
<td>150</td>
<td>1.60</td>
<td>1.62</td>
</tr>
</tbody>
</table>

Table 4-6 – Combination of height and mass in impact tests
4.4. HIGH-SPEED VIDEO FILMING

This part of the experimental work was conducted in order to gain further understanding of the movement of the fragments and their capture. High-speed filming at 8000 frames per second using a Photron camera, model FASTCAM-512PCI 32K was used to study breakage of several types of rock materials. Using Photron Motion Tool, it is possible to measure the velocity of the fragments in the breakage zone. For each type of rock or mineral, 10 particles were broken and the video filming was conducted. The measurement of speed was accomplished on five selected fragments from the other fragments that had more visibility and resolution to track in the Photron Motion Tool. The first step in this measurement is to set a scale of dimension. For this purpose, the anvil diameter, equal to 50mm was set as the scale for the other measurements. In the next step, the fragments that were targeted for velocity measurement were tracked using the manual or automatic tracking tools. In manual tracking, the fragments should be marked in each time step. Deviation of the velocity measurements in the software from the actual measurement can take place due to the distance of the camera from the test setting as well as the three-dimensional movements of the fragments. Samples were impacted by a striker with the drop weight of 1.535 kg from the height of 80mm.
5. MULTI-STAGE BREAKAGE MODEL

5.1. MODEL STRUCTURE

The approach of sequential breakage model is to consider the breakage event as a sequence of primary breakage events; each absorbing only sufficient energy to fragment particles into primary progeny fragments; these are then treated as fresh particles subjected to subsequent breakage stages. This is termed “multi-stage breakage model”.

As such, this model assumes that every event that corresponds to an impact of a particle by strikers is a combination of sequential breakage events (called here stages); which is the first fragmentation (primary breakage), spatial distribution (classification function), and the capture of fragments (selection function) that are repeated in stages until the energy is depleted, to form the final fragmentation product. Figure 5-1 illustrates this sequence. It is important to stress that after each breakage event a fraction of the fragments – given the kinetic energy of the fragments – are ejected from the active comminution zone, thus being not classified for further breakage. The total product after each stage is the summation of the materials that have undergone breakage (classified and selected for breakage) along with the unclassified and unselected materials.
Figure 5.1: Model structure, depicting the sequence of selection, breakage and classification functions
In mathematical terms, the size distribution of the progeny from a single stressing event may be calculated using the following expressions

\[ m_i^{(n)} = \left( \sum_{k=1}^{N} \sum_{j=1}^{i} b_{ij} S_j^{(k)} (C_i^{(k)}) m_i^{*(k-1)} \right) + m_i^{*(n)} \]  
\text{Equation (5.1)}

\[ m_i^{*(n)} = (1 - S_i^{(n)}) m_i^{*(n-1)} + \sum_{j=1}^{i} b_{ij} S_i^{(n)} (1 - C_i^{(n)}) m_i^{*(n-1)} \]  
\text{Equation (5.2)}

where \( m_i^{(n)} \) refers to the mass fraction of fragments in size class \( i \), \( b_{ij} \), \( C_i^{(n)} \) and \( S_i^{(n)} \) are primary breakage, classification, and selection functions, respectively, \( n \) is the number of stages. \( i \) and \( j \) are sieve sizes, so that \( j \geq i \), and \( N \) is the total number of size classes. In each stage, the particles break only once based on their probability of selection. Equations (5.1) and (5.2) should be solved recursively for each size fraction, from \( i = 1 \), which is the size class that contains the original particle, to \( n \), depending on the stressing energy of the striker. For the very first stage, \( m_1^{(0)} = 1 \) and \( m_i^{(0)} = 0 \), for all \( i \neq 1 \). In addition, \( S_i^{(0)} = 1 \), which means that the particle is always captured at the first impact. At this stage, however, the model does not incorporate the probability of fracture and the variability in particle strengths of either the parent particle or its fragments.

5.2. PRIMARY BREAKAGE

The primary breakage function and the corresponding primary fracture energy are the main components of the model. The primary breakage function is defined by the particle size distribution of the fragments after primary breakage. This function, which varies with the parent particle size, may be estimated by breaking a number of particles with an energy that is just enough to fracture the rock particle for the first time, using the impact load cell (ILC) (Tavares and King, 1998), a short impact load cell (SILC) or a compression testing device. However, using a compression testing device for this purpose is advantageous over an impact load cell. It can allow the test to discontinue when the first breakage takes place, whereas this is not practical using a standard procedure of impact load cell. In a typical impact load cell, breakage stops only when the entire input energy of the falling weight is dissipated. Despite this, Bourgeois (1993) was one of the first who measured the primary breakage function in a more direct way by using a metal ring capable of arresting the falling weight (ball) on a drop weight tester right after the particle suffered a primary fracture.
The traditional two-slope breakage function, widely used to model mills (Austin et al., 1984) can be fitted to experimental results to describe primary breakage progeny distributions in a log-log scale, being given by:

\[
B_{ij} = \varphi_j \left(\frac{d_i}{d_j}\right)^\gamma + \left(100 - \varphi_j\right) \left(\frac{d_i}{d_j}\right)^\beta
\]

Equation (5.3)

where \(B_{ij}\) is the cumulative breakage function, \(\beta\) describes the slope at coarser sizes, \(\gamma\) describes the slope at fine sizes and \(\varphi\) defines the percentage of the distribution at fine size, which can vary as a function of parent particle size. In density form it becomes \(b_{ij} = B_{ij} - B_{i-1,j}\). Also, \(d_i\) is the representative of product distribution in different size fractions \(i\) broken from original size fraction \(j\). The relation between the two sizes are called relative size and used for normalisation of size distribution.

The breakage function is essentially a function of the material. For some types of materials, such as single-phase particles, the breakage function approaches a linear relationship in log-log scale, that is, \(\beta\) is approximately equal to \(\gamma\), whereas data from breakage of multiphase materials can be fitted using the two-slope breakage function (Tavares, 2000), given by Equation (5.3).

At this stage, it is assumed that the primary breakage function is independent of particle size, that is, it can be expressed in a size normalizable way. However, for some types of rocks, the appearance of an inflection point should be taken into consideration for modelling purposes, so that the primary breakage function is no longer normalizable (Tavares and das Neves, 2008).

Whenever this inflection point exists in the cumulative size distribution, the accumulation of particles in sizes below this value and a depletion of material in sizes just above it occurs. In this case, the cumulative size distribution plotted against the relative size on a log-log scale does not superimpose. Figure 5-2 illustrates the size-normalizable breakage functions of quartz and fluor-apatite with the fitted parameters.
The energy that corresponds to primary fragmentation is called primary fracture energy, being equal to the amount of strain energy that is stored in a rock particle in the instant of fracture. This is obtained by measurement of fracture energy distribution for particles contained in individual sieve sizes and over a wide range of irregularly shaped particles using, for instance, the impact load cell device (Tavares, 2007). The lognormal distribution (Tavares, 2007) or the Weibull distribution (Weichert, 1992) can be used to describe them. Vogel and Peukert (2003) suggested that the energy corresponding to 50% fracture probability (median particle fracture energy) varies inversely with particle size. Hence, fine particles require a higher amount of energy for breakage per unit weight of particles compared to coarser fragments. Thus, it is of high significance to gain an insight into the fracture energy at very small sizes and how it increases from one size fraction to another. The variation of this mass-specific particle fracture energy $E_{m50}$ with particle size can be described in a more general way using the model proposed by Tavares and King (1998):

$$E_{m50, i} = E_{m, \infty} \left[ 1 + \left( \frac{d_{p,i}}{d_t} \right)^\phi \right]$$

Equation (5.4)
In this model, $E_{m,\infty}$, $d_{p,o}$, and $\phi$ are material constants, where $E_{m,\infty}$ represents the residual fracture energy of the material at coarse sizes; in other words, the specific particle fracture energy of infinitely large particles. $d_{p,o}$ is a characteristic size of the material microstructure and is related to the transition between the two extremes of the size spectrum. $\phi$ is related to the slope of the curve for size fractions smaller than the characteristic size ($d_{p,o}$) and vary between 0.9-2.7. The $d_i$ is the representative size of particles contained in size class $i$. The parameter $E_{m,\infty}$ of minerals is regularly lower than that of rocks and ores. However, the finer microstructure of rocks and ores in comparison with minerals results in lower values of $d_{p,o}$ for this types of materials.

This equation has been fitted to a variety of data from the fracture distribution of a variety of different materials and over a range of particle sizes (Tavares and King, 1998). Figure 5-3 illustrates the variation of the median specific fracture energy of the materials dealt with in the present work (quartz and fluor-apatite) for a range of particle sizes.

Figure 5-3: Median mass-specific fracture energy ($E_{m50}$) of quartz and fluor-apatite as a function of particle size. Curves correspond to Equation (5.4). The data are from impact experiments by Tavares and King (1998)
5.3. CLASSIFICATION FUNCTION

The classification function is defined to describe the spatial distribution of fragments in relation to the breakage zone. It is a function that is meant to describe the amount of material in the breakage zone after breakage that remains in it and that is amenable to be selected for breakage in the next stage, that is, the material that did not move away from the active stressing zone. The spatial distribution of rock particles is related to the system geometry; described by the impactor and the anvil geometries. Figure 5-4 illustrates a schematic diagram of breakage zone to describe the classification function attributed to a curved and a flat impactor. The material that remains in this zone is regarded as classified. Indeed, the curvature of the impactor and the ratio between the diameter of the particle and the striker is influential in determining the extent of the breakage zone. The probability of containment is also high when the diameter of the striker is significantly larger than the diameter of the particle. It is also important to account for the fact that there is a dead area, for which the classification is zero.

In addition to the geometry of breakage environment, the spatial distribution depends on the material properties such as their brittleness. Brittle materials exhibit certain behaviour due to the amount of kinetic energy available within the fragments after each breakage event. According to Tavares (2000), the response of single-phase materials such as mineral crystals and glass is associated with their ability to accumulate damage and hence brittle behaviour. Indeed, the rapid accumulation of damage in crystalline materials (prior to fracture) results from their high brittleness. In contrast to that, polycrystalline materials tend to accumulate damage more gradually. Damage accumulation and therefore the brittle tendency is also affected by the strength of intergranular bonding. Poor intergranular bonding in some rocks and ores can lead to particularly gradual damage accumulation. The brittleness behaviour results in wide scattering of fragments after breakage due to the high velocity of the fragments.
Furthermore, it is reasonable to consider a variable classification function in different stages. As breakage progresses, the spatial distribution of material is affected by the early-broken rock particles that surround the breakage environment, hinder the scattering of the fragments and lead to the containment of fragments inside the breakage zone. A classification of 1 or 100%, which is the maximum amount of containment, means that for all size fractions, all the fragments remain in the active breakage zone and do not leave it when breakage is in progress. Numbers smaller than 1 or 100% suggest that a fraction of particles mass is lost because it escaped from the breakage zone. In the present work, it is assumed that the classification function is a constant for each combination of material and stressing geometry and does not vary with particle size.

5.4. SELECTION FUNCTION

The selection of rock particles during a breakage process is determined by the probability of material that is contained in the active breakage zone in different size fractions that are nipped by the breakage device in that particular stage. The selection probability of the larger particles is higher than that of the smaller particles, from a pure geometrical perspective. The curvature of the striker in relation to the anvil, not only impacts the classification but also is influential in

![Figure 5-4: Breakage zone with (A) Curved strikers (B) Flat impactor](image)
determining the catchment of the progeny. Figure 5-5 depicts a schematic picture to describe various circumstances. A related topic is the capture of particles contained in a bed that is impacted by balls. This has been modelled for impacts of balls of different sizes, and, therefore, different curvatures, by Barrios et al. (2011).

![Figure 5-5: (A) Catchment of a particle by strikers with different curvature (B) Catchment of a particle when it is small in relation to the diameter of the striker](image)

Also, the progeny may break in a single-particle mode without any interaction with one another. On the other hand, the circumstances may favour the preferential selection of smaller particles, thus preventing the selection of coarse particles. This is described in detail in the present work. One empirical function that could be used to describe selection of material is

\[
S_i^{(n)} = 1 - \exp \left[ -\left( \frac{d_i}{\left(2^{\frac{n-1}{4}} \kappa d_1 \right)} \right)^\delta \right]
\]

Equation (5.5)

where \(\kappa\) and \(\delta\) are other model parameters, in which \(\delta\) is responsible for describing the slope of the curve. Higher values of \(\delta\) are associated with the significant difference between the probabilities of selection for coarse and fine particles. Examples of selection functions are given in Figure 5-6. \(d_1\) is the representative size of the original particle. Further, it is assumed that in each stage the impactor will move by one quarter root of 2 of the initial particle size. The key difference between the selection and the classification function is that if fragments are not selected for breakage in one cycle, they may be in subsequent cycles, whereas if a fragment is not classified, then it will no longer suffer additional breakage.
Figure 5-6: Selection functions (Equation 5.5) with different $\delta$ values ($\kappa = 0.68$)

5.5. ABSORBED ENERGY

When particles suffer a primary fracture, they break following a size distribution that was described in the previous section. Also, they require a certain amount of energy to break. The model takes into account the energy that is used by rocks to cause breakage. Hence, due to the loss of energy in impact experiments, it is important to calculate that portion of the total input energy that is absorbed by particles and is responsible for breakage. This allows comparison between the model predictions and experimental results.

Absorbed energy for breakage of a single particle using the ILC is calculated from the force-time signals recorded during the impact event. It takes into account the input energy, the energy consumption for rod deformation and the energy involved in the restitution of the striker. The details surrounding the calculation of absorbed energy is presented previously in Equation (2.28) in Section 2.3.1. Also, the ratio of absorbed energy to input energy is called energy transfer efficiency that is influenced by the brittleness of materials. Higher brittleness leads to lower efficiency to transfer the kinetic energy of the comminution device (Tavares, 1999). Tavares (1997) compared the breakage behaviour of quartz with copper ore in terms of their
energy absorption during an impact event. For particles of both types of materials fractured under similar load, quartz suffered little re-breakage after the first fragmentation in comparison with copper. This was linked to the high brittleness of quartz that caused ejection of fragments outside the breakage zone. As such, a small portion of fragments was captured in the area between striker and anvil that resulted in steel on steel collision. In comparison, less brittle copper suffered successive breakage. Tavares (1999) determined the energy transfer efficiency as influenced by the ratio of impact input energy to mean fracture energy. As the impact energy increases, the energy-transfer efficiency decreases. For instance, in the case of quartz, the energy transfer efficiency of particles in size fraction 1.0-1.18mm was found to decline from over 80% to less than 50%, when the ratio of the input energy to median fracture energy increased from 3 to 20. This is because in impacts at high energies and on crystalline materials such as quartz, a significant fraction of the input energy is consumed in the restitution of the striker. Table 5-1 shows this relationship for different ratios of impact input energy to mean fracture energy for size fraction 1.0-1.18mm of quartz particles.

Table 5-1: Energy transfer efficiency for different ratios of impact input energy to mean fracture energy impact for quartz (1.0 - 1.18 mm) (Tavares, 1999)

<table>
<thead>
<tr>
<th>Impact energy/mean fracture energy</th>
<th>Energy transfer efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>80</td>
</tr>
<tr>
<td>7</td>
<td>71</td>
</tr>
<tr>
<td>20</td>
<td>55</td>
</tr>
<tr>
<td>35</td>
<td>52</td>
</tr>
<tr>
<td>70</td>
<td>49</td>
</tr>
</tbody>
</table>

In a very ideal condition; when the available energy is just enough to cause the first fragmentation, all the energy may be used entirely for breakage. Therefore, in this model, it is assumed that particles that remain in the breakage zone (classified) and undergo selection receive a portion of available energy and break. Also, due to the inhomogeneous nature of mineralogical material, the particle fracture energy can be presented as a distribution. Despite this, for the simplicity, it is assumed in the model that in each stage, particles receive the amount of energy that is equal to the median of this distribution - \( E_{m,50} \) in Equation (5.4)- which varies as a function of particle size.

As such, the amount of energy dissipated in the stressing event can be estimated as a function of the stage number using the expression:
\[ E^{(n)}_{cs} = \sum_{k=1}^{n} \sum_{j=1}^{N} C^{(k)}_j S^{(k)}_j m^{(k-1)}_j E_{m50j} \]  

Equation (5.6)

At the present stage, the model has been proposed with the following assumptions:

- The breakage function is normalizable in respect to parent particle size, that is, \( \varphi_j = \varphi \);
- The classification function is a constant that does not vary with particle size and stage number, only with material and stressing geometry;
- The selection function varies with particle size and stage number, besides material and stressing geometry.
- At this stage, the model does not yet describe the packing effect that takes place at high energy levels due to the bed formation.
- The model does not account for the distribution of particle fracture energies in a lot of material, describing the material response solely as averages. As such, it is unsuitable to describe breakage when the amount of energy is smaller than that required for breakage of all particles in a single blow.

5.6. MODELLING OF DROP WEIGHT TESTING

5.6.1. Materials

In order to demonstrate the modelling approach proposed in the present work, it has been applied to describe breakage of three materials in a drop weight tester: quartz particles contained in the size range 4.00-4.75 mm, fluor-apatite particles contained in the range 2.0-2.80 mm and silicate in the size range of 3.35-4.75mm. Quartz data was collected from the work of Bourgeois (1993) and Tavares (1997), whereas data from fluor-apatite was from Tavares (1997). Silicate data was obtained as described in the previous chapter on “Experimental Procedure”. For three materials, different impact energy levels were reached by using different combinations of drop weight mass and impact height.

In the case of quartz, the measurement of the primary breakage function was possible due to the careful experimentation of Bourgeois (1993), who used steel rings with heights matching the size of parent particles so as to arrest the falling drop weight (steel ball). His result is given in Figure 5-2. Unfortunately, primary breakage data for fluor-apatite and silicate was not
measured in a similar fashion. Instead of using steel ring that could inhibit particles from further breakage (after primary fragmentation), in the case of fluor-apatite and silicate, impact tests at low energy levels were used. Therefore, it was assumed that the progeny size distribution from an impact test at a relatively low impact energy (254 J/kg for apatite and 350 J/kg for Silicate) would give a reasonable approximation of the material primary breakage function. For apatite, however, data was available for single-particle breakage testing at different impact energies but also different contact geometries, namely ball-ball, ball-flat and flat-flat geometries. In the case of silicate, the test was conducted at different impact energies using flat-flat geometries.

A summary of single-particle primary breakage distribution and energy distributions is given in Table 5-2 for quartz and apatite. The experimental procedure for obtaining measurements of primary breakage characteristics for silicate particles was explained in the previous chapter. The data analysis related to these measurements will later be discussed in details in the present chapter.

Table 5-2: Summary of single-particle primary breakage distribution parameters

<table>
<thead>
<tr>
<th>Material</th>
<th>Particle fractures energies; Equation (5.4)</th>
<th>Primary breakage function; Equation (5.3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E_{m,\infty}$ (J/kg)</td>
<td>$d_{p,o}$ (mm)</td>
</tr>
<tr>
<td>Quartz</td>
<td>43.4</td>
<td>3.48</td>
</tr>
<tr>
<td>Fluor-apatite</td>
<td>1.5</td>
<td>19.3</td>
</tr>
</tbody>
</table>

5.6.2. Sensitivity Analysis of Model Parameters

A sensitivity analysis of the model to different selection and classification functions is conducted for quartz to demonstrate the different progeny size distributions that result in response to different function values. One convenient method to summarize data from single particle breakage tests or simulations is to calculate the breakage index, $t_{10}$, from each size distribution and then relate it to the specific input energy. $t_{10}$ can be estimated from the interpolation of the cumulative percentage passing versus size between the upper and the lower size fraction of 1/10th of the original size in the sieve series. The total absorbed energy in each stage in the multi-stage breakage model may be estimated using Equation (5.6). The relationship between the absorbed energy and the resulting $t_{10}$ has been empirically described by Narayanan (1986):
\[ t_{10} = A(1 - \exp(-bEcs)) \]  \hspace{1cm} \text{Equation (5.7)}

\(Ecs\) is in KWh/t and can be converted to J/kg

Parameter \(A\) is the maximum degree of fragmentation than can be achieved. The increase in input energy is expected to increase the degree of fragmentation. Despite this, for certain types of material and depending on the mechanism of breakage, the increase of energy does not necessarily guarantee an increased fragmentation. In the case of experiments in a flat-flat drop weight tester it typically varies from 50 to 60, giving the particles better chance of breakage at coarse to medium size fractions. For some other stressing geometry, it may vary in the wider range from 30 to 80 and limit or improve the achievable degree of fragmentation with the increase in energy. The product of \(A\times b\) is regarded as the indicator of rock amenability to breakage and is a common measure used to compare the strength of various types of rocks.

In the first case of the sensitivity analysis, the classification function was regarded as a constant, equal to 1 in each stage of the model and for all size classes \((C_{i}^{(k)} = 1)\). It translates into “no fragments leave the active breakage zone” as the striker hits the fragments and the selection function was modified to investigate the changes in size distribution. In this case, the probability of selection of coarse particles is assumed to be high. This assumption is justified by their large height, which makes them amenable to be captured when a striker hits a particle. However, as the particles become smaller, the probability of selection by the breakage tool drops dramatically from one size to the next immediate smaller size. The probability of selection increases for all size fractions as the striker continues its compression against the anvil and the gap between them becomes smaller. Situation like this can takes place when, for instance, particles and their progeny break in sequence in a single-particle mode of breakage without the intervention of small-size fragments that are generated from the previous stage. Figure 5-7 (B) illustrates this case. This situation corresponds to the selection function depicted by Figure 5-6, with parameter \(\delta = 1\) in Equation (5.5).
Figure 5-7: Scheme describing various forms of selection function: (A) fresh rock; (B) high probability of coarse particles selection; (C) moderate probability of coarse particles selection and (D) high probability of fine particles selection.

The combination of the selection function with $\delta = 1$ and a classification function equal to 1 resulted in the size distribution presented in Figure 5-8(A) for quartz. It shows that the cumulative passing in the coarse fractions is high in contrast to fine sizes, with a significant disappearance of coarse material in the first few stages and limited production of fine-size fragments. Figure 5-9 shows the $t_{10}$-Ecs relationship for this simulation. The Ecs is calculated by the model using Equation (5.6). Also, the size distribution at different energy levels was obtained by the model (in Equation (5.1) and (5.2)). Then, the value of $t_{10}$ was calculated from size distribution for the corresponding Ecs. Having $t_{10}$ and energy, fitting A and b in $t_{10} = A(1 - \exp(-bEcs))$ is obtained. The value of A corresponds to the maximum degree of fragmentation and b related to the slope of the curve (also see Section 2.2.1). With values of $A = 90$ and $b = 0.92$, this rock type would be regarded a soft ore for which, the maximum fragmentation is significantly high. This large b value is also an indicator of fast depletion of coarse fragments. It is important to state that predictions that lead to exceeding high values of A (typically near 100) are not realistic on the basis of experience from drop weight testing (Napier-Munn et al., 1996). As such, this combination of parameters for the classification and selection functions leads to unrealistic predictions on the basis of the present model.
Figure 5-8: Predicted progeny size distributions for (A) Classification= 1, selection with $\delta = 1$ (B) Classification= 1, selection with $\delta = 0.3$ (C) Classification= 1, selection with $\delta = -0.1$ (D) Classification= 0.82, selection with $\delta = 0.3$ (quartz). Specific energy is calculated from the model in Equation (5.6). Primary breakage function parameters are given in Table 5-2. Primary-Experiment = 84 J/kg.
A second simulation scenario is here considered: one in which the probability of coarse particle selection is relatively high, though relatively finer particles have a higher chance of selection in comparison to the previous scenario. As the fragments become finer, the probability of selection by the breakage tool declines moderately from one size to the next immediate smaller size. This situation could potentially occur when the broken fragments of medium size would lay between the other fragments, making them more amenable for being captured by the striker, in comparison to the previous scenario. Figure 5-7 (C) illustrates this situation. The parameter that can describe this selection function (Equation (5.4)) is $\delta = 0.3$, resulting in the curve shown in Figure 5-6 and the simulated size distributions given in Figure 5-8(B). When the particle break, the progeny appears in different size fractions. As breakage continues, mass of particles move from one size to the other smaller sizes, which means they are depleted from one size and accumulated in other sizes. Figure 5-8(B) depicts the depletion of coarse, medium and fine-size fragments is relatively constant as the breakage develops.

The t10-Ecs relationship is shown in Figure 5-9, which shows that a value of $A^*b=39$, which would represent a hard rock. One outstanding characteristic of this curve is the gradual depletion of coarse fragments as suggested by the $b$ value of 0.45.

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**Figure 5-9**: t10-Ecs relationship for different selection functions and classification functions used in simulations of breakage of quartz particles (lines represent simulations and symbols represent the experimental data from ball drop tests on a flat anvil).
The third simulation scenario considers an opposite situation to the first, given that the probability of fine particles selection is assumed to be large so that the coarse particles have very low chance of capture by the falling drop weight. This is accomplished by using a negative $\delta$ value, which favours the probability of fine particles selection by the drop weight tester when compared to the coarse fragments. Figure 5-6 shows a selection function with $\delta$ of -0.1. This situation could occur whenever medium to fine-size fragments occupy the voids between the coarser debris and cover them, preventing their breakage. Unlike the first scenario, fragments do not break in a single-particle mode as the breakage of each fragment is influenced by the neighbouring particles. Using this selection function would result in the size distribution as is shown in Figure 5-8(C) for quartz. While some coarse fragments are not subsequently broken in the later stages, the finer fragments rapidly deplete, which leads to the rise of the cumulative passing curve at the fine end. The $t_{10}$-$E_{cs}$ relationship is also shown in Figure 5-9, with a value of the product $A^*b$ of 25. In experiments, this value would correspond to an extremely hard ore. The limited fragmentation of coarse particles as indicated by a small $b$ value of 0.45, as well as the coverage of coarse particles by fine ones, lead to a maximum fragmentation ($A^*$ value) of 55.

This last scenario considers the selection function, with $\delta = 0.3$, while the classification is constant at 0.82. It is thought that the value of 0.82 instead of 1 in the model would suggest a better approximation to the experimental data for the breakage of quartz despite limited available experimental data for this purpose. While the selection function is the same as the one considered in the second scenario simulated, the progressive scattering of fragments outside the active breakage zone that takes place in each stage results in more limited fragmentation of the progeny, as is illustrated in Figure 5-8D. This can be a case in which the brittleness of the particles is high and/or the geometry of breakage environment allows a portion of fragments to escape after each stage, such as in the case of a ball drop test. In Figure 5-9, the $t_{10}$-$E_{sc}$ relationship for these simulations is presented, which would result in $A^*b$=40, with a value of $A$ equal to 65. In comparison to the simulation in which the same value of $\delta$ was used (Figure 8B), but classification was equal to 1, the product $A^*b$ is approximately the same, but the $A$ value is more realistic.

As $A$ and $b$ values of different scenarios in Figure 5-9 suggests, by choosing different parameters for selection and classification functions of the fragments, the $A^*b$ values change
very significantly. This is valid even considering that the primary breakage model parameters, which are material characteristics, were maintained unchanged. This emphasizes the crucial role of breakage sub-processes that can be either the properties of rock or breakage environment on the progeny size distribution, energy-size relationship and the hardness index of materials.

5.6.3. Model Fitting and Validation for Quartz

Due to the limited availability of documented observations regarding the classification and selection of fragments during a drop weight test, the choice of these functions is, at this stage, arbitrary. However, fitting appropriate selection and classification function parameters should be carried out to:

(1) Achieve the same shape of the product size distributions, and

(2) Match the total energy consumption at different stages.

Using a proper form of these functions, the progressive fragmentation of particles in the model should correspond to the fragmentation resulted from conducting breakage experiment. Figure 5-10 compares results from single-particle breakage experiments using a ball drop test to simulations using the present model with $\delta = 0.3$ and $\kappa = 0.68$, and classification = 0.82. It is evident that the measured and simulated size distributions match very well, with only three parameters. These parameters are consistent with the high amenability of the very brittle quartz fragments to be projected outside the active breakage zone and the use of a ball as the striker.

The other piece of the puzzle to fit the progeny size distribution is the energy consumption. In this case, the $t_{10}$-Ecs relationship from the simulations should be compared to those measured experimentally. These are compared in Table 5-3, which shows good agreement. The values of $n$ given in the table are those numbers by which the size distribution from the model converge to the size distribution by the experiment, as shown in Figure 5-10. The specific input energy-Ecs for the experiment in the second column of the table is calculated using the mass and height of drop weight as well as the mass and height of the particles. However, the third column is calculated using the factor of energy efficiency presented in Table 5.1.
Figure 5-10: Comparison between simulations (lines) and experimental results (symbols) (Bourgeois, 1993; Tavares, 1997) for quartz particles (Classification: 0.82 and selection function with $\delta = 0.3$ and $\kappa = 0.68$). Size distribution in the model was obtained using Equation (5.1) and (5.2). The energy in the model is calculated using Equation (5.6) for Ecs.

Table 5-3: t10-Ecs for model and experiment for 3.35-4.0 mm quartz particles

<table>
<thead>
<tr>
<th>t10</th>
<th>Input energy-Ecs (Experiment)</th>
<th>Energy-Ecs (Absorbed)</th>
<th>Ecs (Model)</th>
<th>n (model)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>93</td>
<td>---</td>
<td>84</td>
<td>1</td>
<td>--</td>
</tr>
<tr>
<td>6</td>
<td>692</td>
<td>470</td>
<td>433</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>13</td>
<td>1652</td>
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</tr>
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<td>2202</td>
<td>2380</td>
<td>9</td>
<td>8</td>
</tr>
</tbody>
</table>

5.6.4. Effect of Impact Geometry on Fluor-apatite Breakage

The data that served as the basis for this part were a number of impact experiments in an impact load cell with three different impactor geometries on fluor-apatite particles (Tavares, 1997, 2007). Impact breakage of apatite using ball-ball, ball-flat and flat-flat loadings produced progeny size distributions that varied significantly in shape, as depicted in Figure 5-11. It is
evident that nearly no differences in size distributions appeared for the different impact geometries at a low energy impact. On the other hand, significant differences appeared at the very high impact energy. These differences lead to significantly different $t_{10}$ values, thus demonstrating the important role of the geometry of the impactor when characterizing the breakage response of materials in single-particle breakage.

Figure 5-11: Data from breakage of 2.0-2.8 mm fluor apatite at different impact energy levels (Low: 254 J/kg and high: 6488 J/kg) and with three different geometries (Tavares, 2007). Ball diameters were equal to 25.4 mm.

From the mineralogical standpoint, fluor-apatite is regarded as a relatively soft ore, considered to be less brittle than the quartz sample. As such, it is assumed in the model, for simplicity, that the fragments are all classified (classification = 1). Although it is more realistic to incorporate the effect of geometry also in the classification function, the geometry was considered to only impact the selection function. This decision is made because the selection function was already formulated in Equation (5.5).

At first, the ball-flat configuration is considered. Since it is nearly identical to the geometry used for quartz, the value was used for $\delta$ which translates in the moderate selection probability of coarse particles (i.e. $\delta = 0.3$). As discussed previously, the primary breakage function was assumed to be equivalent to the progeny size distribution of fluor-apatite at a low energy level (254 J/kg). This energy level is the experimental value of the incident kinetic energy for producing size distribution close to primary breakage (Refer to Section 4.3 which describes the experimental procedure to obtain the primary breakage function). Predictions using the model
are presented in Figure 5-12 (using equation 5.6) and are compared to the data. Energy estimation of the model was conducted with the same method as calculated for quartz particles. The energy estimation for the model where the size distribution is nearly identical to the size distribution produced by the applied energy of 874 J/kg is about 480 J/kg, which gives the energy efficiency of 55%. However, when the energy increases, the energy absorption decreases. The estimation of absorbed energy for the applied energy of 6488 J/kg is about 2700 J/kg, which translates to the energy efficiency of 42%.

Figure 5-12: The size distribution of fluor-apatite progeny, impacted by ball-flat geometry-produced by the model (lines) and experiments (symbols) (predictions considering a selection function with $\delta = 0.3$ and classification = 1)

To simulate the size distribution by the model for the case of ball drop on an anvil also made up with a half-sphere, the value of the parameter $\delta$ was set to 0.3. This correlates to the high probability of selection of fine particles. Using this selection function results in the progeny size distribution as illustrated in Figure 5-13. The estimation of absorbed energy the model for the applied energy of 874 J/kg is about 570 J/kg (the energy efficiency of 65%) and for the applied energy of 6488 J/kg is about 3000 J/kg, (the energy efficiency of 46%).
Finally, in the case of flat-flat contact, that is, the impact of a rod with a flat end, impacting on a particle sitting on top of a flat anvil, the δ value of the selection function was chosen as 0.7. This represents a moderate probability of coarse particles selection. However, the selection probability of coarse particle is assumed to be higher than the ball-flat geometry due to the change in geometry. Using this selection function results in the progeny size distribution as illustrated in Figure 5-14. The model predicts energy efficiency of 52% for the applied energy of 874 J/kg. However, when the applied energy increases to 6488 J/kg the energy efficiency drops to 34%.

As seen, the flat-flat geometry produces significant amount of fragmentation at relative size of 0.04 to 1. However, for relative size smaller than 0.04, small amount of fragmentation is generated as opposed to the ball-ball geometry in which high amounts of fines are produced. As the generation of fines consume higher amount of energy due to the increased toughness of these particles with size, it may explain the increased energy efficiency for a mechanism that involves a ball-ball geometry in comparison to flat-flat.
The size distribution of fluorapatite progeny, impacted by flat-flat geometry-produced by the model (lines) and experiments (symbols) (predictions using selection function with $\delta = 0.7$ and classification = 1)

The simulations on the different impact geometries are presented in Figure 5-15, which shows that important differences appear in breakage response because of different impactor geometries. As indicated, the t10-Ecs for three different configurations appear in order, being highest for the flat-flat and lowest for the ball-ball geometry. At the relative size of 0.1 (1/10th of the original size), the flat-flat geometry provides a better opportunity for the capture of coarse particles. The ball-flat geometry, on the other hand, due to its curvature may result in the concentration of energy less on the capture of coarse particle in comparison to the flat-flat geometry. Finally, the ball-ball configuration is less likely to distribute the amount of available energy to the breakage of particles as the result of reduced effective comminution area. This strongly supports the need for only comparing such graphs for tests conducted at the same impact geometry. Further, this also shows that the drop weight test should not be interpreted as a fundamental material characterization test, but rather a simple, yet complete comminution process, in which material and system characteristics influence the results from the test.
5.6.5. Effect of Rock Properties on Beaudesert Silicate Breakage

The single particle breakage model also was validated in the case of Beaudesert silicate particles, in addition to quartz and fluor-apatite. As discussed previously, several measurements from breakage of single particles are required for this purpose; generally categorized as primary and secondary breakage characteristics. The following section explains the results of experiments that were carried out to obtain the model parameters.

Primary Breakage Characteristics—Median Specific Fracture Energy ($E_{50}$)

One of the essential measurements related to the single particle breakage model is the characterisation of primary breakage, as obtained in the case of quartz and apatite. The acquisition of the mass-specific particle fracture energy (one of the primary breakage characteristics) takes advantage of the model developed by Tavares and King (1998) in Equation (5.4). Because the model establishes a relationship between the median specific energy of particles and their size, it was required to obtain the distribution of first fracture energy for silicate particles in different size fractions. The experimental procedure was earlier presented in the previous chapter in Section (4.3). The fracture energy of each individual
particle that suffered fracture was calculated using the SILC Excel Spreadsheet-based software. The software requires inputs such as calibration data. Figure 5-16 illustrates an example of calibration data used by the spreadsheet. The accuracy of measurement for the peak voltage is 1.76 Volt ± 0.02 Volt.

![SILC Spreadsheet](image)

**Figure 5-16: Calibration data used by the Excel Spreadsheet**

Other information consists of drop weight mass, its height, particle mass and height. The calculation of fracture energy, absorbed by particles was conducted by the software using the force-time signal up to the fracture point, according to the description provided in chapter 2 in section (2-3). Once the force-time signal is obtained, the particle deformation is calculated using Equation (2.26). Then, the absorbed energy by particle is calculated using force and deformation as a function of time from Equation (2.28). One of the crucial steps in this calculation is determination of first fracture point, determined when the force drops significantly from its maximum value. Three different criteria were defined for determination of this point:

1. The fracture takes place when the force drops by at least 65% from its maximum value
2. The force drop of 65% does not necessarily guarantee the particle main breakage. It is also necessary to check whether particles lost at least 10% of their original mass.

Hence, the fracture of chipped particles was not taken into account.
3. Some particles, depending on their strength may suffer other fractures even when a low level of energy is applied for their breakage. Hence, the successive breakage should not be considered to determine this point.

Figure 5-17 illustrates a sample of the force-time curve for each size fraction.
Figure 5-17: force-time signal resulted from breakage of particles of five size fractions

Figure 5-18, also illustrates the probability of fracture for each size distribution.
Figure 5-18: Specific fracture energy of different size fraction of Beaudesert silicate

Based on the data provided by the six size ranges, the fitting parameters of $E_{50}$ model; $E_{m,\infty}$, $d_{p,o}$ and $\phi$ were determined as 106.7 J/kg, 0.986 mm and 2.18 (Equation (5.4)) and the model results were extrapolated for smaller or larger than the experimental size range. The median specific fracture energy of different size fractions using the fitted model parameters is shown in Figure 5-19. As the graph illustrates, $d_{p,o}$ which is the characteristic size of Beaudesert particles is determined at 0.986 mm; lower than what was obtained for quartz and apatite (quartz: 3.48 mm, apatite: 19.3 mm). This can be linked to the finer microstructure of Beaudesert samples which are classified as rock, as opposed to apatite and quartz which are minerals. The residual fracture energy related to this size is about 106.7 J/kg, higher than that of quartz and apatite (quartz: 43.4 J/kg, apatite: 1.5 J/kg). It is also related to the higher toughness of rocks, as the case for Beaudesert samples in comparison to minerals (Tavares, 1997). The specific fracture energy of silicate starts to grow with the $\phi$ value of 2.18 for size fractions smaller than 986 microns (See Figure 5-19). This is also in agreement with what suggested by Tavares (1997) for the range of this parameter between 0.9 to 2.7.
Primary Breakage Characteristics - Primary Breakage Function

Similar to fluor-apatite, it was assumed that progeny size distribution from an impact test on Beaudesert silicate at a low energy would give a reasonable approximation of the primary breakage function. The primary breakage function is shown in Figure 5-20 for Beaudesert silicate at a low energy of 350 J/kg for model and experiment. The parameters in Austin model, equation (5.3) was obtained as $\phi = 12$, $\beta = 5.1$ and $\gamma = 1.12$. As the parameters suggest, the curve is steeper at the coarse end in comparison to apatite (higher value of $\beta$) that suggest less fragmentation of coarse size fractions. Despite this, the slope of curve is similar at finer sizes for the three types of samples.
Figure 5-20: Progeny size distribution of silicate at a low energy of 350 J/kg, proxy for primary breakage function

*Size Distribution Measurement*

With the applied energy in different levels; described in the previous chapter in section (4.3), the size distribution was obtained. It is depicted by cumulative percentage passing versus the size in Figure 5-21.
Model Fitting

As discussed earlier, the flat-flat geometry was used for this set of experiments at different energy levels. The particles were contained in the breakage environment to allow a classification of 1. This classification means that no fragments leave the breakage environment during breakage. The primary breakage function was obtained from experimentation and modelled (Figure 5-20). The size distribution model in Equation (5.1) and (5.2) can be used to construct the size distribution from the experiment. Therefore, the parameter $\delta$ in selection function required fitting. The best fitting is achieved using two criteria, explained earlier in Section 3.3 for the Methodology 1:

- The shape of the size distribution by the model should match the experimental results. This means that the overall shape of the fragmentation curve (cumulative passing) in log-log scale should match the experimental curve.
- The energy consumption by the particles, calculated in the model also should match the energy consumption calculated from the experimental data.

The least square method of fitting in log-log scale was used to obtain the best values of the parameter $\delta$ that meet the abovementioned criteria. Using log-log scale is to ensure that fitting in fine size fractions is not compromised by the presence of the majority of fragments in the

Figure 5-21: Size distribution of Beaudesert silicate, size fraction 3.35-4.75 mm at various energy levels
coarse size fractions. The video filming in addition to the least square fitting was used to develop further understanding about the fitted parameter $\delta$ in the case of Beaudesert samples. Beaudesert silicate exhibits a brittle behaviour. Based on the study of video filming, broken fragments of Beaudesert silicate tended to escape from the loading zone and scatter in a different direction. During breakage using an impact mechanism, fragments which were not in immediate contact with the compression device escape laterally from the area between the anvil and the striker. Usually, as observed in the case of Beaudesert silicate, large broken pieces leave the area and do not get a proper chance of breakage by the device. Even the fragments that are in close contact with the breakage environment slide on top of the anvil or rotate and do not sustain a firm contact with the compression device. However, as other fragments fly away from the breakage environment, the ones in an immediate contact with the anvil and striker have a better chance of re-capture for breakage. Therefore, the probability of selection of smaller sizes increases, in compared to the coarser sizes which justifies the use of a moderate $\delta$ of 0.44, smaller than that of apatite. Figure 5-22 illustrates the modeling results along with the experiments.
Figure 5-22: Model results using δ=0.44 for Beaudesert silicate in size fraction 3.35-4.75 mm. The experimental results shown by markers are input energy whereas the dashed lines are energy calculated by the model, Equation (5.6).

As the modelling results illustrate, the shape of the size distribution obtained by the model matches closely with the shape of size distribution obtained by the experiments, except at the fine ends. As appears, Beaudesert silicate has an inflection for sizes smaller than 106micron. This means the shape of the size distribution is not normalized below this size. According to (Powell et al., 2014), the issue of non-normalized behavior below a certain size should be taken into consideration for the modeling purposes. Table 5-4 also illustrates an estimation of the model energy consumption and the t10 value.
Table 5-4: t10-Ecs for model and experiment. Selection function with $\delta = 0.44$

<table>
<thead>
<tr>
<th>$t_{10}$ (Experiment and Model)</th>
<th>Input energy-Ecs (Experiment)</th>
<th>Energy-Ecs (Absorbed)</th>
<th>Ecs (Model)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.95</td>
<td>400</td>
<td>---</td>
<td>400</td>
<td>--</td>
</tr>
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<td>2.5</td>
<td>907</td>
<td>650</td>
<td>611</td>
<td>6</td>
</tr>
<tr>
<td>5.9</td>
<td>1840</td>
<td>1170</td>
<td>1071</td>
<td>7</td>
</tr>
<tr>
<td>9.8</td>
<td>3825</td>
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<td>7370</td>
<td>4053</td>
<td>5335</td>
<td>32</td>
</tr>
</tbody>
</table>

As the results show, at moderate energy levels, the model outcomes show a good agreement with the experimental results. However, at the high energy level, the error is high. This can be explained by the following possible reasons.

1. The model does not address the bed formation effect that occurs at high energy levels. When the breakage proceeds, the fine particles cover the space between the coarser particles. However, at very high energy levels, the effect of bed packing due to the excessive generation of fines ceases the breakage of coarser particles. Hence, by the application of more energy, the fragmentation is unlikely to continue.

2. The breakage function is not normalized as the particles become smaller. This means that by breaking the particles with smaller sizes, there is a shift in the inflection point along the relative size scale. Hence, a non-normalised behaviour is observed and therefore, it should be taken into account when improving the model of single particle breakage.

**Error Propagation**

With the current structure of the model, there is a good agreement between the experimental and model results at low to medium energy levels. However, there is always a level of error that exists between these figures. As discussed, one of the sources of error can be due to the shape of the modelled primary breakage function, particularly at the tail end (fine size fractions) where the modelled size distribution does not reflect the experimental results due to the inflection point. In this section, this error is investigated. A three-slope primary breakage
function (instead of a two-slope) was used to fit a model to experimental results. This practice is to minimise the error caused by modelling the primary breakage with a two-slope function after the inflection point. Figure 5-23 shows the experimental and modelled primary breakage function. The fitting error was calculated $1.7659 \times 10^{-14}$ using the least square method. The other stages were modelled using the three-slope primary breakage function and the error was calculated afterward. Then the error was calculated using the least square method at certain stages where the size distribution from the model is nearly matched to the size distribution by the experiment. The error was calculated 55.57 at energy level 907 J/kg. However, it increased to 88 at 1840 J/kg, 127 at 3825 J/kg and 133 at 7370 J/kg. As evident, there is an increasing trend of error as the applied energy increases.

![Figure 5-23: Primary breakage function by the Model and experimental results](image)

**Improving Estimation of Energy Consumption**

As appears, the energy estimation is an integral part of the single particle breakage model. Therefore, improving the energy estimation either by the model and experiments can improve the capability of the model and gaining insight into the fundamental aspects of rock breakage. In this respect, the following opportunities were identified:
• Enhancement of the method to obtain the breakage characteristics of small particles: In the case of silicate particles, calculation of fracture energy distribution was limited by the limitation in the function of short impact load cell. With the current design, the strain gauge attached to the rod is not triggered for the particles with smaller dimension. This is particularly because, for these particles, shorter drop heights are required that translates into the application of lower force on the surface of anvil. Hence, the force is not enough to initiate the voltage registration by the gauge. One solution to confront this problem is an amplification of force signal. However, this would increase the magnitude of the background noise for higher amplification factors. Other possible solutions are using impact load cells with smaller diameters and different sensitivity of strain gauge.

• Enhancement of the method to calculate the breakage transfer efficiency at high energy levels: The number of signal de-convolution increases as the energy increases. Considering that each de-convolution is accompanied with an error, by decreasing the number of de-convolutions, there is an opportunity for increasing the accuracy of measurements. One possible solution to this problem is the increase in the length of the impact load cell that reduces the number of de-convolutions.

5.7. CONCLUSION

A novel framework has been proposed to analyse results from a single particle breakage test that consists of considering that data from it should not be interpreted as material properties, but rather that more fundamental material properties, as well as contact geometry and impact velocity will influence the outcome of the test. Indeed, a single particle breakage event has been described effectively by regarding a breakage event as a multiple-stage process, which includes primary breakage function and fractures energy, selection and classification functions. The thesis first investigated the sensitivity of the model to parameters in the classification and selection function. As the sensitivity analysis demonstrated, the proposed breakage model was responsive to its selection and classification components and changing these functions changed the shape of the cumulative size distribution from shallow to steep when plotted on a log-log scale. It also indicated a strong dependency of A×b value - which generally regarded as an indication of rock strength- to these sub-processes. As the model suggests, the so-called selection and classification functions impact the extent of fragmentation that can be reached in
a breakage event. The rapid or gradual fragmentation, indicated by b value can also be affected by these sub-processes, rather than the materials themselves.

6. THE EFFECT OF STRESSING VELOCITY

6.1. INTRODUCTION

The concept of single particle breakage as a multi-stage process was applied to study the effect of stressing velocity on breakage characterisation of rocks. The following section discusses the results of a preliminary breakage test on the effect of the stressing velocity. Based on the model concept, the impact of the stressing velocity will be discussed in terms of the primary breakage characterisation of particles, such as fracture energy and the appearance function. Then, the other sub-processes such as selection and classification functions will be investigated under the influence of this parameter.

6.2. PRELIMINARY BREAKAGE TESTS

Particles from soft (magnetite, A*b >100, (Napier-Munn et al., 1996) and tough (silicate, A*b <40) components of an iron ore deposit were tested. 20 particles of size 26.5-31.5mm of each component were tested using the Drop Weight Tester (DWT) and a slow compression machine at a fixed input energy of 0.3 kWh/t. Figure 6-1 shows that the progeny size distributions of magnetite are finer than those of silicate, which is consistent with their different A×b values. The results also show that the DWT produced a slightly finer product for both soft (Magnetite) and tough (Silicate) components. The mass retained in each size fraction indicated that the compression breakage has more particles of +10mm fractions while the impact breakage produces more particles in -10+3mm sizes. Although, at the finer end (-1 mm), there is no significant difference in the size distributions generated by the two mechanisms of breakage, particularly for silicate. The coarser product resulting from compression could be explained by the fact that, unlike impact, the broken fragments at the early stages of compression had enough time to leave the breakage environment, resulting in less chance of rebreakage in the compression test, resulting in coarser fragmentation. Also, the size distribution of magnetite fragments from the test was finer than that of silicate, as expected.
Figure 6-1: Effect of breakage mechanism on the size distribution of progeny (a) cumulative passing and (b) percentage retained (stressing energy: 0.3 kWh/t, stressing velocity: compression: 0.00004 m/s, impact: 4 m/s)

Observations that were made during the experiments demonstrated that fragments resulting from the breakage of magnetite mainly tended to remain close to the parent particle after breakage whereas silicate fragments are ejected from the loading zone due to their high kinetic energy, forming a wide spatial distribution. Figure 6-2 clearly shows that the progeny of a
silicate particle is widely distributed while that of a magnetite particle remained closer to the parent particle. These differences suggest that it is an oversimplification to solely characterize the breakage response of these materials by their $A*b$ value.

![Figure 6-2: Spatial distribution of a) magnetite fragments b) silicate fragments, after breakage (12 cm cylinder at 500 mm/min applied strain rate using a Drop Weight Tester). Marked distance represents 50 mm.](image)

Observations from this study, which agree with those in the literature (Bergstrom et al., 1962; Schönert, 1996; Tavares, 2007), confirm that breakage should be perceived as a process. The finer size distribution of fragments from breakage of magnetite and the coarser distribution of fragments from silicate should not simply be explained on the basis of their difference in “hardness” or toughness. Rather, a closer look into the entire breakage event and the sub-processes involved is required. The observations indicate that a breakage process includes the phenomenon of fracture, the spatial distribution of progenies, and the manner fragments are captured under various loading conditions. In this context, the selection of fragments under loading and their spatial distribution, which determines which fragments will remain in the breakage environment and which fragments will leave the area, are significant sub-processes and their contribution in characterisation tests should receive appropriate attention. This is addressed in the following section.

### 6.3. FIRST FRACTURE CHARACTERISTICS

Experimental test work was conducted in the Instron compression tester and the SILC device to carefully establish accurate comparative data of first fracture for two rock types. Figure 6-3(a) depicts the distribution of specific fracture energy of magnetite and silicate particles for both compression and impact mechanisms for the size fraction 6.7-8 mm. The fracture energy distribution can also be described by a sigmoid shape error function such as Weibull or log-
normal distribution (Tavares, 2007). The normal distribution, as given in following, is used to
describe variability within the fracture energy of rocks, given by:
\[ P(E_m) = \frac{1}{2} [1 + \text{erf}\left(\frac{\ln E_m - \ln E_{m50}}{\sqrt{2} \sigma E}\right)] \]
(Equation 6.1)
where \(E_{m50}\) and \(\sigma_E\) are the median and standard deviation of the distribution.

Considering the variability within each individual particle and their microstructure in a certain
size fraction, scattered values result from this type of test, emphasising the distribution of cracks
in various sizes and quantities. As the distribution of fracture energies of silicate particles
illustrates, there is no significant difference between the distributions in compression and
impact mechanisms. The fracture energy distribution of magnetite in an impact test shows
slightly larger values (average: 0.012 kWh/t, median: 0.010 kWh/t, standard deviation: 0.008
kWh/t) in comparison to the compression test (average: 0.010 kWh/t, median: 0.009 kWh/t,
standard deviation: 0.006). However, performing a statistical Z-test with 95% confidence limit
suggested no significant difference in first fracture energy that resulted from conducting the test
in slow compression or impact. Despite the elimination of flaky and elongated particles for
experiments, the fracture energy of particles for both types of rocks varies in a relatively broad
range. This is between 0.01-0.11 kWh/t for Beaudesert silicate and 0.001-0.04 kWh/t for LKAB
magnetite (Table 6-1).

<table>
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<th>Magnetite-Impact</th>
<th>Silicate-Compression</th>
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</tr>
<tr>
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<td>Not significant</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 6-3: First fracture characteristics for magnetite and silicate under compression and impact loading: (a) Distribution of first fracture energy (b) Primary breakage fragment distribution

Cumulative percentage passing versus the size of fragments in log-log scale was used to illustrate the progeny size distribution of magnetite and silicate particles. As Figure 6-3b shows, primary breakage functions are only material-dependent and do not vary if the particles were stressed by impact or compression. This demonstrates that the primary breakage function is
independent of applied strain rate in the range that experiments were conducted. Also, as the graph suggests the signatures of primary fragmentation are remarkably distinctive for two types of components; magnetite and silicate. In the case of silicate, the slope of the curve is far steeper for size fraction between 100 and 2000 micron. This emphasises the unique fracture and fragmentation patterns of various types of rocks that distinguishes them from one another in terms of breakage, regardless of the mechanisms applied for fracturing them. Also, these findings are in agreement with the previous studies of fracture probability of brittle materials and fracture energy that are not significantly affected by the applied strain rate. It can be implied from this experiment that the differences between the fragmentation through impact in the SILC and Compression tester products of the same material (illustrated in Figure 6-1) cannot be linked to their primary fracture properties such as fracture probability, first fracture energy and size distribution as it is nearly identical for two mechanisms with the same rock type. Therefore, the differences may be related to the influence of other material properties that influence other sub-processes that rock particles undergo in a breakage event such as the classification and selection of the fragments.

6.4. INTERACTION OF APPLIED STRAIN RATE AND BREAKAGE SUB-PROCESSES

Given that the difference between the product of impact and compression in Figure 6-1 cannot be explained by the first fracture properties of materials under these two mechanisms, differences in material response in other breakage sub-processes should contribute to the observed differences. The following experiments aimed to investigate the effect of applied strain rate on the spatial distribution of the fragments, which causes classification of the fragments in a breakage zone. To investigate this effect, the rock samples were tested under contained and uncontained conditions. Containment of the fragments does not allow fragments to classify and leave the breakage zone.

6.4.1. Contained Condition

The t10-Ecs relationship was used to explain the progeny size distribution of fragments resulting from breakage of magnetite and silicate using impact and compression mechanisms at various energy levels, as is illustrated in Figure 6-4(a) and Figure 6-4(b). In the case of impact mechanism, the specific energies are absorbed energies, which are calculated from de-convolution of signals obtained by the short impact load cell. The experiments were performed
in contained conditions, where fragments were not allowed to escape from the zone of breakage. As illustrated by this relationship, there is no significant difference between the size distributions of fragments generated by compression and impact mechanisms under this condition.

Figure 6-4: t10-Ecs (a) magnetite (b) silicate under impact and compression mechanism for different stressing energies in which fragments were contained.
Containment of particles inside the breakage environment does not allow the broken fragments to leave the breakage zone before compression is complete. Also, the experiments have shown that the further fragmentation of magnetite and silicate beyond the primary fracture follows the signature of primary fracture at energy levels larger than first fracture energy. Subsequent fragmentations take place by not only the gradual breakage of coarse fragments but mainly from the breakage of medium to fine-size particles that are captured for breakage. This leads to an accumulation of debris in the fine size fractions. The complete depletion of coarse fragments takes place at high energies, and is achieved gradually. At about 0.49 kWh/t, the compression mechanism produces increased fragmentation at coarse size fraction (Figure 6-5).

Figure 6-5: Size distribution of silicate in contained conditions for compression and impact mechanisms. Solid lines represent impact whereas dashed lines represent compression.

The increased fragmentation of coarse fraction can be explained in terms of higher probability of coarse particles selection, which remained in the area of breakage (between platen and anvil) during compression. As the fresh particle was surrounded by a plastic film in contained condition, the broken fragments had no chance of escaping from the breakage environment. Therefore, when the particles impact the plastic wall, they return back inside the zone of breakage, where they have the chance of being captured by the compression anvil and are further broken. Figure 6-6 illustrates this phenomenon.
6.4.2. Uncontained Condition

Magnetite and silicate particles were also tested in an uncontained condition, so that they were free to escape from the active breakage zone, since no barrier existed. These experiments allowed investigating the effect of applied strain rate and the spatial distribution of fragments on breakage. The results are shown in Figure 6-7 for silicate particles at low and high energy levels under impact and compression mechanisms and in Figure 6-8 for magnetite under compression.

As Figure 6-7 shows, in environments with identical geometries of breakage zone (flat-flat), where a natural distribution of fragments is allowed (without containment) and limited (with...
containment), the size distribution of fragments is affected by their spatial distribution under a compression mechanism at coarse size fractions and higher energy level. As shown in this figure, the particle size distribution resulted from the compression tests in contained condition; energy level 0.490 kWh/t and uncontained condition; energy level 0.504 kWh/t are significantly different. This difference is justified by measurement of error at different size fractions. In this case, the error of measurements varies from coarse to small sizes being lower for coarse particles; for instance, it is 3.55% at 4.75 mm and increases toward 9.77% at 75 microns. This is due to the high kinetic energy of fragments that is stored in particles in the form of strain energy prior to fracture. In the instant of fragmentation, the released strain energy is converted into other forms of energy such as heat, sound and kinetic energy (Atkinson, 1989; Lawn, 1993). This is apparent in the coarse size fraction when a large fraction of particle mass leaves the breakage environment at the early stages of compression. However, this effect was less apparent as the compression progressed and particles lost their original mass and became weaker. Gilvarry (Gilvarry, 1961) suggested that free crushing of a single glass sphere in slow compression contains a significant amount of energy after fracture and if this energy is directed in a proper way it can serve the purpose of propagating further breakage. Also, Bergstrom and Sollenberger (1962) developed a relationship between the material properties and the speed of the fragments while leaving the breakage environment.

In Figure 6-8, the compression results are shown for magnetite in contained and uncontained conditions. As illustrated, the progeny size distributions resulting from contained and uncontained conditions in compression tests were nearly identical, unlike those observed for silicate particles. This emphasises the effect of the material properties on the spatial distribution of fragments under a slow rate of applied strain. Silicate particles demonstrate more brittle behaviour in comparison with magnetite particles which results in scattering of the fragments during the mechanism of fracturing when the speed is slow, resulting in coarser fragmentation of silicate. Despite this, no significant difference between the size distributions of silicate particles was obtained under the impact mechanism in contained and uncontained conditions, as illustrated in Figure 6-7, due to higher velocity of impactor preventing fragments from escape on the impact zone. The magnetite particles were not tested in uncontained conditions under an impact test as no difference was expected, due to the less brittle behaviour in comparison with silicate particles. This also can be explained by the manner the fragments are captured during experimentation. When a particle breaks by impact, the fragments of primary fragmentation
may tend to escape from the breakage zone, due to the strain energy that is stored in particle before failure. However, a portion of fragments in the immediate contact with the impactor are captured by the striker, and re-break and the other fragments move laterally. The distance travelled by fragments also depends on their kinetic energy. However, due to the short time of an impact mechanism, the subsequent fragmentation results in the accumulation of debris in the breakage environment. In a typical impact, fragments move laterally, upward, or even rotate several times before they come to a complete rest on the compression anvil. Many fragments come to a complete rest long after the impact is completed. In this sense, the containment of particle inside the breakage zone, in comparison with an uncontained condition does not cause the re-breakage of the fragments that tend to move away from the breakage area. The high-speed video filming in Figure 6-9 depicts the movements of fragments, broken at a low energy level inside the plastic covering. The same behaviour also was observed at high energy level.

![Diagram](image.png)

**Figure 6-8:** Comparing the size distribution of progeny of compression tests in contained and uncontained conditions for magnetite
Given the interesting results obtained with the more brittle material (silicate), quartz particles were additionally tested under compression and impact mechanisms. Quartz is regarded as an extremely brittle material, and its brittleness index was estimated as 0.97 on the basis of response of riverbed quartz particles contained in size fraction 4.0-4.75 mm (Tavares, 1997). This was estimated on the basis of its amenability to accumulate damage only as fractures propagated while it failed, in the form of a damage accumulation coefficient. This coefficient

6.5. QUARTZ SAMPLES

Figure 6-9: High speed video filming of an impact mechanism and the movement and rotation of the fragments during breakage, at a low energy level, impact velocity: 1.17 m/s
was found to vary according to material microstructure, being higher for materials with complex microstructures, such as quartz (Tavares, 2009).

Initially, quartz particles of size 6.7-8.0 mm were tested in the Instron at a very slow rate of 0.5 mm/min and its exceptionally brittle behaviour was confirmed. In comparison to silicate samples, fragments of broken quartz had significantly less stability under the slow rate of compression. Large fragments of broken quartz tended to leave the breakage zone at the very slow rate of strain of 0.5 mm/min. Due to the instability of quartz particles at this rate, the catchment of the fragments inside the breakage zone was not possible, making it impossible to reach higher levels of energy. Due to this reason, samples of quartz were tested at a higher rate of 500 mm/min to allow a better capture of fragments. At this rate, better capture of fragments by comminution device was achieved. Figure 6-10 illustrates the behaviour of quartz at two different loading rates.

Figure 6-10: Illustration of the response of quartz particles (a) before breakage (b) breakage at 0.5 mm/min (c) breakage at 500 mm/min

Progeny size distributions from stressing quartz particles with the piston compressing for 3 mm under compression, both from contained and uncontained conditions, are shown in Figure 6-11. Breakage of quartz particles was also investigated under impact, using high speed video filming. High-speed filming of the breakage of particles demonstrated that with flat-flat geometry using impact mechanism, a part of fragmentation takes place before fragments disintegrate and start to detach themselves from the area between striker and anvil. Also, filming of rock samples during impact breakage demonstrates progressive breakage of particles, as can be clearly observed in the footage of primary fragmentation. Once the first failure of rock takes place, subsequent fragmentation of daughter particles continues towards forming a bed of fragments. The brittle nature of fragments and their kinetic energy lead to the bed disintegration; in which the debris tend to escape from the vicinity of each other and move in different directions. Figure 6-12 illustrates the sequence of quartz breakage under impact mechanism.
with 2 ms time steps. In Figure 6-12(c), two clear lines of first fracturing of the quartz particle are seen. Further fracturing or secondary breakage of particle in Figure 6-12(d) takes place before the fragments fall apart. In Figure 6-12(e), fragments start to fall apart and move towards the outside. In Figure 6-12(f), the fragments start to leave the breakage environment. In Figure 6-12(g), fragments escape from the breakage zone while the striker is still moving downwards. In Figure 6-12(h), the contact of the fragments with the anvil is lost. As Quartz particles break, the daughter fragments start to fall apart and form a movable, loose bed in which the fragments move laterally with high velocity, while rotating and relocating a number of times before they reach the edge of the anvil.

Figure 6-11: Size distribution of quartz particles (6.7-8 mm) in contained and uncontained conditions at 3mm displacement using compression
6.6. DISCUSSION

The observed invariability of the primary breakage function with strain rate in the case of brittle material supports the assumption in several approaches used in modeling ball mills (Austin et al., 1984; Herbst and Fuerstenau, 1980) which considers that every breakage event in mills, even operating under different conditions, results in the same size distribution, called the breakage function.

As the methodology of interpreting results from breakage tests in the present work allows decoupling material properties from the system of breakage, there is an opportunity to enhance the quality of developing mechanistic models and their predictability. This methodology also can benefit the area of ore breakage characterisation, as the need for characterisation under various stressing environment and conditions (affected by parameters such as applied strain rate, geometry and particle shapes) may become unnecessary. In this case, the primary breakage
characteristics can be employed to represent the material properties. The study of applied strain rate with the new approach allows a better understanding of the breakage details and the sub-processes that contribute to the overall size distribution from a single breakage event.

6.7. SUMMARY

- The characteristics of primary breakage, which are the first fracture energy and the primary breakage function, were found to be nearly identical for both compression and impact mechanisms.
- However, these characteristics of primary breakage were found to depend on material properties rather than on stressing conditions, so they should be a central goal for ore breakage characterisation purposes.
- The spatial distribution of fragments of more brittle materials (Beaudesert silicate and quartz) under compression loading showed more scatter than that for magnetite, which behaved as a less brittle material.
- Little scatter in the spatial distribution was found for impact, due to the higher applied strain rate which prevents the escape of the fragments. Indeed, although the primary fracture characteristics are similar for compression and impact in the case of magnetite and silicate, the final size distributions are affected by the manner in which particles are distributed during the fragmentation process; particularly at coarse sizes.

6.8. IMPLICATION OF SINGLE PARTICLE BREAKAGE MODEL FOR INDUSTRY

The concept of single particle breakage as a process has its own potential implication in the comminution process modelling. At first, it eliminates the need to conduct a large number of breakage characterisation tests for each comminution device available in the industry. The rock competency can be determined using the primary breakage characteristics as it is solely depend on the material. By knowing the properties of the breakage characterisation environment such as selection and classification functions and applying them as active sub-processes, it is possible to predict the breakage product of a comminution device. From the modelling perspective, it enhances the prediction capabilities of the comminution models used for plant optimisation. An ideal comminution model separates the contribution of the breakage environment from that of the ore. Hence, the breakage system characteristics are not lumped into particles breakage behaviour and key factors from the ore or the system would be taken into account.
SUMMARY AND CONCLUSION

The breakage of a single particle was analysed under the light of a new approach; multi-stage breakage process, proposed in this project. Breakage of a single particle as a multi-stage process was modelled based on its main components, namely primary fragmentation, selection and classification. Each component was also modelled separately, with their combination constructing the size distribution. The primary breakage function, which is regarded as a material property, is the first and main component of this model. A two-slope normalised primary breakage function was used to describe the primary fragmentation of single particles with three fitting parameters. The selection function was defined as the probability of capture of the fragments under loading which changes based on the fragments size. A size-dependant function was used to model this phenomenon with a fitting parameter determining the selection probability of fragments from one size fraction to the other, changing the system geometry and properties of materials. The classification function was defined as a function depending on materials brittleness and system properties. The model also accounts for the energy consumption in each stage of breakage when the materials are captured and undergo breakage. The model assumes that selected particles consume the amount of energy that is equal to their median specific fracture energy in each size fraction.

A sensitivity analysis was conducted to variation in selection and classification functions, while the primary breakage function was kept constant for all the scenarios. The model was responsive to all the variations in its main components, changing the shape of the size distribution curve accordingly. When the probability of coarse particle selection was high, a sharp size distribution with a small portion of fines was generated. When the probability of fine particle selection was high, a shallow size distribution with a high portion of fines was produced. Also, the sensitivity analysis to classification was determined to limit the maximum fragmentation of particles that results in some changes in the shape of the size distribution curve.

The model was validated for three types of materials under drop weight testing mechanism. The validation was performed with two main criteria. The shape of the size distribution curve in the model matched to the shape of the size distribution obtained by experiments. Also, the energy consumption was matched accordingly. In the case of quartz, the classification was limited due to the extreme properties of fragments and a moderate selection
function was used to describe the capture of fragments. A good agreement was obtained between the model and experimental results in respect to the shape of the size distribution curve as well as the energy consumption.

In the case of apatite, the size distribution was simulated for three different geometries of impactor and anvil; flat-flat, ball-flat and ball-ball. By changing the selection function, for different combinations, the shape of the size distribution was affected and good agreement was obtained in terms of the shape of the size distribution curve.

In the case of silicate, the selection probability was changed based on the properties of materials and the knowledge gained from the study of high-speed video filming. A good agreement was found between the experimental and model results at moderate energy levels. However, improvements in the model require a better understanding of the multi-stage breakage model at higher energy levels when the fragments form a bed. The development of single particle breakage model and its validation for three types of material using drop weight tester proved the first hypothesis of this project which was “A characterisation test is a sequence of breakage events that can be modelled in a way to decouple the fundamental properties of rock from the geometry of the testing system”.

The effect of applied strain rate (stressing velocity) was evaluated using the concept of multi-stage breakage. In this regard, compression and impact mechanisms were compared to each other for two different types of ore; magnetite and silicate. The Impact load cell was used to conduct impact experiments, whereas Instron compression device was used for compression. The results indicated that compression and impact mechanism produce nearly identical primary fragmentation under the contained condition when the fragments were forced to stay in the breakage environment. The fracture energy distributions of silicate and magnetite were also identical for these two mechanisms. The identical primary breakage functions and fracture energies under impact and compression mechanisms demonstrated the second hypothesis defined as “Primary breakage characteristics (i.e. appearance function and fracture energy) are not affected by the applied strain rate”.

The test also demonstrated that at identical energy levels, in contained conditions, both mechanisms generated similar size distributions for both types of ore. However, in uncontained conditions; when fragments were allowed to escape from the breakage environment, the results were affected by classification of silicate particles, ejecting themselves from the breakage zone.
An extreme material such as quartz was also examined under the compression mechanism and it was found that its size distribution is affected by the way that fragments are captured under contained and uncontained conditions. Further, high-speed video filming provided detailed insight into the breakage of quartz particles and the manner the fragmentation proceeded. The observations and the experimental results in contained and contained conditions at different applied strain rates confirmed the third hypothesis stated as “Applied Strain rate changes the classification and selection functions”.

CONTRIBUTION TO KNOWLEDGE

In overall, this project contribution to knowledge can be specified as:

- The breakage of a single particle was modelled as a sequence of primary breakage events.
- A single-particle breakage was determined as a process, consisted of different sub-processes, and the effect of applied strain rate on breakage sub-processes was investigated.
- The primary breakage function was determined to be independent of applied strain rate.
- The contributions of the ore and of the stressing conditions on the outcome of the breakage event was decoupled.
RECOMMENDED FURTHER WORK

As part of the investigation on a single breakage event as a multi-stage process, in both compression and impact mechanisms, it is recommended that this work is extended to higher energy levels to study the selection probability of the broken fragments. It is hypothesised that selection probability of particles is also affected by the stressing velocity which may manifest itself at higher energy levels.

It is envisaged that brittleness index of materials also affects the selection probability of particles, based on the preliminary study of ore breakage behaviour using high speed video filming. Hence, it is recommended that this work continues to study this effect in greater detail using different types of natural ores. Also, it is recommended to conduct an analytical estimation of the velocity ratio which explains the interaction between fragment velocity and stress velocity. This would help to see the differences of slow compression and fast impact stressing.

Due to the importance of geometry in the breakage response of the rock in tumbling milling environments, it is recommended to study this effect in more detail, with different types of materials, and with different curvatures of the ball. It is hypothesised that there is an interaction between the brittleness index of materials and the geometry of impact. The extent that materials respond to the effect of geometry may be influenced by their brittleness behaviour.

It is recommended to investigate the multi-stage breakage process at high energy levels, when the packing of fragments lead to bed formation and limit the classification and selection of the particles.

Because the model prediction is influenced by the non-normalised appearance function of some materials (after a certain size) as the result of rock micro-structure, it is recommended to investigate this effect in future work. It is believed that the study of rock texture using tomography and other tools provide insight into this phenomenon. Hence, the development of the present model for non-normalised breakage behaviour will be possible.

Finally, the major shortcoming of the approach proposed in this work is that the damage accumulation due to the repeated stressing (fatigue) cannot be taken into account. Hence, it is recommended to conduct further studies to apply the fundamentals and theories of damage accumulations into the framework of the present model.
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