Analytical Error Propagation in
Four-Step Transportation Demand Models

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A thesis submitted for the degree of Doctor of Philosophy at
The University of Queensland in 2017
School of Civil Engineering
Abstract
Transportation demand models currently lack a rigorous and analytic treatment to quantify the error propagation from different sources through the models. The error of traffic forecasts is attributed to two main sources: the model specification error and the input variable measurement error. Since Four-Step Transportation Demand Model (FSTDM) is commonly used in practice but its error is not well-studied, the first part of the current study illustrates how the errors of the input variables as well as of the model specification are propagated analytically step by step and how these errors interact to result in inaccurate traffic forecasts.

The proposed approach is able to quantify separately and collectively the share of different sources of error in the traffic forecast error. The proposed procedure is an efficient method that is less time consuming than existing simulation-based methods. This enables the proposed procedure to analyse the sensitivity of the traffic forecast to the input measurement error and the quality of modelling in large scale networks. Moreover, comparing the output errors using the proposed approach with the acceptable ranges of error specified in transportation guidelines, decision makers will have a clear opportunity to realise the credibility of a point traffic forecast and its associated variance.

The proposed approach derives the variance from calibrated models in each of the four steps, to obtain the variance of the output based on the variance of inputs. The resulting variance formula provides an analytical expression to estimate the forecast errors from the input errors. In addition, the model specification error of each step of the FSTDM is added to the propagated input measurement errors. The proposed approach is applied to the city of Brisbane as a case study spanning the four-step models for eight different trip purposes.

As an example, a measurement error of 10 percent for the input variables of the Brisbane FSTDM (BFSTDM) as well as the specification errors of models calibrated for the Home Based Work - Blue collar (HBWB) trip purpose were explored. The model specification error produces variances of 1760.77 (trip/h)², 976.72 (trip/h)², 0.01082 (trip/h)² and 0.001327 respectively for trip production, trip attraction, trip distribution and modal split steps. Subsequently, the variance of output errors for the same steps are respectively, on average, 2885.50 (trip/h)², 7218.70 (trip/h)², 0.25 (trip/h)² and 0.18. The variance of output error in the traffic assignment step is calculated to be 2097.20 (veh/h)² for all trip purposes, while the model specification error of the same step is 1056 (veh/h)². Having the existing
868 traffic zones, from the first to the third step, a reduction in the variance of trips per origin-destination (O-D) pair is observed. At the same time, in the traffic assignment step, considering all trip purposes, the size of the forecast error variance per link increases.

In the second part of the present study, the specification error of a user equilibrium traffic assignment (UETA) is measured using validation techniques. Moreover, the propagation of O-D demand measurement errors to the UETA output is investigated using two different methods: the proposed analytical sensitivity-based method and a simulation-based method. The analytical method uses the results of a sensitivity analysis (SA) on the UETA mathematical program, while the simulation-based method runs a Monte Carlo Simulation (MCS).

The proposed method for error propagation is applied to an illustrative example to address three main questions: the number of samples that ensure a reasonably accurate result for the MCS method; the size of the O-D demand measurement error for which the analytical method is valid; and, the share of the path flow rate variance and covariance from the variance of the O-D demand measurement error.
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Publications during candidature

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No publications are included.

Contributions by others to the thesis
No contributions by others.

Statement of parts of the thesis submitted to qualify for the award of another degree
None.
Acknowledgements
Firstly, I would like to express my sincere gratitude to my principle advisor, Dr. Mahmoud Mesbah, and my associate advisor, Prof. Mark Hickman, for the continuous support of my PhD study and related research, for his patience, motivation, and immense knowledge. His guidance helped me in all the time of research and writing of this thesis report.

Besides my advisor, I would like to thank my thesis review panel: Prof. David Lockington, and Prof. Carlo Prato, for their insightful comments and encouragement, but also for the hard questions which helped me to widen my research from various perspectives.

Last but not the least, I would like to thank my family: my lovely wife, Zeinab, and to my dear parents and sister for supporting me spiritually throughout my studying and writing this thesis report.
Keywords
Error Propagation, Model Specification Error, Input Measurement Error, Four-Step Transportation Demand Model, Sensitivity Analysis, Monte Carlo Simulation

Australian and New Zealand Standard Research Classifications (ANZSRC)
090507, Transport Engineering, 100%

Fields of Research (FoR) Classification
0905, Civil Engineering, 100%
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List of abbreviations

\( \varepsilon = \) Residual term,
\( \varepsilon_0(z) = \) Residual value created by trip production model for zone \( z \in Z \),
\( \varepsilon_D(z) = \) Residual value created by trip attraction model for zone \( z \in Z \),
\( \varepsilon_{0D}(z,w) = \) Residual value created by trip distribution model for \( z, w \in Z \),
\( \varepsilon_{0D}^\text{DF}(z,w) = \) Residual term of probability of mode car for \( z, w \in Z \),
\( \xi_x = \) Random variable due to an error in measuring or estimating \( x \),
\( \eta_0 = \) Model specification error for trip production model,
\( \eta_D = \) Model specification error for trip attraction model,
\( \eta_{0D} = \) Model specification error for trip distribution model,
\( \eta_{0D}^\text{car} = \) Model specification error for modal split model,
\( \eta_{0D}^\text{traf} = \) Model specification error for traffic assignment,
\( \theta = \) Parameter of route-based traffic assignment (RTA),
\( \delta_{I,R_m}(z,w) = 1, \) if link \( i \) belongs to set of links creating route \( r_m \); otherwise, 0 for \( i \in I \) and \( r_m \in R(z, w) \),
\( \omega(z,w) = \) Dual variable of conservation equality constraint in traffic assignment for O-D pair of \( (z, w) \) for \( z, w \in Z \),
\( \mu_{r_m}(z,w) = \) Dual variable of non-negativity inequality constraint in traffic assignment for route \( r_m \) for \( r_m \in R(z, w) \) and \( z, w \in Z \),
\( \Sigma = \) Variance-covariance matrix,
\( A(z,w) = \) O-D specific parameter to travel from origin \( z \) to destination \( w \) for \( z, w \in Z \),
\( \text{AMWAET} = \) AM-peak access and egress time to public transport for O-D pairs in BFSTDM,
\( \text{AMWIVT} = \) AM-peak in-vehicle time in public transport for O-D pairs in BFSTDM,
\( \text{AMWWAT} = \) AM-peak waiting time in public transport for O-D pairs in BFSTDM,
\( a_i = \) Estimated parameter for \( i \)th input variable in trip production model for \( i \in S_0 \),
\( b_i = \) Estimated parameter for \( i \)th input variable in trip attraction model for \( i \in S_D \),
\( \text{BLUHH} = \) Average number of blue-collar workers per household in traffic zones in BFSTDM,
\( c_{0D}(z,w) = \) Travel impedance for travelling from origin \( z \) to destination \( w \) for \( z, w \in Z \),
\( \text{CYTIME} = \) Cycling time for O-D pairs in BFSTDM,
\( \text{DepA}_{i} = \) Share of households with \( i \) dependent A from total households in traffic zones in BFSTDM for \( i \in \{1,2,3,4\} \),
\( \text{DepB}_{i} = \) Share of households with \( i \) dependent B from total households in traffic zones in BFSTDM for \( i \in \{1,2,3,4\} \),
\( \text{DepC}_{i} = \) Share of households with \( i \) dependent C from total households in traffic zones in BFSTDM for \( i \in \{1,2,3,4\} \),
\( f(z,w) = \) Friction function value to travel from origin \( z \) to destination \( w \) for \( z, w \in Z \),
\( G_{ij} = \) Set of joint links between route \( i \) and route \( j \) that is identical with \( G_{ji} \),
\( g_{r_m}(z,w) = \) Non-negativity inequality constraint in traffic assignment for route \( r_m \) for \( r_m \in R(z, w) \) and \( z, w \in Z \),
\( h(z,w) = \) Conservation equality constraint in traffic assignment for O-D pair of \( (z, w) \) for \( z, w \in Z \),
\( \text{HAMCST} = \) Highway AM-peak costs for O-D pairs in BFSTDM,
$HH(z) =$ Number of households in traffic zone $z \in Z$ in BFSTDM,

$HH_{\text{Size}} =$ Average household size in traffic zones in BFSTDM,

$HOPCST =$ Highway off-peak costs for O-D pairs in BFSTDM,

$HWYAMT =$ Highway AM-peak time for O-D pairs in BFSTDM,

$HWYOPT =$ Highway off-peak time for O-D pairs in BFSTDM,

$HTPR =$ Household trip production rate in traffic zones in BFSTDM,

$I =$ Set of network links,

$Industry =$ Number of industry workers in traffic zones in BFSTDM,

$Industry_B =$ Number of blue-collar industry workers in traffic zones in BFSTDM,

$Industry_W =$ Number of white-collar industry workers in traffic zones in BFSTDM,

$K(z, w) =$ Set of routes between origin $z$ and destination $w$,

$L_i =$ Set of links creating route $i$,

$n =$ Number of observations,

$p =$ Number of estimated parameters,

$Pr(i) =$ Probability of alternative $i$ for $i \in S$,

$P_{00}^{\text{Car}}(z, w) =$ Probability of mode car after adding residual term for $z, w \in Z$,

$P_{00}^{\text{Car}}(z, w) =$ Probability of mode car calculated based on logit model for $z, w \in Z$,

$p_{r_i}^{\text{Car}}(z, w) =$ Probability of choosing route $r_i$ for $r_i \in R(z, w)$ and $z, w \in Z$,

$PKCOST =$ CBD all-day parking cost in traffic zones in BFSTDM,

$PKOPC =$ Off-peak parking cost in traffic zones in BFSTDM,

$PKOPCO =$ Off-peak parking cost in traffic zones for non-home based trips in BFSTDM,

$Population =$ Population in traffic zones in BFSTDM,

$PrePrimary =$ Number of pre-primary students in traffic zones in BFSTDM,

$Professional =$ Number of professional workers in traffic zones in BFSTDM,

$Professional_B =$ Number of blue-collar professional workers in traffic zones in BFSTDM,

$Professional_W =$ Number of white-collar professional workers in traffic zones in BFSTDM,

$PTFARE =$ Zonal fare in 2006 cents to travel by public transport for O-D pairs in BFSTDM,

$Q_{r_i}^{\text{Car}}(z, w) =$ Flow on route $r_i$ for $r_i \in R(z, w)$ and $z, w \in Z$,

$Q^{\text{Car}} =$ Vector of route flow rates, $Q^{\text{Car}}_{r_i}(z, w)$,

$R(z, w) =$ Set of routes connecting origin $z$ to destination $w$ for $z, w \in Z$,

$Retail =$ Number of retail workers in traffic zones in BFSTDM,

$Retail_B =$ Number of blue-collar retail workers in traffic zones in BFSTDM,

$Retail_O =$ Number of retail workers excluding wholesale in traffic zones in BFSTDM,

$Retail_W =$ Number of white-collar retail workers in traffic zones in BFSTDM,

$S =$ Set of available alternatives,

$S_O =$ Set of specific independent variables in trip production model,

$S_D =$ Set of specific independent variables in trip attraction model,

$Secondary =$ Number of secondary students in traffic zones in BFSTDM,

$Service =$ Number of service workers in traffic zones in BFSTDM,

$Service_B =$ Number of blue-collar service workers in traffic zones in BFSTDM,

$Service_W =$ Number of white-collar service workers in traffic zones in BFSTDM.
\(T_0(z)\) = Number of observed trips produced in zone \(z \in Z\),
\(T_p(z)\) = Number of observed trips attracted in zone \(z \in Z\),
\(T_{0o}(z)\) = Number of distributed trips from origin \(z\) to destination \(w\) after adding residual term,
\(T_{0w}(z)\) = Number of trips distributed between origin \(z\) and destination \(w\) for \(z, w \in Z\),
\(T_{0d}^{car}(z, w)\) = Travel demand of mode car in travelling from origin \(z\) to destination \(w\) for \(z, w \in Z\),
\(T_{0d}^{par} = \) Vector of O-D travel demand of mode car, \(T_{0d}^{par}(z, w)\),
\(t_{i}^{car}(z, w)\) = Travel time on route \(r_i\) for \(r_i \in R(z, w)\) and \(z, w \in Z\),
\(t_{i}^{car} = \) Travel time on link \(i\) for \(i \in I\),
\(t_{i}^{car} = \) Derivative of travel time on link \(i\) with respect to \(v_{i}^{car}\) for \(i \in I\),
\(t_{ij}(z, w)\) = Travel time on \(j\)th link of route \(r_i\) for \(r_i \in R(z, w)\) and \(z, w \in Z\),
\(t_{ijn}(z, w)\) = Travel time on \(n\)th joint link between route \(r_i\) and route \(r_j\) for \(r_i, r_j \in R(z, w)\) and \(z, w \in Z\) that is identical with \(t_{ijn}(z, w)\),
\(t_{ij}^{car}(z, w)\) = Travel time in minutes by mode car to travel from origin \(z\) to destination \(w\) for \(z, w \in Z\),
\(t_{ij}^{bus}(z, w)\) = Travel time in minutes by mode bus to travel from origin \(z\) to destination \(w\) for \(z, w \in Z\),
\(t_{ij}^{bike}(z, w)\) = Travel time in minutes by mode bike to travel from origin \(z\) to destination \(w\) for \(z, w \in Z\),
\(T_{r}(z, w)\) = Sum of travel times on specific links of route \(r_i\) for \(r_i \in R(z, w)\) and \(z, w \in Z\),
\(T_{r}^{j}(z, w)\) = Sum of travel times on joint links between route \(r_i\) and route \(r_j\) for \(r_i, r_j \in R(z, w)\) and \(z, w \in Z\),
\(T_{r}^{ij}(z, w)\) = Number of tertiary students in traffic zones in BFSTDM,
\(T_{IL}(z, w)\) = Travel impedance to travel from origin \(z\) to destination \(w\) in BFSTDM,
\(U_{i}\) = Utility function of alternative \(i\) for \(i \in S\),
\(U_{0d}^{car}(z, w)\) = Utility function of mode car to travel from origin \(z\) to destination \(w\) for \(z, w \in Z\),
\(U_{0d}^{bus} = \) Utility function of mode bus to travel from origin \(z\) to destination \(w\) for \(z, w \in Z\),
\(U_{0d}^{bike}(z, w)\) = Utility function of mode bike to travel from origin \(z\) to destination \(w\) for \(z, w \in Z\),
\(v_{i}^{car}\) = Volume of vehicle on link \(i\) after adding residual term for \(i \in I\),
\(v_{i}^{car}\) = Volume of vehicle on link \(i\) for \(i \in I\),
\(WATIME\) = Walking time for O-D pairs in BFSTDM,
\(WHTHH\) = Average number of white-collar workers per household for O-D pairs in BFSTDM,
\(Wkers\ Blu_i\) = Share of households with \(i\) blue-collar worker from total households in traffic zones in BFSTDM for \(i \in \{1,2,3\}\),
\(Wkers\ Wht_i\) = Share of households with \(i\) white-collar worker from total households in traffic zones in BFSTDM for \(i \in \{1,2,3\}\),
\(x_{oi}(z) = \) \(i\)th specific independent variable in trip production model for \(i \in S_o\) and \(z \in Z\),
\(x_{di}(z) = \) \(i\)th specific independent variable in trip attraction model for \(i \in S_p\) and \(z \in Z\),
\(x_{oi}(z)\) = Number of agricultural workers residing in zone \(z\) as the first specific independent variable in the trip production model of the demonstrating example for \(z \in Z\),
\(x_{o2}(z)\) = Number of industrial workers residing in zone \(z\) as the second specific independent variable in the trip production model of the demonstrating example for \(z \in Z\),
\(x_{o3}(z)\) = Area of schools and educational centres in hundred square meters as the first specific independent variable in the trip attraction model of the demonstrating example for \(z \in Z\),
$x_{D2}(z) = \text{Number of industrial workers working in zone } z \text{ as the second specific independent variable in the trip attraction model of the demonstrating example for } z \in Z,$

$x, y = \text{Random variable},$

$Y = \text{Vector of KKT triples of O-D pairs, } Q_{\text{Car}}^{z}(z, w), \omega(z, w), \mu_{\text{Car}}(z, w),$

$Z = \text{Set of traffic zones},$

$ZONEAH = \text{Average zonal adults per household in traffic zones in BFSTDM},$

$ZONEVH = \text{Average zonal vehicles per household in traffic zones in BFSTDM},$

$\text{var}(\cdot) = \text{Variance of a random variable around expected value or a set of values around mean},$

$\text{cov}(\cdot, \cdot) = \text{Covariance between two random variables or two sets of values},$

$E[\cdot] = \text{Expected value of a random variable},$

$(\cdot)^{T} = \text{Transpose of a matrix}.$
Introduction

Over the past decades, the difference between transport model estimates and actual values observed after project implementation has showed that the models are inaccurate in forecasting. Many efforts have been made by researchers to decrease the model inaccuracy through proposing new methods to model the travel behaviour better; however, the traffic forecasts are still erroneous (Flyvbjerg et al., 2005). In a recent study undertaken by Salling and Leleur (2015), the demand forecast inaccuracy (in traffic volumes) among 79 road projects was on average 22.3% underestimated with a relatively high standard deviation of 44%.

Without explicit statistical recognition of error in transportation forecasts, transportation planning takes an unnecessary risk (Zhao and Kockelman, 2002). de Jong et al. (2007) expressed that, in order to make an informed decision on infrastructure projects, it is essential to estimate not only the most plausible outcome, but also the possible range of outcome variation. Deterministic forecasting procedures or point estimate methods that have no systematic methodologies to implement the error analysis are the main reasons of neglecting error in traffic forecasts (Yang et al., 2013; de Jong et al., 2007). The point estimate methods are not able to provide a confidence interval for the model forecasts, even when a small sensitivity analysis compares several plausible scenarios. Therefore, it is important to provide a comprehensive framework to quantify errors generated by the current transportation modelling approaches.

Transportation demand models currently lack a rigorous and analytic treatment to quantify the error propagation from different sources through the models. Since the Four Step Transportation Demand Model (FSTDM) is commonly used in practice but its error is not well-studied, the current study illustrates how the errors of the input variables as well as of the model specification are propagated analytically step by step through an FSTDM and how these errors interact to result in inaccurate traffic forecasts.

In order to keep travel demand models credible for forecasting, it is necessary to recognize and analyse major sources of error. The error of forecasted demand is attributed to at least two main sources: the model specification error and the error in measuring (or forecasting) the input variables. In the calibration stage, the difference between what has been observed and what is estimated by the model is considered as the residual. In the prediction stage, where the calibrated model is applied to forecast demand, this residual term results in the model specification error. In this study, the variance of the model
specification error is obtained from the calibration process. Along with the model specification error, the measurement error of input variables might be intensified or diminished, depending on the value of covariance between the measurement errors of the input variables and the type of mathematical operation such as addition/subtraction, multiplication, or exponentiation in which the input variables are involved.

The proposed approach measures the variance of model specification error of the FSTD steps using two different methods, employing the goodness-of-fit criteria and using the model validation technique. Moreover, the relationship between the variance of input measurement errors, the variance of model specification error, and the output error variance of each step is derived by taking the variance of both sides of the calibrated model of the step.

The main advantages of using the proposed approach compared to the existing error analysis approaches in the literature are as follows:

- A procedure that is able to quantify separately and collectively the share of different sources of error in the traffic forecast error, and
- An approach that is able to analyse the sensitivity of the traffic forecast accuracy to the input measurement error and the model specification error.

This thesis contains three main chapters. In the first chapter, relevant literatures are reviewed to determine gaps in existing knowledge. In the second chapter, a general and analytical framework for quantifying the variance of error in FSTDMs is proposed, and the propagation of error through a user equilibrium traffic assignment (UETA) is investigated. Finally in the third chapter, within a demonstrative example, the propagation of error from inputs towards the final output is followed step by step. In the third chapter, the proposed approach is also applied to the city of Brisbane as a real case study. In the last case study of the third chapter, the error propagation through a typical UETA example is quantified using an illustrative network example. Finally, conclusions and recommendations from this research are given in a conclusions chapter.
Chapter 1     Literature Review

This chapter reviews the literature on propagation of error through sequential transportation demand models and particularly Four-Step Transportation Demand Models (FSTDM). Based on available evidence in the literature, the importance of error propagation in transportation demand models is investigated. In addition, the dominant types of contributing sources to traffic forecast error are classified. The main methods in the literature to address the error propagation in the transportation demand model are also introduced by listing advantages and disadvantages of each method.

As the main gap in the literature, transportation demand models currently lack a rigorous and analytic treatment to quantify the error propagation from different sources through the models. The current study proposes an analytical approach to overcome the main problems of the existing methods in the literature. The proposed analytical approach is efficient and accurate in the error propagation, and is also able to quantify separately and collectively the share of different contributing sources of error to the traffic forecast error.
1-1-Error in the Traffic Forecasts

Over the past decades, the difference between model estimations and actual values observed afterwards showed that the models are inaccurate in forecasting. Many efforts have been made by researchers to decrease the model inaccuracy through proposing new methods to model the travel behaviour better; however, the traffic forecast is still erroneous (Flyvbjerg et al., 2005). For instance, according to Page et al. (1981), 273 road projects indicated a mean absolute percentage error of 19.5% in the number of vehicles per hour with a percentage error range of +56.9% to -59.9%. Among the studies, Hartgen (2013) mentioned that the range of error in traditional traffic model forecasts to calculate the number of vehicles per hour is estimated to be in the order of 15–30% for a 20-year horizon. Moreover, in a more recent study undertaken by Salling and Leleur (2015), the demand forecast inaccuracy (number of vehicles per hour) among 79 road projects was on average 22.3% underestimated with a relatively high standard deviation of 44%.

The forecast inaccuracy is even more serious for some toll roads. In a review of seven Australian toll roads opened since 2005, Bain (2013) found out that in all seven cases, the counted traffic volumes were less than the predicted volumes by 40–60%. A review has also been undertaken by the US National Cooperative Highway Research Program (NCHRP) (2006) to evaluate the forecasting errors for 15 US toll roads opened between 1986 and 1999. NCHRP (2006) stated that the actual number of vehicles per hour was 35% below the predicted traffic on average. Similarly, Naess et al. (2006) reached a result that 13 of 14 US toll roads had over-predicted number of vehicles per hour by an average of 42%.

1-2-Importance of Error Analysis in the Traffic Forecasts

In order to keep travel demand models credible for forecasting, it is needed to recognize and analyse major sources of error. Yang et al. (2013) stated that a systematic analysis of error can provide an overall understanding about the level of confidence of the transport forecasts and also enable practitioners to determine critical sources of error. Additionally, according to Pickrell (1989) and Richmond (1998), the accuracy of transport forecasts plays a significant role in making decisions on the financial acceptability of transport projects. Transport plans made based on this type of forecast are inaccurate and even misleading. This may make the transport facility investments poorly directed.

To make private financing sectors more interested to invest in transport projects, it is essential to provide more reliable forecasts or to represent at least the risk of
investment. In this light, it is required to provide a tool to measure forecast errors. Lemp and Kockelman (2009) mentioned that as the participation of the private sector in investing in the transport project increases, measuring the error associated with such projects becomes critical. Hartgen (2013) also expressed that the rapid increase in toll-based project financing as well as the natural risk of investing in this type of project for private investors has increased the requests for a more accurate traffic forecasts. In the case of providing a possible range of error for the projections, transportation planners, policy makers, and investors may choose a project with more certain benefits rather than one with a greater predicted benefit but along with a larger degree of error (Fagnant and Kockelman, 2012).

Without explicit statistical recognition of error in transportation forecasts, transportation planning takes an unnecessary risk (Zhao and Kockelman, 2002). de Jong et al. (2007) expressed that in order to make an informed decision on infrastructure projects, it is essential to estimate not only the most plausible outcome, but also the possible range of outcome variation. Deterministic forecasting procedures or point estimate methods that own no systematic methodologies to implement the error analysis are the main reasons of neglecting error in traffic forecasts (Yang et al., 2013; de Jong et al., 2007). The point estimate methods are not able to provide a confidence interval for the model forecasts even by producing several scenarios. Therefore, it is important to provide a comprehensive framework to quantify the variance of errors generated by the current transportation modelling approaches.

1-3-Source of Traffic Forecast Errors

Sources of error are classified differently by different studies. In the literature, the significant sources of error are classified into two error groups: measurement and specification errors (Zhao and Kockelman, 2002; de Jong et al., 2007; Fagnant and Kockelman, 2012; Yang et al., 2013; Manzo, Nielsen, and Prato, 2015). In the present study, the effect of the measurement error as well as the model specification error is taken into consideration. Although other classification have also been suggested, they share the fact that propagation of all error types should be investigated in a systematic way.

1-3-1-Input Measurement Error

The measurement error includes errors in forecasting and projecting the values of input variables (Stopher and Meyburg, 1975; Rodier and Johnston, 2002; Rasouli and
Based on a general definition provided by Stopher and Meyburg (1975), measurement error comes from inaccuracy in assessing a magnitude. More specifically, Rodier and Johnston (2002) expressed that the measurement error includes error in estimating the value of an input variable, which usually arises from sampling error. Rasouli and Timmermans (2012) also added survey design error, and incomplete information, to the main reasons measurement error occurs in the inputs.

Inaccurate projection of transportation demand model inputs could be considered as an example of incomplete information of variables. Hartgen (2013) has listed a series of possible technical reasons of this type of inaccuracy such as inaccurate demographic forecasts, rising vehicle availability to more households, changing lifestyles, and unverified assumptions about the stability of household travel relationships. Mackie and Preston (1998) explained that because of budgetary limitations on the data collection and modelling costs, study areas are quite tightly defined that may lead to input measurement errors. In this light, as data are so costly to be collected, transportation studies are conducted often based on outdated data that commonly result in a huge input measurement error.

Regarding the relative contribution of input measurement errors to an output error, Rodier and Johnston (2002) and Harvey and Deakin (1995) demonstrated that errors in socioeconomic forecasts and other input data are the greatest sources of error in travel demand models. According to MacKinder and Evans (1981), the main part of the prediction error of conventional aggregate models comes from the prediction errors of exogenous input variables. In the same way, Flyvbjerg et al. (2005, 2006) associated much of the model output error to the input measurement errors. In the US federal government guidelines, FHWA (2010b) identified the socioeconomic forecasts, and particularly population and employment forecasts, and housing trends and costs as the substantial error sources.

1-3-2-Specification Error

The specification error occurs when a model fails to take into account the effect of key explanatory variables (Mackie and Preston, 1998). Tadi and Khasnabis (1990) pointed out more causes for the specification error such as lack of understanding and simplification of the relationship between variables involved in the model.

Rodier and Johnson (2002) recognized the specification error as one of the significant contributors to the forecasting error. Specification error comes from a failure to
identify the true model or a simplification of the true model. Tadi and Khasnabis (1990) pointed out some causes for specification error such as lack of understanding or a deliberate simplification of the relationship between the variables contained in the model. A simple instance is the representation of a nonlinear relationship by a linear equation. In other words, Mackie and Preston (1998) explained that the specification error occurs when a model fails to take into account the impact of key explanatory variables or when a model misspecifies the effect of an explanatory variable.

As discussed by many researchers (e.g., Antoniou et al., 2011; Rodier, 2005; Schiffer et al., 2005), another possible reason to have an inaccurate traffic forecast might be an induced travel demand. The induced or latent travel demand is that part of demand that cannot be captured under the current market conditions and probably appears under more favourable market situations. This part of travel demand might be shifted from other travelling options or be completely new additional demand added to the existing demand by the model. Generally speaking, the induced travel demand would be considered as a part of demand which may not be taken into account by all types of models due to existing specification errors.

Parthasarathi and Levinson (2010) stated that while research efforts have focused on improving the technical aspects of conventional FSTDMs, few studies have evaluated the conventional FSTDM accuracy by comparing the traffic forecasts with the actual traffic counts in a validation process. The point mentioned by Parthasarathi and Levinson (2010) is the main technique employed in the present study to find out the statistical characteristics of the specification error PDFs in the FSTDM steps.

The current study introduces a method to measure the variance of model specification error based on techniques regularly used in the model validation step. Since the variance of model specification error measured by the proposed method uses the information provided in the validation process, a realistic figure of the predictive power of the model is represented. This improves the common approach in the literature (Clay and Johnston, 2005, 2006; Zhao and Kockelman, 2002; Yang et al., 2013) in which a predetermined standard deviation of error (e.g. 10% of the mean value) is attributed to the calibrated parameters and may lead to an incorrect representation of the total model error.

1-3-3-Correlation between inputs/estimated parameters

The covariance between input measurement errors is a source of error that contributes to the variance of model output error depending on the sign and the magnitude of correlation.
Ignoring the existing covariance between the error sources causes an undocumented propagation of error to outputs. In this light, a positive covariance between the input measurement errors intensifies the output error variance; while a negative covariance reduces the output error variance. Flyvbjerg et al. (2005) mentioned that neglecting covariance between the input error sources is similar to assuming they are independent, as ignoring error in analysis is similar to considering a deterministic situation.

Krishnamurthy and Kockelman (2003) proposed a simulation method in which all possible covariance between parameter errors and input measurement errors are taken into account to provide a more realistic result. Assuming a univariate normal distribution for the input measurement errors ignores available covariance between the errors. According to de Jong et al. (2007), consideration of covariance between input measurement errors might be performed by drawing values from a multivariate distribution including all input variables. Furthermore, as Lemp and Kockelman (2009) stated, considering a multivariate distribution between the errors usually lead to a wider bound for the output error rather than when a univariate input or parameter error distribution is considered.

1-3-4-Available methods to represent error

Pradhan and Kockelman (2002) expressed that the most popular methods to represent errors are fuzzy theory, interval mathematics, and probabilistic analysis. According to Pradhan and Kockelman (2002), fuzzy theory is employed to represent errors in a continuous format rather a set of discrete values. In this way, de Jong et al. (2007) stated that the use of fuzzy theory for traffic forecast errors is not applicable as the main reason to have erroneous traffic forecast is the inability to know the exact value of the traffic forecasts, not the definition format of the traffic forecasts. On the other hand, the interval mathematics method estimates an upper and a lower limit on model forecasts that contains limited information about an output error. However, in the probabilistic method, error associated to inputs, parameters and outputs can be described by a statistical measure related to the error Probability Distribution Function (PDF), such as mean, variance, and Coefficient of Variation (CV), that provides more comprehensive information about the error characteristics.

In this way, several practical measures to quantify error in inputs or outputs were identified in the literature (e.g., Bendtsen, 1975; de Jong et al., 2007), such as the difference between observed and predicted mean value, root mean square error, standard deviation of forecast error, variance of forecast error, coefficient of multiple correlation,
coefficient of multiple determination, and 95-percent limit for deviation. Among these measures, standard deviation and variance of error were widely used in the literature (e.g., Tadi and Khasnabis, 1990; Clay and Johnston, 2005; Zhao and Kockelman, 2002) to represent error.

Among the available methods to represent error, in the present study, error is defined as the difference between estimations and corresponding actual values. A PDF can be drawn based on observed errors. The variance of the drawn PDF is considered as the variance of error. In each FSTDM step, the current study derives a formula that calculates variance of output error based on the variance of input and model specification errors. Using the derived formulas, the variance of input and model specification errors is propagated toward outputs called as error propagation in this thesis.

1-4-Error in the Four-Step Transportation Demand Model (FSTDM)

It has been demonstrated by Pradhan and Kockelman (2002) and Krishnamurthy and Kockelman (2003) that various modelling frameworks propagate error differently and are affected differently by errors in inputs and model specification. As indicated by Pradhan and Kockelman (2002) and Rodier et al. (2002), in a sequential model, the main part of variance of resulting error in traffic forecasts comes from the propagated error from the previous steps; hence, in order to have a realistic evaluation about the variance of resulting error, it is required to consider all steps of a sequential model together to correctly estimate the variance of final output error.

One of the most popular approaches to model travel demand is the Four-Step Transportation Demand Model (FSTDM) that includes four steps: trip generation, trip distribution, modal split, and trip assignment. There exists evidence in the literature concerning the inaccuracy and validity of the FSTDM. For example, Hartgen (2013) stated that while the FSTDM is generally adequate to analyse major investment proposals, the FSTDM is not suitable for an accurate financial analysis that is needed for a toll road evaluation.

As another issue discussed in section (1-3-2), the inability of FSTDM to consider induced travel demand leads to the model specification error that is one of the main reasons of FSTDM traffic forecast errors. Schiffer et al. (2005) reviewed a large number of studies on modelling induced demand and concluded that conventional FSTDMs do not completely address induced travel demand. Similarly, Rodier (2005) stated that the
conventional FSTDMs do not adequately consider induced demands and hence tend to overestimate the benefits of new highway projects.

Additionally, according to Hartgen (2013), due to fundamental weaknesses, FSTDMs are not able to provide detailed outputs to distinguish a dominant policy among a series of policies with high competition which have only a marginal output differences on average. Hartgen (2013) expressed that recent developments to the FSTDMs have added new complexity without providing reliable methods for verification. Generally speaking, the new improvements first need to be completely established and verified to prevent the new FSTDMs from contributing a new source of error to outputs. In this light, error propagation through a basic FSTDM is investigated in the current study to find the largest contributing sources of error in the basic FSTDM; however, the framework of the current study is also applicable to more advanced forms of the FSTDM. The outcomes of error propagation in the basic FSTDM can be used to show the FSTDM weaknesses and to recognize the most effective treatments and improvements to the FSTDM.

When measuring the error propagated by a travel demand model, someone may ask what range of output error is considered acceptable for the traffic forecasts. Different guidelines have introduced different acceptable ranges for having a credible traffic forecast. According to FHWA (2010a), an acceptable range for the variance of output error should be determined specifically for each model based on its application; the required accuracy for a calibrated model is strongly dependent on the possible use of the model. As an example, if a model is employed to compare several competing alternatives, a close match between the model estimates and the corresponding observations is needed.

In the relevant literature, guidelines have not proposed an acceptable range of output error for all types of travel demand models, including all steps of a FSTDM. Among the available guidelines, FHWA (2010a) presented acceptable ranges for different measures in the trip distribution step. For instance, the modelled average trip lengths are proposed to be within 5% of observed values, and as another measure, the estimated intrazonal trip percentages need to be within 3% of observed values.

FHWA (2010a) also suggests the target accuracies in the traffic assignment step. The general target values for percent differences in estimated and actual volumes are proposed at 5% and 10% respectively for screen-lines and cut-lines in Michigan. Additionally, FHWA (2010a) provided acceptable/preferable percent of differences between estimated and observed vehicle miles travelled in traffic assignment for the US states including Ohio, Florida and Michigan. The percent differences are proposed based on the road functionality and area type. For example, the acceptable percent differences
for principle arterials are ±10%, ±15% and ±7% in Ohio, Florida and Michigan, respectively.

As a guideline for Australia about the acceptable/desirable accuracy during model validation, ATAP (2016) has provided some recommendations on the traffic assignment output error. For example, ATAP (2016) mentions that it is desirable to achieve a GEH of less than 5 in 90% and 75% of street links located respectively inside and outside a project study area. Moreover, the guideline has proposed a reasonable margin of ±20% and ±10% for hourly traffic volumes on regular street links and screen-lines, respectively.

1-5-Error Propagation Methods

In the literature, the most well-known methods to quantify propagation of error from inputs to outputs are Sensitivity Analysis (SA), moment, analytical, and Model Validation (MV) methods. The major deficiency of these methods is the inability of the methods to correctly measure the propagation of the error that is introduced only by the inputs or the model specification. In most of the employed methods, the variation of output caused by the variation of inputs is considered as the propagation of the error generated only by the inputs. However, due to the imperfection of the models, all estimations from the models involve a model specification error that is ignored in a typical error propagation method. Indeed, the variance of error measured as the propagated error of the inputs is mixed with the model specification error.

A description of the employed methods as well as the advantages and the disadvantages of the methods are discussed in the following sections.

1-5-1-Sensitivity Analysis (SA) Methods

SA measures the contribution of input errors to the output error variance. There exist several SA methods such as scenario-based, set-based, and more generally sampling-based methods.

In the set-based and the scenario-based SA methods, a limited number of discrete values are considered as the most probable values of input variables, and the corresponding outputs are calculated. In the sampling-based methods, a random sample from the distribution of inputs, and successive model runs are taken until a statistically significant distribution for outputs is obtained. Compared to the set-based and the scenario-based SA methods, the sampling-based method can extract an unlimited number of values from the input PDF.
Bonsall (1977) is one of the first studies that proposed a systematic approach to measure the sensitivity of output forecasts to the inputs through a SA method without specifying any particular PDF for the input measurement errors. The set-based or scenario-based methods are the simplest and the most practical methods among the SA methods. NCHRP (2010) suggested the definition of scenarios as optimistic, average, and pessimistic to deal with variations rooted in input measurement errors. SA methods might be also implemented to measure error coming from the model specification. In this light, Hugosson (2005) expressed that the model specification error can be quantified through implementing an SA method to compare the elasticity of the model output for different model specifications.

Univariate SA methods take into consideration the variation of only a single input. In addition to univariate SA methods, there exist more accurate and sophisticated multivariate SA methods that consider variation of several inputs simultaneously. In this way, Krishnamurthy and Kockelman (2003) introduced a multivariate SA method to examine the effect of input errors on the output error. The multivariate SA method is also able to measure the sensitivity of output to changes in one input while controlling for the variations of other inputs. In a multivariate SA method, as one of the most critical problems, it is essential to take a realistic value for the correlation between the input error sources.

In terms of differences between a typical SA method and a SA method that is conducted for an error analysis, Clay and Johnston (2005) provided a distinction based on the number of input variables for which the marginal effect on the output is calculated. Clay and Johnston (2005) believed that in a typical SA method, all available input variables and estimated parameters are employed without considering any initial hypothesis, while in a SA-based error analysis method, only a subset of specific input variables or estimated parameters that have the most contributions to the error are selected. However, it is always unknown which inputs have the most contribution before implementing an initial SA.

In the SA methods conducted for error analysis, a common strategy to find the greatest contributing inputs is to make a regression analysis between the output and the inputs. For instance, Zhao and Kockelman (2002) pointed out that a simple regression of the output on the inputs provides a very high predictive power to identify the main sources of forecast errors. Similarly, Fagnant and Kockelman (2012) identified the key inputs by regressing outputs on the set of input variables. There also exist other methods to find inputs with the largest contributions. For example, Pell (1984) proposed two thresholds to
find out the most important error sources: the sensitivity of traffic forecasts to the input errors, as measured by the elasticity, and the magnitude of traffic forecast errors, as measured by CV.

In the set-based and the scenario-based SA methods, as the most practical types of SA methods, a limited number of discrete values are considered as the most probable values of erroneous input variables. According to de Jong et al. (2007), in the scenario-based SA method, due mainly to not specifying a probability for the scenarios, it is not possible to estimate the PDF or any other statistical measures of the output errors. Similarly, Lemp and Kockelman (2009) and Kriger et al. (2006) expressed that the scenario-based SA methods are only able to determine a lower and an upper bound on model outputs, but certainly not a PDF for the output error. Moreover, according to Lemp and Kockelman (2009), as one of the main drawbacks of the scenario-based SA method, the number of defined scenarios is typically constrained to three or four.

1-5-2-Sampling-based Methods

The most common method to quantify the output error is a sampling-based method (de Jong et al., 2007). Pradhan and Kockelman (2002) described that sampling-based methods involve random sampling from the distribution of inputs and/or estimated parameters, and successive model runs are performed until statistically significant distributions for outputs are obtained. Compared to the set-based and the scenario-based SA methods, the sampling-based method can obtain a more realistic result of the output error through extracting an unlimited number of values from the input error PDF. According to Clay and Johnston (2006), the input and parameter combinations in the sampling-based methods are varied to reflect the plausible amount of error variance considered for the input variables and the estimated parameters.

The sampling-based methods require many simulations leading to great computational effort and time (Zhao and Kockelman, 2002; Lemp and Kockelman, 2009). Moreover, such techniques require assumptions about the input error PDF and the covariance of inputs, which are often unknown (Lemp and Kockelman 2009; Clay and Johnston, 2006). On the other hand, Krishnamurthy and Kockelman (2003) state that although sampling-based methods require substantial computer run time, they produce more accurate and realistic results compared to other methods. As a negative point, Yang et al. (2013) mentioned that it is always unclear how many samples are sufficient to conduct an error analysis with sampling-based methods.
Based on strategies implemented to draw values from the PDF of the input errors or to select the suitable outputs from the collected outputs, there exist some specific sampling-based methods such as Monte Carlo Simulation (MCS), Jackknifing, Bootstrap, Bayesian Melding, and Antithetic Sampling methods. In the current study, sampling-based methods, with the exception of the MCS method, are categorized as “other sampling-based methods”.

1-5-2-1-Monte Carlo Simulation (MCS) Method

The most widely used method among the sampling-based methods to quantify the error propagation is the Monte Carlo Simulation (MCS) (de Jong et al., 2007). Many researchers (e.g. Ashley, 1980; Lowe et al., 1982; Lam and Tam, 1998; Boyce and Bright, 2003; Zhao and Kockelman, 2002; Hugosson, 2005) examined the sensitivity of traffic volume forecasts to the model inputs and parameters using MCS. MCS, as a more sophisticated SA method, considers a PDF for the measurement error of each input variable. Sampling values from the PDF of the input errors, and running the model for each sample, it becomes possible to find the variance or the PDF of the model outputs collected across the samples. The variance or the PDF obtained from the collected outputs is interpreted as the statistical characteristics of the output error. Zhao and Kockelman (2002) and Pradhan and Kockelman (2002) employed this MCS method to provide a relationship, linear or nonlinear, between the variance of resulting output error and the input error variances.

One of the most important features of the MCS method is the possibility of simulating error from a variety of sources simultaneously, along with considering the existing correlations in complex urban systems (Pradhan and Kockelman, 2002). It was mentioned by Krishnamurthy and Kockelman (2003) that the MCS method is able to draw input values from multivariate distributions which are more accurate than a univariate distribution. Drawing values for input variables from a specified set of scenarios instead of a predetermined PDF is also possible as expressed by Krishnamurthy and Kockelman (2003).

1-5-2-2-Other Sampling-based Methods

Among the available sampling-based methods, there exist methods employing different strategies compared to the MCS methods in drawing values from the PDFs of the input errors or in choosing desirable outputs. These are discussed as ‘other’ sampling-based methods. For instance, according to Hugosson (2005), the jackknifing method estimates
the statistical characteristics of model output using statistical information obtained from different samples extracted from a single reference output database. The idea basically developed by Quenouille (1956) is to use jackknifing to reduce the bias of an estimator. Armoogum (2003) also employed jackknifing to create different samples to estimate the variance of the forecast of trip frequency and daily distance travelled. Additionally, according to Sarndal et al. (1992), the jackknife technique provides the possibility to estimate confidence intervals.

In comparison with the jackknifing method, the bootstrap method is employed to draw randomly with replacement from the reference output database to calculate the statistical characteristics of the model output errors (Efron and Tibshirani, 1993; Hjorth, 1994). The bootstrap method was also employed by Hugosson (2005) to obtain statistical measures of outputs produced by the forecasting system and to estimate the sampling error of a travel forecasting system.

As other developed sampling-based methods, Bayesian melding and antithetic sampling methods were employed respectively by Sevcikova et al. (2007) and Kockelman et al. (2008). These methods are able to sample thoughtfully and perform estimation rapidly. As an example, Sevcikova et al. (2007) developed a Bayesian melding method to measure errors associated to any outputs using urban simulation models. According to Sevcikova et al. (2007), the Bayesian melding method considers all available information about model inputs and outputs including PDFs and likelihoods, and uses Bayes’ theorem to obtain the resulting PDF of any function of model inputs and outputs. Lemp and Kockelman (2009) expressed that developed sampling-based methods such as Bayesian melding and antithetic sampling are helpful to obtain output error PDFs in complex models with a relatively higher speed compared to the basic MCS methods.

1-5-3-Moment and Model Validation (MV) Methods

As another well-known way to quantify the forecasting error, Krishnamurthy and Kockelman (2003) explained the method of moments in which a Taylor-series expansion is employed to approximate the first and the second moments of output (mean and variance) or even higher-order moments. In the method of moments, it is required to specify the output as an explicit function of the inputs; however, it is always a challenge to derive a closed-form and direct relationship between inputs and output to quantify the generated error.
The MV method compares what is estimated by a model for a certain period of time with what is observed in the same period. This type of comparison might assist researchers to provide an estimate of the error contributed by the model specification itself. In this way, Rodier (2007) considered the MV method to quantify error by comparing the model forecasts with the observed data that is not used in the model calibration process. Rodier (2007) also suggested the MV method as a valuable complement to other SA methods. In the same way, Armoogum (2003) made a regression between the survey observations and the model estimates at the finest level to compare the observations with the result of estimations to provide a measure of model performance. Rodier (2005) also conducted a study illustrating how validation tests can be used to quantify the prediction capabilities of a model. In the current study, the model specification errors of FSTDM steps are measured comparing observations of a calibration dataset with the corresponding model estimations to find the statistical characteristics of the model specification errors.

1-5-4-Analytical Methods

In contrast to the MCS methods, analytical methods require relatively less computational effort and time. Moreover, in the case of the error originating simultaneously from different sources, the MCS method compounds the specific share of error sources (measurement and specification errors) with the share of their interaction when measuring the final output error. In this light, the MCS method is not able to determine the shares of the sources and their interactions in a single MCS run. As the main advantage, employing the analytical method enables one to quantify separately and collectively the share of errors originating simultaneously from the inputs besides the model and to capture the share of their interactions in the final output error.

According to de Jong et al. (2007) and Yu (2013), the challenge of obtaining a direct relationship between the variance of input errors and the output error variance in the error propagation process is a serious issue when an analytical method is employed. de Jong et al. (2007) believed that only if the relationship is relatively straightforward, an analytical method can be utilized. Among the recent studies, Yang et al. (2013) used the results of a sensitivity analysis within an analytical method to develop a systematic framework for quantitative error analysis of a sequential travel demand model. However, the proposed approach by Yang et al. (2013) has only been tested over a small range of input variation due to using the result of a local sensitivity analysis.
The present study for the first time investigates an analytical method for error propagation in a complete FSTDM model with all four steps included. The conventional models for trip generation, trip distribution, modal split, and traffic assignment are considered in the FSTDM process.

1-5-5-Analytical Sensitivity-based Error Propagation in UETAs

There exist other types of traffic assignments such as a User Equilibrium Traffic Assignment (UETA) that can be considered as the fourth step of a typical FSTDM model. In a UETA model, the only available input variable that contributes to the variance of traffic assignment output error would be the origin-to-destination (O-D) flows (demands).

Analytical propagation of O-D demand measurement errors in a UETA can be undertaken using the results of an SA method implemented on the UETA model. The main issue in the SA of an UETA is to find an accurate approximation of the new optimal solution under O-D demand measurement errors. Early studies on the SA of nonlinear programs and variational inequalities (e.g., Fiacco, 1976, 1983; Tobin, 1986; Dafermos, 1988) made it possible to propose methods for the SA on UETA. However, the direct application of these proposed methods onto UETA is not feasible since the path flows resulting from UETA programs are not unique (Tobin and Friesz, 1988; Cho et al., 2000). The uniqueness of a solution is an essential condition to use the SA methods proposed in the literature on nonlinear programs.

Early on, Tobin and Friesz (1988) developed an SA method for UETA in which an equivalent restricted problem was provided that satisfied the required uniqueness conditions of the original problem. The restricted problem involved only those paths that were active and positive in a nondegenerate extreme solution point. The path flow rates of a nondegenerate extreme solution point are calculated using a linear programming formulation (Tobin and Friesz, 1988). Alternatively, Cho et al. (2000) proposed a reduction method employing a minimum-distance technique to select a unique equilibrium path flow vector. Yang and Bell (2007) and more recently Du et al. (2012) showed that under mild conditions, the equilibrium link flow derivatives can be obtained by the formula obtained from the implicit function theorem by constructing a restricted problem. Their method also provided a left and right derivative for the link flows when there is a degenerate and non-differentiable equilibrium point.

In a different way, Patriksson (2004) represented a method to conduct the SA of UETA using directional derivatives, gradients and sub-gradients. Other studies (e.g., Qiu
and Magnanti, 1989; Patriksson and Rockafellar, 2003) employed the same method to perform an SA. Josefsson and Patriksson (2007) then improved the Patriksson (2004) results by establishing a new method in which demand sensitivities are obtainable regardless of the directional differentiability of equilibrium link flows. To perform the SA of an UETA, Lu (2008) calculated semi-derivatives under general and weaker conditions and the derivatives under more restrictive conditions that is equivalent to the strict complementarity assumed in Cho et al. (2000) and Yang and Bell (2007). Lu (2008) calculated the semi-derivatives by solving a linear traffic user equilibrium problem and the derivatives by matrix multiplication together with the solution of a linear equation.

Chung et al. (2014) addressed some misconceptions relating to the application of the method proposed by Tobin and Friesz (1988) by introducing four regularity conditions: continuous differentiability, strong monotonicity, strict complementarity, and a non-degeneracy assumption, which need to be satisfied when applying the Tobin and Friesz (1988) method on any type of UETA. In this method, violation of the regularity conditions, especially the fourth condition, may result in incorrect outcomes. Regarding the fourth condition, Tobin and Friesz (1988) and Chung et al. (2014) assumed that the unperturbed equilibrium path flow rates need to correspond to a non-degenerate extreme point to avoid the issue of non-uniqueness in the path flows.

The method employed in section (2-2-3) to perform SA on the UETA is fundamentally based on Fiacco (1983). The employed method also takes into consideration the points of the SA method explained by Tobin and Friesz (1988) and the method and discussions provided by Chung et al. (2014).

1-6- Gaps in the Literature

After reviewing the relevant literature, the main gaps can be listed as follows:

- The methods employed in the literature are mostly simulations (of sampling-based type) which are considerably time-consuming. Providing an efficient method which is less time consuming and more practical is needed,
- There is a lack of an analytical method to quantify the separate share of model specification and input measurement errors in the outcomes,
- Obtaining a reliable and accurate estimate of output error variance from the sampling-based methods requires too many simulations. The number of simulations is never known in advance. Considering only a small number might lead to inaccurate results, while a large number can be computationally challenging and
time-consuming. Therefore, a more reliable method which is not dependent on the number of simulations is needed,

- Available approximation-based methods commonly simplify the relationship between inputs and output by employing different methods such as Taylor series expansion. Using this type of simplification might be suitable in some situations, while it may create biased results in complicated transport demand models. For this reason, providing a general approach which is not dependent on specific model context is required,

- There is always a big concern about the storage and the memory that computers need to run a complex model with a large number of simulations. Therefore, it is required to find an optimized method in terms of needed storage and memory on a computer, and

- Recent analytical methods like analytical sensitivity-based methods have a serious validity range problem due to the local application of SA results. This problem might lead to a limitation on the range of error variance that can be considered for the input variables. In this light, finding a generic analytical method without any short validity range is required.

In this study, the error of traffic forecasts is attributed to two main sources, the model specification error and the input variable measurement error. Transportation demand models currently lack a rigorous and analytic treatment to quantify the error propagation from different sources through the models. Since FSTDM is commonly used in practice but its error is not well-studied, the current study illustrates how the error variance of the input variables as well as the model specification are propagated analytically step by step and interact to result in an inaccuracy in the traffic forecasts.

The present study proposes an analytical approach to provide a solution for the abovementioned problems of simulation and approximation bases methods. This analytical approach is efficient and accurate in the error propagation, and is also able to quantify separately and collectively the share of different contributing sources of error to the variance of traffic forecast error. The proposed approach is also applied to the city of Brisbane as a real case study.
Chapter 2 Methodology

The purpose of this section is to propose a framework for propagation of error from inputs to the output in the prediction stage of an FSTDM as a sequential type of transportation demand model. The FSTDM models travel behaviour of travellers via mathematical formulas which relate the inputs to the outputs. The input variables include a variety of socioeconomic, land use, and transportation network attributes. The output variables in the FSTDMs are the number of trips generated, distributed, assigned to modes, and assigned to routes/links. The considered FSTDM model includes a linear regression and a gravity model for the trip generation and trip distribution steps respectively, and a logit model for both modal split and traffic assignment steps.

The error variance of forecasted demand is attributed to at least two main sources: the model specification error and the error in measuring the input variables. In the calibration stage, the difference between what has been observed and what is estimated by the model is considered as the residual. In the prediction stage, where the calibrated model is applied to forecast demand, this residual term results in the model specification error. In this study, the probability distribution function (PDF) of the model specification error is obtained from the calibration process.

Along with the model specification error, there is the measurement error of input variables. Input variables can be classified into two categories: internal and external. The internal input variables and the corresponding measurement errors are those projected by the FSTDM steps, while the external input variables are assumed to be sourced from other econometric models. The error variance of the input variables might be intensified or diminished by the FSTDM depending on the value of covariance between the measurement errors of the input variables and the type of mathematical operation such as addition/subtraction, multiplication, or exponentiation in which the input variables are involved. In the proposed approach, it is assumed that during the calibration process, the input variables are errorless and they are not exposed to any sampling error; hence, there is no error in the estimated parameters.

The proposed framework involves two main stages: measuring the variance of model specification error of an FSTDM and quantifying the error propagation through the FSTDM. First, a method to measure the variance of model specification error in each step of the FSTDM is proposed; then, the propagation of the input measurement error along with the model specification error through the steps of the FSTDM is investigated. Employing a UETA model for the traffic assignment step of FSTDMs is a common
alternative to a logit route choice model. In this light, an analytical method to propagate error through a UETA is also explained in this chapter. The definition of parameters in the following equations is provided in the list of abbreviations.

2-1-Measuring the Variance of Model Specification Error

The difference between what is revealed in reality and what is estimated by a model indicates that a model is unable to represent the reality perfectly which is the main cause for the model specification error in prediction. In other words, the difference is a residual value that cannot be explained by the model. The residual value for all observations during the calibration stage enables one to derive a PDF for the residual term. In the current study, the derived PDF for the residual term is considered the PDF of the model specification error.

The proposed approach adopts validation techniques to determine the PDF of the residual term. Comparing the actual values with those estimated by a model, the predictive power of the model is evaluated. The actual values may consist of an observation dataset or any externally supplied and independent datasets. One part of the calibration dataset is typically left unused in model development to be used for validation. Since no validation is required in an error propagation study, within the framework explained here, it is also possible to use the full dataset in a calibration process to better estimate the PDF of the residual term. In the present study, this second approach is followed as provided in Figure (2-1).
The calibration (Input/Output) dataset mentioned in Figure (2-1) is assumed errorless; therefore, in the model validation section, any difference between the estimated outputs and the actual outputs can be attributed to the specification error of the calibrated model. In the prediction stage, the input measurement errors including the covariance matrix of the errors are inserted into a calibrated model to quantify the propagated measurement error of the input variables. Then, the variance of model specification error from the calibration stage is added to the propagated measurement error to quantify the variance of total forecast error from the prediction stage.

According to Figure (2-1), the variance of model specification error is calculated using a given calibration dataset. This calibration dataset can also be the same as the one originally employed to calibrate models in the base year. The database used to measure the variance of model specification error might be obtained from expanding a sample observation database. The database expansion definitely creates an inherent sampling error in the resulting database.

On the other hand, a model always has a specification error regardless of the size of the dataset sampling error. Since the model specification error is defined as the inability of a model to represent the observed outputs given in the calibration dataset, the size of
sampling error does not influence the size of model specification error. The statistical characteristics of a model specification error are only dependent on the calibration dataset characteristics and the model performance. Hence, the sample size of inputs and outputs in the calibration dataset has no effect on the specification error of a calibrated model.

According to Figure (2-1), the measurement errors in the input variables, like population and employment forecast errors, are taken into account during the prediction stage where the variance of input measurement errors are propagated to the outputs.

2-1-1-Statistical Characteristics of Model Specification Error

The PDF of the residual term depends on the initial assumptions of a given estimation method. For instance, in a linear or non-linear Ordinary Least Square (OLS) regression method, it is commonly assumed that the residual term follows a normal distribution. This normal distribution has a mean of zero and the variance is related to the goodness-of-fit measure (e.g. R-squared). Eq. (2-1) shows how the variance of the residual term is calculated using the R-squared measure and the variance of the dependent variable over actual observations.

\[
R^2 = 1 - \frac{\text{var}(e)}{\text{var}(y)} \cdot \frac{n - p - 1}{n - 1}
\] (2-1)

Alternatively, the PDF or the statistical characteristics of the residual term can be obtained by comparing the actual values with the estimated ones in the calibration stage. This approach can be employed for all types of models and for all types of estimation methods. The difference between the actual value and the corresponding estimation gives the residual value with a distribution over all observations used in the calibration process. By this approach, the statistical characteristics of the residual term are obtainable without having any knowledge about the goodness-of-fit measure. In the case that both above mentioned approaches are applicable, a similar result is achieved. In the current study, for a linear model, the variance of the residual term is calculated using the R-squared measure, while for other types of the models, the latter approach is employed.

2-2-Error Propagation in the Prediction Stage

This section investigates how the measurement error of input variables is propagated towards the output in the prediction stage. The measurement error variance of an input variable may be intensified or diminished through a model, depending on the type of
mathematical operation (addition/subtraction, multiplication, or exponentiation) in which the input variable is involved. The compound effect of the propagated measurement error of input variables and the model specification error on final outputs is discussed in the following.

To find how much the measurement error of input variables and the model specification error contribute to the output error variance, we drive a direct relationship between the inputs and the output based on the calibration stage. By taking the variance of both sides of the calibrated model, a relationship between the expected value and variance of the input variables as well as the variance of the residual term is established. In the prediction step, we replace the variances of the input variables and the residual term with the corresponding input variable measurement error variances and the model specification error variance, respectively, to obtain a formula for the propagation of error.

2-2-1-Correlation Coefficient

A challenging point in the proposed approach is to capture the correlation between different sources of error in the prediction stage. There exist two alternatives: the first is to assume that the correlation between the measurement errors of the input variables and the model specification error in the prediction stage is the same as what is observed for the correlation of the input variables and the residual term during the calibration stage. As a second alternative, the correlation coefficients may be either given or reasonably assumed for the prediction stage.

To make reasonable assumptions about the correlation values, it is helpful to consider the properties of the estimation method employed in the model calibration stage. For instance, in OLS, the correlation between the residual term and the independent (input) variables in a linear model is considered to be zero. Accordingly, in a nonlinear model, the correlation between the residual term and the whole calibrated model as one term is also considered to be zero. This property satisfies the exogeneity condition in the OLS regression method (Greene, 2012). Hence, the covariance between the measurement errors of the input variables and the model specification error in the prediction stage can be similarly considered zero.

In the prediction stage, input variables may be determined using different models. In the case of using independent models, the resulting measurement errors of the input variables are uncorrelated. However, the proposed framework is also able to take into account any possible correlation between different sources of error.
2-2-2-Propagation of Error in the Prediction Stage of FSTDM

The FSTDM conventionally includes four sequential sub-models: trip generation, trip distribution, modal split and traffic assignment. It is possible to obtain a closed-form formula to relate the outputs directly to the inputs, in the case of using, respectively, a linear/nonlinear regression for trip generation, a gravity model for trip distribution, and a logit model for modal split and traffic assignment. In the prediction stage, the error propagation from internal and external input variables through the FSTDM is schematically presented in Figure (2-2).

Figure (2-2). Schematic Figure of Error Propagation in the Prediction Stage of an FSTDM

2-2-2-1-Propagation of Error in the Prediction Stage of Trip Generation

The number of generated trips consists of produced and attracted trips from and to traffic zones. The trip generation can be modelled as a linear mathematical equation in which the number of generated trips is a function of socio-economic and land use attributes. Two typical models for trip production and trip attraction are presented respectively in Eq. (2-2) and Eq. (2-3). With the R-squared values, and the variance of the dependent variable in the population, the variance of the residual term can be calculated using Eq. (2-1).
Taking the variance of both sides of the calibrated linear models in Eq. (2-2) and Eq. (2-3), a formula to propagate error from the inputs to the output is created. As an example, the formula for the trip production model is presented in Eq. (2-4).

\[
\text{var}(T_O(z)) = \sum_{i=1}^{s_O} a_i^2 \cdot \text{var}(x_{Oi}(z)) + \sum_{i=1}^{s_O} \sum_{j \neq i} a_i \cdot a_j \cdot \text{cov}(x_{Oi}(z), x_{Oj}(z)) + 2 \cdot \sum_{i=1}^{s_O} a_i \cdot \text{cov}(x_{Oi}(z), \epsilon_O) \quad \forall z \in Z
\]

Eq. (2-4) can be also used to compute the output error variance created by the model specification error and the input variable measurement errors in the prediction stage. In the prediction stage, the variance of the model specification error, \(\text{var}(\eta)\), is considered equal to the variance of the residual term, \(\text{var}(\epsilon)\), in the calibration stage. If we assume no correlation between the measurement errors of the external input variables and the model specification error, Eq. (2-5) and Eq. (2-6) are derived respectively to propagate error for the trip production and the trip attraction models in traffic zone \(z\).

\[
\text{var}(\xi_{T_O(z)}) = \sum_{i=1}^{s_O} a_i^2 \cdot \text{var}(\xi_{x_{Oi}(z)}) + \sum_{i=1}^{s_O} \sum_{j \neq i} a_i \cdot a_j \cdot \text{cov}(\xi_{x_{Oi}(z)}, \xi_{x_{Oj}(z)}) \quad (2 - 5)
\]

\[
\text{var}(\xi_{T_D(z)}) = \sum_{i=1}^{s_D} b_i^2 \cdot \text{var}(\xi_{x_{Di}(z)}) + \sum_{i=1}^{s_D} \sum_{j \neq i} b_i \cdot b_j \cdot \text{cov}(\xi_{x_{Di}(z)}, \xi_{x_{Dj}(z)}) \quad (2 - 6)
\]

In Eq. (2-5) and Eq. (2-6), the variance of resulting error for the number of trips produced and attracted in traffic zone \(z\) stems from two main sources: the model specification error, \(\text{var}(\eta)\), and the propagated measurement error of the external input variables. In this way, the variance of the residual term (\(\text{var}(\epsilon)\)) is considered as the model specification error in the output error, and the remaining terms are considered the propagation of the input variable measurement errors.
The number of produced or attracted trips is an internal input variable for the trip distribution model. It is assumed that there is no correlation between the measurement errors of the inputs and the model specification error, and also between the model specification errors of the trip production and of the trip attraction models. Therefore, the covariance between the errors of produced trips in zone $z$ and the number of attracted trips in zone $w$ is calculated using Eq. (2-7).

\[
\text{cov}(\xi_{T_D(z)}, \xi_{T_D(w)}) = \sum_{i=1}^{s_o} \sum_{j=1}^{s_p} a_i * b_j * \text{cov}(\xi_{xO_i(z)}, \xi_{xD_j(w)}) + \sum_{i=1}^{s_o} a_i * \text{cov}(\xi_{xO_i(z)}, \eta_D) + \sum_{j=1}^{s_p} b_j * \text{cov}(\eta_D, \xi_{xD_j(w)})
\]

\[+ \text{cov}(\eta_D, \eta_D) = \sum_{i=1}^{s_o} \sum_{m=1}^{s_p} a_i * b_j * \text{cov}(\xi_{xO_i(z)}, \xi_{xD_j(w)}) \quad \forall z, w \in Z \tag{2-7}\]

The covariance is used in the next section to measure the propagation of error in the trip distribution step.

2-2-2-2-Propagation of Error in the Prediction Stage of Trip Distribution

The well-known gravity model is considered for trip distribution (Ortuzar and Willumsen, 2011). This model takes the form in Eq. (2-8).

\[
T'_{00}(z, w) = A(z, w) * T_O(z) * T_D(w) * f(z, w) \quad \forall z, w \in Z \tag{2-8}
\]

The calibration of this model is subject to satisfying both the production- and the attraction-specific constraints. The constraints ensure the total numbers of produced and attracted trips in each traffic zone are equal. The PDF of the residual term in the trip distribution model could be determined by comparing the number of trips estimated by the model with the corresponding actual values for all O-D pairs during the calibration process. In the prediction stage, the specification error of the trip distribution model is considered identical to the residual term in the calibration stage. Considering the residual term, a modified model is presented in Eq. (2-9).

\[
T_{00}(z, w) = T'_{00}(z, w) + \varepsilon_{00}(z, w) \quad \forall z, w \in Z \tag{2-9}
\]

In the case of having the measurement error of the inputs and the specification error for the trip distribution model, the expected value of the distributed trips takes the form of Eq. (2-10) assuming no correlation between $\xi_{T_O(z)} * \xi_{T_D(w)}$ and $\xi_f(z, w)$.
\[ E[\xi_{T_{OD}(z,w)}] = A(z, w) \ast \left( E[\xi_{T_{O}(z)}] \ast E[\xi_{T_{D}(w)}] + \text{cov}(\xi_{T_{O}(z)}, \xi_{T_{D}(w)}) \right) \ast E[\xi_{f(z,w)}] + E[\eta_{OD}] \quad \forall z, w \in Z \quad (2 - 10) \]

The correlation between the model specification error and the input variable measurement error is assumed to be zero in Eq. (2-10). This assumption is similar to the assumption made for error propagation in the trip generation model.

For the error propagation in the prediction stage of trip distribution step, we take the variance of both sides of Eq. (2-8). The resulting formula is shown in Eq. (2-11).

\[
\begin{align*}
\text{var} \left( \xi_{T_{OD}(z,w)} \right) &= A^2(z,w) \ast \text{var}(\xi_{T_{O}(z)} \ast \xi_{T_{D}(w)} \ast \xi_{f(z,w)}) \\
&= A^2(z,w) \ast (E[\xi_{T_{O}(z)}^2] \ast E[\xi_{T_{D}(w)}] + \text{cov}(\xi_{T_{O}(z)}, \xi_{T_{D}(w)}) + 4 \\
&\ast E[\xi_{T_{O}(z)}] \ast E[\xi_{T_{D}(w)}] + \text{cov}(\xi_{T_{O}(z)}, \xi_{T_{D}(w)}) + \left( E[\xi_{f(z,w)}] + \text{var}(\xi_{f(z,w)}) \right) \\
&\ast \left( \left(E[\xi_{T_{O}(z)}] \ast E[\xi_{T_{D}(w)}] + \text{cov}(\xi_{T_{O}(z)}, \xi_{T_{D}(w)}) \right) \ast E[\xi_{f(z,w)}] \right) \ast E[\xi_{f(z,w)}]^2 \right) \quad \forall z, w \in Z \quad (2 - 11)
\end{align*}
\]

An expanded form of Eq. (2-11) is provided in Eq. (2-12), describing the propagation of error from the inputs toward the output in the prediction stage of the trip distribution model.

\[
\begin{align*}
\text{var} \left( \xi_{T_{OD}(z,w)} \right) &= A^2(z,w) \\
&\ast \left( \left(E[\xi_{T_{O}(z)}] + \text{var}(\xi_{T_{O}(z)}) \right) \ast \left(E[\xi_{T_{D}(w)}] + \text{var}(\xi_{T_{D}(w)}) \right) + 2 \ast \text{cov}^2(\xi_{T_{O}(z)}, \xi_{T_{D}(w)}) + 4 \\
&\ast E[\xi_{T_{O}(z)}] \ast E[\xi_{T_{D}(w)}] + \text{cov}(\xi_{T_{O}(z)}, \xi_{T_{D}(w)}) + \left( E[\xi_{f(z,w)}] + \text{var}(\xi_{f(z,w)}) \right) \\
&\ast \left( \left(E[\xi_{T_{O}(z)}] \ast E[\xi_{T_{D}(w)}] + \text{cov}(\xi_{T_{O}(z)}, \xi_{T_{D}(w)}) \right) \ast E[\xi_{f(z,w)}] \right) \ast E[\xi_{f(z,w)}]^2 \right) \quad \forall z, w \in Z \quad (2 - 12)
\end{align*}
\]

To consider the model specification error, the variance from both sides of Eq. (2-9) is taken, yielding Eq. (2-13).

\[
\begin{align*}
\text{var}(\xi_{T_{OD}(z,w)}) &= \text{var}(\xi_{T_{OD}(z,w)}) + \text{var}(\eta_{OD}) \quad \forall z, w \in Z \quad (2 - 13)
\end{align*}
\]

Again, the correlation between the model specification error and the input variable measurement error is assumed to be zero in Eq. (2-13).

**2-2-2-3-Propagation of Error in the Prediction Stage of Modal Split**

The third step of the FSTDM, modal split, is typically a well-known model in discrete choice modelling called the logit model. In the logit model, the probability of each alternative is a function of the utilities for those alternatives. The utility function of the alternatives might be a linear or a nonlinear function of the parameters. The exponential function of the utility realized from each alternative determines the share of that alternative among others, as shown in Eq. (2-14).


To estimate the parameters in Eq. (2-14), a Maximum Likelihood Estimation (MLE) method is employed. The residual term of the logit model in the MLE method is not considered as a parameter to be estimated. Therefore, for the residual term of each modal alternative, the estimations corresponding to that alternative should be first separated from all estimations; then, the PDF of the residual term can be found by comparing the actual probabilities with the ones estimated by the logit model.

In the calibration stage of a logit model, the residual term of utility functions is assumed to follow a Gumbel distribution. In the prediction stage, as long as the calibrated logit model is used to predict modal travel demands, the calibration assumptions are valid and the PDF of the input measurement errors do not violate the calibration assumptions. Therefore, during the prediction stage, there is no restriction on the PDF type of input measurement errors.

In the current study, three different modal alternatives (car, bus, and bike) are considered. The modal split step here contains two sub-steps: estimating the probability of the alternatives based on the logit model and multiplying the probability by the distributed travel demand calculated in the trip distribution step. To be consistent with the previous steps, the residual term is quantified in the logit model itself. Taking the residual term into consideration, the modified probability is shown in Eq. (2-15).

\[
P_{OD}(z, w) = P_{OD}^{'}(z, w) + \epsilon_{OD}^{'}(z, w) \quad \forall z, w \in Z
\]  

(2-15)

where \( P_{OD}^{'}(z, w) \) is calculated in Eq. (2-16)

\[
P_{OD}^{'}(z, w) = \frac{P_{OD}(z, w)}{e_{OD}^{'}(z, w) + e_{OD}(z, w) + e_{OD}(z, w)} \quad \forall z, w \in Z
\]  

(2-16)

The residuals generated for mode car in Eq. (2-15) should be first extracted from the residuals of other modes. Then, the statistical characteristics of the extracted residuals are calculated. The resulting distribution has no predetermined characteristics and differs case by case. It means, in contrast to the previous steps, the mean might be different from zero and the distribution different from a normal distribution. By considering alternative-specific constants for the utilities during the calibration stage, the average of alternative shares across O-D pairs weighted by the O-D demands should be equal to the market share that represents a mean error of zero for each alternative. In this light, calculating a
regular mean error for each alternative in the present study without considering the existing differences between the O-D demands might lead to a mean different from zero. By taking the variance of both sides of Eq. (2-15), a new formula is derived in Eq. (2-17).

\[
\text{var}(P_{OD}^{\text{Car}}(z,w)) = \text{var}(P_{OD}^{\text{Car}}(z,w)) + \text{var}(\xi_{OD}^{\text{Car}}) + 2 \cdot \text{cov}(P_{OD}^{\text{Car}}(z,w), \xi_{OD}^{\text{Car}}) \quad \forall z,w \in Z \tag{2-17}
\]

Eq. (2-17) is employed to quantify the magnitude of the variance of forecast error that is a combination of the variances of propagated measurement error of the input variables and the model specification error, as presented in Eq. (2-18).

\[
\text{var}(\xi_{P_{OD}^{\text{Car}}(z,w)}) = \text{var}(\xi_{P_{OD}^{\text{Car}}(z,w)}) + \text{var}(\eta_{OD}^{\text{Car}}) + 2 \cdot \text{cov}(\xi_{P_{OD}^{\text{Car}}(z,w)}, \eta_{OD}^{\text{Car}}) \quad \forall z,w \in Z \tag{2-18}
\]

In the prediction stage, the model specification error (\(\eta_{OD}^{\text{Car}}\)) and the propagated measurement error (\(\xi_{P_{OD}^{\text{Car}}(z,w)}\)) are assumed uncorrelated, and the resulting formula appears in Eq. (2-19). If such correlations are provided, the proposed approach is also able to take them into account.

\[
\text{var}(\xi_{P_{OD}^{\text{Car}}(z,w)}) = \text{var}(\xi_{P_{OD}^{\text{Car}}(z,w)}) + \text{var}(\eta_{OD}^{\text{Car}}) \quad \forall z,w \in Z \tag{2-19}
\]

A suitable formula to propagate the measurement error of the inputs to the output in the prediction stage is established by taking the variance of both sides of Eq. (2-16). The logit model contains a sum of the exponential functions and a division operation that make it a complex model. The complexity of the logit model makes it impossible to find a closed-form formula to analytically compute the propagation of error without using any approximation method. In this light, an approximation method that is based on the Taylor series expansion is used to derive a closed-form formula to calculate the variance and the expected value of the logit model.

**2-2-2-3-1-Propagation of Error in the Probability of Mode Car**

In the propagation of error via the logit model, there is a sum of exponential functions in the denominator of Eq. (2-16) that might be correlated. If the measurement errors of the input variables in the utility functions are all normally distributed, the exponent of these measurement errors in the utility functions is log-normally distributed. The expected value and the variance of the sum of the resulting log-normally distributed variables are calculated readily using the existing principles in statistics. In the next stage, to calculate the statistical characteristics of the logit model, an approximation method proposed by
Blumenfeld (2009) is employed. The approximation method uses the Taylor series expansion to provide a closed-form formula for the statistical calculations. Referring to Blumenfeld (2009), in the case of having two correlated random variables, x and y, the proposed formulas are provided in Eq. (2-20) and Eq. (2-21) to calculate respectively the expected value and the variance of the ratio of x and y. The provided formulas have no restriction on the PDF type of x and y variables. The approximations are obtained about $E[x]$ and $E[y]$ considering up to second order terms of the Taylor series expansion.

$$E \left[ \frac{x}{y} \right] = \left( \frac{E[x]}{E[y]} \right) \left( 1 + \frac{\text{var}(y)}{E^2[y]} \right) - \frac{\text{cov}(x,y)}{E^2[y]} \quad \forall z, w \in Z \quad (2-20)$$

$$\text{var} \left( \frac{x}{y} \right) = \left( \frac{E[x]}{E[y]} \right)^2 \frac{\text{var}(x)}{E^2[x]} + \left( \frac{E[x]}{E[y]} \right) \frac{\text{var}(y)}{E^2[y]} - \frac{2\text{cov}(x,y)}{E[x]E[y]} \quad \forall z, w \in Z \quad (2-21)$$

Following the method explained above, in the case of assuming a normal distribution for the measurement error of the input variables in Eq. (2-16), the expected value and the variance of $\xi_{P_{OD}^{Car}(z,w)}'$ are calculated explicitly using this approximation-based method. Calculating the statistical characteristics of $\eta_{OD}^{Car}$ as explained under Eq. (2-14), the expected value and the variance of $\xi_{P_{OD}^{Car}(z,w)}'$ are estimated using respectively Eq. (2-15) and Eq. (2-19). $\text{var} \left( \xi_{P_{OD}^{Car}(z,w)}' \right)$ contains the variance of propagated measurement error of the input variables as well as the variance of model specification error.

2-2-2-3-2-Propagation of Error to Travel Demand of Mode Car

After finding the probability of choosing mode car, the probability should be multiplied by $T_{OD}^*(z,w)$, the number of distributed trips from the previous step, as provided in Eq. (2-22).

$$T_{OD}^*(z,w) = P_{OD}^{Car}(z,w) \times T_{OD}(z,w) \quad \forall z, w \in Z \quad (2-22)$$

Assuming no correlation between the sources of error, the expected value of $\xi_{T_{OD}^{Car}(z,w)}$ is presented in Eq. (2-23).

$$E \left[ \xi_{T_{OD}^{Car}(z,w)} \right] = E \left[ \xi_{P_{OD}^{Car}(z,w)} \right] \times E \left[ \xi_{T_{OD}^{Car}(z,w)}^* \right] = E \left[ \xi_{P_{OD}^{Car}(z,w)} \right] \times E \left[ \xi_{T_{OD}^{Car}(z,w)}^* \right] + \text{cov} \left( \xi_{P_{OD}^{Car}(z,w)}, \xi_{T_{OD}^{Car}(z,w)}^* \right)$$

$$= E \left[ \xi_{P_{OD}^{Car}(z,w)} \right] \times E \left[ \xi_{T_{OD}^{Car}(z,w)}^* \right] \quad \forall z, w \in Z \quad (2-23)$$

By taking the variance of both sides of Eq. (2-22), a relationship between the variance of the demand of mode car and the variance of the input variables is shown in Eq. (2-24).
\[
\var(\tilde{T}_{OD}(z,w)) = \var(P_{OD}^{Car}(z,w) \ast T_{OD}^{'}(z,w)) = E\left[ P_{OD}^{Car}(z,w) \ast T_{OD}^{'}(z,w) \right] - E^2\left[ P_{OD}^{Car}(z,w) \ast T_{OD}^{'}(z,w) \right] \\
= \left( E\left[ P_{OD}^{Car}(z,w) \right] \ast E\left[ T_{OD}^{'}(z,w) \right] + \text{cov}\left( P_{OD}^{Car}(z,w), T_{OD}^{'}(z,w) \right) \right) \\
- \left( E\left[ P_{OD}^{Car}(z,w) \right] \ast E\left[ T_{OD}^{'}(z,w) \right] + \text{cov}\left( P_{OD}^{Car}(z,w), T_{OD}^{'}(z,w) \right) \right)^2 \quad \forall z, w \in Z \quad (2 - 24)
\]

The variables are replaced correspondingly by their measurement errors to derive a generic formula to propagate the error from the inputs towards the output. The proposed formula takes into account the correlation between different sources of error. In the case of having no information about the correlation between the input measurement errors, the proposed formula for the error propagation in the prediction stage of modal split reduces to Eq. (2-25), that ignores \( \text{cov}\left( P_{OD}^{Car}(z,w), T_{OD}^{'}(z,w) \right) \) and \( \text{cov}\left( P_{OD}^{Car}(z,w), T_{OD}^{'}(z,w) \right) \).

\[
\var(\tilde{T}_{OD}(z,w)) = \left( \var\left( P_{OD}^{Car}(z,w) \right) + E^2\left[ \tilde{T}_{OD}(z,w) \right] \right) \ast \left( \var\left( \tilde{T}_{OD}(z,w) \right) + E^2\left[ \tilde{T}_{OD}(z,w) \right] \right) - E^2\left[ \tilde{T}_{OD}(z,w) \right] \quad \forall z, w \in Z \\
(2 - 25)
\]

**2-2-2-4-Propagation of Error in the Prediction Stage of Traffic Assignment**

Traffic assignment, as the fourth step of the FSTDM, is implemented in two sub-steps, first by assigning traffic to routes and then by assigning traffic to links. For the first sub-step, a route-based traffic assignment (RTA) is employed that is similar to the model used for modal split, except that the method assigns traffic to competing routes instead of on competing modes. The main reason to employ the logit model to assign traffic is to find a formula that relates the traffic demand as the input to the flows of the routes as the outputs. The formula is essential to analytically quantify the propagation of error from the inputs to the outputs in the prediction stage of traffic assignment.

In most cases, the actual vehicle volumes are observed on the links to validate traffic assignment; accordingly, we also need to calculate the link volumes. In the second sub-step of the traffic assignment, the link flows are calculated from route flows based on the network layout. The distribution of the residual term in traffic assignment is extracted by comparing the actual link volumes with the estimated ones during the validation stage. The resulting distribution will be the distribution of the model specification error in the prediction stage.
2-2-2-4-1-Propagation of Error to Route Flows

The assumptions of the RTA in the fourth step of the FSTDM are similar to those in the modal split step. The competing options in traffic assignment involve the routes connecting the same O-D pairs. Similar to the mode choice, the main assumption in the traffic assignment is to consider a normal distribution for the input measurement errors.

The route utility in the RTA depends on the route travel time in most cases. The travel times of the routes connecting the same O-D pair, due to overlapping links, are dependent and consequently correlated. For instance, in the case of considering two parallel routes, the travel time of the routes can be defined as the sum of travel times on route-specific links and joint links between the routes. Mathematically, the travel times of two parallel routes, \( t_{r_i}^{Car}(z,w) \) and \( t_{r_j}^{Car}(z,w) \), are defined respectively as \( T_{r_i}(z,w) + T_{r_j}(z,w) \) and \( T_{r_i}(z,w) + T_{r_j}(z,w) \) in which \( T_{r_i}(z,w) \) and \( T_{r_j}(z,w) \) are identical. The expanded forms of \( T_{r_i}(z,w) \) and \( T_{r_j}(z,w) \) are shown respectively in Eq. (2-26) and Eq. (2-27), and indicated schematically in Figure (2-3).

\[
T_{r_i}(z,w) = t_{i_1}(z,w) + t_{i_2}(z,w) + \cdots + t_{i_{ul}}(z,w) \quad \forall z, w \in Z, \forall r_i \in R(z,w) \tag{2-26}
\]

\[
T_{r_j}(z,w) = t_{j_1}(z,w) + t_{j_2}(z,w) + \cdots + t_{j_{ul}}(z,w) \quad \forall z, w \in Z, \forall r_j \in R(z,w) \tag{2-27}
\]

![Figure (2-3). Structure of Parallel Routes Connecting Origin z to Destination w](image)

Considering routes \( r_i \) and \( r_j \), the covariance between the route travel times is shown in Eq. (2-28).
In traffic assignment, it is common to ignore the interaction between link performance functions. It means the travel time of a link is only dependent on the flow of that link (Sheffi, 1985). Therefore, it is assumed that the covariance between the link travel times in Eq. (2-28) is zero. Hence, as a general rule, the covariance between two route travel times is taken as equal to the sum of the variance of travel time on the joint links. In the same way, the covariance between the measurement errors of the route travel times is calculated as Eq. (2-29).

\[
\text{cov} \left( t_{ij}^{\text{car}}(x,w), t_{ij}^{\text{car}}(z,w) \right) = \text{cov} \left( T_{r_i}(z,w) + T_{r_j}^{f}(z,w), T_{r_j}(z,w) + T_{r_j}^{f}(z,w) \right) \\
= E \left[ \left( \sum_{m=1}^{L_i} (t_{im} - E[t_{im}]) + \sum_{n=1}^{L_j} (t_{jn} - E[t_{jn}]) \right) \right] * \left( \sum_{k=1}^{L_j} (t_{jk} - E[t_{jk}]) + \sum_{n=1}^{L_j} (t_{jn} - E[t_{jn}]) \right) \\
= \sum_{m=1}^{L_i} \sum_{k=1}^{L_j} \text{cov}(t_{im}, t_{jk}) + \sum_{n=1}^{L_j} \text{var}(t_{jn}) \\
\forall z, w \in Z, \forall r_i, r_j \in R(z,w) \quad (2-28)
\]

The resulting formula to assign traffic on route \( r_i \) is shown in Eq. (2-30). Compared to the modal split model that includes distinctive utility functions for the competing modes, the utility functions of competing routes are similar, having not only the same structure but also the same parameters.

\[
P_{r_i}^{\text{car}}(z,w) = \frac{e^{\theta + t_{ij}^{\text{car}}(z,w)}}{\sum_{j=1}^{R(z,w)} e^{\theta + t_{ij}^{\text{car}}(z,w)}} \\
\forall z, w \in Z \quad (2-30)
\]

Assuming the measurement error of the link travel time is normally distributed, the error in measuring the route travel time, and the product of the route travel time and any constant value are also normally distributed. Therefore, the exponential function of the route travel times is log-normally distributed. The expected value and the variance of the sum of the resulting log-normally distributed variables are calculated using the existing principles in statistics. Referring to Eq. (2-30), we again employ the approximation method used in modal split to find explicitly the statistical characteristics of the logit model in the RTA. The approximation method uses the Taylor series expansion to provide Eq. (2-20) and Eq. (2-21) respectively for the expected value and the variance of a ratio of two random variables. Following this method, the propagation of the measurement error of the
link travel times via the logit model, presented in Eq. (2-31), is analytically quantified in the prediction stage of the RTA.

\[
\xi_{\text{Car}}(z,w) = \frac{e^{\theta z \xi_{\text{Car}}(z,w)}}{\sum_{j=1}^{n} e^{\theta z \xi_{\text{Car}}(z,w)}} \quad \forall z, w \in Z
\]  

(2 – 31)

Concerning the magnitude of demand assigned to route \( r_i \), \( P_{r_i}^{\text{Car}}(z,w) \) is multiplied by the demand obtained for mode car in the modal split step, \( T_{OD}^{\text{Car}}(z,w) \), as shown in Eq. (2-32).

\[
Q_{r_i}^{\text{Car}}(z,w) = P_{r_i}^{\text{Car}}(z,w) \ast T_{OD}^{\text{Car}}(z,w) \quad \forall z, w \in Z
\]  

(2 – 32)

Using Eq. (2-32) and replacing the variables with the measurement errors, the expected value of \( \xi_{Q_{r_i}^{\text{Car}}(z,w)} \) is provided in Eq. (2-33).

\[
E \left[ \xi_{Q_{r_i}^{\text{Car}}(z,w)} \right] = E \left[ \xi_{P_{r_i}^{\text{Car}}(z,w)} \right] \ast E \left[ \xi_{T_{OD}^{\text{Car}}(z,w)} \right] \quad \forall z, w \in Z
\]  

(2 – 33)

By taking the variance of both sides of Eq. (2-32), a relationship between the variances of the input variables and the output is established. Replacing the variables with the corresponding measurement errors and without considering any correlation between \( \xi_{P_{r_i}^{\text{Car}}(z,w)} \) and \( \xi_{T_{OD}^{\text{Car}}(z,w)} \), a formula to propagate the measurement error of the inputs to the output is shown in Eq. (2-34).

\[
\text{var} \left( \xi_{Q_{r_i}^{\text{Car}}(z,w)} \right) = \left( \text{var} \left( \xi_{P_{r_i}^{\text{Car}}(z,w)} \right) \right) + \left( E^2 \left[ \xi_{P_{r_i}^{\text{Car}}(z,w)} \right] \right) \ast \left( \text{var} \left( \xi_{T_{OD}^{\text{Car}}(z,w)} \right) \right) + E^2 \left[ \xi_{P_{r_i}^{\text{Car}}(z,w)} \right] - E^2 \left[ \xi_{P_{r_i}^{\text{Car}}(z,w)} \right]\] \ast E^2 \left[ \xi_{T_{OD}^{\text{Car}}(z,w)} \right] \quad \forall z, w \in Z
\]  

(2 – 34)

Ignoring the correlation between \( \xi_{P_{r_i}^{\text{Car}}(z,w)} \) and \( \xi_{T_{OD}^{\text{Car}}(z,w)} \) creates the simplest state of Eq. (2-34); however, in the case of existing a correlation between \( \xi_{P_{r_i}^{\text{Car}}(z,w)} \) and \( \xi_{T_{OD}^{\text{Car}}(z,w)} \), Eq. (2-34) can be updated to take the correlation into account.

2-2-2-4-2-Propagation of Error to Link Volumes

As the variance of model specification error of traffic assignment is measured on the links in the calibration stage, one should calculate the link volumes from route flows:

\[
\psi_{i}^{\text{Car}} = \sum_{w=1}^{Z} \sum_{z=1}^{Z} \sum_{m=1}^{\delta(z,w)} Q_{r_i}^{\text{Car}}(z,w) \ast \delta_{\nu_{m}}(z,w) \quad \forall i \in I
\]  

(2 – 35)
Taking the variance of both sides of Eq. (2-35) and replacing the variables with the corresponding measurement errors, a relationship between the variance of measurement errors of the link volume and the route flows is shown in Eq. (2-36). The variance of the error propagated to the link volume should be calculated by considering the existing correlation between the errors of the route flows as presented in Eq. (2-36). The covariance in Eq. (2-36) also includes the variance, when the selected routes are identical.

\[
\text{var} \left( \xi_{\text{Car}} \right) = \sum_{w=1}^{Z} \sum_{x=x}^{Z} \sum_{z=1}^{Z} \sum_{r_m}^{R(z,w)} \sum_{r_n}^{R(p,q)} \text{cov} \left( \xi_{\text{Car}m}^{(z,w)}, \xi_{\text{Car}n}^{(p,q)} \right) \ast \delta_{i,r_m}(z, w) \ast \delta_{i,r_n}(p, q) \quad \forall i \in I \quad (2-36)
\]

In order to calculate the covariance between the measurement errors of the route flows, one needs the covariance between the measurement errors of the probabilities of the routes. Eq. (2-37) shows how this covariance can be calculated using the probability defined by the logit model in Eq. (2-30).

\[
\text{cov} \left( \xi_{p_{r_m}^{\text{Car}}(z,w)}, \xi_{p_{r_n}^{\text{Car}}(p,q)} \right) = E \left[ \xi_{p_{r_m}^{\text{Car}}(z,w)} \ast \xi_{p_{r_n}^{\text{Car}}(p,q)} \right] - E \left[ \xi_{p_{r_m}^{\text{Car}}(z,w)} \right] \ast E \left[ \xi_{p_{r_n}^{\text{Car}}(p,q)} \right] \\
= E \left[ \frac{e^{\theta \left( \xi_{p_{r_m}^{\text{Car}}(z,w)} + \xi_{p_{r_n}^{\text{Car}}(p,q)} \right)}}{\sum_{j=1}^{R(z,w)} e^{\theta \left( \xi_{p_{r_m}^{\text{Car}}(z,w)} + \xi_{p_{r_n}^{\text{Car}}(p,q)} \right)}} \right] - E \left[ \frac{e^{\theta \xi_{p_{r_m}^{\text{Car}}(z,w)}}}{\sum_{j=1}^{R(z,w)} e^{\theta \xi_{p_{r_n}^{\text{Car}}(z,w)}}} \right] \\
= E \left[ \frac{e^{\theta \xi_{p_{r_n}^{\text{Car}}(p,q)}}}{\sum_{j=1}^{R(p,q)} e^{\theta \xi_{p_{r_n}^{\text{Car}}(p,q)}}} \right] \quad \forall z, w, p, q \in Z, \forall r_m \in R(z, w), \forall r_n \in R(p, q) \quad (2-37)
\]

The expected value of \( \xi_{p_{r_m}^{\text{Car}}(z,w)} \ast \xi_{p_{r_n}^{\text{Car}}(p,q)} \) in Eq. (2-37) takes into account the correlation between the measurement errors of the route travel times, and can be calculated approximately by following the discussion under section (2-2-2-3-1) and Eq. (2-20).

The covariance between the measurement errors of the route flows is presented in Eq. (2-38). As the covariance term is expanded, the measurement errors of the probabilities and the O-D demands also appear in this formula. Similar to the discussion under Eq. (2-32), there is no correlation assumed between the measurement errors of the probability and the O-D demand; therefore, the formula only depends on the expected values, \( \text{cov} \left( \xi_{p_{r_m}^{\text{Car}}(z,w)}, \xi_{p_{r_n}^{\text{Car}}(p,q)} \right) \) and \( \text{cov} \left( \xi_{p_{r_m}^{\text{Car}}(z,w)}, \xi_{p_{r_n}^{\text{Car}}(p,q)} \right) \).
Substituting Eq. (2-38) and Eq. (2-33) into Eq. (2-36), a formula is created to propagate the measurement error of the route flows into the link volumes. Since the variance of model specification error of the traffic assignment is quantified on the links, it is possible to add the model specification error variance \( \sigma^2_{\eta_i^{\text{car}}} \) to the propagated measurement error \( \sigma^2_{\hat{\xi}_{i^{\text{car}}}} \) of the link volumes, as in Eq. (2-39). The variance of model specification error of a traffic assignment can be quantified by comparing the actual link volumes with the corresponding volumes estimated by the model during the calibration process.

\[
\sigma^2_{\eta_i^{\text{car}}} = \sigma^2_{\hat{\xi}_{i^{\text{car}}}} + \sigma^2(\eta_i^{\text{car}}) \quad \forall i \in I \quad (2 - 39)
\]

In Eq. (2-39), it is assumed that there is no correlation between \( \eta_i^{\text{car}} \) and \( \hat{\xi}_{i^{\text{car}}} \). Therefore, Eq. (2-39) only contains the sum of the variances of the two sources of error. Ignoring the correlation between \( \eta_i^{\text{car}} \) and \( \hat{\xi}_{i^{\text{car}}} \) creates the simplest state of Eq. (2-39); however, in the case of existing a correlation between \( \eta_i^{\text{car}} \) and \( \hat{\xi}_{i^{\text{car}}} \), Eq. (2-39) can be updated to take the correlation into account. The expected value of the link volume can be also calculated using Eq. (2-40).

\[
E[\hat{\xi}_{i^{\text{car}}}] = E[\hat{\xi}_{i^{\text{car}}}] + E[\eta_i^{\text{car}}] \quad \forall i \in I \quad (2 - 40)
\]

It should be noted that \( \eta_i^{\text{car}} \) can also be calculated specifically for each type of network link. The propagation of error in the prediction stage of the FSTDM is briefly described in Figure (2-4). According to Figure (2-4), due to lack of observations in reality,
there is assumed no correlation between the propagated input measurement errors and
the model specification errors. However, in the case of having relevant observations to
measure this correlation, the correlation between these two error sources can be easily
included in the proposed framework.

Figure (2-4). Analytical Propagation of Error in the Prediction Stage of an FSTDM
2-2-3-Analytical Error Propagation through an UETA

This section indicates how the measurement error of O-D demand is propagated through a UETA program along with the specification error of traffic assignment using an analytical sensitivity-based error propagation method. The analytical sensitivity-based method includes two main parts: calculating the derivatives of the Karush-Kuhn-Tucker (KKT) parameters in the UETA program, including route flow rates, and propagating the O-D demand measurement errors to output using a Taylor series expansion.

2-2-3-1-Derivatives of KKT Triples

The mathematical program defined in the current study for an UETA is given in Eqs. (2-41) – (2-43).

\[
\text{Min} \sum_{i \in I} \int_{0}^{t_{i}^{\text{car}}} t_{i}^{\text{car}}(\alpha) d\alpha
\]

Subject to:

\[
h(z,w): \sum_{r_{m} \in R(z,w)} Q_{r_{m}}^{\text{Car}}(z,w) - T_{OD}^{\text{Car}}(z,w) = 0 \quad \forall z,w \in Z
\]

\[
g_{r_{m}}(z,w): Q_{r_{m}}^{\text{Car}}(z,w) \geq 0 \quad \forall z,w \in Z, \forall r_{m} \in R(z,w)
\]

The corresponding dual variables of Eq. (2-42) and Eq. (2-43) are \(\omega(z,w)\) and \(\mu_{r_{m}}(z,w)\), respectively. The link volumes subsequently are defined from the route flow rates using Eq. (2-35). To find the optimal solution of a mathematical program subject to the existing constraints, the Lagrangian of the mathematical program is determined in Eq. (2-44).

\[
L[Y(T_{OD}^{\text{Car}}), T_{OD}^{\text{Car}}] = \sum_{i} \int_{0}^{t_{i}^{\text{car}}} t_{i}^{\text{car}}(\alpha) d\alpha - \sum_{z,w,r_{m}} \mu_{r_{m}}(z,w) * Q_{r_{m}}^{\text{Car}}(z,w)
\]

\[
+ \sum_{z,w,r_{m}} \omega(z,w) * \left( \sum_{r_{m}} Q_{r_{m}}^{\text{Car}}(z,w) - T_{OD}^{\text{Car}}(z,w) \right)
\]

In the UETA program, \(T_{OD}^{\text{Car}}(z,w)\) is the only parameter that leads to a change in the KKT triples (route flow in addition to the dual variables). The KKT first-order necessary
conditions of Eqs. (2-41) – (2-43), Eqs. (2-45) – (2-47) are calculated which includes the gradient of the Lagrangian function.

\[
\nabla Q_{\text{Car}} * L \equiv \nabla Q_{\text{Car}} * \sum_{i=0}^{\nu^\text{car}} \int_0^{t^\text{car}_i} a(z) \, da + \nabla Q_{\text{Car}} * \sum_{z,w,r_m} \left( \omega(z,w) - \mu_{r_m}(z,w) \right) * Q^\text{Car}_{r_m}(z,w) - \nabla Q_{\text{Car}} * \sum_{z,w} \omega(z,w)
\]

\[
\ast \nabla^\text{Car}_0 (z,w) = 0 \quad (2 - 45)
\]

\[
\mu_{r_m}(z,w) * Q^\text{Car}_{r_m}(z,w) = 0 \quad \forall z,w \in Z, \forall r_m \in \mathcal{R}(z,w) \quad (2 - 46)
\]

\[
\sum_{r_m} Q^\text{Car}_{r_m}(z,w) - \nabla^\text{Car}_0 (z,w) = 0 \quad \forall z,w \in Z \quad (2 - 47)
\]

Eqs. (2-45) – (2-47) include non-negativity constraints on \( \mu_{r_m}(z,w) \) and \( Q^\text{Car}_{r_m}(z,w) \). Each element of the resulting vector in Eq. (2-45) is calculated using Eq. (2-48).

\[
\frac{\partial L}{\partial Q^\text{Car}_{r_m}(z,w)} = \sum_i t^\text{car}_i * \delta_{r_m}(z,w) + \omega(z,w) - \mu_{r_m}(z,w) \quad (2 - 48)
\]

Referring to Fiacco (1983), Eq. (2-49) is calculated for the partial derivatives of KKT triples by defining \( M \) and \( N \), respectively, as the Jacobian matrix of derivations in Eqs. (2-45) – (2-47) with respect to \( Y \) and \( T^\text{Car}_{OD} \). As \( M \) is nonsingular for small changes in \( T^\text{Car}_{OD} \), \( M \) is invertible in Eq. (2-49).

\[
\nabla^\text{Car}_Y (T^\text{Car}_{OD}) = -M(t^\text{car}_{OD})^{-1}, N(t^\text{Car}_{OD}) \quad (2 - 49)
\]

The \( M \) matrix takes the general form provided in Eq. (2-50) and includes the first- and second-derivatives with respect to route flow rates, \( Q^\text{Car} \).

\[
M = \begin{bmatrix}
\nabla Q^2_{\text{Car}} * L & \cdots & - (\nabla Q_{\text{Car}} * g_{r_m}(z,w))^T & \cdots & \nabla Q_{\text{Car}} * h(z,w))^T & \cdots \\
\mu_{r_m}(z,w) * \nabla Q_{\text{Car}} * g_{r_m}(z,w) & g_{r_m}(z,w) & \cdots & 0 & \cdots & 0 \\
\nabla Q_{\text{Car}} * h(z,w) & 0 & \cdots & 0 & \cdots & 0
\end{bmatrix} \quad (2 - 50)
\]

Considering the elements of \( \nabla Q_{\text{Car}} * L \) vector provided in Eq. (2-48), each element of \( \nabla^2 Q_{\text{Car}} * L \) matrix in Eq. (2-50) would be calculated using Eq. (2-51).
\[
\frac{\partial}{\partial Q_{cn}(p,q)} \left( \sum_i t_i^{car} \ast \delta_{lr_m}(z,w) + \omega(z,w) - \mu_m(z,w) \right) = \sum_i \frac{\partial t_i^{car}}{\partial Q_{cn}(p,q)} \ast \delta_{lr_m}(z,w)
\]
\[
= \sum_i \frac{\partial t_i^{car}}{\partial Q_{cn}(p,q)} \ast \frac{\partial v_i^{car}}{\partial Q_{cn}(p,q)} \ast \delta_{lr_m}(z,w)
\]
\[
= \sum_i t_i^{car} \ast \delta_{lr_m}(p,q) \ast \delta_{lr_m}(z,w)
\]
\[
(2 - 51)
\]

\[\nabla Q_{car} \ast g_{rm}(z,w) \text{ and } \nabla Q_{car} \ast h(z,w) \text{ in Eq. (2-50) are vectors of Eq. (2-52) and Eq. (2-53), respectively.}\]

\[
\nabla Q_{car} \ast g_{rm}(z,w) = \begin{bmatrix} 0, 0, \ldots, \frac{\partial}{\partial Q_{rm}^c(z,w)} Q_{rm}^c(z,w) = 1, 0, \ldots \end{bmatrix}_{1 \times \Sigma_{zw} \setminus K(z,w)}
\]
\[
(2 - 52)
\]

\[
\nabla Q_{car} \ast h(z,w) = \begin{bmatrix} 0, 0, \ldots, \frac{\partial}{\partial Q_{1}^c(z,w)} Q_{1}^c(z,w) = 1, \frac{\partial}{\partial Q_{2}^c(z,w)} Q_{2}^c(z,w) = 1, \ldots \end{bmatrix}_{1 \times \Sigma_{zw} \setminus K(z,w)}
\]
\[
(2 - 53)
\]

Referring to Eq. (2-49), the \( N \) matrix takes the general form of Eq. (2-54). The \( N \) matrix involves the first- and second-derivatives with respect to input O-D demands, \( T_{OD}^{Car} \).

\[
N = \left( \begin{array}{ccc}
\left( T_{OD}^{Car} Q_{Car} \ast Q_{Car} \right) & \ldots & \left( \mu_{rn}(z,w) \ast \nabla_{od}^{T} g_{rn}(z,w) \right) \\
\vdots & \ddots & \vdots \\
\left( \nabla_{od}^{T} g_{rm}(z,w) \right)^{T} & \ldots & \left( \nabla_{od}^{T} h(z,w) \right)^{T} \\
\end{array} \right)^{T}
\]
\[
(2 - 54)
\]

Since the resulting formula in Eq. (2-48) is not dependent on any O-D demands, \( T_{OD}^{Car} Q_{Car} \ast L \) is a zero matrix. Similarly, \( \nabla_{od}^{T} g_{rn}(z,w) \) is a zero vector, while \( \nabla_{od}^{T} h(z,w) \) is a vector as indicated in Eq. (2-55).

\[
\nabla_{od}^{T} h(z,w) = \begin{bmatrix} 0, 0, \ldots, \frac{\partial}{\partial T_{OD}^{Car} (z,w)} (-T_{OD}^{Car}(z,w)) = 1, \ldots, 0 \end{bmatrix}_{1 \times z \times z}
\]
\[
(2 - 55)
\]

Having \( M(T_{OD}^{Car}) \) and \( N(T_{OD}^{Car}) \) in hand, the partial derivatives of the KKT triples are obtained using Eq. (2-49) in this analytical method.
2-2-3-2-Propagation of O-D Demand Measurement Errors

To calculate the new value of an output from the current value while an input changes in a small range, an approximation method provided by the first-order Taylor series expansion is used as presented in Eq. (2-56).

\[ Y = Y_0 + \nabla_{y_{o_{0}}} Y (T^\text{Car}_{o_{0},0}) \ast (r^\text{Car}_{o_{0},0} - r^\text{Car}_{o_{0},0,0}) \]  

(2 – 56)

where, the index of zero shows the current values. Taking the variance from both sides of Eq. (2-56), the variance of the output is calculated using the variance of input as well as the result of SA as presented in Eq. (2-57).

\[ \text{var}(Y) = \left( \nabla_{y_{o_{0}}} Y (T^\text{Car}_{o_{0},0}) \right)^2 \ast \text{var}(r^\text{Car}_{o_{0},0}) \]  

(2 – 57)

In this light, a generic formula provided in the matrix format in Eq. (2-58) is employed to propagate the O-D demand measurement errors through an UETA program via the proposed analytical method. Eq. (2-58) can consider the existing correlation between the input measurement errors and the results of SA on the UETA. In the current study, it is assumed that an error creates randomness in the value of a variable. The randomness would be measured by taking the variance of the corresponding variable.

\[ \Sigma_{\text{output}} = \left( \nabla_{y_{o_{0}}} Y (T^\text{Car}_{o_{0},0}) \right)^T \ast \left( \Sigma_{\text{input}} \right) \ast \left( \nabla_{y_{o_{0}}} Y (T^\text{Car}_{o_{0},0}) \right) \]  

(2 – 58)

The input variance-covariance matrix shows the characteristics of O-D demand measurement errors including variances and covariance, while the corresponding output matrix indicates the statistical characteristics of the resulting errors of route flows and the UETA dual variables.

The method explained above quantifies the propagation of O-D measurement errors to the route flow rates. Measuring the variance of UETA specification error on links as explained in section (2-1), it is required to calculate the propagated error on the links using Eq. (2-35). Taking the variance from both sides of Eq. (2-35), a relation between the variances of route flows and link volumes is established as shown in Eq. (2-36). The UETA specification error would also be taken into consideration when calculating the propagated O-D demand measurement errors on links through an UETA. Assuming no correlation between the propagated measurement error and the UETA specification error, the variance of the specification error is directly added to the calculated variance in Eq. (2-36).

2-2-4-Simulation-based Error Propagation through an UETA

The analytical method proposed in section (2-2-3) to propagate error in an UETA is compared with a simulation-based method, particularly an MCS method. This comparison
determines the efficiency and accuracy of the proposed analytical method in calculating the output error variance. In the MCS method, the UETA specification error is attributed to the randomness of the residual term in a calibrated UETA.

In the MCS method, the input measurement errors and the UETA specification errors are taken into account by sampling from multivariate probability distributions assumed for the inputs and the residual term. The multivariate probability distribution can consider the correlation between the random variables. The statistical characteristics of the residual term, mean, and variance are calculated as described in section (2-1-1). The statistical characteristics of the measurement error of the input variables are given in the prediction stage. The number of model iterations that is equal to the number of samples extracted from the probability distribution needs to be specified. The model is run for the number of extracted samples and provides an output for each run. The variance of output error is then determined by taking the variance over all collected outputs (Zhao and Kockelman, 2002; Hugosson, 2005).
Chapter 3  Case Study

In this chapter, the approach proposed in the methodology chapter is applied to three different case studies: two demonstrating examples and one real case study. The first case study as an illustrative example displays step by step the propagation of error in an FSTDM in a small case. In the second case study, the error propagation process is implemented for an FSTDM for the city of Brisbane, Australia as a real case study. The third case study only includes a User Equilibrium Traffic Assignment (UETA), and the error propagation is undertaken using the analytical sensitivity-based and the MCS methods as explained in section (2-2-3) and section (2-2-4) respectively. Furthermore, the validity and the efficiency of the proposed analytical sensitivity-based method are addressed in comparison with the MCS method. In all case studies, the propagation of input measurement errors as well as the model specification errors are investigated.

3-1-Case Study 1

The current section presents an example to show how the measurement error of the input variables is propagated and combined with the model specification errors through the sequential steps of an FSTDM model. Following the proposed methodology, this section has two main stages, calibration and prediction. In the calibration stage, the estimated parameters as well as the model specification error of each step of the FSTDM are presented. In the prediction stage, by considering the model specification errors obtained from the calibration process, the propagation of the input measurement error towards the final step output is quantified. The type of models considered here for the trip generation and the trip distribution steps are respectively a linear and a gravity model, while logit models are employed for modal split and traffic assignment.

3-1-1-The Calibration Stage

This section demonstrates the calibration of an FSTDM for the first case study. The four-step model in this example is calibrated on a synthetic database, and the schematic network layout of the calibrated model is provided in Figure (3-1).
The error propagation is investigated for a single O-D pair connecting origin 1 to destination 2 as presented in Figure (3-2).

3-1-1-1-Calibration of Trip Generation

The models calibrated here for the trip generation step are both linear. The dataset used for calibration is provided in Appendix-A1. The calibrated trip production and trip attraction models are presented respectively in Eq. (3-1) and Eq. (3-2).

\[ T_o(z) = 0.799 + 0.982 \times x_{o1}(z) + 0.429 \times x_{o2}(z) + \varepsilon_o(z) \quad R_o^2 = 0.90 \quad \forall z \in Z \tag{3-1} \]

\[ T_d(z) = 0.851 + 1.212 \times x_{d1}(z) + 0.679 \times x_{d2}(z) + \varepsilon_d(z) \quad R_d^2 = 0.91 \quad \forall z \in Z \tag{3-2} \]
Where, the definition of variables is provided in the list of abbreviations at the beginning of thesis report. The residual term in the calibration process is considered as the model specification error in the prediction stage. The PDF of the residual term is a normal distribution whose mean value is zero and its variance is calculated using the provided R-squared and Eq. (2-1). The resulting variance for the residual terms of the trip production and the trip attraction models are respectively 7.62 and 9.73 with a correlation of 0.07.

3-1-1-2-Calibration of Trip Distribution

A gravity model is utilized to distribute trips between the destinations based on the attractiveness and the travel impedance. The output of this step is a matrix containing the actual trips distributed between the destination zones. The input variables are the number of produced and attracted trips in zones, and the travel impedance between zones. In order to calibrate the gravity model provided in Eq. (2-8), the O-D specific parameters as well as the friction function parameters are estimated using VISSUM software package. In the current example, the friction function is estimated as the exponential function of the travel impedance as provided in Eq. (3-3). Travel impedance is defined as a combination of travel distance and travel time by mode car. The travel impedances used for the calibration are provided in Appendix-A2, Table (A2-2).

\[ f(z,w) = e^{-0.003506 \cdot \mid_{od}(z,w)} \quad \forall z,w \in Z \quad (3 - 3) \]

Through comparing the distributed trips estimated by the gravity model with the actual ones (provided in Appendix-A2, Table (A2-1)), the distribution of error originating from the model specification is obtained. The mean and the variance of the obtained PDF for the model specification error are respectively zero and 4.486.

3-1-1-3-Calibration of Modal Split

In the current example, among the most popular models in discrete choice modelling, a logit model is calibrated for the modal split step. Three modes of car, bus, and bike are competing alternatives whose utility functions are only dependent on travel time. In order to estimate the utility function parameters, the LIMDEP software package is employed through using the Maximum Likelihood Estimation (MLE) method. The modal utility functions calibrated by the software package are shown in Eq. (3-4) to Eq. (3-6). The LIMDEP outputs (e.g., goodness-of-fit value and estimated parameters) have been also presented in Figure (A4-1).
The database used for the calibration is provided in Appendix-A3 (Table (A3-1) to Table (A3-6)). The mean and variance of the residual term in the probability of choosing mode car in the modal split model are respectively 0.011 and 0.0085 as shown in Table (3-1). The statistical characteristics of the residual term of mode car are calculated by comparing the actual probabilities of mode car with the estimated ones. After calculating the probability of mode car, the probability is multiplied by the demand obtained from the previous step to find out the travel demand assigned to mode car.

3-1-1-4-Calibration of Traffic Assignment

The traffic assignment model as a Route-based Traffic Assignment (RTA) employs a logit model in which the competing alternatives are the parallel routes connecting an O-D pair. The utility function of the routes is assumed to be only dependent on the route travel time. Employing the LIMDEP software as used in the modal split step, \( \theta \) in Eq. (2-30) is estimated -0.1 as shown in Figure (A4-2).

The current example investigates the propagation of error through three parallel routes connecting origin 1 to destination 2. The routes respectively involve links 1 and 2, links 1 and 3, and link 4. In the RTA, the estimated link volumes by the model should be compared with the actual ones in order to find the PDF of the model specification error. In the present example, the mean and the variance of the PDF are respectively zero and 0.16. In summary, the mean and the variance of the specification errors of all steps in the current example are presented in Table (3-1).

Table (3-1). The Mean and Variance of Model Specification Errors in the FSTDM steps of the Case Study 1

<table>
<thead>
<tr>
<th>Step</th>
<th>Mean</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trip generation</td>
<td>0</td>
<td>7.62</td>
</tr>
<tr>
<td>Attraction</td>
<td>0</td>
<td>9.73</td>
</tr>
<tr>
<td>Trip distribution</td>
<td>0</td>
<td>4.486</td>
</tr>
<tr>
<td>Modal split</td>
<td>0.011</td>
<td>0.0085</td>
</tr>
<tr>
<td>Traffic assignment</td>
<td>0</td>
<td>0.16</td>
</tr>
</tbody>
</table>

* Exception of modal split step, the units of mean and variance are the number of trips per hour (trip/h) and the number of trips per hour squared ((trip/h)^2) respectively. In the modal split step, the mean and variance are unitless.
3-1-2-The Prediction Stage

In this section, the propagation of error is quantified from the input variables and the model specification in the trip generation step towards the selected link volumes in the traffic assignment.

3-1-2-1-Error Propagation in the Prediction Stage of Trip Generation

Taking the variance of both sides of calibrated trip production and trip attraction models in Eq. (3-1) and Eq. (3-2), two formulas are derived as shown in Eq. (3-7) and Eq. (3-8) that are based on Eq. (2-5) and Eq. (2-6) respectively.

\[
\begin{align*}
\text{var}(\xi_{T_D(z)}) &= 0.982^2 \cdot \text{var}(\xi_{x_{Q_1(z)}}) + 0.429^2 \cdot \text{var}(\xi_{x_{Q_2(z)}}) + \text{var}(\eta) + 0.421 \\
&\times \text{cov}(\xi_{x_{Q_1(z)}}, \xi_{x_{Q_2(z)}}) \quad \forall z \in Z \\
\text{var}(\xi_{T_D(z)}) &= 1.212^2 \cdot \text{var}(\xi_{x_{D_1(z)}}) + 0.679^2 \cdot \text{var}(\xi_{x_{D_2(z)}}) + \text{var}(\eta) + 0.823 \\
&\times \text{cov}(\xi_{x_{D_1(z)}}, \xi_{x_{D_2(z)}}) \quad \forall z \in Z
\end{align*}
\]

(3 – 7)

(3 – 8)

The expected value and the variance of measurement error of the input variables are given in Table (3-2). In addition to the individual error considered for the input variables, it is also assumed that the measurement errors of the input variables are correlated. This correlation is assumed between the measurement errors of \(x_{Q_1}\) and \(x_{Q_2}\) (in the same model) and between the measurement errors of \(x_{Q_2}\) and \(x_{D_2}\) (in different models). The correlation values are -0.5 and 0.7, respectively. Without loss of generality of the proposed approach, correlation between the measurement errors of other couples of the input variables (e.g., \(x_{Q_1}\) and \(x_{D_2}\)) is ignored in the current example.

<table>
<thead>
<tr>
<th>Table (3-2). Expected Value of Inputs and Variance of Measurement Errors in the Trip Generation of the Case Study 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected value</td>
</tr>
<tr>
<td>Expected value</td>
</tr>
<tr>
<td>Variance of Measurement error</td>
</tr>
</tbody>
</table>

Based on Eq. (3-7) and Eq. (3-8) and the measurement errors assumed for the inputs, the expected value and the variance of the resulting errors for the trips produced and attracted in zone 1 and 2 are respectively (94.039, 57.020) and (117.231, 83.798).
3-1-2-2- Error Propagation in the Prediction Stage of Trip Distribution

The expected value of the number of distributed trips from zone 1 to zone 2 can be estimated using Eq. (2-10). The O-D specific parameter to travel from zone 1 to zone 2, \( A(1,2) \), is 0.000894. The expected value assumed for the travel impedance between zone 1 and zone 2, \( c_{OD}(1,2) \), is equal to 28, and the corresponding variance of measurement error is 36.

Assuming \( \xi_{COD(1,2)} \) is normally distributed, the measurement error of the friction function of Eq. (3-3) would be log-normally distributed. The mean and variance of the resulting distribution for \( \xi_{f(1,2)} \) would be \( \mu = -0.003506 \times 28 = -0.0982 \) and \( \sigma = 0.003506 \times 6 = 0.021 \). Having the parameters of the resulting log-normal distribution in hand, the expected value and the variance of \( \xi_{f(1,2)} \) are shown respectively in Eq. (3-9) and Eq. (3-10).

\[
E[\xi_{f(1,2)}] = e^{\mu + \sigma^2/2} = 0.907 \quad (3-9)
\]
\[
\text{var}(\xi_{f(1,2)}) = (e^{\sigma^2} - 1) \times e^{2\mu + \sigma^2} = 0.000364 \quad (3-10)
\]

There is also a covariance between the measurement error of the produced trips in zone 1, \( \xi_{T_D(1)} \), and the measurement error of the attracted trips in zone 2, \( \xi_{T_D(2)} \), which comes from having correlated variables in their formulas (\( \xi_{XO(1)} \) and \( \xi_{XD(2)} \)). Meanwhile, it is also assumed that there is no correlation between the model specification errors of the trip generation models. Hence, the magnitude of covariance could be calculated based on Eq. (2-7) as shown in Eq. (3-11).

\[
cov(\xi_{T_D(1)}, \xi_{T_D(2)}) = 0.429 \times 0.679 \times 0.7 \times \sqrt{169} \times \sqrt{81} = 23.857 \quad (3-11)
\]

Referring to Eq. (2-10) and Eq. (2-13) and substituting all calculated values including the mean and variance of the trip distribution specification error, 0 and 4.486, the expected value and the variance of \( \xi_{T_{OD}(1,2)} \) can be calculated as follows:

\[
E[\xi_{T_{OD}(1,2)}] = 0.000894 \times (94.039 \times 117.231 + 23.857) \times 0.907 = 8.955 \text{ trip/h} \quad (3-12)
\]
\[
\text{var}[\xi_{T_{OD}(1,2)}] = 0.000894^2 
\times \left( (94.039^2 + 7.551^2) \times (117.231^2 + 9.154^2) + 2 \times 23.857^2 + 4 \times 94.039 \times 117.231 \times 23.857 \right) 
\times (0.0907^2 + 0.000364) - \left( (94.039 \times 117.231 + 23.857) \times 0.907 \right)^2 + 4.486 
= 5.873 \text{ (trip/h)}^2 \quad (3-13)
\]
3-1-2-3-Error Propagation in the Prediction Stage of Modal Split

In the modal split step, the error of predicting the probability of choosing mode car is first quantified; then, the error of predicting the demand of mode car is calculated. The measurement errors introduced into the input variables to travel from zone 1 to zone 2 are as presented in Table (3-3).

<table>
<thead>
<tr>
<th>Table (3-3). Statistical Characteristics of Measurement Errors in the Modal Split of the Case Study 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>t_{OD}^{Car} (1,2)</td>
</tr>
<tr>
<td>Expected value (minute)</td>
</tr>
<tr>
<td>Variance of Measurement error</td>
</tr>
</tbody>
</table>

Using Eq. (2-16) and the values calculated in Eq. (3-10) and Eq. (3-11), the expected value and the variance of the numerator and the denominator of the logit model are respectively (0.462, 0.010) and (0.885, 0.023). Furthermore, the covariance between the numerator and the denominator of the logit model is 0.01. Using Eq. (2-20), the expected value of the probability of travelling from zone 1 to zone 2 that are only subject to the measurement error of the input variables is calculated in Eq. (3-14). Using Eq. (2-21), the variance of the resulting error is also calculated in Eq. (3-15).

\[
E \left( \xi \right) = \frac{0.462}{0.885} \left( 1 + \frac{0.023}{0.885^2} \right) - \frac{0.01}{0.885^2} = 0.525
\]

\[
var \left( \xi \right) = \left( \frac{0.462}{0.885} \right)^2 \left( \frac{0.01}{0.462^2} + \frac{0.023}{0.885^2} - \frac{2 \times 0.01}{0.462 \times 0.885} \right) = 0.0075
\]

Similar to the previous steps, it is assumed that there is no correlation between the model specification error and the error propagated by the logit model. The expected value and the variance of the residual term calculated in the calibration stage were 0.011 and 0.0085 (see section (3-1-1-4)) which should be added directly to 0.525 and 0.0075. Consequently, the resulting expected value and variance for the probability of choosing mode car, \( \xi_{p_{OD}}^{Car} (1,2) \), will be 0.536 and 0.016. Referring to Eq. (2-23) and Eq. (2-25), the expected value and the variance of \( \xi_{t_{OD}}^{Car} (1,2) \) are respectively calculated as shown in Eq. (3-16) and Eq. (3-17).

\[
E \left( \xi \right) = 0.536 \times 8.955 = 4.8 \text{ trip/h}
\]

\[
var \left( \xi \right) = (0.016 + 0.536^2) \times (5.873 + 8.955^2) - 0.536^2 \times 8.955^2 = 3.064 \text{ (trip/h)^2}
\]
Error propagation in the traffic assignment step contains two sub-steps. First, the error is attributed to route flows by using the RTA. Second, the calculated route flows are converted into the link flows by considering the existing correlation between the route flows. Link travel times are taken to provide an equilibrium situation in the network. The expected value of the link travel times as well as the final values obtained for the variance of measurement errors are presented in Table (3-4). The initial values assumed for the measurement error of link travel times become updated using the volume delay functions of links in the iterative process of error propagation through the RTA.

<table>
<thead>
<tr>
<th>Table (3-4). Expected Value of Link Travel Times and Final Values Calculated for Variance of Measurement Errors in the RTA of the Case Study 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected value (minute)</td>
</tr>
<tr>
<td>-------------------------</td>
</tr>
<tr>
<td>Variance of Measurement error</td>
</tr>
</tbody>
</table>

Based on given information in the calibration stage, the expected value and the variance of travel time on all three parallel routes are identical and equal to 28 and 25 respectively, as the network is under an equilibrium situation. Therefore, referring to Eq. (2-30), the expected values of all three probabilities are similar and expected to be 0.333. To calculate the variance of the resulting error of the route probability, the covariance between any couple of the route travel times that are located in the denominator of Eq. (2-30) should be first specified.

The expected value and the variance of the route probability model are calculated using respectively Eq. (2-20) and Eq. (2-21). Using Eq. (2-36), the covariance matrix of the errors of the route flows is presented in Table (3-5). The variance values on the main diagonal are considered as the error propagated from the traffic assignment input variables towards the route probabilities. Route 3 has no common link with other routes; therefore, the correlation of the measurement error of route 3 with routes 1 and 2 comes from the conservation constraint that exists for each O-D pair in traffic assignment step.

<table>
<thead>
<tr>
<th>Table (3-5). Covariance Matrix of Route Probability Errors in the RTA of the Case Study 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{r_{1}}^{car}(1,2)$</td>
</tr>
<tr>
<td>-------------------------</td>
</tr>
<tr>
<td>$p_{r_{1}}^{car}(1,2)$</td>
</tr>
<tr>
<td>$p_{r_{2}}^{car}(1,2)$</td>
</tr>
<tr>
<td>$p_{r_{3}}^{car}(1,2)$</td>
</tr>
</tbody>
</table>
Referring to the logit formula in Eq. (2-30), the sum of the probabilities should be equal to one. Subsequently, the sum of the variance of the probabilities and the covariance between the probabilities is equal to zero. This can be easily verified by the reader from the values shown in Table (3-5).

The variance of the resulting error as well as the expected value of the flow of all routes is calculated in Table (3-6) using respectively Eq. (2-34) and Eq. (2-33) as shown in Eq. (3-18) and Eq. (3-19) respectively.

$$
\text{var} \left( \xi_{Q_{1\text{car}}^{\text{Car}(1,2)}} \right) = (0.0136 + 0.333^2) \times (3.064 + 4.8^2) - 0.333^2 \times 4.8^2 = 0.695 \, \text{(trip/h)}^2 
$$

$$
E \left[ \xi_{Q_{1\text{car}}^{\text{Car}(1,2)}} \right] = 0.333 \times 4.8 = 1.6 \, \text{trip/h}
$$

Table (3-6). Error Propagation from Inputs to Route Flows in the RTA of the Case Study 1

<table>
<thead>
<tr>
<th></th>
<th>$Q_{r_1\text{car}}^{\text{Car}(1,2)}$</th>
<th>$Q_{r_2\text{car}}^{\text{Car}(1,2)}$</th>
<th>$Q_{r_3\text{car}}^{\text{Car}(1,2)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected value</td>
<td>1.600</td>
<td>1.600</td>
<td>1.600</td>
</tr>
<tr>
<td>Variance of error</td>
<td>0.695</td>
<td>0.695</td>
<td>0.811</td>
</tr>
</tbody>
</table>

* The units of expected value and variance of error are the number of trips per hour (trip/h) and the number of trips per hour squared ((trip/h)^2) respectively.

In order to calculate the variance of the error in estimating the volume on links, it is also required to calculate the existing covariance between the flows of the routes (see Eq. (2-37) and Eq. (2-36)), which results in a covariance of Eq. (3-20) and the covariance matrix of Table (3-7).

$$
\text{cov} \left( \xi_{Q_{1\text{car}}^{\text{Car}(1,2)}}, \xi_{Q_{1\text{car}}^{\text{Car}(1,2)}} \right) = \left( 0.111 + \text{cov} \left( \xi_{Q_{r_1\text{car}}^{\text{Car}(1,2)}}, \xi_{P_{r_1\text{car}}^{\text{Car}(1,2)}} \right) \right) \times 26.104 - 2.56 \quad \forall r_m, r_n \in R(1,2) 
$$

Table (3-7). Covariance Matrix of Route Flow Errors in the RTA of the Case Study 1

<table>
<thead>
<tr>
<th></th>
<th>$Q_{r_1\text{car}}^{\text{Car}(1,2)}$</th>
<th>$Q_{r_2\text{car}}^{\text{Car}(1,2)}$</th>
<th>$Q_{r_3\text{car}}^{\text{Car}(1,2)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_{r_1\text{car}}^{\text{Car}(1,2)}$</td>
<td>0.695</td>
<td>0.222</td>
<td>0.106</td>
</tr>
<tr>
<td>$Q_{r_2\text{car}}^{\text{Car}(1,2)}$</td>
<td>0.222</td>
<td>0.695</td>
<td>0.106</td>
</tr>
<tr>
<td>$Q_{r_3\text{car}}^{\text{Car}(1,2)}$</td>
<td>0.106</td>
<td>0.106</td>
<td>0.811</td>
</tr>
</tbody>
</table>

The sum of all variance and covariance values in Table (3-7) is equal to 3.064 that is consistent with Eq. (3-17). Having the variance and the covariance of route flow errors in hand, the expected value of the link volumes as well as the variance of the resulting errors are calculated with referring respectively to Eq. (2-34) and Eq. (2-35) as shown for route 1 in Eq. (3-21) and Eq. (3-22) respectively. The results of the calculations are given in Table (3-8). The variance of resulting errors in Table (3-8) are only subject to the measurement error of the input variables including link travel times and car travel demand.

$$
\nu_{1\text{car}}^{\text{Car}} = 1.6 + 1.6 = 3.2 \, \text{veh/h}
$$

(3 – 21)
\[ \text{var} \left( \xi_{\text{car}} \right) = 0.695 + 0.222 + 0.695 + 0.222 = 1.834 \text{ (veh/h)}^2 \] 

(3 – 22)

**Table (3-8). Propagated Error from Inputs to Link Volumes in the Traffic Assignment of the Case Study 1**

<table>
<thead>
<tr>
<th></th>
<th>( v_{1\text{car}}' )</th>
<th>( v_{2\text{car}}' )</th>
<th>( v_{3\text{car}}' )</th>
<th>( v_{4\text{car}}' )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Expected value</strong></td>
<td>3.200</td>
<td>1.600</td>
<td>1.600</td>
<td>1.600</td>
</tr>
<tr>
<td><strong>Variance of error</strong></td>
<td>1.834</td>
<td>0.695</td>
<td>0.695</td>
<td>0.811</td>
</tr>
</tbody>
</table>

* The units of expected value and variance of error are the number of vehicles per hour (veh/h) and the number of vehicles per hour squared ((veh/h)^2) respectively.

In the final stage of the error propagation in the traffic assignment, the model specification error should also be taken into consideration. The model specification error is provided in the calibration stage on the links. The expected value and the variance of the model specification error are zero and 0.16, respectively. Due to not having any correlation between the model specification error and the propagated measurement error in Table (3-8), the variance of the model specification error is directly added to the variances in Table (3-8). The expected value besides the variance of resulting error on the link volumes are provided in Table (3-9) considering all sources of error.

**Table (3-9). Resulting Expected Value and Error Variance of Link Volumes in the Traffic Assignment of the Case Study 1**

<table>
<thead>
<tr>
<th></th>
<th>( v_{1\text{car}}' )</th>
<th>( v_{2\text{car}}' )</th>
<th>( v_{3\text{car}}' )</th>
<th>( v_{4\text{car}}' )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Expected value</strong></td>
<td>3.200</td>
<td>1.600</td>
<td>1.600</td>
<td>1.600</td>
</tr>
<tr>
<td><strong>Variance of error</strong></td>
<td>1.994</td>
<td>0.855</td>
<td>0.855</td>
<td>0.971</td>
</tr>
</tbody>
</table>

* The units of expected value and variance of error are the number of vehicles per hour (veh/h) and the number of vehicles per hour squared ((veh/h)^2) respectively.

The propagation of input measurement errors as well as model specification errors from the trip generation step to the link volume in traffic assignment step is indicated in Figure (3-3). Values in the brackets show the variance of error and coefficient of variation (CV), respectively. As presented in Figure (3-3), the method proposed by the current thesis is able to take into account the effect of correlation between error sources, and to show the share of the error sources in each part of the FSTDM. Additionally, Figure (3-3) shows the CV of the steps increases continuously from the trip generation towards the link volume. Tracking back from the error of link 1 volume in Figure (3-3), it is revealed that the most significant contributing error source is the specification error of the trip distribution step that is more than three times greater than the propagated error to the trip distribution step.

Referring to Figure (3-3), the CV of trip generation input variables are reduced over the trip generation model, while the overall CV is then intensified continuously over the trip
distribution, the modal split and the traffic assignment steps along with introducing some new input variable measurement errors in the steps. In the trip generation step, the effects of trip generation input errors, covariance between the input errors, and trip generation specification error are included. In the second step, the effects of trip generation error, travel impedance error, covariance between trip production and trip attraction errors, and trip distribution specification error are taken into account. In the modal split step, the effects of trip distribution error, car travel time error, and modal split specification error are involved. In the fourth step, the effects of modal split error, route travel time error, covariance between the route travel time errors, and traffic assignment specification error are engaged.
Figure (3-3). Results of Error Propagation from the Trip Generation towards the Volume of Link 1 through the FSTDM of the Case Study 1
3-2-Case Study 2

This section investigates the error propagation in the prediction stage of an FSTDM calibrated for the city of Brisbane, Australia. The model is calibrated in 2006 and applied to the horizon year of 2011. The models used in the first three steps are a linear regression model, a gravity model, and a nested logit model respectively. These calibrated models are provided by the Department of Transport and Main Roads (TMR). In the present study, an RTA is calibrated for the last step of the Brisbane FSTDM (BFSTDM).

3-2-1-Brisbane Four-Step Transportation Demand Model (BFSTDM)

BFSTDM is calibrated within a bigger FSTDM model called the South East Queensland (SEQ) model. In the BFSTDM, the Brisbane city is partitioned into 868 traffic zones, and the network includes 27157 links. Figure (3-4) and Figure (3-5) respectively shows the Brisbane traffic zones and the Brisbane network. BFSTDM classifies travel demands into 8 different trip purposes follows:

1. Home Based Work – Blue collar (HBWB),
2. Home Based Work – White collar (HBWW),
3. Home Based Education – primary and secondary only (HBE),
4. Home Based Education – tertiary only (HBET),
5. Home Based Shopping and personal business (HBS),
6. Home Based Other (HBO),
7. Other Non-Home Based – excluding WBW (ONHB), and
8. Work Based Work (WBW).

The travel demand is forecasted specifically for each trip purpose in trip generation, trip distribution, modal split and time-of-day steps and then is assigned on the Brisbane network for all trip purposes together as presented schematically in Figure (3-6).
Figure (3-4) – Brisbane Traffic Zones in BFSTDM

Figure (3-5) – Brisbane Network in BFSTDM
Figure (3-6) – Schematic Figure of Travel Demand Modelling in the BFSTDM

*HBWB = Home Based Work (Blue Collar)
3-2-1-1-BFSTDM Trip Generation

As presented in Figure (3-6), BFSTDM trip generation step includes two models, trip production and trip attraction. The calibrated model for trip production is a linear equation that estimates the average rate of produced trips per household. The calibrated trip production model is dependent on the attributes describing the household type such as the household size, the number of blue collar and white collar workers in the household, and the number of different types (A, B and C) of dependants in the household. The number of workers and the dependents are stratified into discrete variables. The coefficients estimated by TMR for the trip production model for different trip purposes are indicated in Table (3-10).

Table (3-10) – Estimated Coefficients by TMR for Household Trip Production in BFSTDM

<table>
<thead>
<tr>
<th>Attribute</th>
<th>HBWB</th>
<th>HBWW</th>
<th>HBE</th>
<th>HBET</th>
<th>HBS</th>
<th>HBO</th>
<th>WBW</th>
<th>ONHB</th>
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</thead>
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<tr>
<td>HH_Size*</td>
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<td>0</td>
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<td>WTHH</td>
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<tr>
<td>Wkers Blu_1</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0.44</td>
<td>0.25</td>
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<td>0</td>
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<td>0.67</td>
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<tr>
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<td>0</td>
<td>0</td>
<td>1.08</td>
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<td>0.11</td>
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<td>1.25</td>
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<td>0</td>
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<tr>
<td>Wkers Wht_3p</td>
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<td>0.15</td>
<td>1.85</td>
<td>1.65</td>
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<td>DepA_1</td>
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<td>2.56</td>
<td>1.44</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

*Definition of attributes is provided in the list of abbreviations.

The calculated average rate of produced trips per household is then multiplied by the number of households as provided in Eq. (3-23).

\[ T_o(z) = HTPR \times HH(z) \quad \forall z \in Z \]  

(3 – 23)

59
Household trip production rate as a coefficient in Eq. (3-23) is calculated for 2011 and is considered errorless during the error propagation in the prediction stage. In the prediction stage, the trip production specification error is attributed to the residual term, and the only input variable with a measurement error would be the number of households. The trip attraction model of BFSTDM is calibrated to estimate the total number of trips attracted to a traffic zone. A linear relationship between the attributes is calibrated by TMR for each trip purpose. The attributes are all aggregated at the zone level. Table (3-11) displays the estimated coefficients for the trip attraction model.

Table (3-11) – Estimated Coefficients by TMR for Zonal Trip Attraction in BFSTDM

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Retail_B</th>
<th>Service_B</th>
<th>Professional_B</th>
<th>Industry_B</th>
<th>Retail_W</th>
<th>Service_W</th>
<th>Professional_W</th>
<th>Industry_W</th>
<th>Retail</th>
<th>Service</th>
<th>Professional</th>
<th>Industry</th>
<th>Retail_O</th>
<th>PrePrimary</th>
<th>Secondary</th>
<th>Tertiary</th>
<th>Population</th>
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</thead>
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<tr>
<td></td>
<td>HBWB</td>
<td>HBWW</td>
<td>HBE</td>
<td>HBET</td>
<td>HBO</td>
<td>WBW</td>
<td>ONHB</td>
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</tr>
</tbody>
</table>

*Definition of attributes is provided in the list of abbreviations.

At the end of the trip generation step, the total number of trips produced from and attracted to the all traffic zones is balanced to a target value for each trip purpose. The target value in each trip purpose is the average of produced and attracted trips. In this way, a balancing factor is obtained for each trip purpose that is used to balance the calculated errors of generated trips in the traffic zones.
3-2-1-2-BFSTDM Trip Distribution

The model calibrated for the trip distribution step of the BFSTDM in each trip purpose is a gravity model. The friction function used in the gravity models is presented in Eq. (3-24).

\[ f(z, w) = T(I(z, w)^a * e^{\beta*I(z, w)}) \quad \forall z, w \in Z \quad (3 - 24) \]

The friction function parameters estimated by TMR for different trip purposes are presented in Table (3-12). Travel impedances in HBWB and HBWW trip purposes are obtained from the utilities calculated for travelling modes in the modal split step. The composite disutility is defined as the logsum of mode disutilities.

<table>
<thead>
<tr>
<th>Trip Purpose</th>
<th>Travel Impedance</th>
<th>Coefficients</th>
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</thead>
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<tr>
<td></td>
<td></td>
<td>Alpha</td>
</tr>
<tr>
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<td>HBWB All Modes Composite Disutility</td>
<td>-5</td>
</tr>
<tr>
<td>HBWW</td>
<td>HBWW All Modes Composite Disutility</td>
<td>-6</td>
</tr>
<tr>
<td>HBE</td>
<td>Highway AM Peak Distance</td>
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<td>Highway AM Peak Distance</td>
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</tr>
<tr>
<td>HBS</td>
<td>Highway Off Peak Distance</td>
<td>-1.2</td>
</tr>
<tr>
<td>HBO</td>
<td>Highway Off Peak Distance</td>
<td>-1.5</td>
</tr>
<tr>
<td>ONHB</td>
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<td>-1.15</td>
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<tr>
<td>WBW</td>
<td>Highway Off Peak Distance</td>
<td>-1</td>
</tr>
</tbody>
</table>

Referring to Eq. (2-8), in addition to the friction function parameters; the OD specific parameter needs to be calculated for the O-D pairs. To this aim, the OD specific parameter of each O-D pair is obtained through substituting the number of distributed trips between that O-D pair, the number of generated trips in the origin and the destination traffic zones, and the friction value corresponds to that O-D pair into Eq. (2-8). The matrix of the OD specific parameters is calculated specifically for each trip purpose using the information of travelling in 2011.

3-2-1-3-BFSTDM Modal Split

A logit model is employed by TMR for the modal split step of the BFSTDM in each trip purpose. The calibrated model includes 5 different modes of travelling such as car driver (CD), car passenger (CP), public transport (PT), cycling (Cyc) and walking (Wk). In the BFSTDM, the utility functions associated to the modes have a linear
structure. The utility function coefficients of the modes for trip purpose of HBWB are shown as an instance in Table (3-13). The utility function coefficients for other trip purposes are provided in Table (A5-1).

Table (3-13) – Estimated Coefficients for Utility Functions in HBWB Trip Purpose in BFSTDM

<table>
<thead>
<tr>
<th>Attributes</th>
<th>CD</th>
<th>CP</th>
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*Definition of attributes is provided in the list of abbreviations.

3-2-1-4-BFSTDM Time-Of-Day

Following the approach described in Figure (3-6), after calculating the daily travel demand of mode car in all trip purposes, it is required to find the share of AM-peak period. Additionally, the type of expressing the travel demand needs to be changed from the PA (Production/Attraction) format to the OD (Origin/Destination). To this aim, the AM-peak PA and AP time-of-day coefficients are multiplied by the daily travel demand to find the AM-peak PA and AP travel demand matrices for each trip purpose. Then, the matrix of AP travel demand is transposed and added to the PA travel demand matrix to have a single matrix for each trip purpose. The summation of the resulting demand matrices of all trip purposes is considered as the AM-peak OD demand matrix. In the next step, this single matrix is introduced to the traffic assignment step.

3-2-1-5-BFSTDM Traffic Assignment

In the traffic assignment step, an RTA that is only dependent on the route travel time, and has a single calibrating parameter is selected. As discussed in the methodology chapter, the RTA model can be calibrated as a logit model. The only calibrating
parameter is the coefficient of route travel time that is estimated in a trial-and-error process. The coefficient that provides the closest estimations to the actual link volumes in the AM-peak period is selected as the calibrated coefficient. In the case of BFSTDM, the routes connecting the OD pairs are extracted from the Brisbane network coded in the Emme software by considering a maximum of three parallel routes for each OD pair.

Following the trial-and-error process for each coefficient, the total travel demand including all trip purposes is assigned on the Brisbane network incrementally in 15 steps. At the end of each step, the route travel times are updated, and subsequently, the shares of the parallel routes from the next demand increment are determined. The root mean square error (RMSE) between the estimated and the actual volumes is chosen as a measure to find the most suitable coefficient for the route travel time in the RTA. The change of RMSE versus travel time coefficient is illustrated in Figure (3-7) which shows a coefficient of 1.4 creates the closest estimations to the actual link volumes in the RTA model.

![Figure (3-7) – Change of RMSE against Travel Time Coefficient in Calibration of RTA Model in BFSTDM](image)

The route probability is calculated using the calibrated RTA model. The route flow rates are determined by multiplying the route probabilities to the travel demand of mode car. The traffic volume is then determined on the network links using an incidence matrix that shows the relationship between the route flow rates and the link volumes. The incidence matrix is extracted from the Brisbane network coded in the Emme software.
3-2-2-Introducing Input Measurement Errors to BFSTDM

The input measurement errors considered for the trip generation of the BFSTDM are obtained through comparing the demographic information projected by TMR for 2011 with the actual observations provided in the ABS website for 2011. It is observed that TMR overestimated the population, the number of households, and the number of workers in the city of Brisbane by 9.29, 4.51 and 9.02 percent respectively. In this way, the STD of measurement errors assumed in the current study for the population and other demographic variables are 5 and 10 percent respectively. In the trip distribution step, the STD of measurement error of friction of travelling between a specific O-D pair is assumed 10 percent of the expected value of the friction. The existing errors in the trip distribution model are rooted in the error propagated from the trip generation step.

In the modal split step, a STD of 10 percent of the mean with a normal distribution is assumed for the measurement error of each input variable. Following Eq. (2-19), the resulting choice probability error of mode car is multiplied by the error of the distributed trips calculated in the trip distribution step.

Since the RTA model possesses a logit structure, the propagation of the route travel time error in the prediction stage of RTA is similar to the propagation of the input measurement errors in the modal split step. Assuming a measurement error with a STD of 10 percent of the mean and a normal distribution for the route travel times, the error variances of route probabilities are calculated using Eq. (2-34). The resulting route probability errors are multiplied by the error of mode car trips to find the route flow rate errors. The error variance is then determined on the network links using the incidence matrix of Brisbane network.

In addition to the input measurement errors, the correlation between the input errors can influence the output error variance. The correlation between the input variable errors in 2011 is taken equal to the existing correlation between the original input variables in 2006. The input measurement error correlation used for the trip generation and the modal split steps of the BFSTDM are provided in in Table (3-14) and Table (3-15) respectively.

The accuracy and the validity of the proposed analytical method for the error propagation in an FSTDM are not dependent on the size of input measurement errors. In contrast with the available sensitivity-based methods in the literature, the
proposed method in the current study is generic and without any applicability limitation. Since the proposed method is based on some driven mathematical formulas, it can provide accurate and valid outcomes for any given input measurement errors and correlation matrices.
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<tr>
<td>HWYOPT</td>
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<td>0.95</td>
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<td>0</td>
<td>0.83</td>
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<td>1</td>
<td>0</td>
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<tr>
<td>ZONEAH</td>
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<td>0</td>
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<td>0</td>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.93</td>
<td>1</td>
<td>0</td>
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<td>PKOPCO</td>
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<td>0</td>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>BLUHH</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.60</td>
<td>0.61</td>
<td>0</td>
</tr>
<tr>
<td>WHTHH</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
<td>0.68</td>
<td>0.66</td>
<td>0</td>
</tr>
</tbody>
</table>

*Definition of attributes is provided in the list of abbreviations.*
3-2-3-Model Specification Errors in BFSTDM

The variance of model specification error is calculated for each step based on the base year (2006) observations. Following the method explained in the methodology chapter, the variance of the model specification errors in the BFSTDM steps are calculated and presented in Table (3-16).

Table (3-16) – Variance of Model Specification Errors of BFSTDM Steps Based on the Base Year (2006) Observations

<table>
<thead>
<tr>
<th>Trip Purpose</th>
<th>BFSTDM Steps</th>
<th>Traffic Assignment (Across Links)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Trip Generation (Across Zones)</td>
<td>Trip Distribution (Across O-D pairs)</td>
</tr>
<tr>
<td></td>
<td>Trip Production</td>
<td>Trip Attraction</td>
</tr>
<tr>
<td>HBWB</td>
<td>1760.77</td>
<td>976.72</td>
</tr>
<tr>
<td>HBWW</td>
<td>45802.24</td>
<td>51842.7</td>
</tr>
<tr>
<td>HBE</td>
<td>7329.87</td>
<td>7312.39</td>
</tr>
<tr>
<td>HBET</td>
<td>2269.53</td>
<td>829.96</td>
</tr>
<tr>
<td>HBS</td>
<td>14373.05</td>
<td>19952.45</td>
</tr>
<tr>
<td>HBE</td>
<td>13072.41</td>
<td>8419.4</td>
</tr>
<tr>
<td>ONHB</td>
<td>8231.86</td>
<td>11013.2</td>
</tr>
<tr>
<td>WBW</td>
<td>114.78</td>
<td>177.41</td>
</tr>
</tbody>
</table>

* The unit of errors in trip generation and trip distribution is the number of trips per hour squared ((trip/h)^2). The unit of traffic assignment error is the number of vehicles per hour squared ((veh/h)^2). Variance of modal split specification error is unitless.

3-2-4- Error Propagation in the Prediction Stage of BFSTDM

An overall outcome of error propagation in the prediction stage of BFSTDM is presented for the 2011 forecasts in two different scenarios in Table (3-17) and Table (3-18) respectively. In the first scenario, the final error variance is estimated considering the input measurement errors only, while in the second scenario, both the input measurement errors and the BFSTDM specification errors are combined.
According to Table (3-17), it is observed that the variance of error in different BFSTDM steps is located in a different domain. The main reason to have a different domain for the variance of error in the trip generation models compared to the second and the third steps of BFSTDM comes from having a different number of elements in each step. In the trip generation step, the model outcomes are calculated for 868 traffic zones, while in the next steps, the created errors in the first step are distributed among 868x868 O-D pairs. Therefore, the share of an O-D pair from the total variance of error would be considerably less than a traffic zone. Changes in the variance of error from the trip generation step to the modal split step are almost the same across trip purposes in the first scenario. In Table (3-17), for each trip purpose, the overall trend of variance of error is decreasing across the BFSTDM steps with an exception in the trip attraction step.

### Table (3-17) – Overall Outcomes of Error Propagation in BFSTDM in the First Scenario in 2011

<table>
<thead>
<tr>
<th>Trip Purpose</th>
<th>BFSTDM Steps</th>
<th>Trip Generation</th>
<th>Trip Distribution</th>
<th>Modal Split</th>
<th>Traffic Assignment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Trip Production</td>
<td>Trip Attraction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HBWB</td>
<td>1124.80</td>
<td>6242</td>
<td>0.18</td>
<td>0.13</td>
<td>361.2</td>
</tr>
<tr>
<td>HBWW</td>
<td>5674.10</td>
<td>56272</td>
<td>1.26</td>
<td>0.70</td>
<td></td>
</tr>
<tr>
<td>HBE</td>
<td>6059.40</td>
<td>32608</td>
<td>8.29</td>
<td>0.33</td>
<td></td>
</tr>
<tr>
<td>HBET</td>
<td>139.02</td>
<td>11154</td>
<td>0.17</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>HBS</td>
<td>10033</td>
<td>63802</td>
<td>15.12</td>
<td>5.90</td>
<td></td>
</tr>
<tr>
<td>HBO</td>
<td>9259.30</td>
<td>16453</td>
<td>8.10</td>
<td>2.63</td>
<td></td>
</tr>
<tr>
<td>ONHB</td>
<td>4668.86</td>
<td>22970.83</td>
<td>4.74</td>
<td>1.39</td>
<td></td>
</tr>
<tr>
<td>WBW</td>
<td>392.43</td>
<td>2264.80</td>
<td>0.07</td>
<td>0.04</td>
<td></td>
</tr>
</tbody>
</table>

* The unit of presented errors in trip generation, trip distribution and modal split steps is the number of trips per hour squared ((trip/h)²). The unit of traffic assignment error is the number of vehicles per hour squared ((veh/h)²).

### Table (3-18) – Overall Outcomes of Error Propagation in BFSTDM in the Second Scenario in 2011

<table>
<thead>
<tr>
<th>Trip Purpose</th>
<th>BFSTDM Steps</th>
<th>Trip Generation</th>
<th>Trip Distribution</th>
<th>Modal Split</th>
<th>Traffic Assignment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Trip Production</td>
<td>Trip Attraction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HBWB</td>
<td>2885.50</td>
<td>7218.70</td>
<td>0.25</td>
<td>0.18</td>
<td>2097.2</td>
</tr>
<tr>
<td>HBWW</td>
<td>51476</td>
<td>108110</td>
<td>5.90</td>
<td>2.23</td>
<td></td>
</tr>
<tr>
<td>HBE</td>
<td>13389</td>
<td>39920</td>
<td>16.85</td>
<td>0.88</td>
<td></td>
</tr>
<tr>
<td>HBET</td>
<td>2408.50</td>
<td>11984</td>
<td>0.46</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>HBS</td>
<td>24406</td>
<td>83755</td>
<td>20.56</td>
<td>8.59</td>
<td></td>
</tr>
<tr>
<td>HBO</td>
<td>22332</td>
<td>24873</td>
<td>9.88</td>
<td>3.62</td>
<td></td>
</tr>
<tr>
<td>ONHB</td>
<td>16772.03</td>
<td>39757.56</td>
<td>7.71</td>
<td>2.23</td>
<td></td>
</tr>
<tr>
<td>WBW</td>
<td>507.21</td>
<td>2442.20</td>
<td>0.08</td>
<td>0.04</td>
<td></td>
</tr>
</tbody>
</table>

* The unit of presented errors in trip generation, trip distribution and modal split steps is the number of trips per hour squared ((trip/h)²). The unit of traffic assignment error is the number of vehicles per hour squared ((veh/h)²).
Table (3-18) shows the combined propagation of the input measurement errors as well as the model specification errors, the variance of combined error is obviously larger than the corresponding steps in Table (3-17). Any difference between the variance of error of a specific step in the first scenario and the second scenario comes from two sources: the specification error of that step and the propagation of the specification errors of the previous steps. As the model specification errors are given in Table (3-16), any difference between the results of the two scenarios at a specific step that is further than the specification error of that step is attributed to the propagation of specification error of its previous steps.

Comparing the overall results of the two investigated scenarios in Table (3-17) and Table (3-18), it is observed that taking the model specification errors into account in the second scenario has the severest effect on the traffic assignment step. The shares of different error sources in the total variance of traffic assignment error in the second scenario are 361.20 and 1056 (veh/h)^2 for the propagated measurement errors and the traffic assignment specification error respectively. These values result in a share of 680 as the cumulative effect of propagation of model specification errors of all previous steps.

In addition to Table (3-17) and Table (3-18) that provide an overall outcome of applying the proposed approach to the BFSTDM for all trip purposes and for both scenarios, the following subsections present specific graphical outputs for each BFSTDM step and trip purpose.

### 3-2-4-1-Error Propagation in the Prediction Stage of Trip Generation

Considering the first scenario, Figure (3-8) and Figure (3-9) display respectively the errors of the produced trips and the attracted trips for the trip purpose of HBWB. In the figures, the variance of error is shown relatively by the circle size. In terms of the variance of output error, Figure (3-8) shows that the traffic zones with the largest trip production errors are usually located far from the Brisbane CBD, while according to Figure (3-9), the largest trip attraction errors occur in traffic zones closer to the Brisbane CBD.
Outcomes of adding the variance of HBWB trip production and attraction specification errors from Table (3-16) to the propagated error in the second scenario are presented in Figure (3-10) and Figure (3-11) respectively. In comparison with the first scenario, the results of error propagation are relatively similar due to having a constant model specification error across traffic zones.
In the trip distribution step, the errors generated in the traffic zones are distributed between the O-D pairs. Figure (3-12) shows the result of distributing errors for the trip purpose of HBWB in the first scenario where the propagation of input measurement errors are only included. In Figure (3-12), the top 100 O-D pairs that possess the largest variance of error and include the intrazonal travel demands are presented. As observed in Figure (3-12), a
considerable part of the top 100 O-D pairs are the intrazonal trips. The top 100 O-D pairs are spread almost uniformly in the city of Brisbane. Considering Figure (3-8) and Figure (3-9) about the created trip generation errors, many of O-D pairs in Figure (3-12) connect the origins and the destinations that don’t have a high trip generation error; therefore, the mathematical operations involved in the trip distribution gravity model intensify the variance of errors associated to these O-D pairs.

Figure (3-12) – Variance of Error of Distributed Trips \((\text{trip/h})^2\) for HBWB in the First Scenario in BFSTDM

In the second scenario that the variance of trip distribution specification error is added to the propagated input measurement error, all O-D pairs experience an increase in the variance of error. Figure (3-13) displays the result of distributing errors for the trip purpose of HBWB in the second scenario. Concerning the location of the top 100 O-D pairs in Figure (3-13), it is observed that the location of top 100 O-D pairs have changed in comparison with the first scenario in Figure (3-12). In the second scenario, more O-D pairs among the top 100 are observed in the north of Brisbane city.
Figure (3-13) – Variance of Error of Distributed Trips ((trip/h)²) for HBWB in the Second Scenario in BFSTDM

3-2-4-3-Error Propagation in the Prediction Stage of Modal Split

In the prediction stage of the BFSTDM modal split step, the propagation of error from the trip distribution step toward mode car is investigated. Figure (3-14) presents the top 100 O-D pairs with the highest variance of error for the travel demand of mode car in the first scenario. According to Figure (3-14), the share of intrazonal trips from the top 100 O-D pairs has increased compared to the Figure (3-12) that presents the variance of error in the trip distribution step. In comparison with Figure (3-12), the location of top 100 O-D pairs has changed significantly, and has moved from the north toward the CBD and the east. Totally, the top 100 O-D pairs are spread more uniformly across the city of Brisbane.

It is also revealed that the O-D pairs have experienced different changes in the variance of error compared to Figure (3-12) such that some circles become bigger, while others become smaller. In the first scenario, the differences between Figure (3-12) and Figure (3-14) in terms of the variance of error is contributed by the measurement errors of the input variables involved in the utility functions of modes in the modal split step.
Figure (3-14) – Variance of Error of Mode Car ((trip/h)^2) for HBWB in the First Scenario in BFSTDM

Figure (3-15) – Variance of Error of Mode Car ((trip/h)^2) for HBWB in the Second Scenario in BFSTDM

Figure (3-15) shows the top 100 O-D pairs in the second scenario for trip purpose of HBWB in the AM-peak period. The location of top 100 O-D pairs has changed slightly compared to Figure (3-14).

3-2-4-4-Error Propagation in the Prediction Stage of Time-Of-Day

After propagating error from the inputs to the travel demand of mode car in all trip purposes, the matrices of the OD demand error of mode car in different trip purposes in
the AM-peak period are added to reach a single final matrix. Figure (3-16) provides the top 100 OD pairs that have the largest error variances in the final matrix in the first scenario.

The variance of error in Figure (3-16) has changed incredibly compared to the outcome of the modal split step in Figure (3-14). Comparing the AM-peak period outcome with the HBWB modal split outcome, it is revealed that share of intrazonal trip errors has decreased considerably. Additionally, it seems the number of traffic zones involved in the set of top 100 OD pairs in Figure (3-16) has decreased compared to the HBWB modal split outcome in Figure (3-14). In Figure (3-16), the largest error variances occur in travel demand between two traffic zones in the north and in the southwest that are far from the Brisbane CBD, and the eastern traffic zones and the Brisbane CBD.

Considering the second scenario, the result of error propagation to the top 100 OD pairs for all trip purposes in the AM-peak period is provided in Figure (3-17). Comparing with Figure (3-16) in the first scenario, the selected OD pairs for the top 100 set experience a slight change. The variance of error deals with a big change compared to the first scenario.

Figure (3-16) – Variance of Error of Mode Car ((trip/h)²) for All Trip Purposes in First Scenario in BFSTDM in AM-Peak Period
3-2-4-5-Error Propagation in the Prediction Stage of Traffic Assignment

In the traffic assignment step, the matrix of the O-D demand errors in the AM-peak period including all trip purposes is assigned on the Brisbane network. Figure (3-18) shows the result of assigning the O-D demand error matrix in the first scenario that involves only the propagation of the input measurement errors from all BFSTDM steps. According to Figure (3-18), the largest variance of error occurs usually on the main roads like motorways and highways; however, the size of variance of error decreases once approaching toward the Brisbane CBD. In this light, following a trip starts from an origin far from the CBD and ends to the CBD, it is revealed that the width of bars decreases.

Considering the second scenario, Figure (3-19) presents the result of assigning the O-D demand error matrix that involves the effects of BFSTDM input measurement errors and the model specification errors of all BFSTDM steps. In this scenario, the effects of traffic assignment specification error as well as the propagation of the specification error of the previous steps are added to the results of the first scenario. For this reason, the variance of link volume errors becomes more than three times larger, while the traffic volumes on the links remain unchanged. The variance of traffic assignment specification error is constant across the links; therefore, any dissimilar increases in the variance of link volumes in Figure (3-19) compared to Figure (3-18) are rooted in the propagation of specification errors of the previous BFSTDM steps.
Comparing the error propagation through BFSTDM in the first scenario with the second scenario, the contributing share of model specification error to the outcome error variance of each step is revealed. In each step, this contributing share contains the shares of model specification of that step in addition to the propagation of model specification errors of previous steps. This implies the significance of improving the quality of model calibration. Based on the contributing share of input measurement errors to the outcome error of each BFSTDM step in the first scenario, the likely improvement of outcome due to an improvement in the input measurement accuracy is found.

Figure (3-18) – Variance of Link Errors ((veh/h)^2) in the First Scenario in BFSTDM
Figure (3-19) – Variance of Link Errors ((veh/h)²) in the Second Scenario in BFSTDM
3-3-Case Study 3

This section presents an example of how the O-D demand measurement errors are propagated through a UETA and combined with the UETA specification error. The layout of the network is provided in Figure (3-20). The network selected for this section connects two origins, 1 and 2, to two destinations, 3 and 4, respectively. Each O-D pair is connected by three parallel routes, namely, $F_1$, $F_2$ and $F_3$ for the first O-D pair, and $F_4$, $F_5$ and $F_6$ for the second O-D pair. Figure (3-20) also shows two parameters for the network links: capacity and free flow time. The O-D demand from origin 1 to destination 3 is 20, and from origin 2 to destination 4 is 40. Moreover, the used link volume delay function is based on the Bureau of Public Roads’ definition with $\alpha=0.15$ and $\beta=4.0$. To perform the MCS method, a combination of Emme and Matlab software packages is employed.

![Network Layout and Connecting Routes in the Case Study 3](image)

There are three main questions addressed in this example. First, it is determined how many samples from the distribution of the O-D demand measurement error can provide a reasonably accurate result for the MCS method. Second, since the proposed analytical method is valid in the neighbourhood of the current equilibrium, it is of interest to determine how large the O-D measurement error variances can be to retain a valid analytical sensitivity-based answer compared to the MCS one. Third, it is worthwhile to investigate the effect of different parameters on the shares of the variance and the covariance of route flow rates from the variance of an O-D demand measurement error.

3-3-1-Performing the Analytical Sensitivity-based Method on the UETA

Following the process explained in section (2-2-3-1), the matrices M and N are needed. Referring to Eq. (2-50), since there are two O-D pairs in addition to six connecting routes,
the resulting matrix \( M \) is a 14x14 square matrix, and subsequently, matrix \( N \) is a 14x2 matrix. Using Eq. (2-48), the partial derivatives of route flow rates with respect to the O-D demands are provided in Table (3-19). In the next step, the measurement error variances considered for the O-D demands are propagated through an UETA program using Eq. (3-23), and the calculated partial derivatives shown in Table (3-19). The values assumed for the variances of the O-D demand measurement errors and the corresponding correlation coefficient are 1, 4 and 0.2. The result of error propagation via the analytical method is presented in the last column of Table (3-19).

### Table (3-19) - Partial Derivatives with respect to O-D Demands and Analytically Calculated Variances

<table>
<thead>
<tr>
<th>Routes</th>
<th>Partial Derivatives</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>O-D1</td>
<td>O-D2</td>
</tr>
<tr>
<td>( F_1 )</td>
<td>0.256</td>
<td>0.011</td>
</tr>
<tr>
<td>( F_2 )</td>
<td>0.056</td>
<td>0.029</td>
</tr>
<tr>
<td>( F_3 )</td>
<td>0.688</td>
<td>-0.040</td>
</tr>
<tr>
<td>( F_4 )</td>
<td>0.033</td>
<td>0.451</td>
</tr>
<tr>
<td>( F_5 )</td>
<td>0.050</td>
<td>0.052</td>
</tr>
<tr>
<td>( F_6 )</td>
<td>-0.083</td>
<td>0.497</td>
</tr>
</tbody>
</table>

### 3-3-2-Sensitivity of the MCS Results to the Number of Samples

It is challenging to determine the number of samples requires from the distribution of O-D demand measurement error to obtain an accurate result in the MCS method. To address the effect of sample size on the accuracy of the output variances, the variance of route flow rates was calculated for different sample sizes. The variances and correlation coefficient considered for the O-D demand measurement errors are identical to those in section (3-3-1). For instance, to show how the MCS result converges to the corresponding analytical outcome, a comparison was made for the variance of \( F_4 \) for five different sets of O-D demand variances and covariance. To provide the possibility of comparison across the results of the sets, the difference between the MCS results and the corresponding analytical outcomes was calculated in percentage. Figure (3-21) presents the convergence trend of the MCS results. In the legend of Figure (3-21), the values respectively show the STD of the first and the second O-D pairs as well as the correlation coefficient.
As shown in Figure (3-21), as the number of samples increases, the estimated variances from the MCS method converge to the variances calculated by the analytical method. The MCS method took about 900 minutes on average to complete which included more than 5,000 UETA runs by the Emme software package. However, the fluctuation of the obtained results indicates that the MCS method still needs more samples to provide reasonably accurate results for the route flow rate variances. This issue definitely leads to a great computational effort in real case studies.

3-3-3-Validity of the Analytical Sensitivity-based Method

The validity of the analytical method was investigated for different parameters. For a given O-D demand matrix, since the proposed analytical method is valid in the neighbourhood of the current equilibrium, it is of interest to know in what range of variance and covariance of the O-D demand measurement errors, the analytical method remain valid to quantify the error propagation through an UETA program. The investigation was performed by comparing the resulting route flow rate variances in the analytical method with the MCS method.

To evaluate the validity of the analytical method in terms of the variance of O-D demand measurement error, the MCS method was performed for a variety of STD from 5% to 50% of the expected value of the O-D demand in 10 different scenarios. The obtained variances for the route flow rates were then compared with the analytical calculations in 5 selected scenarios as shown in Figure (3-22). In the legend of Figure (3-22), the values respectively show the STD of the first and the second O-D pairs as well as
the percentage of the STDs to the corresponding expected values. The gradient of linear trend lines in Figure (3-22) may be considered as a measure of the validity of the proposed analytical method compared to the MCS method. The expected values of the O-D demands and the correlation coefficient are 20, 40 and 0.2, respectively.

![Figure (3-22) - Comparison of Route Variances (trip/h²) in the MCS and Analytical Methods for Different STDs of the O-D Demand Measurement Errors](image)

To find out for what range of correlation coefficient the analytical method remains valid, 11 scenarios were defined with different correlation coefficients varying from -0.5 to 0.5 in 11 equal steps, and with similar variances, 1 and 4, for the O-D demand measurement errors. The MCS method was performed for each defined scenario, and then the resulting route flow rate variances were compared with the corresponding values in the analytical method as shown in 6 selected scenarios in Figure (3-23).

![Figure (3-23) - Comparison of Route Variances (trip/h²) in the MCS method with the Analytical Method for Different Correlation between the O-D Demand Measurement Errors](image)
As observed in Figure (3-22) and Figure (3-23), the variances of route flow rates are more sensitive to the variance of the O-D demand measurement errors than the correlation coefficients. The ideal results in the analytical method provide a gradient close to 1 for the linear trend lines. In the current study, the range in which the analytical method remains valid is considered between 0.9 to 1.1. Figure (3-22) shows that the validity of the analytical method decreases significantly after the 6-12 set that has STDs six times larger than the basic scenario with values of 1 and 2 as the STDs of the O-D demand measurement errors. In contrast, in Figure (3-23), the analytical method provides a consistent result with an acceptable gradient for most of correlation coefficients.

3-3-4-Shares of the Variance and the Covariance of Route Flow Rates

Considering the conservation constraint in an UETA program, the variation or the measurement error of an O-D demand is distributed between two groups: the variance and the covariance of the route flow rates connecting the O-D pair. When a large part of the O-D demand measurement error is attributed to the covariance group, it means the parallel route flow rates change similarly most of the time. In other words, it means in the case of an O-D demand increase/decrease, both parallel routes experience a relatively similar increase/decrease in flow rate, which leads to a large covariance.

The similarity of the routes behaviour in the current study is dependent on the similarity of the routes traffic conditions. For example, if two parallel routes have links with the same level of service, in the case of gradually increasing O-D demand, a corresponding continuous increase in both flow rates is expected. A corresponding decrease might also occur once there is a decrease in O-D demand. For this example, there is a positive large correlation between the route flow rates.

On the other hand, for two parallel routes with different level of services, congested and uncongested, the route flow rates increase with two different patterns: one before becoming congested and one when congested. Before the congestion, the uncongested route attracts more demands than when congested during which there is a competition between the two routes, and the uncongested route would attract less demand. The different increase patterns lead to different behaviour from the route flow rates that subsequently result in lower correlation between the route flow rates.

As a result, when the variance of the O-D demand measurement errors increases, the occurrence of a larger O-D demand is more likely and subsequently the congestion on both routes increases. The congested parallel routes behave similarly which leads to a
high positive correlation between the routes. High correlation between the route flow rates results in attributing a smaller share to the variance group as observed in Figure (3-24). In Figure (3-24), the values on the horizontal axis shows the STDs of the first and the second O-D demand measurement errors. Moreover, as shown in Figure (3-25), the increase of correlation between the two O-D demand measurement errors makes a higher correlation between the errors of the flow rates of the nonparallel routes that connect different O-D pairs. The STDs of the O-D demand measurement errors in Figure (3-25) are as assumed in section (3-3-1). In Figure (3-24) and Figure (3-25), the average variances and correlation coefficients are calculated respectively from the parallel and the nonparallel route flow rates. It is also observed that the average correlation between nonparallel routes changes linearly against the correlation between the O-D demand measurement errors referred to Figure (3-25). However, in Figure (3-24), the average parallel route variances change less and approach a constant value as the STD of O-D demand measurement error increase.

![Figure (3-24) - Average of Parallel Path Variances ((trip/h)²) versus STDs of O-D Demand Measurement Errors](image-url)
Referring to section (2-1), the variance of an UETA specification error is measured on links in the validation step. The propagated measurement error on a link is calculated using Eq. (2-35) and the obtained route flow rate errors. Calculating the variance of link volume errors by either the analytical or the MCS methods, the variance of UETA specification error may also be added to the propagated measurement errors. In the case of considering 20 and 1, and 40 and 4, respectively, as the mean and the variance of error in the first and the second O-D demands with a correlation of 0.2, the variance of the resulting propagated measurement errors for links (9,10) and (1,5) are, for example, provided in Eq. (3-25) and Eq. (3-26), respectively.

\[
\begin{align*}
\text{var}(x_{9,10}) &= \text{var}(F_3) + \text{var}(F_6) + 2 \times \text{cov}(F_3,F_6) = 0.453 + 0.912 + 2 \times 0.016 = 1.397 \\
\text{var}(x_{1,5}) &= \text{var}(F_3) + \text{var}(F_2) + 2 \times \text{cov}(F_3,F_2) = 0.453 + 0.008 + 2 \times 0.042 = 0.545
\end{align*}
\] (3 – 25) (3 – 26)

The assumed specification error for the present illustrative example has a mean of zero and a STD of 0.4. It is assumed that there is no correlation between the propagated measurement error and the UETA specification error; therefore, the variance of the resulting error of links (9,10) and (1,5) in Eq. (3-25) and Eq. (3-26) increases to 1.557 and 0.705, respectively, to take into account the UETA specification error.
3-3-6-Conclusion

In this case study, the variance of specification error of an UETA program was measured using validation techniques. Moreover, the propagation of input measurement error from O-D demands to the output of an UETA program was investigated using two different methods: an analytical sensitivity-based method and a simulation-based method. The simulation-based method ran a MCS employing Matlab and Emme software packages, while the analytical method used the results of a SA on an UETA program to measure the propagation of error.

The proposed method for error propagation was applied to an illustrative example to address three main issues:

1. How many samples from the O-D demand measurement error distribution provide reasonably accurate results for the MCS method,
2. How large the O-D measurement error variances can be to retain validity of the analytical sensitivity-based method, and
3. How different the variance of an O-D demand measurement error is distributed between the variance and the covariance of path flow rates.

Further investigations are required concerning the validity of the proposed analytical sensitivity-based method such as finding a more accurate error propagation formula and finding a SA method that provides more generic results. The application of the proposed method on a real case study can also show the complexity of matrix calculations in the proposed analytical method.
Conclusions

The present study proposes an analytical approach to quantify the propagation of error in an FSTDM. The proposed approach is able to analytically quantify the output error variance of each step as a result of the input measurement errors as well as model specification errors. The proposed approach recommends various methods to determine the variance of model specification error of each of the four steps. The variance of model specification error is determined by employing the goodness-of-fit criteria or using the model validation process.

For each step, the relationship between the variances of input measurement errors, the model specification error and the output error of each step is derived by taking a variance from both sides of the calibrated model.

The proposed approach is applied to the FSTDM for the city of Brisbane as a real case study in two different scenarios: 1) how the measurement error of the input variables is propagated step by step; and, 2) how the combination of input error with the model specification error is propagated through the steps. As an example, a measurement error with the STD of 10 percent for the input variables of Brisbane FSTDM (BFSTDM) as well as specification errors of models calibrated for the Home Based Work - Blue collar (HBWB) trip purpose were explored. The specification error variance is 1760.77 (trip/h)², 976.72 (trip/h)², 0.01082 (trip/h)² and 0.001327 respectively for trip production, trip attraction, trip distribution and modal split steps. With these inputs, the variance of output errors in the second scenario for the same steps are respectively, on average, 2885.50 (trip/h)², 7218.70 (trip/h)², 0.25 (trip/h)² and 0.18 which outweigh the first-scenario results. Moreover, with a specification error of 1056 (veh/h)² for the traffic assignment step, the variance of output error in the same step of the second scenario (the scenario with model specification error) is calculated as 2097.20 (veh/h)² for all trip purposes, that is almost six times greater than the first scenario (with input errors only).

Considering the existing 868 traffic zones, from the first to the third step, a reduction in the created error variance per origin-destination (O-D) pair in both scenario is observed, while, in the traffic assignment step, with considering all trip purposes, the size of created error per link increases. This increase is more considerable for the second scenario due to the effect of model specification errors.

Obviously, the proposed approach is computationally more efficient than the simulation-based methods in terms of both result accuracy and computation time. The
main advantages of using the proposed approach compared to the existing error analysis approaches in the literature are:

- An efficient approach that is less time consuming which makes the proposed approach practical for large scale networks,
- A procedure that is able to quantify separately and collectively the share of different sources of error in the traffic forecast error, and
- An approach that is able to analyse the sensitivity of the traffic forecast accuracy to the input measurement error and the quality of modelling.

Using the proposed framework, the error propagation in the prediction stage of a User Equilibrium Traffic Assignment (UETA) is also addressed. In the third case study, the variance of specification error of a UETA is measured using validation techniques. Moreover, the propagation of input measurement errors from O-D demands to the output of a UETA is investigated using two different methods: an analytical sensitivity-based method and a simulation-based method. The simulation-based method runs a Monte Carlo simulation, while the analytical method uses the results of a sensitivity analysis on the UETA to measure the propagation of error. Application of the proposed method to the third case study provides three main outcomes:

1. More than 5000 samples are required from the O-D demand measurement error distribution to provide reasonably accurate results from the MCS method,
2. The STDs of O-D measurement errors can be up to six times larger than the basic scenario, while still retaining the validity of the analytical sensitivity-based method, and
3. The variance of the O-D demand measurement error is distributed between the variance and the covariance of path flow rates depending on the congestion level of the paths. The more the paths are congested, the more the error can be attributed to the path flow rate covariance.

In the current study, the error propagation is examined only in a typical FSTDM, while the proposed approach is also applicable to other types of transportation demand models including: FSTDMs with different types of models in the steps; tour-based travel demand models; and, activity-based models. Each of these modelling approaches needs further research. Within the proposed framework, it is also possible to consider more complicated models like a non-linear model and a nested logit model for the trip generation and the modal split steps respectively. In this case, it is required to employ approximation-
based approaches, like a Taylor series expansion, to calculate the variance of error in the nested logit model.

The accuracy of measuring the model specification error depends on the number of available observations in each of the different steps of an FSTDM. In this light, a large amount of information collected automatically from road and public transport networks can be employed to improve the accuracy of measurement of model specification errors. Furthermore, due to a lack of observations, the correlation between different input error sources is taken as an assumption; however, in future studies, the correlation needs to be measured using relevant observations to provide a more accurate result. Comparing the total output errors measured using the proposed approach with the acceptable ranges of error specified in transportation guidelines, decision makers now have a clear opportunity to realise the credibility of a point traffic forecast and its associated variance based on the four step model properties.
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### Table (A1-1). The actual trips produced from origin zones

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### Table (A2-2). Travel impedance matrix

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## Appendix-A3

### Table (A3-1). The actual values of distributed trips of mode car between OD pairs

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### Table (A3-2). The actual values of distributed trips of the mode bus between OD pairs

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Figure (A4-1). Output table of LIMDEP software containing the estimated parameters in the modal split step
Appendix-A4

|-> IMPORT;FILE="C:\Users\uqhrezae\Desktop\RTA.txt"$
| Last observation read from data file was 2394
|-> read;nvar=3;nobs=2394;file=C:\Users\uqhrezae\Desktop\RTA.txt$
|-> nlogit;lhs=PROPORTI;choices=r1,r2,r3
 ;model:
 u(r1)= B0*ROUTE1/
 u(r2)= B0*ROUTE2/
 u(r3)= B0*ROUTE3
 ;Wts=WEIGHT
 ;Prob=RoutePr$

Normal exit: 5 iterations. Status=0, F= 11041.013

Discrete choice (multinomial logit) model
Dependent variable Choice
Weighting variable WEIGHT
Log likelihood function  -11041.0134
Estimation based on N = 798, K = 1
Inf.Cr.AIC  = 58793.7 AIC/N = 73.6763
Model estimated: Jul 03, 2014, 10:16:46
R2=1-LogL/LogL* Log-L fncn R-sqrd R2Adj
Constants only must be computed directly
Use NLOGIT ;...;RHS=ONE$
Response data are given as proportions.
Number of obs. = 798, skipped 0 obs

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Note: ***, **, * ==> Significance at 1%, 5%, 10% level.

|-> SAVE;file="C:\Users\uqhrezae\Desktop\RTA.lpj"$

Figure (A4-2). Output table of LIMDEP software containing the estimated parameters in the traffic assignment step
#### Table (A5-1) – Estimated Coefficients for Utility Functions across Trip Purposes in BFSTDM

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*Definition of attributes is provided in the list of abbreviations.*