Acceleration-Displacement Crash Pulse Optimisation –
A New Methodology to Optimise Vehicle Response for Multiple
Impact Speeds

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Previous crash pulse optimisation studies have involved optimisation of a vehicle crash pulse at a single impact speed. Whilst the results of such a study may lead to significantly improved crash performance at the speed considered, design for vehicle safety requires satisfactory performance across a range of impact speeds. A method of optimising vehicle response across a range of impact speeds is required, balancing safety requirements across each potential impact scenario. This paper describes a new methodology developed to achieve this aim.

It was proposed that the front structure of the vehicle – which crushes during impact, thereby decelerating the remaining structure – could be modelled as a non-linear spring. Assuming that the force-displacement curve of this spring was independent of impact speed, it followed that the deceleration of the remaining vehicle structure was also a function of crush displacement. Hence, using a vehicle’s unique acceleration-displacement profile during a crash event, the dynamic response of the vehicle could be derived for any impact speed.

Optimisation of an acceleration-displacement curve was performed using a Simulated Annealing algorithm. At each iteration, acceleration-time crash pulses were derived for the standard crash test speeds of 28, 48 and 56 km/hr. Occupant response was simulated in MADYMO, and a harm metric was used to estimate injury cost at each speed. A function weighting harm across all three speeds was used as the optimisation objective function.

Comparing optimised results with results from a set of real vehicle crash pulses, it was found that occupant harm was significantly reduced for each impact speed. As expected, improvements were not as great as those achieved for crash pulses optimised for a single impact speed only, highlighting the trade-off required for optimised performance across multiple speeds. However, the methodology demonstrated that a set of significantly improved acceleration-time crash pulses could be generated, each being consistent with a single vehicle acceleration-displacement profile, and therefore a single structural design.

NOTATION

\[ x \]  
vehicle crush displacement (m)

\[ v_{\text{max}} \]  
maximum design impact velocity (m/s)

\[ s^1, s^2 \]  
non-dimensional displacement (-)

\[ a \]  
vehicle acceleration (m/s^2)

\[ a^1, a^2 \]  
non-dimensional acceleration (-)

\[ a_{\text{sf}} \]  
acceleration scaling factor (m/s^2)

\[ m_{\text{sf}} \]  
mass of structure aft of front firewall (kg)

\[ s_0 \]  
initial crush displacement (m)

\[ s_f \]  
final crush displacement (m)

\[ s_{\text{max}} \]  
maximum crush displacement (m)

\[ s_{\text{sf}} \]  
displacement scaling factor (m)

\[ s_{\text{max}} \]  
maximum crush displacement (m)

\[ v \]  
vehicle velocity (m/s)

\[ t \]  
time (s)

\[ k, k(s) \]  
vehicle front structure stiffness (N/m)

\[ F(s) \]  
vehicle front structure reaction force (N)

\[ v_0 \]  
initial vehicle impact velocity (m/s)

\[ v_f \]  
final vehicle impact velocity (m/s)

\[ v_{i,j} \]  
any impact velocity, i (m/s)

\[ m_{\text{fr}} \]  
mass of structure aft of front firewall (kg)

INTRODUCTION

Applied to full frontal collisions, the crash pulse describes the forward deceleration of the vehicle occupant compartment as it crushes into a barrier during a collision. Usually defined
by a deceleration-time plot, the shape of the crash pulse has a direct effect on the resulting interaction of occupants within the vehicle, and therefore directly affects occupant injury risk during a collision. As such, several studies have previously been conducted to study the effect of crash pulse shape on occupant injury risk.

Previous studies have focused on crash pulse optimisation at a single impact speed. Whilst such studies have yielded various degrees of improved occupant response, they are subject to a severe limitation – a vehicle must protect its occupants across a range of impact speeds. Several authors have suggested solutions to this problem. Complex adaptive structures have been proposed which would permit optimal deceleration response at any speed [1], however their complexity and expense will disallow such designs in the foreseeable future. Others have proposed non-adaptive structures which aim to produce crash pulses which are desirable across a range of speeds [2]. Certainly, this is the only viable alternative available at present. The problem is identifying what the optimal stiffness distribution of such structures should be.

This paper describes a new methodology to optimise vehicle response for multiple impact speeds, through optimisation of the vehicle acceleration-displacement curve. Preliminary results suggest that unlike previous studies, this new methodology can be used to lower injury risk of a non-adaptive structure across several impact speeds. This research is part of a broader crash pulse optimisation study. Another paper has been written on optimisation of vehicle crash pulses for single impact speeds [3]. Reference to this paper will provide more detailed description of some of the concepts and methodology common to both studies.

THEORY

Acceleration-displacement crash pulse

Considering the vehicle during a full frontal crash, prior to the crash the vehicle is travelling with initial velocity, \( v_0 \), towards a rigid barrier. As the vehicle ploughs into the barrier, the front structure crushes, transforming the vehicle’s kinetic energy primarily into strain energy within the vehicle’s deforming front structure, thereby bringing the vehicle to rest.

The displacement of the vehicle during the crash is not constant across the structure. The front of the vehicle slows down immediately as it hits the barrier, whilst the rest of the structure decelerates at a slower rate. Modern vehicles are generally designed so that the front of the structure, forward of the front firewall, will absorb the collision energy of a full frontal crash. The remaining rearward structure generally remains structurally intact. Consequently, this rearward structure displaces approximately as a whole system. A useful definition of the vehicle crush displacement during a crash, denoted by \( s \), may therefore be taken to be the displacement of the vehicle rearwards of the front firewall, as illustrated in Figure 1(a).

![Figure 1](image_url)

Figure 1. (a) Measurement of crush displacement. (b) Idealisation of front structure.
Referring to Figure 1(a), structure rearward of the front firewall is considered to act like a rigid body during a full frontal crash. As the front structure crushes, it applies a decelerating force to the rest of the vehicle, which varies with crush displacement. The vehicle frontal structure can be likened to a nonlinear spring, with stiffness $k$, which varies with spring compression (the crush displacement, $s$). This concept is illustrated in Figure 1(b).

As the vehicle crushes, the spring produces a resisting force, $F(s)$, which decelerates the rigid body portion of the vehicle (with mass denoted by $m_{RB}$). Figure 2(a) below illustrates this rigid body approximation at the initiation of the collision event. At any given impact speed, a force-displacement curve may be used to describe the variation of this force as the vehicle front structure crushes. An example of such a curve is illustrated in Figure 2(b) below:

$$s(0) = 0$$
$$v_0$$
$$F(s)$$
$$m_{RB}$$

(a)  

$$F(s)$$  

(b)  

Figure 2. (a) Force of deforming front structure. (b). Example of a force-displacement curve

In order to proceed, an important simplifying assumption is made about the force-displacement characteristics of the vehicle front structure. It is assumed that the force-displacement behaviour of the front structure is a constant property of the system, independent of collision velocity, or any other factor. This assumption implies that for any collision speed, at a given crush displacement, the applied force on the rigid body portion of the vehicle is the same. This assumption is only approximately true – to varying degrees, rate of deformation does affect the stiffness response of a structure. The validity of this assumption is considered later in the discussion section of this paper.

Referring to Figure 2(b), $F(s)$ acts on rigid body mass, $m_{RB}$. Summing forces in the direction of initial motion, $v_0$:

$$-F(s) = m_{RB} \cdot a$$  \[1\]

where $a$ is the longitudinal acceleration of the rigid body mass during the collision event. Equation [1] may be rearranged as follows:

$$a = -\frac{F(s)}{m_{RB}}$$  \[2\]

Observing Equation [2], if the rigid body mass remains constant during the collision, and if the force is a function of displacement only, then it must follow that vehicle acceleration is also a function of displacement only:

$$a = a(s)$$  \[3\]

Equation [3] presents a significant proposition, with implications as follows:

1. Any given structural configuration will have a unique acceleration-displacement function, $a(s)$.
2. $a(s)$ is a constant property of the system, independent of impact speed or any other consideration.
3. Given any initial impact speed, \(v_0\), \(a(s)\) may be used to derive the deceleration response (crash pulse) of the system.

4. There is a maximum impact velocity, \(v_{0,\text{max}}\), for which a given \(a(s)\) will produce a complete deceleration of the vehicle. If this impact velocity is exceeded, the vehicle will still have velocity once all of the available crush space has been used.

**Crash pulse requirements**

Some constraints were imposed on the crash pulse variable generated by the optimiser. At each iteration of the optimisation, the following crash pulse characteristics were enforced:

1. The change in velocity across the acceleration-displacement crash pulse was set equal to the maximum allowable impact speed, \(v_{0,\text{max}}\).
2. The total crush displacement available was set equal to a maximum length, \(s_{\text{max}}\).
3. The acceleration was forced to be negative or zero for the duration of the impact.
4. The initial acceleration was set equal to zero.

These requirements are outlined in Figure 3 (a) and (b) below.

![Figure 3. (a) General crash pulse model. (b) Crash pulse requirements](image)

**Crash pulse generation**

The variable crash pulse had to be expressed in terms of a discrete set of optimisation variables. A set of \(n\) discrete control point variables was used to define a non-dimensional acceleration-displacement crash pulse shape. The coordinates of these control points were defined in terms of non-dimensional acceleration, \(\hat{a}\), and non-dimensional displacement, \(\hat{s}\):

\[
a^*(s) = (a_1^*, a_2^*, ..., a_n^*)^T \quad [4] \\
s^* = (s_1^*, s_2^*, ..., s_n^*)^T \quad [5]
\]

The set of non-dimensional displacement control points, \(s^*\), was kept constant for any given optimisation task. The set of non-dimensional acceleration control points, \(a^*\), was defined as the set of optimisation variables. A cubic spline was interpolated through the control points, resulting in a set of data points defining an approximately smooth function:

\[
a^*(\hat{s}) \Rightarrow a^*(s^*) \quad [6]
\]

This interpolated function represented the non-dimensional crash pulse shape, and required scaling so that displacement and velocity criteria were met. The continuous non-dimensional
crash pulse shape was therefore scaled to meet the requirements outlined in Figure 3(b). Two scaling factors were derived to scale \( a' (s') \) into a fully dimensional crash pulse:

\[
\{s\} = s_{sf} \times \{s'\} \quad [7] \\
\{a\} = a_{sf} \times \{a'\} \quad [8]
\]

Referring to Equations [7] and [8], it can be shown that:

\[
s_{sf} = \frac{s_{\text{max}}}{s_f} \quad [9] \\
a_{sf} = \frac{-v_{0,\text{max}}^2}{2 \int_{t_0}^{t_{\text{max}}} a \cdot ds} \quad [10]
\]

Hence, a methodology was developed to generate an acceleration-displacement crash pulse which satisfies maximum crush displacement and maximum impact velocity criteria. For any impact speed \( v_{0,f} < v_{0,\text{max}} \), the acceleration-time response of the vehicle may then be derived.

**OPTIMISATION PROCESS**

An optimisation process was developed to optimise an acceleration-displacement crash pulse across the three industry standard and regulatory impact velocities: 28 km/hr (ADR), 48 km/hr (ADR) and 56 km/hr (NCAP). The acceleration-displacement crash pulse was scaled to satisfy a maximum impact speed of 56 km/hr, and a maximum crush displacement of 600 mm.

Occupant response was simulated using a generic driver-side MADYMO restraint model supplied by Holden Ltd. The optimisation was performed with the robust Simulated Annealing global search algorithm, via the commercial optimisation code iSIGHT. Occupant injury was estimated using a harm metric of the following form:

\[
\text{Harm} = \sum_{\text{all injuries considered}} \left( \text{Probability of injury} \times \text{cost of injury} \right) = f \left( \text{HIC, } N_j, \text{ CTI, Femur} \right) \quad [11]
\]

The harm metric used injury criteria from MADYMO to predict the probable cost associated with these injuries. The derivation of this harm metric is discussed in another paper [4]. Harm was calculated for each impact speed, and weighted by the relative frequency of crashes at each given velocity. Weighting values were derived from MUARC [5]. A total weighted harm was then calculated, incorporating all three impact speeds. This total weighted harm was used as the optimisation objective function:

\[
\text{Harm} = 0.430 \times \text{Harm}_{28 \text{ km/hr}} + 0.377 \times \text{Harm}_{48 \text{ km/hr}} + 0.193 \times \text{Harm}_{56 \text{ km/hr}} \quad [12]
\]

Figure 4(a) below shows an illustration of the driver-side restraint system used for the analysis. The general optimisation model used in the analysis is presented in Figure 4(b). It is noted that the process shown is a simplified schematic of the more complex system developed:
OPTIMISATION RESULTS

An acceleration-displacement crash pulse, defined by 21 control points, was optimised for minimum weighted occupant harm, across the three impact velocities of 28, 48 and 56 km/hr. The optimised harm results were compared to harm results calculated for a set of real-life crash pulses representative of a mid-sized sedan crash, provided by Holden Ltd for comparative purposes. It was found that optimisation approximately halved the weighted occupant harm compared to the representative crash pulses. Figure 5(a) below shows the harm calculated at each individual velocity, and the total weighted harm, for both the representative and optimised crash pulses. Figure 5(b) provides a corresponding summary of calculated injury criteria for representative and optimised crash pulses. It is clear that optimisation of the acceleration-displacement crash pulse across the three velocities produced reductions in occupant injury risk at each velocity. Figure 5(b) demonstrates that significant reductions in harm correspond to significant reductions in most of the injury criteria:

![Graph showing harm comparison](image1)

Figure 5. Results for harm optimised vs. representative crash pulses. (a) Comparison of estimated occupant harm. (b) Comparison of injury criteria

Figure 6(a) below compares optimised and representative acceleration-displacement curves. The three acceleration-time crash pulses derived from the optimised acceleration-displacement curve are shown in Figure 6(b-d), for each of the three velocities. These are compared to each of the corresponding representative crash pulses for each speed.

![Graph showing acceleration-displacement comparison](image2)

(a) (b) (c) (d)

Figure 6. Optimised vs. representative vehicle crash pulses. (a) Acceleration-displacement crash pulses. (b) 56 km/hr acceleration-time crash pulses. (c) 48 km/hr acceleration-time crash pulses. (d) 28 km/hr acceleration-time crash pulses.
The occupant responses were analysed at each impact speed, for both representative and optimised crash pulses. Figure 7(a) shows the resultant seatbelt loads for the representative crash pulses. Figure 7(b) shows the resultant seatbelt loads for crash pulses derived from the optimised acceleration-displacement curve.

![Graphs showing resultant seatbelt loads for representative and optimised crash pulses.](a) (b)

Figure 7. Resultant seatbelt loads. (a) Representative crash pulses. (b) Optimised crash pulses.

DISCUSSION

The results demonstrate that the acceleration-displacement crash pulse optimisation methodology can be used to produce a set of vehicle crash pulses which outperform the real-life crash pulses provided by Holden Ltd. However, unlike previous crash pulse optimisation studies, this new methodology ensures that these optimised crash pulses are compatible with one another, both in terms of structural design, and the use of vehicle crush displacement.

Observing the results, it is clear that reductions in injury are significant, particularly at higher speeds. Referring to Figure 7(a,b), it appears that the common trend between optimised crash pulses is that occupants are loaded up faster, courtesy of the high initial deceleration spikes observed in each of the crash pulses (Figure 6). The balanced reductions in injury levels at each speed also validate the harm metric used. The absolute values of the harm estimates are not of great relevance – what is important is that the dollar cost allocated to different sets of injuries is properly weighted so that an automated optimisation process will be effective.

It is suspected that the three localised deceleration spikes observed in Figure 6(a) have evolved during the optimisation routine in order to provide specific performance advantages at the three impact velocities considered. It is possible that the sequence of these spikes is important, and that at other impact velocities these deceleration spikes may actually be harmful to the occupant. In terms of improving performance in industry crash tests, the crash pulse only has to be optimised for the speeds which the vehicle will be tested at. However, this pragmatic option does not responsibly address real world design requirements. In reality, a whole spectrum of velocities should be considered in order to obtain a true “overall” optimised acceleration-displacement pulse. It is expected that as the number of impact velocities considered is increased, the performance of the optimised pulse for any of these individual speeds may suffer, but will still provide a tangible improvement over current designs.

Finally, the proper value of these results depends upon the validity of the key assumption underpinning the methodology: that vehicle structural stiffness does not vary with impact velocity. The proper verification of these results would be achieved via structural simulation and crash testing. However, at this conceptual stage, several factors are noted. Firstly, it is noted that strain-rates change the stiffness of structural materials. For example, high strain rates increase the strength and energy absorption of mild steel [6]. However, strain rate effects usually only vary significantly over several orders of magnitude of strain rate. Serious vehicle impacts occur over a comparatively narrow range of impact velocities (considering this analysis, the difference between maximum and minimum velocities is 50%). Further, an
increasingly popular automotive structural material, aluminium, has been shown to have mechanical properties which are relatively insensitive to strain rate effects [6, 7].

Secondly, material mechanical properties are not the only structural properties which are sensitive to loading rate. The dynamic response of the structure (i.e. the resulting vibrations and oscillations which result after impact) is not independent of the loading rate.

Thirdly, current numerical simulations of vehicle collisions do not model strain rate phenomena with much accuracy, often under-predicting the dynamic stiffness of vehicles. Despite this inadequacy, current numerical simulations are capable of producing meaningful predictions of the structural response.

Hence, the assumptions made in this analysis should nonetheless provide meaningful and relevant results. However, it is expected that this methodology would have less value as a detailed design tool. It is more likely to have use as a means of identifying an approximately optimal structural response, which can then be used to guide decisions about structural layout and design at the preliminary design stage.

CONCLUSION

Based on the assumption that the vehicle force-displacement curve is independent of impact velocity, the methodology introduced in this paper allows the vehicle structural response to be optimised across multiple speeds. This analysis has shown that the methodology is viable – that it can be successfully implemented and optimised using commercially available software packages. The methodology yields sufficient improvement in occupant response to justify further investigation.

Key factors which require addressing include:

- Verification of the assumptions via structural testing and computer simulation;
- Consideration of a greater number of impact speeds.

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