Population Estimates and Projections for Australia’s Very Elderly Population at State and National level

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Abstract

As in many developed countries, the very elderly population (ages 85+) is the fastest growing age group in Australia, with far-reaching economic and social consequences. To effectively plan and budget for the income, aged care and health care needs of the very elderly, accurate estimates and projections are required. There are, however, several obstacles relating to the availability and accuracy of very elderly data. Official population estimates at very high ages in Australia have been found to be too high and fluctuate implausibly over time. International and local studies have found large errors in projected very elderly populations, stemming from inaccurate mortality rate forecasts.

This thesis aims to create accurate estimates and projections for Australia’s very elderly population at a state and national level. Various methods for estimating very elderly populations from death counts were assessed for accuracy at both a national and state level. While the Human Mortality Database uses such methods to create estimates for many countries, their accuracy has never been assessed for Australia. Furthermore, little is known, locally or internationally, about their performance at a sub-national scale. In this study, the accuracy of various nearly-extinct-cohort methods were assessed at the Australian national and state level by retrospectively applying them to extinct cohorts and comparing the results against those obtained from applying the Extinct Cohort method. Suitable methods were applied to create very elderly population estimates and death rates for Australia from 1972-2012 by sex, state and at single ages 85-110+. The growth, changes in the age-sex composition and the demographic drivers of growth of Australia’s very elderly population were analysed. Based on these estimates, more reliable death rates were calculated, allowing a detailed study of the changing patterns and trends in Australian adult mortality. Finally, a number of mortality forecasting methods were retrospectively evaluated for their accuracy in projecting adult death rates for Australia over 10 and 20 years ending in 2012. An appropriate method was applied to create projections for the next three decades.

It was found that, compared to the official census-based estimates, more plausible and accurate estimates, especially for ages 95+, could be derived from death counts. The Survivor Ratio (SR) method with results constrained to official estimates for ages 85+ produced accurate very elderly population estimates for Australia across the sexes and ages at both a national and state level. Internal migration is sufficiently minor to be ignored. Very accurate
state-level estimates could also be derived using a simpler method of apportioning national single-age estimates between the states. Adult death rates in Australia were found to show consistent and regular patterns of decline since the 1970s, with rates of decline decreasing with age. These patterns support the use of simple direct extrapolation methods for forecasting. The Geometric, Ediev and Lee-Carter BMS methods were all very successful in projecting adult death rates and very elderly populations, and differences between them were small.

Australia’s very elderly population increased more than four-fold between 1981 and 2012, from 105,000 to 430,000, or from 0.7% to 1.9% of the total population. Accompanying this growth was an ageing of the very elderly population itself, as well as increasing sex ratios. Improvements in survival beyond age 65 and especially beyond age 85 were the main drivers of growth in nonagenarian and centenarian numbers. Australia’s very elderly population is expected to continue growing rapidly over the next 30 years to almost 1.5 million in 2042, or 4.2% of the total population. Centenarian numbers are expected to increase from almost 3,500 in 2012 to over 15,000. South Australia’s very elderly population is projected to grow the least and Western Australia’s the most. Projected very elderly numbers in 2042 are 13% higher than official projections. Official projections of centenarians were, however, 55% higher. These differences stem from overstated official population estimates and understated death rates for the high ages as well as lower assumed declines in death rates in official projections.

This thesis presents more plausible and detailed estimates and projections of Australia’s very elderly population at a state and national level based on methods found to be accurate. It is shown that estimation methods based on deaths can be reliably applied at a sub-national level. The analyses improve our understanding of the demographic drivers of growth of nonagenarians and centenarian populations in Australia, nationally and in the separate states. Insights are gained on how the patterns and trends of change in adult mortality in Australia compare to those internationally. This study furthermore contributes new insights into the accuracy of both old and new mortality forecasting methods. An improved understanding of past trends based on accurate population estimates and death rates, combined with more accurate projections based on accurate data and appropriate methods, facilitates effective planning and budgeting for infrastructure, services, aged pension, aged care and health care for the very elderly.
Declaration by author

This thesis is composed of my original work, and contains no material previously published or written by another person except where due reference has been made in the text. I have clearly stated the contribution by others to jointly-authored works that I have included in my thesis.

I have clearly stated the contribution of others to my thesis as a whole, including statistical assistance, survey design, data analysis, significant technical procedures, professional editorial advice, and any other original research work used or reported in my thesis. The content of my thesis is the result of work I have carried out since the commencement of my research higher degree candidature and does not include a substantial part of work that has been submitted to qualify for the award of any other degree or diploma in any university or other tertiary institution. I have clearly stated which parts of my thesis, if any, have been submitted to qualify for another award.

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Contributions by others to the thesis

None

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<td>Australian Bureau of Statistics</td>
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<td>ACT</td>
<td>Australian Capital Territory</td>
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<td>AIHW</td>
<td>Australian Institute of Health and Welfare</td>
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<td>DA</td>
<td>Das Gupta Advanced method</td>
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<td>Das Gupta method</td>
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<td>ERP</td>
<td>Estimated Resident Population</td>
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<td>Kannisto-Thatcher Database</td>
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<td>MD</td>
<td>Mortality Decline method</td>
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Abbreviations
Chapter 1. Introduction

1.1 Background

*The growing significance of the very elderly in Australia*

The ageing of populations in developed countries, a consequence of the demographic transition, is a well-known phenomenon that has been receiving increasing attention over the last three decades. Stage four of the demographic transition, with death rates and birth rates stabilising at low levels, is characterised by low population growth and an age structure becoming more weighted to older people (Goldstein, 2009). In Australia, the population aged 65+ (the elderly) grew from 160,000 in 1901 (4% of the total population) to 1.1 million in 1970 (8.3%) and further to 3.2 million in 2012 (14.2%) (ABS, 2012a). Those aged 85+ (the very elderly) comprise the fastest growing segment of the population, increasing from 67,000 or 0.5% of the total population in 1970, to almost double that at 154,000 or 0.9% in 1992, and then more than double again to over 400,000 or 1.9% in 2012 (ABS, 2009a, 2012b). This reflects a 160% increase in very elderly numbers over the two decades from 1992 to 2012, compared to total population growth of 30% (ABS, 2012a). Large cohorts from the baby boom decades, combined with declining mortality rates, are driving expectations of continued significant growth in elderly numbers in the next two decades and in very elderly numbers for several decades thereafter (ABS, 2012b; Wilson, 2012b).

The demographic characteristics, health status, living arrangements and care needs of the very elderly differ significantly from that of other age groups. As a result, their growing numbers have widespread economic and social consequences, for federal and state governments, business and industry, families of the very elderly and the very elderly themselves. Cognitive, physical and psychological functional limitations increase with age, with resulting increases in the need for care. In 2009, 75% of Australian females aged 90+ and 50% aged 85-89 suffered from severe or profound disabilities which meant they required daily assistance. These proportions represent a significant increase compared to 31% at ages 80-84 and 19% at ages 75-79 (ABS, 2010). Unfortunately, trends in the prevalence of disability among very elderly Australians, especially severe and profound restrictions in the ability to perform core activities, have not been decreasing with increasing longevity (AIHW, 2000, 2008, 2014). This means that the ageing of the 85+ age group has resulted in an overall
increase in the prevalence of severe core activity restrictions (AIHW, 2000, 2008). In 2006, 31% of those aged 85+ were living in cared accommodation, compared to only 7% of those aged 75-84 years (ABS, 2006). Increased longevity for both men and women is expected to result in couples surviving together for longer, possibly providing care that would otherwise have to be sought elsewhere. On the other hand, smaller families and increased workforce participation among women may mean children may be less available than before to provide care for ageing parents, resulting in greater demands for community-provided services or aged-care institutions (ABS, 2012b). Significant increases in government spending on aged care are expected, driven both by growth in spending on residential aged care and costs associated with community care (ABS, 2012b). Good quality data and insight into the reasons for the growth in very elderly numbers are thus becoming increasingly important.

The consequences of numerical and structural ageing of the population for the costs and fiscal impact of income support, aged care and health care is becoming a major focus for policymakers in Australia, as it is in many countries. Along with climate change, population ageing and health care were the main themes of the Treasurer’s 2010 Intergenerational Report (IGR) (Treasury, 2010). The focus of the 2015 Intergenerational Report was largely on the fiscal implications of expected changes in Australia’s population structure under various policy scenarios (Treasury, 2015). The Commonwealth government is the primary funder of Medicare, whilst public hospital services are jointly funded by the Commonwealth and State governments. Contributions are also made to the funding of non-government organisations for services such as residential aged care and community care (Treasury, 2004). Government costs related to ageing and health care as a percentage of GDP are expected to increase rapidly in the future. The 2010 IGR expected this to result in fiscal gaps from the early 2030s unless either tax rates are increased, spending is curtailed, or labour force productivity increases significantly. In the absence of significant changes, the Treasury in 2010 expected government spending on health, age-related pension and aged care to increase from a quarter to half of total government spending over the next 40 years (Treasury, 2010). The Treasury proposed a number of changes to social security benefits in 2015 in order to improve the expected fiscal position. Accurate data, especially at very elderly ages where a large portion of the total social security cost is incurred, are vital for ensuring these measures are effective.
In June 2006, 66% of Australians over the qualifying age (65 years for men and 63 for women at 30 June 2006) received the means-tested Age Pension, which is funded by the Commonwealth government on a pay-as-you-go basis (AIHW, 2007). In the last twenty years, various policy decisions were taken aimed at reducing reliance on, or the cost of government-funded income. These include the introduction of a superannuation guarantee in 1992, changes in the taxation of superannuation benefits and an increase in the qualifying age (AIHW, 2007). Accurate data on very elderly estimates and projections are essential for assessing whether these measures are sufficient.

**Very elderly demographic data**

Given the challenges posed by increasing very elderly numbers, accurate estimates and projections are vital for policy development, budgeting and strategic planning for services. The Australian Institute of Health and Welfare (AIHW 2007: iv) states, “The availability of high-quality data and accompanying analysis that paints a meaningful picture of ‘older’ Australia, and which reflects this complexity and diversity, is therefore fundamental to improving understanding of the situation as a whole and the many possible situations within it.” However, few detailed analyses have been undertaken of the changing demography of Australia’s very elderly population. This is surprising given the fact that Australians’ life expectancy at birth is the fourth highest in the world (UNPD, 2012), and that the growth rate of Australia’s elderly population over the period 2005 to 2010 was surpassed only by that in Japan (Stokes & Preston, 2012).

Several obstacles relating to the availability and accuracy of data on the very elderly in Australia prevent a detailed understanding of the growth in numbers, changing age and sex composition and drivers of the growth. The Australian Bureau of Statistics (ABS) has provided official population estimates for Australia at a national level from 1921 and at state level from 1961. Official statistics on population estimates and projections inform policy decisions and facilitate the monitoring of existing government programs, both at federal and state or territory level. However, in common with countries without population registers, these estimates suffer from a number of shortcomings. The first problem is that the ABS derives population estimates from census counts. It is widely recognised that at very high ages official population estimates and mortality rates derived from census counts suffer from inaccuracies due to incorrect age information provided in census forms and errors introduced
by census processing and editing (Coale & Kisker, 1986; Hill, Preston, & Rosenwaike, 2000; Kannisto, Lauritsen, Thatcher, & Vaupel, 1994; Preston, Elo, & Stewart, 1999; Thatcher, 1999a). As a result, population estimates at the highest ages are often inflated and mortality rates understated, with the degree of inaccuracy increasing with age (Preston et al., 1999; Thatcher, 1992).

According to Wilson and Bell (2004), official Australian population estimates provided by the ABS for ages 90 and older appear to be too high and fluctuate implausibly over time. The ABS’ estimated numbers of centenarians in Australia seem especially problematic, and McCormack (2010: 102) has highlighted the fact that there “are no exact or validated figures on the number of centenarians in Australia”. Preston et al. (1999) showed that any form of age misstatement at the highest ages, whatever the bias, results in an understatement of mortality rates. The degree of inaccuracy increases with age due to the steep slope of the mortality curve at these ages, and the increasing tendency for age to be misreported (Andreev, 1999; Hill et al., 2000; Preston et al., 1999). A further limitation of official population estimates is that the ABS publish single year of age population estimates up to age 99 only, with the final open-ended age group being 100+ (ABS, 2013a) (ABS 2013). Rapidly growing numbers of centenarians make single year of age mortality rates at ages above 100 increasingly important for robust projections.

To overcome the shortcomings of census-based population figures, indirect estimation methods have been widely used to produce alternative estimates based on deaths data (Bourbeau & Lebel, 2000; Coale & Kisker, 1990; Hill et al., 2000; Kannisto, 1988; Thatcher, 1992, 2001). Because dates of birth are typically verified at death, ages on death certificates are generally regarded as more accurate than those given on census forms. Population estimates for the very elderly derived from deaths data are thus considered to be more reliable than official population estimates (Coale & Caselli, 1990). Such estimates have been found to exhibit more plausible patterns over time and across ages, especially at ages above 90, and are very accurate when compared with estimates from population registers (Jdanov, Scholz, & Shkolnikov, 2005). The use of deaths data to create population estimates also ensures consistency between numerator and denominator in the calculation of mortality rates, resulting in more accurate rates than those derived from deaths and census-based population estimates (Thatcher, 1992).
Such indirect estimation methods have been tested for a number of countries. The Kannisto-Thatcher Database (KTD) and the Human Mortality Database (HMD) were established in the 1990s and contain indirectly estimated population numbers at ages 80 and older for a large number of countries, including Australia. The method used to derive these estimates has been shown to produce relatively accurate estimates for nine large European countries. Indirect estimation methods have not been tested for Australia, however. Also, only national-level estimates are produced and indirect estimates have not been widely applied at a sub-national level. The availability of accurate estimates for this population age group at a state or regional scale is essential to effectively plan and budget for services.

**Understanding the drivers of growth of very elderly numbers in Australia**

Since the establishment of the KTD and HMD, extensive analyses have been undertaken on very elderly populations in a large number of European countries, Japan, the US, Canada and China (Rau, Soroko, Jasilionis, & Vaupel, 2008; Robine & Paccaud, 2005; Robine & Saito, 2009b; Vaupel, 1997b, 2001; Yi & Vaupel, 2003). Some studies included Australian data from 1965 or 1970 (Jdanov, Jasilionis, Soroko, Rau, & Vaupel, 2008; Kannisto, 1994; Kannisto et al., 1994; Rau et al., 2008), but a detailed analysis of the growth of very elderly numbers in Australia has not recently been undertaken and no such analysis exists at a sub-national level. International studies indicated that, while trends in very elderly numbers are similar, there are significant differences between countries in terms of timing, patterns and rates of change in the numbers, age and sex composition, and mortality rates (Rau, Muszynska, & Vaupel, 2013; Robine & Paccaud, 2005; Robine & Saito, 2009a; Vaupel, 1997b, 2001).

Analyses undertaken by organisations such as the Australian Bureau of Statistics (ABS), the Australian Institute of Health and Welfare (AIHW), the Department of Health and Ageing, the Treasury Department and other governmental bodies use ABS population estimates. Problems relating to very elderly estimates derived from census-counts were highlighted earlier. In addition, ageing and longevity in Australia is generally discussed in terms of aggregated numbers for ages 65+ or 85+, or in the context of structural ageing of the population as a whole. These are typically based on measures such as life expectancy at birth, median age, the old-age dependency ratio, and proportions of the elderly relative to the total population. While these measures are important, they conceal compositional changes within
the elderly group, such as changes in the age and sex structure. Furthermore, cross-country comparisons of structural measures may be misleading. For example, large net international migration gains in Australia compared to other developed countries resulted in a significantly lower median age and lower elderly proportion, creating the impression that ageing may be a less significant issue for Australia. However, unless the number of immigrants increases every year, immigration tends to make populations younger in the short term but older in the longer term as migrants age with the population (Goldstein, 2009; UNPD, 2001). Therefore, in addition to structural measures, it is important to have available and analyse accurate numbers of the very elderly. Numerical ageing (as opposed to structural ageing measures) is also more relevant to policy formulation and planning relating to aged care, health care and residential care (Wilson, 2012b).

Studies relating to the demographic characteristics of Australia’s very elderly population were limited to centenarians and were based on ABS data (McCormack, 2010; Richmond, 2008). There were, however, significant differences between these ABS estimates and those in the KTD. For example, the estimated numbers of centenarians quoted by Richmond (2008) of 443 in 1981 (at 30 June), 1,268 in 1991, and 2,297 in 2001 differ significantly from those sourced from the KTD (at 1 January) by Rau et al. (2008), namely 412 in 1980, 955 in 1990 and 1,465 in 2000. While the KTD and HMD estimates are derived from death counts and have been found to be more reliable than the official estimates, there also seem to be problems with earlier estimates in the KTD and HMD. The HMD’s number of centenarians per million of the total population in Australia on 1 January 1960 of 17 (Rau et al., 2008) seems implausibly high compared to that in other low-mortality countries, for example, 4.1 for Denmark, 8.1 for France and 5.4 for the Netherlands (Kannisto, 1994).

These differences highlight a need for assessing the accuracy of the methods used for deriving population estimates for Australia in the KTD and HMD, as well as of official population data. More accurate population estimates and mortality rates will allow a better understanding of the demographic drivers of historical growth in numbers and will facilitate more accurate projections. There is furthermore a need to develop more accurate very elderly estimates at a sub-national (e.g. state) level, where planning and budgeting for aged and healthcare services and infrastructure, catering for the needs of the very elderly, takes place.
Understanding mortality patterns and trends in Australia

Very elderly population growth is largely driven by mortality declines, so that an understanding of mortality trends is essential for creating accurate projections. Changes in the age profile of death rates as well as death frequency distributions have been studied extensively in many low-mortality countries. While general trends are similar, experience varies in terms of the timing and extent of mortality improvement at different ages. Studies considering changes in the age profile of adult or elderly death rates include Kannisto (1994; 1996), Bongaarts (2005), Rau et al. (2008) and Thatcher (1999b). Other studies have analysed past trends in modal ages at death (the age at which the highest concentration of deaths occur) and the concentration of deaths around the modal age. These include, for example, Canudas-Romo (2008), Kannisto (2000, 2001) and Cheung and Robine (2007); Cheung, Robine, Paccaud, and Marazzi (2009); Cheung, Robine, Tu, and Caselli (2005); Robine, Cheung, Horiuchi, and Thatcher (2008); Thatcher, Cheung, Horiuchi, and Robine (2010). These studies found that during the first half of the twentieth century substantial declines in death rates at younger ages resulted in the greater concentration of deaths at the modal age. The decline of death rates at older ages during the second half of the twentieth century caused a shift in death frequency distributions with increasing modal ages. This was accompanied by a greater compression of deaths into smaller age bands. In some countries a slow-down in the pace of compression has been observed in recent years.

There is, however, little understanding of how patterns and trends in Australian mortality compare with those of other low-mortality countries. It is not known whether and to what extent compression, shifting or expansion in the death frequency distribution has occurred. Furthermore, very few studies have considered relationships between measures relating to death frequency distributions and the age profile of death rates e.g. Thatcher (1999b). A greater understanding is needed as to how information on past trends in the age profile of death rates and death frequency distributions by age can be combined to produce better forecasts.

Projected very elderly population numbers

While official population estimates at ages 90 and older in Australia have been found to be too high, a retrospective comparison of ABS projections by state, age and sex to actual Estimated Resident Population (ERPs) revealed that projected numbers for the 85+ age group
were generally too low (Wilson, 2012a). Wilson (2007) found that mean absolute errors of age-sex population projections for Australia were largest at the youngest and oldest ages, with high errors at older ages attributed to both inaccurate mortality rate forecasts and numbers by cohort moving into these age groups. Furthermore, official national and state projections are only provided for single ages to 99 and 84 and in aggregate for ages 100+ and 85+ respectively. Given the increasing care needs with advancing age, more state-level detail is needed at single ages above 85.

The historical under-projection of very elderly populations partly stemmed from an over-projection of death rates, or, more specifically, under-projecting the extent to which death rates would decline in future. This issue is not unique to Australia and is related to the use of expectation methods (Keilman, 1997; Lee & Carter, 1992; Murphy, 1995; Olshansky, 1988; Shaw, 2007). Expectation methods are a variation of pure extrapolation methods but incorporate an assumed target to be reached by a specified future date, based on expert opinion (Olshansky, 1988). Targets include, for example, future improvements in mortality rates or life expectancy, or a future level of life expectancy (Andreev & Vaupel, 2006; Olshansky, 1988; ONS, 2012a; Tabeau, 2001; Willekens, 1990). The ABS’ projections are based on assumed future improvements in life expectancy at birth (ABS, 2013c). Organisations such as the World Bank and the United Nations, and official agencies such as the UK’s Office for National Statistics and the US Social Security Administration have all used expectation methods (Andreev & Vaupel, 2006; Olshansky, 1988; ONS, 2012a; Willekens, 1990). However, mortality projections based on subjective views of future survival improvements or mortality declines have often turned out to be too high because mortality declined more than expected (Lee & Carter, 1992; Murphy, 1995; Olshansky, 1988).

The Lee-Carter method (LC) is a widely-used direct extrapolation mortality forecasting method acclaimed for its limited subjective judgement involved in setting assumptions relating to future mortality decline (Booth, Maindonald, & Smith, 2002; Lee & Miller, 2001; Tuljapurkar, Li, & Boe, 2000; Wilmoth, 1993). Median forecasts from the LC method typically produce significantly higher forecasts of life expectancies than official forecasts, and this has also been found to be the case for Australia (Booth & Tickle, 2003; Tuljapurkar et al., 2000). Over the last 20 years the LC model has gained popularity as an alternative to the approach of targeting a future level of life expectancy, mortality rates or rates of change
in mortality applied by many official agencies. However, the application of the method is complex in practice and there is scope for exploring simpler options. The Lee-Carter method is furthermore based on an assumption of constant rates of change in death rates over time as well as stable relationships between age-related rates of decline, which may not be borne out in reality.

Alternative direct mortality extrapolation methods include, for example, the Geometric method (Pollard, 1987), Ediev’s adjusted version of the Geometric method (Ediev, 2008) and a Relational model (Brass, 1974; Himes, Preston, & Condran, 1994). Mortality can also be forecast indirectly by extrapolating the parameters of functions fitted to the age profile of death rates (Keyfitz, 1982; McNown & Rogers, 1989). While there is a large literature on the appropriateness of fit of different mathematical functions to the age profile of mortality at a point in time, there are only a few studies on projecting mortality by forecasting the parameters of mathematical functions. These include, for example, Bongaarts (2005) and Gavrilova and Gavrilov (2011). Few recent studies exist of the relative accuracy of different extrapolative mortality forecasting methods. Keyfitz (1991) compared results from the Geometric and Brass methods as well as from a number of fitted mathematical models. Stoeldraijer, van Duin, van Wissen, and Janssen (2013) compared the life expectancy produced by different mortality forecasting methods. Neither study considered the accuracy of the methods. Another example is a study by (Hansen, 2013), where the accuracy of the LC, Geometric and Brass methods were compared based on simulated life tables. Booth, Hyndman, Tickle, and de Jong (2006) assessed the accuracy of death rates forecasted by the original LC method and four variants or extensions. Little information is available, however, on the relative accuracy of the LC methods compared to other methods, and the accuracy of various methods in forecasting very elderly populations (ages 85+) in particular is not well understood. There is thus considerable scope for investigating and comparing different mortality forecasting methods, both conceptually in terms of criteria such as simplicity and objectivity, and empirically in terms of validity and plausibility.

**Statement of the research problem**

In conclusion, accurate population estimates and death rates are fundamental to understanding changes in the demographic characteristics and the drivers of the significant historical growth of Australia’s very elderly population. The status quo (official estimates) is
an overstatement of population estimates and an underestimation of death rates at very high ages (90+), resulting in projections that are too low. The Human Mortality Database contains estimates based on death counts at the national level for Australia but accurate estimates are also needed at a sub-national level. An improved understanding of past trends, based on accurate estimates and death rates, will better inform the likely future. More accurate projections based on accurate data and appropriate mortality forecasting methods will facilitate more effective planning and budgeting for infrastructure, services, aged pension, aged care and health care.

1.2 Aims and objectives

This research aims to create accurate estimates and projections of Australia’s very elderly population at a state and national level. (Refer to Appendix B for a map of Australia’s States and territories.) To meet this aim the following objectives are addressed:

Objective 1: To identify appropriate very elderly population estimation methods

In order to meet the aim of creating accurate very elderly population estimates, appropriate estimation methods need to be identified. The following research questions are addressed:
- Based on a retrospective assessment of accuracy, which estimation methods are the most appropriate for estimating Australia’s very elderly population?
- How can these methods be adjusted for application at a sub-national level and how accurate are state-level estimates?
- How do official estimates provided by the ABS compare with indirect estimates?

The accuracy of the nearly-extinct-cohort methods are assessed by retrospectively applying them to create estimates for cohorts that were extinct or very nearly extinct by 31 December 2012. These cohorts were aged 90+ before 1996. National level data allows the accuracy of these methods to be considered from 1976 and state level data from 1981. The methods found to be most appropriate at a national and state level respectively are used to create national and state population estimates and death rates at single ages 85-110 for Australia from 1971 to 2012. Although applied to Australian data, the methods and insights gained are applicable in other geographical contexts.
Meeting this objective provides reliable and detailed estimates of Australia’s very elderly population at the state and national level. Accurate estimates are vital inputs for reliable projections. Insights are gained regarding the applicability of the methods at a sub-national level. Methodological contributions include a new method to allow explicitly for survival improvement and a method for incorporating interstate migration when applying the methods at the sub-national level.

**Objective 2: To understand changes in the age and sex distribution of Australia’s very elderly population and the demographic drivers of growth**

The second objective is to understand the historical changes in the age- and sex composition of Australia’s very elderly population and the demographic drivers of the growth in numbers. The data used for these analyses are the population estimates created under the first objective. This objective strives to answer the following research questions:

- How did the age-sex composition of Australia’s very elderly population at the state and national level change since 1981?
- To what extent was the growth in Australia’s very elderly population over the last three decades due to increases in birth cohort sizes, declines in mortality and changes in international migration?

This objective thus contributes substantive knowledge regarding changes in the age-sex distribution and the demographic drivers of very elderly population growth in Australia, which will allow better planning for the future. This also enhances our understanding of experience in Australia in the international context.

**Objective 3: To understand changes in adult mortality**

The third objective is to develop an understanding of the patterns and trends of change in adult mortality in Australia at a national level. An understanding of past trends will better inform the likely future. In particular, it will provide insights as to appropriate mortality forecasting methods and assumptions, as well as explain the behaviour of different methods. The following research questions are addressed:

- How did the age profiles of death rates and death frequency distributions in Australia change over time?
- Which mathematical functions best fit the age profile of adult death rates, and how did the parameters of these functions change over time?
- How did modal ages and concentrations of death by age change over time and is compression and/or shifting of ages at death observed?
- What relationships can be identified between changes in the age profile of death rates and the features of death frequency distributions?

The analysis covers the period from 1921 to 2012. Data used for these analyses are death rates from 1921 to 1970 obtained from the Human Mortality Database. Data for 1971-2012 are death rates created under the first objective for ages 85-110 and death rates based on death data and ABS official estimates (ABS, 2013).

This objective contributes substantive knowledge of changes in death rates and death frequency distributions in Australia. Australian evidence adds further insights to the international debate on compression and shifting mortality profiles. It furthermore makes a theoretical contribution by proposing a method for decomposing changes in modal ages and concentrations of deaths into changes in the level and slope of the age profile of death rates.

**Objective 4: To identify appropriate methods for forecasting mortality and create very elderly population projections**

In order to meet the aim of creating accurate very elderly population projections, appropriate mortality forecasting methods first need to be identified. The methods and assumptions found to be most appropriate were used to create reliable population projections at single ages 85-110 for Australia at a national and state level over the next 30 years. Given that the focus of this research is on projections of the very elderly population over the next 30 years, only adult death rates are considered. The research questions addressed are:

- Which mortality forecasting methods are most appropriate for projecting adult death rates and the very elderly population in Australia?
- What are the projected very elderly populations for Australia at a state and national level over the next 30 years?
- What are the drivers of future expected growth of Australia’s very elderly population?
- How do these projected numbers compare with ABS projections?
This objective makes a contribution to knowledge with respect to the accuracy of different extrapolative mortality forecasting methods for projecting adult death rates and very elderly populations. The outcomes from meeting this objective are reliable projections of Australia’s very elderly population at a state and national level, which will assist in appropriate planning and the management of longevity risk.

1.3 Thesis outline

This thesis is structured as follows. Chapter 2 contains a description of the literature on the different issues addressed in this thesis, namely: analyses of growth of very elderly populations worldwide, very elderly population estimation methods based on death counts, decomposition methods to dissect total growth into proximate drivers, the measures used and findings relating to the analyses of age profiles of death rates and death frequency distributions and, finally, mortality forecasting methods.

Chapter 3 presents the results of retrospectively testing population estimation methods (objective 1). Based on the results of Chapter 3, population estimates and death rates by state, sex, and single ages from 85-110 were created and used in the analyses presented in the subsequent chapters. Chapter 4 presents the analysis of very elderly population growth and changes in age-sex composition, as well as the decomposition of the growth of nonagenarian and centenarian populations into demographic drivers (objective 2). Chapter 5 contains an analysis of the patterns and trends of change in various mortality measures (objective 3). Objective 4 is presented in two chapters. Chapter 6 sets out the conceptual and empirical comparison of mortality forecasting methods and Chapter 7 presents population projections for Australia at single ages 85-110 at national and state level for the next 30 years. Chapter 8 contains a summary discussion of results and the conclusion.
Chapter 2. Literature review: very elderly population estimates and projections

2.1 Introduction

In this literature review chapter, a description of the explosion in very elderly numbers worldwide over the last six decades will provide some context for Australian trends. Problems relating to official very elderly population estimates, mentioned in the introduction, are discussed in more detail. Methods for estimating very elderly populations which are more reliable than conventional approaches based on census counts are described in section 2.3. This is followed by a review of the demographic drivers of growth in very elderly populations worldwide and the methods used to decompose total growth into these drivers. A description of the features and temporal changes in the age profile of death rates and death frequency distributions specific to adult ages is provided. An overview of mortality forecasting approaches is then given, together with a discussion of the criteria to be applied for choosing appropriate methods. This is followed by a detailed description of mortality extrapolation methods considered to be the most appropriate for the purpose of this thesis. Unless otherwise indicated, the term ‘very elderly’ refers to those aged 85+. In earlier studies, as reflected in some of the literature, the ‘very elderly’ referred to those aged 80+.

2.2 Explosion in the numbers and proportions of very elderly populations

The increase in the size of very elderly populations worldwide has been the topic of numerous international studies since the 1980s. A study by Kannisto (1994) revealed that, over the 40 years from 1950 to 1990, the number of octogenarians (ages 80-89) in twelve countries with reliable data (Austria, Belgium, Denmark, England and Wales, Finland France, West-Germany, Italy, Japan, Norway, Sweden and Switzerland) grew four-fold, nonagenarians (ages 90-99) eight-fold and centenarians (ages 100+) more than 20-fold.

While similar general trends were observed in many developed countries, the rates of growth of very elderly numbers varied significantly. For example, over the period 1950 to 1990, populations aged 80+ multiplied by a factor of 7.7 in Japan, by around 5.1 in Finland and Switzerland, by around 3.9 in Sweden and Italy and by 3.4 in France (Kannisto, 1994). More recently, diverse trends have continued to be observed, with the proportion of octogenarians
and nonagenarians increasing between 1990 and 2004 by a factor of 2 in Japan, 1.5 in Italy, 1.3 in Sweden, and 1.2 in France and Switzerland (Rau et al., 2008).

Expressed as a proportion of the total population, populations aged 80+ in Europe and Japan (on average for 32 countries) increased from 1.1% in 1950 to 3.9% in 2004 (Rau et al., 2008). In Australia the increase was from 1.2% to 3.3%. This compared to proportions in France increasing from 1.6% to 4.3%, in Japan from 0.4% to 4.5%, and in Sweden from 1.5% to 5.3%. Australia’s increase, while significant, was thus slightly less than in Europe and Japan.

The number of centenarians in low mortality countries roughly doubled every ten years since the 1950s (Kannisto, 2001; Kannisto et al., 1994; Robine & Saito, 2009b; Vaupel, 2001; Vaupel & Jeune, 1995). This equates to average annual growth rates of around 7%, which is significantly higher than overall population growth rates of around 1%-2% per year. For example, from 1950 to 2004, the number of centenarians in France increased from 195 to 11,020, from 265 to 7,778 in England and Wales, and from 46 to 1,191 in Sweden (Rau et al., 2008). The number of centenarians in Japan increased from 111 in 1950 to 21,713 in 2004 (Rau et al., 2008) and further to 40,399 in 2009 (Robine et al., 2010). Robine and Saito (2009b) identified regional differences in the centenarian increase rate over the period from 1996 to 2006, with the number of centenarians in Japan multiplying by a factor of 4 compared to 2.1 or more in Southern European countries (such as Italy, Portugal and, Spain) and 1.7 or less in Northern European countries (such as Sweden, England and Wales, Denmark, Norway and Iceland).

Rau et al. (2008) found that, on average for 32 countries with good quality data, the number of centenarians per million of the total population increased from 13 in 1950 and 15 in 1960, to 71 in 1990, and further to 131 in 2004. Kannisto (1994) observed that the highest ratios of centenarians per million of the total population in 1990 – around 60-70 per million – were observed in countries in which the demographic transition took place early and old age mortality has been low for a longer time. In Australia the number of centenarians increased from 56 per million of the total population in 1990 to 94 in 2004 (Rau et al., 2008). Australia’s proportion of centenarians was lower than the average of 131 across all 32 countries in the study, and significantly lower compared to countries such as France (178),
Japan (169), the US (154), England and Wales (138), Sweden (130) and Canada (122) (Rau et al., 2008).

Other, more well-known, structural ageing measures such as the median age and elderly proportions are also low for Australia compared to many other developed countries. For example, the median age of Australia’s population was 37.4 in 2012 (ABS, 2012a, 2012b), compared to 44.5 for Japan, 44.3 for Italy, 42.8 for Hong Kong, 41.7 for Sweden and 40.5 for the United Kingdom (UNPD, 2012). The proportion of elderly (65+) in Australia (13.5% in June 2010) is comparable to countries such as New Zealand (13.0%) and the US (13.1%), but low compared to Japan (22.7%) and Western European countries such the UK (16.6%), France (16.8%) and Italy (20.4%) (UNPD, 2012).

Different degrees of structural ageing reflect variations in demographic factors such as high net international migration and fertility rates. Similar to New Zealand and the US, Australia’s net international migration rates are high. The lower extent of structural ageing of a population due to high net international migration is only a short-term phenomenon, however, which cannot be sustained unless immigration numbers continue to increase (Goldstein, 2009). Lower structural ageing in Australia is also partly the result of an extended baby boom with birth rates falling only in the mid-1970s (Wilson, 2012b). According to Wilson (2012b), this is a demographic dividend, to be repaid in the form of increased structural and numerical ageing over the next three decades. Therefore, while Australia appears to be relatively young compared to other developed countries based on structural measures, very elderly numbers are expected to grow rapidly over coming decades. In order to appropriately plan for the aged care, health care and residential care needs of the very elderly we need to have accurate information on numerical ageing (Wilson, 2012b).

Very elderly numbers in Australia are on a comparable scale to many other countries for which significantly more extensive analyses have been undertaken. For example, based on data obtained from the Human Mortality Database and quoted in Rau et al (2008), it can be seen that in absolute terms there were greater numbers of people aged 80+ in Australia than in countries such as Denmark, Finland, the Netherlands, Norway, Sweden and Switzerland, and Australia’s very elderly population has also grown faster in recent years (Rau et al., 2008). For example, in 1980 there were 241,168 people aged 80-99 and 412 centenarians in Australia – of a similar scale as in Sweden at that time (254,080 and 308 respectively). By
2004, Australian numbers aged 80-99 had increased by a factor of 2.8 (to 670,137) compared to 1.9 in Sweden (to 474,661) (Rau et al., 2008). From 1980 to 2004 the number of centenarians in Australia increased by a factor of 4.7 (to 1,916) compared to Sweden’s 3.9 (to 1,191). However, comparatively little is known about the demographic characteristics of, or changes in Australia’s very elderly population.

2.3 Reliability of very elderly population data

Before performing detailed analyses of historical growth in Australian very elderly numbers, changes in the age and sex composition of this population group and the demographic drivers of growth, reliable data are required. In countries with population registers, such as Sweden, Denmark, the Netherlands and Belgium, the quality of data has been found to be very high and very elderly population numbers are accurate (Jdanov et al., 2008). However, it has been found that in countries without population registers, official population estimates and death rates at very high ages are problematic, largely due to the tendency for ages to be over- or understated on census forms, or errors in census editing and processing (Coale & Kisker, 1986; Kannisto, 1994; Kannisto et al., 1994; Thatcher, 1992). Factors contributing to data problems at the high ages often relate to shortcomings in the information regarding date of birth which is linked to the period of existence of a birth registration system (Hill et al., 2000; Jdanov et al., 2005; Kannisto, 1988). Robine and Saito (2009b) found that the number of centenarians in a country appeared to decrease before they started to increase, with the timing of the decrease corresponding to when the system of national vital statistics was ‘modernised’.

The Australian Bureau of Statistics (ABS) provides official demographic information for Australia, including population estimates, which are based on data from the latest census. There are, however, a number of problems inherent in the official data, the first of which relates to methodology changes resulting in inconsistencies when comparing estimates over time. After the 1981 census the ABS introduced the concept of Estimated Resident Population (ERP), which refers to the number of people who usually reside in Australia and is derived by adjusting the de jure census count for net census undercount, residents temporarily overseas, and overseas visitors (ABS 2009b). Annual population estimates have been provided for Australia at a national level from 1921 and at a state level from 1961, but only estimates from 1971 are based on the ERP definition (ABS 2009b). The method of
estimating the census undercount was furthermore improved in 2013, resulting in a “recasting” of ERPs from September 1991 to June 2011.

A further problem with official estimates in Australia is the lack of single year of age estimates above age 100. Up to 1970, the ABS provided only aggregated figures for estimated populations aged 85+, so that analyses of the demographic characteristics of the very elderly at single ages from earlier periods cannot be undertaken. Population estimates from 1971 are by single age up to 99, with estimates for ages 100+ being aggregated. The lack of data at individual ages above 100 in Australia means that age-specific death rates cannot be determined. Growing numbers of centenarians make mortality rates at individual ages above age 100 increasingly important for accurate projections.

2.4 Population estimation methods

The first objective of this study is to identify appropriate very elderly population estimation methods. This will be based on an assessment of the accuracy of methods for estimating very elderly populations from death counts for Australia. This section details the methods assessed for a number of other countries, including the methods used by the HMD.

The Extinct Cohort method is used to calculate historical population numbers for cohorts for which all members have died. A population estimate for any year and age of the cohort is obtained by summing all subsequent cohort deaths and it is assumed that net migration is negligible (Vincent, 1951). For cohorts that are not fully extinct, methods collectively described as nearly-extinct-cohort methods, are employed. These either estimate the cohort population at the most recent date based on survivor ratios, or do so indirectly by estimating future cohort deaths. A set of population estimates can then be obtained by summing deaths along the cohort as in the Extinct Cohort method. Nearly-extinct-cohort methods proposed in the literature include:

- the Survivor Ratio method, which estimates a cohort’s population from the survivorship of earlier cohorts (Dépoid, 1973),
- Das Gupta’s method, based on estimating future cohort deaths (Das Gupta, 1990),
- the method of Coale and Caselli (1990), which estimates very elderly populations from death counts and estimated rates of change in death rates by age,
- the Mortality Decline method, which estimates the survivorship of a cohort allowing for age-specific mortality decline over time (Andreev, 1999),
the Meslé and Vallin (2002) approach which, similar to that of Das Gupta, estimates future deaths for the youngest cohort from ratios observed among older cohorts,

- the Das Gupta Advanced method which estimates future cohort deaths but allowing for mortality decline (Andreev, 2004).

These methods are described below.

2.4.1 Extinct-Cohort method

The Extinct Cohort method estimates a cohort’s population for any year and age by summing subsequent cohort deaths (Kannisto et al., 1994). Algebraically, the population of the cohort aged $x$ last birthday on 31 December of year $t$ ($P_{x,t}$) for cohort $c$ is:

$$P_{x,t} = \sum_{i=1}^{w-x} D_{t+i}^c$$  \hspace{1cm} (2.1)

where $w$ is the age of extinction, and $D_{t+i}^c$ is the number of deaths in year $t+i$ from cohort $c$, which are people born in the year $t-x$. Thatcher, Kannisto, and Andreev (2002) estimated $w$ as the highest age at which there was expected to be only one survivor. The Extinct Cohort method is illustrated in Figure 2.1, which shows data graphically by age (vertical axis), time period (horizontal axis) and cohort (diagonal axis).

Deaths data are typically available by single year of age and single calendar year, so that the allocation between cohorts is approximate. Furthermore, when applying population estimation methods based on death counts, an implicit assumption is made that deaths are the only source of population flows, and thus that international migration at these ages is negligible and can be ignored (Thatcher, 1992). If this method is applied at sub-national level, allowance will need to be made for internal migration, if shown to be significant.
**Figure 2.1: The Extinct Cohort method of estimation**

Source: Kannisto (1994)

Notes: Numbers in shaded parallelograms are cohort-period deaths ($D_{t+i}$); the thick vertical line represents the end of year population ($P_{x,t}$), which is the sum of cohort-period deaths in future years.

### 2.4.2 Survivor Ratio method

The Survivor Ratio (SR) method, developed by Dépoid (1973), is an extension of the extinct cohort method. It estimates the size of a nearly-extinct cohort for a recent date and age on the basis of the survivorship of older cohorts to the same age. The population is obtained via a survivor ratio, defined as the ratio of a cohort’s population at the calculation date to its size $k$ years ago. It may be expressed as:

$$R_x = \frac{P_{x,t}}{P_{x-k,t-k}}$$ (2.2)

In line with the extinct-cohort method, the number of survivors from a particular cohort $k$ years earlier can be written as:

$$P_{x-k,t-k} = P_{x,t} + \sum_{t=0}^{k-1} D_{t-i}$$ (2.3)

so that the survivor ratio for this cohort over $k$ years can be expressed as:
\begin{equation}
R_x = \frac{p_{x,t}}{p_{x,t} + \sum_{i=0}^{k-1} n_{t-i}^c} \tag{2.4}
\end{equation}

The estimated population aged \( x \) at 31 December of year \( t \) can be obtained by solving for \( P_{x,t} \):

\begin{equation}
P_{x,t} = \frac{R_x}{1-R_x} \times \sum_{l=0}^{k-1} D_{t-l}^c \tag{2.5}
\end{equation}

The cohort’s population in earlier years and younger ages may then be obtained by summing deaths as in the Extinct Cohort method. To smooth out random variations, the estimated survivor ratio in equation 2.5 is usually based on the average experience of \( m \) older cohorts:

\begin{equation}
R_x = \frac{\sum_{j=1}^{m} p_{x,t-j}}{\sum_{j=1}^{m} p_{x-k,t-k-j}} \tag{2.6}
\end{equation}

2.4.3 Das Gupta’s method

The Das Gupta (1990) method involves first estimating future cohort deaths, based on death ratios, instead of starting from the assumed proportion of a cohort that is still alive at a certain date (Andreev, 1999). A death ratio is the number of deaths experienced by a cohort at a particular age relative to deaths at the previous age. Death ratios are typically averaged over a number of older cohorts, as illustrated in Figure 2.2. The death ratio at age \( x \), based on observed deaths among \( m \) older cohorts is calculated as:

\begin{equation}
dr_x = \frac{\sum_{i=1}^{m} d_{t-i}^c}{\sum_{i=1}^{m} d_{t-i-1}^c} \tag{2.7}
\end{equation}

The numbers of deaths from a cohort at future dates and at higher ages are then derived by applying these death ratios successively:

\begin{equation}
D_t^c = dr_x \times D_{t-1}^c \tag{2.8}
\end{equation}

Cohort population estimates are then derived in the same way as in the Extinct Cohort method by summing cohort deaths (equation 2.1). A very similar approach was applied by Meslé and Vallin (2002).
Figure 2.2: Das Gupta's death ratios
Source: Andreev (2004)
Notes: Numbers in shaded parallelograms are period-cohort deaths. A death ratio for a single cohort is calculated as the ratio of deaths in year $t$ to deaths in year $t-1$. Shown above are cohort-period deaths for three cohorts (c-3 to c-1) used to estimate deaths for cohort c.

2.4.4 Methods allowing for decline in mortality

In many countries mortality rates at high ages are declining over time (Kannisto, 1994; Rau et al., 2008). By applying survivor ratios or death ratios based on the observed deaths among older cohorts to younger cohorts without adjustment may therefore result in underestimating populations (Andreev, 2004). This decline in mortality rates over time can be allowed for directly by adjusting either the survivor ratios or death ratios, or indirectly by adjusting the resulting population estimates. Thatcher et al. (2002) and Wilmoth et al. (2007) indirectly allowed for mortality improvements by increasing the estimated populations produced by the Survivor Ratio and Das Gupta methods by a factor such that the total estimated population at ages 90 and older equal the total of official estimates at the calculation date. This also ensures a smooth transition into official population estimates.

With the Mortality Decline and Das Gupta Advanced (DA) methods, variants of the SR and DG methods respectively, explicit allowance is made for declines in mortality rates by adjusting survivor ratios and death ratios. These methods are not dependent on the existence
of reliable official population estimates for ages 90+. The Mortality Decline (MD) method involves modelling temporal declines in mortality rates for a given age by fitting a loglinear function to mortality rates observed for a number of older cohorts (Andreev, 1999). This function is then used to extrapolate age-specific mortality rates and survival ratios to the youngest cohort, after which the survivor ratio method is applied. The DA method is based on an assumption that mortality decline reduces linearly with increasing age (Andreev, 2004). In this case, mortality decline is expressed as a function of age rather than time. A linear function is fitted to observed mortality decline over ages 80 to 89 for a particular group of cohorts and extrapolated to older ages. This is similar in concept to a method used by Coale and Caselli (1990). Andreev applied the DA method to the US population with good results.

2.4.5 Retrospective testing of estimation methods

Over ten years ago, Thatcher et al. (2002) published the results of an empirical comparison of several variations of the Survivor Ratio, Das Gupta and Mortality Decline methods applied to nine countries (Denmark, England and Wales, Finland, France, Japan, Netherlands, Norway, Sweden and Switzerland). The methods were applied retrospectively to estimate populations aged 90 and older over the period 1960-95, and the results compared to estimates derived solely from the Extinct Cohort method. Based on average rankings of absolute deviations of estimated numbers from observed numbers at ages 95+ and ages 100+, they concluded that the Survivor Ratio method with survivor ratios based on 5 cohorts (m) and measured over 5-year (k) age bands and with results constrained to official 90+ population estimates, produced the most accurate results. Together with the Extinct Cohort method, this method is now used for producing estimates in the Human Mortality Database (HMD) at ages 80 to 110+ for 37 countries, including Australia (Wilmoth et al., 2007), and is also applied by the UK’s Office for National Statistics (ONS, 2012b).

Andreev (2004) compared the relative percentage errors at ages 90+ from the DA method against those from unconstrained SR and DG variants for the same countries as above and also the United States. He found that for large populations such as England and Wales, France and the US, the DA method, which explicitly allows for mortality decline, produced more accurate estimates than the SR and DG methods where no allowance was made for declining mortality rates. A comparison of results from the DA method against those from the SR and DG methods with results constrained to official totals was, however, not made. The
greater accuracy of the DA method was furthermore less conclusive for the smaller populations of Denmark, Finland, the Netherlands, Norway, Sweden and Switzerland. Australia’s very elderly population is somewhat larger than those of these countries but significantly smaller than those of France, England and Wales, Japan and the US (Rau et al., 2008).

Both the studies by Thatcher et al. (2002) and Andreev (2004) made very important contributions to the literature. However, they are now more than ten years old, and very elderly population numbers have grown substantially since then, with the possibility that the best method may have changed over time. Moreover, neither study included Australia. There is thus a need to retrospectively test a number of nearly-extinct-cohort methods for estimating very elderly populations in Australia in order to determine which is most accurate. This is the first objective of this thesis.

Furthermore, while the Survivor Ratio method has been widely applied to estimate very elderly populations at a national level, there are very few examples of its application for estimating very elderly populations at a sub-national level. At a sub-national level, it may not be appropriate to ignore migration and the Survivor Ratio method may therefore not produce reliable results. Alternatively, it may have to be adjusted to allow for internal migration. Kannisto (1990) demonstrated that the Extinct Cohort method can be successfully applied at sub-national level and to lower ages by applying it to derive estimates for the provinces of Finland from age 65. While this study made an important contribution to the literature, it was carried out more than 20 years ago and applied to a much earlier era, namely the late 19th and early 20th century. Furthermore, it was applied to provinces in Finland where migration rates are much lower than in Australia. In the UK, the Survivor Ratio method has been used since 2010 to derive official population estimates for ages 90 to 105+ at a sub-national level namely for England and Wales (combined), Scotland and Northern Ireland (ONS, 2012b). However, it is not known how the accuracy of nearly-extinct-cohort methods, applied at a sub-national level, compares to that of census-based estimates. Given the importance of very elderly populations at a sub-national level for policy and research purposes, better methods are needed to estimate these fast-growing populations.

This thesis makes a substantive contribution to knowledge by assessing estimation methods based on death counts for Australia and applying the most appropriate method to create
reliable very elderly estimates. This includes consideration of alternative methods to allow for survival improvement, thereby making methodological contributions to the international literature. The new proposed method for making explicit allowance for survival improvement is simpler than other methods discussed in the literature and remove reliance on official estimates. The evaluation of these methods at a sub-national level is a novel contribution providing insight into the reliability of the methods at different geographical scales.

2.5 Demographic drivers of very elderly population growth

Accurate population estimates and death rates will allow the demographic drivers of very elderly population growth to be identified, which is part of the second objective of this thesis. The size of any population is a function of the births, deaths and migrants into and out of the population. Very elderly or centenarian populations thus need to be traced back to births more than 85 or 100 years ago, migrants into and out of these cohorts and the mortality experience of the cohorts over their lifetimes. Investigations into the demographic drivers of growth in very elderly European and Japanese populations have revealed upward trends in births in the late 19th century and significant improvements in survival, especially at very elderly ages, to be the main causes (Dini & Goldring, 2008; Festy & Rychtarikova, 2008; Kannisto, 1994, 1999; Robine & Saito, 2009b; Thatcher, 1999a; Vaupel & Jeune, 1995). International migration played a minor role in these countries. The next section sets out the findings of international studies into the demographic drivers of very elderly population growth. This is followed by a discussion of the gaps in knowledge with regards to Australia, and a decomposition method which will be used to perform such a decomposition.

2.5.1 International studies of demographic drivers

Survival improvement at different ages

Significant declines in infant and child mortality in the first half of the twentieth century, followed by declines in mortality at adult and older ages since the 1950s, were the main drivers of the growing very elderly numbers in developed countries in the second half of the twentieth century (Thatcher, 1981; Vaupel, 1997a, 2001; Wilmoth, 2000). Changes in death rates at different ages represent different stages in the epidemiological transition (Omran, 1971). This is the process by which chronic and degenerative diseases such as heart disease, cancer and stroke replaced infectious and parasitic (or acute) diseases, peri natal and maternal disorders and nutritional deficiencies as the primary cause of death (Omran, 1971; Wilmoth,
This transition was brought about by general improvements in nutrition, sanitation, hygiene, maternal care, living standards and health care (Omran, 1971).

Mortality decline at ages 80+ has been found to be the most significant driver of the substantial growth in the centenarians populations in developed countries – more significant than increases in birth cohort size or mortality declines at younger ages (Kannisto, 1994; Kannisto et al., 1994; Rau et al., 2008; Robine & Paccaud, 2005; Robine & Saito, 2003; Thatcher, 1999a; Vaupel, 1997a; Vaupel & Jeune, 1995; Wilmoth, Deegan, Lundström, & Horiuchi, 2000). Thatcher (1999a) found that the 20-fold increase in female centenarians in England and Wales between 1951 and 1996 could be explained by a 56% increase in births, 130% increase in survival to age 80, and 441% increase in survival from age 80 to 100, with survival improvement beyond age 100 contributing a further 16%. Similar results emerged from analyses by Festy and Rychtarikova (2008) and Rau et al. (2008) for European countries and Japan. Kannisto (1994) and Kannisto et al. (1994) also found that in Western European countries the degree of mortality decline for females at ages 80+ decreased with age and mortality declines were generally lower for males than for females.

**Survival improvement by sex**

On average, women live longer than men and this gap in survival increases with age (Kannisto, 1994). For example, in 2006 in England and Wales there were over seven females for each male among centenarians, compared to a ratio of three females for each male in the age group 90-99 (Dini & Goldring, 2008). The sex ratio has also widened over the course of the twentieth century, from only three female centenarians for each male at the start of the century to seven at the end (Thatcher, 1999a). This was the result of different rates of mortality decline for males and females, ascribed to, for example, war casualties among men in the World Wars, more men taking up smoking, a reduction in deaths among women associated with childbirth, and different rates of migration (Dini & Goldring, 2008; Thatcher, 1999a; Tomassini, 2005). However, since around the 1980s a narrowing has been observed in the gap between male and female mortality, consistent with changes in the prevalence of smoking among women and men (Tomassini, 2005). Janssen, Kunst, and Mackenbach (2007) and Staetsky (2009) found that differences in earlier smoking behaviour provide some explanation for the divergence in temporal trends between countries and the sexes from around the 1970s.
Temporal trends in survival improvement

The pace of improvement in survival at very elderly ages also changed over time. Resulting from fewer deaths caused by chronic diseases, the pace of mortality decline above age 80 accelerated from the 1950s in many developed countries, especially from the 1970s (Robine, 2006; Wilmoth, 2000). On average in European countries and Japan, the pace of mortality decline increased from an average of about 1% per year in the 1950s to more than 2% per year in the 1990s (Kannisto et al., 1994; Vaupel, 2001). Dini and Goldring (2008) found that the number of centenarians in England and Wales increased more during the period 1951 to 1976 than the period 1976 to 2001. This was due to greater increases in both birth numbers and survival probabilities to age 80. However, survival probabilities from age 80 to 100 improved more in the second period, especially for males. Similarly, Robine and Saito (2003) found that in Japan the rate of survival improvements accelerated from the 1970s to the end of the century, resulting in the centenarian doubling time reducing from 10 years in 1973 to 5 years in 2000.

Survival improvement: comparison of countries

It was also found that the timing of mortality decline at very elderly ages varied significantly between countries (Brunborg, 2012; Kannisto, 1994; Kannisto et al., 1994; Rau et al., 2008; Vaupel, 2001). According to Kannisto (1994), sustained mortality decline for the very elderly in Australia started in 1970 for females and in 1975 for males. This was much later than in countries such as France, Switzerland, Belgium and Sweden where it started around 1955 for females, in 1967 for males in Switzerland, but only in 1978 for males in Belgium and Sweden. In England and Wales very elderly mortality started declining around 1964 for females and 1977 for males, while in Italy the timing was similar to Australia’s - 1970 and 1978 for females and males respectively. Consistent declines in Japan started around 1966, at a significantly faster pace than in any other country (Kannisto, 1994; Rau et al., 2008). Countries like Japan, France and Italy, Switzerland and Australia have continued to experience substantial reductions in very elderly mortality since the 1980s, while mortality decline slowed or stagnated in the US, Canada, England and Wales, New Zealand and Sweden, Norway, Denmark, and the Netherlands (Rau et al., 2008; Vaupel, 2001).
The impact of international migration

The contribution of net migration to the significant increase in very elderly numbers in European countries has been found to be relatively immaterial (Festy & Rychtarikova, 2008). However, high proportions of the Australian elderly were born overseas: 32% of those aged 85+ and 36% of those aged 65+ (ABS, 2012a). This suggests that migration played a significant role in the numbers and composition of the very elderly population.

Stokes and Preston (2012) decomposed the growth rates of elderly populations (aged 65+) over the period 2005 to 2010 for a number of countries, including Australia. They found that the growth rate of the Australian elderly population over 2005-2010 (2.55%) was the second highest after Japan (2.69%). This high rate was ascribed to high rates of migration into these cohorts and a high probability of survival to age 65, which more than compensated for the relatively low birth rates in Australia during 1940-1945. Cohorts born in 1940-1945 in Australia had, by age 65, increased by a factor of 1.61 due to migration. The next highest was France with a factor of 1.27. The increase in Australia was significantly higher than the average factor of 1.06 across Europe, North America, Australia, New Zealand and Japan. Furthermore, Australia’s probability of survival from birth to age 65 of 0.82 was exceeded only by Sweden’s 0.83. This analysis thus indicates that both high rates of survival and net migration played a significant role in the growth of Australia’s very elderly populations. While insightful, this study only considers the factors contributing to the growth of the elderly population, and only over a very short time period (2005 to 2010). A decomposition of very elderly growth in Australia has not been undertaken. Furthermore, no such analysis has been undertaken at a state level, in Australia, or anywhere else that the author is aware of.

2.5.2 Demographic drivers of very elderly population growth in Australia

Little is understood of the drivers of growth of Australia’s very elderly population. In an attempt to analyse the demographic drivers of increasing numbers of centenarians in Australia during the twentieth century, McCormack (2010) applied Thatcher’s (1999a) decomposition method to data and life tables published by the ABS. However, it appears as if the survival probabilities used in these calculations were based on period life tables, which does not give a true reflection of survival experience of particular cohorts. It is also not clear what the period of analysis is as reference is made to a 40-fold increase in the number of centenarians from 1911 to 2001 but a comparison is made of numbers of births in 1860 and
1899. Furthermore, reference is made to reductions in probabilities of death at age 100 from 1953 to 1995. The impact of migration was also not separately shown.

In addition to analysing how the age- and sex composition of Australia’s very elderly population changed over the last few decades, the second objective of this thesis is to undertake a decomposition of the growth at both national and state levels. This will provide insight into the relative contributions of growth in births, improvement in survival and changes in net migration to very elderly population growth in Australia. What follows is a decomposition method to quantify the contributions of increases in births, survival and net migration to growing very elderly numbers and a description of the drivers of growth in countries for which such analyses have been performed in other countries.

2.5.3 Decomposition methods

This section provides a description of the methods used for decomposing the growth in very elderly numbers into proximate drivers. A cohort aged x last birthday on 31 December of year t is a function of the births in year t-x, their survival to age x and net migration. This can be expressed as (Thatcher, 1999a):

\[ P_{x,t} = B_{t-x} x P_0^c x n^c \]  

where:

\( P_{x,t} \) is the population aged x on 31 December of year t;

\( B_{t-x} \) is the number of births in year t-x (cohort c);

\( x P_0^c \) is the probability of survival from birth to age x for cohort c, obtained from a cohort life table.

\( x n^c \) is the net migration ratio, allowing for the impact of net migration into and out of cohort c, born in year t-x, and their survival to age x. This factor is typically derived as a residual and thus also includes the effect of any errors in estimating the other factors. Similarly, the population at age x at a time t-h can be written as:

\[ P_{x,t-h} = B_{t-h-x} x P_0^{c-h} x n^{c-h} \]  

The increase factor in numbers aged x from 31 December year t-h to 31 December year t is given by the ratio (2.9)/(2.10), and the total factor-increase can thus be written as the product of factors relating to increases in births, survival and migration ratios:
In order to compare the impact of improvement in survival at different ages, the probability of survival can be split into smaller age groups. Thatcher (1999a) quantified the contribution of each of these factors to the increase in the centenarian population in England and Wales between 1951 and 1996 (comparing 1850 and 1895 cohorts). He considered survival from birth to age 80, from age 80 to 100, and approximately allowed for survival beyond age 100. Vaupel and Jeune (1995) and Robine and Paccaud (2005) applied similar methods to explain the increase in the number of centenarians in Europe. This is also similar to the approach followed by Stokes and Preston (2012) to explain the growth in population numbers attaining age 65 over the period 2005 to 2010 in a number of developed countries.

2.6 Mortality patterns and trends at high ages

Mortality decline is one of the main drivers of very elderly population growth. An understanding of the patterns and trends in mortality thus forms an essential component of accurate projections. The third objective of this thesis is to develop insights into the changes in adult mortality in Australia, based on the death rates created under the first objective. Both the age profile of death rates and death frequency distributions will be studied. In many respects mortality patterns are very similar in low-mortality countries. Therefore, existing knowledge on, for example, appropriate mathematical functions for describing the age profile of death rates will be a useful starting point for this thesis. It will furthermore be useful to know how Australia compares to other countries in terms of changes in death frequency distributions, and whether compression, shifting or expansion has been observed. Also, while there is extensive literature on both age profiles of death rates and features of the death frequency distribution, there is little on identifying or defining the relationships between these. The literature discussed in this section provides a foundation for the results presented in chapter 5.

2.6.1 Age profile of mortality rates

Apart from problems relating to age misreporting mentioned earlier, there are two issues in particular that are relevant to the analysis and estimation of mortality rates at high ages. The first relates to large stochastic variations in mortality rates resulting from small numbers at the highest ages (Boleslawski & Tabeau, 2001; Condran, Himes, & Preston, 1991; Horiuchi
Volatility of rates at the highest ages

Because of large random fluctuations at high ages, mortality rates at these ages are typically adjusted and smoothed (Benjamin & Pollard, 1986). One method of graduation involves fitting an appropriate mathematical function using only data up to an age where rates increase smoothly with age. The fitted function can then be used to estimate rates at higher ages. Thatcher, Kannisto, and Vaupel (1998) and Thatcher (1999b) fitted functions to data at ages 80-98, and used the parameters to obtain estimates at ages 99-109 and 99-120 respectively. Yi and Vaupel (2003) used data for ages 80-96 to estimate death rates for Han Chinese up to age 105. Coale and Kisker (1990) derived mortality rates between ages 85 and 110 based on assumed values at, and an assumed pattern of change in the rates between these two ages.

Another approach is to calculate rates for high ages based on pooled data for a number of countries with high quality data (Thatcher et al., 1998). The Human Mortality Database contains deaths data and population data estimated on a consistent basis for a large number of countries at single ages up to 109, which may be used for this purpose. Data gathered in the International Database on Longevity may be used to calculate mortality rates between ages 105 and 115 (Robine & Vaupel, 2002). Himes et al. (1994) employed a combination of methods by fitting a logistic curve on a standard set of mortality rates at ages 45 to 99 derived from the smoothed observed rates based on pooled data for 16 low-mortality countries over the period 1948 to 1985, and extrapolating to age 115.

Mathematical functions fitted to the age profile of mortality rates

Mathematical functions used to describe the age pattern of mortality rates are referred to as ‘laws of mortality’, ‘parameterization functions’ or ‘mortality age schedules’. Apart from forecasting, they may also be used for life table construction, to smooth data, eliminate errors and aid inferences from incomplete data (Keyfitz, 1982). Over the last two centuries, researchers have experimented with fitting various mathematical functions to the observed age profile of mortality rates (Boleslawski & Tabeau, 2001; Bongaarts, 2005; Keyfitz, 1991; Tabeau, 2001; Thatcher, 1999b; Thatcher et al., 1998). What follows is a description of mathematical functions found to describe mortality at older ages well.
Exponential functions

The first and probably best known law of mortality is that of Gompertz (1825). According to this law, mortality increases exponentially with age. This is based on the hypothesis that a person’s ability to resist death decreases with age at a constant rate (Thatcher et al., 1998):

$$\mu_x = \alpha e^{\beta x}$$

(2.12)

where $\alpha$ is the initial level of mortality and $\beta$ is the rate of increase in mortality with age, or the rate of senescence (Curtsinger, Gavrilova, & Gavrilov, 2005).

Gompertz noted that, in addition to senescence, death may also be due to chance and Makeham (1860) added a term to Gompertz’s law representing non-senescent (or chance or accidental) deaths which does not vary with age. This results in an improved fit to observed rates at younger ages (Beard, 1971). Tabeau (2001) refers to this as a shifted exponential function, and the constant term ($\gamma$) is referred to as background mortality (Gavrilov & Gavrilova, 1991):

$$\mu_x = \alpha e^{\beta x} + \gamma$$

(2.13)

Logistic functions

Perks (1932) experimented with modifications to the Gompertz and Makeham functions. His aim was to provide a better fit for mortality at the older ages and in particular, to allow for the observed phenomenon of a slowing down of the rate of increase in the risk of death with age (Beard, 1971). These were later identified by Beard (1971) as logistic functions, who experimented with further modifications. Thatcher (1999b) and Thatcher et al. (1998) used the following generalised logistic function:

$$\mu_x = \frac{K\alpha e^{\beta x}}{1+\alpha e^{\beta x}} + \gamma$$

(2.14)

The first term relates to senescent mortality and the second term ($\gamma$) to background mortality. When $\alpha$ is small (ages up to 70), this produces rates in line with the Makeham function. At higher ages, rates increase at a decelerating rate with increasing age (Thatcher, 1999b). The parameter $K$ in equation 2.14 has been found to be close to 1 for low-mortality countries (Bongaarts, 2005; Thatcher, 1999b). Furthermore, at high ages background mortality is an immaterial portion of overall mortality and can be ignored (Bongaarts, 2005; Thatcher, 1999b). Therefore, for the purpose of modelling mortality of the very elderly, a simplified,
two-parameter logistic function is appropriate, referred to as the ‘Kannisto model’ (Bongaarts, 2005; Thatcher, 1999b; Thatcher et al., 1998):

\[
\mu_x = \frac{ae^{\beta x}}{1 + ae^{\beta x}} \tag{2.15}
\]

A logit transformation of the force of mortality (\(\mu_x\)) modelled by this two-parameter logistic function is a two-parameter linear function of age (Himes et al., 1994; Thatcher, 1999b; Thatcher et al., 1998):

\[
\text{logit}(\mu_x) = \ln \left( \frac{\mu_x}{1 - \mu_x} \right) = \ln \left( \frac{ae^{\beta x}}{1 + ae^{\beta x}} \right) - \ln \left( 1 - \frac{ae^{\beta x}}{1 + ae^{\beta x}} \right)
\]

which simplifies to:

\[
\text{logit}(\mu_x) = \ln(\alpha) + \beta x \tag{2.16}
\]

Given the approximate relationship, \(m_x = u_{x+\frac{1}{2}}\), this function may also be applied to the central death rate (Himes et al., 1994):

\[
\text{logit}(m_x) = \ln(\alpha) + \beta x \tag{2.17}
\]

Heligman and Pollard (1980) combined three functions in order to represent mortality across all ages:

\[
\frac{q_x}{p_x} = A(x+\beta)^C + De^{-D(x+\ln x-\ln F)^2} + GH^x \tag{2.18}
\]

The first two terms aim to respectively model infant mortality and the accident hump at young adult ages, which are not relevant for the purpose of this thesis and will thus not be considered further. The third term (GH\(^x\)), aimed at modelling adult mortality, is an exponential function similar to the Gompertz function, with \(G = \alpha\) and \(H = e^\beta\), but is applied to \(\frac{q_x}{p_x}\), rather than \(q_x\) (or \(m_x\) or \(u_x\)). Considering the term that represents adult mortality only and rearranging the terms, this can be re-written in terms of \(q_x\) as follows:

\[
\frac{q_x}{p_x} = ae^{\beta x} \tag{2.19}
\]

which simplifies to a logistic function applied to the probability of death:

\[
q_x = \frac{ae^{\beta x}}{1 + ae^{\beta x}} \tag{2.20}
\]

and

\[
\text{logit}(q_x) = \ln(\alpha) + \beta x \tag{2.21}
\]
Equations 2.16 and 2.21 are not equivalent, however. A logistic function produces a value between 0 and 1. The force of mortality (\( \mu_x \)) modelled as a logistic function of age (equation 2.16) will approach an asymptote of 1 with increasing age. Using the approximation \( q_x = 1 - e^{-\mu_x} \) will result in the probability of death approaching an asymptote of less than 1 (around 0.632) with increasing age. Under equation 2.21, however, the probability of death approaches an asymptote of 1 with increasing age.

**Power functions**

Weibull (1951) proposed a model representing the failure of technical systems due to wear and tear, implying that the force of mortality is proportional to a power of age (Thatcher et al., 1998).

\[
\mu_x = ax^b
\]  \hspace{1cm} \text{(2.23)}

**Quadratic function**

An exponential quadratic function, referred to in the literature as the Coale-Kisker method, was applied by Coale and Kisker (1990) to derive mortality rates at single ages between 85 and 110. The method was first applied by Coale and Guo (1989) to 5-year age groups up to age 100 for model life tables. It is an adjustment of the Gompertz function, such that the rate of increase in the risk of death decreases with increasing age, rather than remaining constant as modelled by Gompertz.

According to the Gompertz law, mortality at age x can be written as a function of mortality at age x-1 as follows:

\[
m_x = m_{x-1} e^{k_x}
\]  \hspace{1cm} \text{(2.24)}

where \( k_x \) is the rate of increase in the mortality rate at age x:

\[
k_x = \ln \left( \frac{m_x}{m_{x-1}} \right)
\]  \hspace{1cm} \text{(2.25)}

and according to Gompertz, this rate of change remains constant across the age spectrum \((k_x = k)\). Mortality at ages above a certain threshold age, for example 85, can be written as a function of mortality at age 85:

\[
m_x = m_{85} e^{k(x-85)}
\]  \hspace{1cm} \text{(2.26)}

Coale and Kisker (1990) proposed that \( k \) decline in a linear way above the threshold age of 85, so that mortality rates above age 85 can be determined as:
\[ m_x = m_{x-1} e^{k_x} \]  
with \( k_x \), also referred to as the ‘life-table aging rate’ (LAR) (Horiuchi & Wilmoth, 1998), calculated as follows for \( x > 85 \):

\[ k_x = k_{85} + s(x - 85) \]  
Therefore:

\[ m_x = m_{85} e^{k_{85} + s(x - 85)} \]

Coale and Kisker calculated the slope \( s \) assuming \( m_{110} = 1 \) for males and \( m_{110} = 0.8 \) for females. Applying these assumptions and solving for \( s \), gave:

\[ s = -\frac{\ln \left( \frac{m_{84}}{m_{110}} \right) + 26k_{85}}{325} \]

Wilmoth (1995) has shown the Coale-Kisker model implies an exponential-quadratic function for mortality rates with respect to age:

\[ m_x = e^{a + bx + cx^2} \]

which can be transformed to:

\[ \ln (m_x) = a + bx + cx^2 \]

Wilmoth (1995) furthermore suggested that instead of the fitting the function only to mortality rates at ages 85 and 110 as suggested by Coale and Kisker, that this function be fitted to data at all ages from 85 to 100 and death rates at higher ages be derived via extrapolation.

**Decelerating pattern of age-related mortality rates**

A number of studies have found that, although an exponential pattern of increase is observed over most of the adult ages, age-specific mortality rates (the force of mortality or central death rates) increase at a decelerating rate with increasing age above a certain threshold age (Gavrilov & Gavrilova, 1991; Horiuchi & Coale, 1990; Horiuchi & Wilmoth, 1998; Vaupel, 1997a; Vaupel et al., 1998). Furthermore, empirical studies of mortality rates at very high ages indicate that they reach a peak at around age 110 with roughly constant rates at higher ages (Robine, 2006; Robine & Vaupel, 2002; Vaupel, 1997b).

A popular theory for explaining the decelerating rate of increase in mortality is the heterogeneity hypothesis, or “frailty” theory (Beard, 1971; Vaupel, Manton, & Stallard, 1979). According to this theory, mortality for individuals of the same age is described by an exponential function, but due to population heterogeneity, individuals have different genetic
‘frailties’, resulting in different probabilities of death that remain fixed throughout their lives (Curtsinger et al., 2005; Horiuchi & Coale, 1990; Vaupel et al., 1998). The different frailties of individuals in a cohort thus result in the composition of the very elderly population becoming less frail due to the selection effect of the more frail individuals dying sooner. The frailties are assumed to vary according to a Gamma distribution at birth, so that the average value of $\mu_x$ for survivors to age $x$ will take the logistic form. This is also referred to as the Gamma-Makeham model (Andreev, 1999; Yashin, Vaupel, & Iachine, 1994).

Le Bras (1976) explained the decelerating pattern of mortality with age as a stochastic process by which individuals in a cohort, starting off homogeneous, become more heterogeneous by moving progressively through deteriorating states of health in a random way (Thatcher, 1999b; Thatcher et al., 1998). These random changes in individual frailty “occur due to environmental influences or stochastic processes of physical deterioration” (Yashin et al. 1994: 4). Progressive stages “have both higher risks of failure and higher risks of further transition” (Horiuchi and Wilmoth 1998: 392). This theory can also be modelled with a logistic function (Yashin et al., 1994).

**Which mathematical function best describes this pattern?**

The Gompertz, Makeham, and Weibull functions, the adult mortality term of the Heligman-Pollard (HP) model and logistic functions all produce estimated rates close to the observed rates at ages where most deaths are concentrated (Thatcher, 1999b). However, the Gompertz and Makeham functions increase exponentially with age and have been found to overestimate mortality rates at ages above 95 or 100. The Weibull and HP functions model a decelerating rate of mortality increase with increasing age but have also been found to overstate mortality rates at high ages, albeit to a lesser extent (Horiuchi & Coale, 1990; Horiuchi & Wilmoth, 1998; Robine & Vaupel, 2002; Thatcher et al., 1998).

The logistic and exponential quadratic functions were found to produce the best estimates for age-specific mortality rates at very elderly ages (Boleslawski & Tabeau, 2001; Horiuchi & Coale, 1990; Robine, 2006; Thatcher, 1999b; Thatcher et al., 1998; Wilmoth, 1995; Yi & Vaupel, 2003). Horiuchi and Coale (1990) have shown that while the Gompertz, Makeham and logistic functions all fitted observed death rates at ages 55-95 well, only the logistic function produced the bell-shaped curve of the rate of change in the force of mortality with
age (denoted by $k_x$ as in equation 2.25), with a peak at age 75, exhibited by observed death rates. These functions represent observed mortality well up to age 95 or 100, but diverge at higher ages because they tend to different asymptotes. The force of mortality modelled with the logistic function tends towards an asymptote of one; under the HP model it moves towards a linear asymptote which increases with age; whilst rates produced by the Weibull function are between those of Gompertz and the HP model (Thatcher, 1999b; Thatcher et al., 1998).

Estimates produced by the logistic and exponential quadratic functions diverge only from around age 115. Due to a lack of empirical data above this age it is difficult to say which is more appropriate (Robine & Vaupel, 2002). However, many researchers favour the logistic function due to its parsimony (only two parameters), its generality (it can be applied to a wide age range), and its explanatory backing (heterogeneity of frailty with the resulting demographic selection effect) (Thatcher et al., 1998). For these reasons, the simplified (Kannisto) logistic model has been used to smooth observed death rates for ages 80 and older in the Human Mortality Database (Wilmoth et al., 2007). Actual death rates are replaced by smoothed death rates at ages above which the number of deaths is less than 100, which is between age 80 and age 95, depending on the population.

The logistic function can be used to describe mortality from around age 30. In comparison, the exponential quadratic model suggested by Coale and Kisker (1990) applies only from the age where the rate of increase in mortality starts to decelerate (Thatcher et al., 1998). This age will need to be established before the model can be applied. Different threshold ages have been mentioned in the literature e.g. age 75 (Boleslawski & Tabeau, 2001; Horiuchi & Coale, 1990), age 80 (Thatcher, 1999b; Vaupel, 1997b) and 85 (Coale & Kisker, 1990; Robine, 2006; Robine & Vaupel, 2002; Wilmoth, 1995). Horiuchi and Wilmoth (1998: 393) argued that the age at which mortality deceleration starts may be expected to shift to older ages as the level of adult mortality declines. It can thus be expected to vary between countries and over time. Horiuchi and Wilmoth (1998) studied the shape of $k_x$ curves over time for Swedish and Japanese cohorts born between 1880 and 1905 and found that the age at which deceleration starts shifted to older ages as the level of senescent mortality declined. Bongaarts (2005) and Horiuchi and Wilmoth (1998) ascribed a shift over time in the ascending part of the bell-shaped curve to reductions in background mortality, while shifts in the descending part of the curve at high ages are due to changes in the senescent component of mortality.
Coale and Kisker’s (1990) exponential-quadratic method incorporates assumptions about the age after which the mortality rate is flat as well as the level of mortality at this age. They assumed central death rates at age 110 of 1 for males and 0.8 for females. Gampe (2010) and Robine and Vaupel (2002) found that the probability of death after age 110 is roughly flat at around 0.5, with observed mortality from age 114 becoming very volatile due to sparse data. A probability of death of 0.5 translates into a central death rate or force of mortality of approximately 0.7 if deaths are evenly distributed. Given these empirical findings it seems that lower assumptions would now be appropriate when applying the Coale-Kisker model.

It is important to note that different conclusions about the shape of age profile of mortality rates at the highest ages may be drawn depending on whether the probability of death ($q_x$), the force of mortality ($u_x$) or central death rates ($m_x$) are analysed. It is preferable to use the force of morality, and then derive the probability of death, because $q_x$ is bounded above by one (Gavrilov & Gavrilova, 2011; Vaupel et al., 1979). As $u_x$ does not have an upper limit, it can greatly exceed $q_x$ at very high ages, and factors resulting in substantially reducing $u_x$ may have much less effect on $q_x$ (Vaupel et al., 1979). Where continuous data are not available, $m_x$ can be used instead of $u_x$.

**Empirical studies of temporal change in age profiles of mortality rates**

Empirical studies involving time series of mortality rates or parameters of fitted mathematical functions show that the level of the age profile of mortality reduced significantly across all ages and for both sexes over the last century (Bongaarts, 2005; Gavrilov & Gavrilova, 1991; Gavrilova & Gavrilov, 2011; Himes et al., 1994; Thatcher, 1999b). Some studies considered the split between senescent ($\propto$) and background ($\gamma$) mortality and found that both types reduced significantly. Bongaarts (2005) found that there was greater variation between countries in the levels of senescent mortality compared to background mortality and furthermore that background mortality fell for all countries between 1950 and 1975, remaining roughly level thereafter, while the level of senescent mortality continued to fall until 2000. Background mortality is now a very small portion of total mortality at high ages and can be ignored (Bongaarts, 2005; Thatcher, 1999b).
Some empirical analyses indicate that the age profile of adult mortality has changed very little over the last century, in line with Brass’ findings (1972:128): “the average β is found to be near one in observed life tables whatever the level of mortality”, and “moves steadily with falling death rates while β fluctuates about 1 but has a strong tendency to return to this central value”. Varying findings were made by other researchers. McNown and Rogers (1989) found the slope of the last term of the HP model (parameter H, equation 2.18) increased in the US from 1970; Thatcher (1999b) found the slope (parameter β) of the logistic function (equation 2.15) rose slightly, and Bongaarts (2005) found the slope parameter changed very little over the period 1950-2000 for all countries analysed.

2.6.2 Death frequency distributions

While the previous section contains a discussion of the age profile of death rates, this section considers another, related aspect of the life table, namely the death frequency distributions. What follows is a summary of analyses and hypotheses in the literature relating to trends in frequency distributions of life table deaths, and the relationship between these hypotheses and likely changes in the age profiles of mortality.

Theories on human life span limits

Increases in elderly and very elderly populations raised the question of whether there is an absolute biological limit to human life span. This continues to be a subject of debate and disagreement among biologists, demographers and actuaries (e.g. Fries 1980; Kannisto, 2000; Oeppen and Vaupel 2002; Olshansky, 1988; Vaupel, 2001). Discussion is typically set in the context of the shape of the survival curve, death frequency distributions and maximum ages attained.

Fries (1980) and Olshansky (1988), for example, promoted the hypothesis that there was a fixed biological limit to life span. Fries (1980: 130) claimed that “despite a great change in average life expectancy, there has been no detectable change in the number of people living longer than 100 years or in the maximum age of persons dying in a given year”. He suggested that a continuation of the decline in premature deaths will result in morbidity being compressed between a higher age of onset of chronic illness, and a fixed age at death, with the result that more deaths will occur within a shorter time span (four years) around a modal age of 85 years. Both the morbidity curve and mortality (survival) curve will therefore
become more rectangular, and this hypothesis is commonly referred to as ‘the compression of mortality’ hypothesis, as the ‘rectangularization of the survival curve’ hypothesis (Wilmoth & Horiuchi, 1999). Fries concluded that human life span had a ‘natural limit’ and predicted an ‘ideal’ average life span of 85 years. This was based on the observation that increases in life expectancy were the result of mortality declines at younger ages, with life expectancy at older ages not showing similar improvements. Olshansky (1988: 486), in a discussion on mortality forecasting, noted: “in the case of mortality, there are biologically determined upper limits to life that will eventually end the downward trend in mortality rates....”. In contrast, researchers such as Vaupel (1997a), Wilmoth (1998) and Oeppen and Vaupel (2002) argued that the substantial survival improvements observed at older ages since the 1970s does not support the notion that death rates at older ages or life expectancy are approaching a biological limit.

**Compression, shifting and expansion of mortality**

Related to the debate about human life span limits, is whether death frequency distributions are shifting, compressed or expanding. Before the epidemiological transition, the highest concentration of deaths occurred in the first year of life. Significant reductions in infant mortality, followed by survival improvements at adult ages, contributed to the emergence of the oldest-old and hence greater proportions of deaths occurring at progressively higher ages (Canudas-Romo, 2008; Kannisto, 2000; Wilmoth & Horiuchi, 1999). The rate and timing of increases in modal ages (the age at which the death frequency is the highest) have varied between countries. In Sweden, for example, the modal age for females reached 80 in around 1900, where it remained until 1950 and then increased again to reach 90 in 2000, while the modal age for females in France only reached 80 in the 1950s, and then 90 in 2000 (Robine, 2011). In 2007 modal ages at death in developed countries varied between 83 and 87 for males and between 88 and 92 for females (Robine, 2011). In various empirical studies researchers confirmed that increases in modal ages at death during the transition from high to low mortality over the last decade were accompanied by significant compression of mortality (Kannisto, 2000; Wilmoth & Horiuchi, 1999; Yashin, Begun, Boiko, Ukraintseva, & Oeppen, 2001).

Kannisto (2000) suggested that mortality can be considered as ‘fully compressed’ and rectangularity of the survival curve completed once mortality in the early years is very low.
and the age range within which 90% of deaths occur (denoted by C90, where ‘C’ refers to compression), has declined to one year. While strong declining trends in C90 (smaller age range) have been observed in many developed countries, in none of these populations was the age range within which 90% of deaths took place less than 35 years, indicating that survival curves are far from being fully rectangularized.

This ‘shifting mortality hypothesis’ refers to a move of death frequency distributions towards higher ages, with no change in the concentration of deaths around the modal age at death (Kannisto, 1996). In this case, the survival curve shifts towards the right, but retains its shape (Bongaarts, 2005; Bongaarts & Feeney, 2002; Canudas-Romo, 2008). From around the 1970s, acceleration in the declines in death rates at higher ages resulted in the pace of compression slowing down in many countries while modal ages continued to increase (Canudas-Romo, 2008; Kannisto, 2000; Wilmoth & Horiuchi, 1999). According to Yashin et al. (2001), the rectangularization trend was replaced by a near parallel shift of the survival curve to the right for countries like France and Japan during the second half of the last century, and an increase in the tail of the death frequency distribution (referred to as expansion). According to Canudas-Romo (2008), the slowing down of the reductions in standard deviations around the modal age at death in many countries suggests that countries are transitioning from a rectangularization regime to a shifting mortality regime. Differences in the patterns of this transition between countries reflect varying rates of reduction in death rates over time and at different ages.

*What is the link between human life span theories and mathematical functions of mortality rates?*

The appropriate mathematical function to use for forecasting of mortality at the highest ages depends on one’s views on whether there is a fixed limit to human life span and therefore also whether the death frequency distribution will be compressed, decompressed or shift going forward. A fixed upper limit to the length of human life implies that the probability of death is 1 at a finite age, and that further falls in death rates will result in greater compression of deaths into a narrower age band and hence a more rectangular survival curve (Thatcher, 1999b). If the probability of death does not reach one at a finite age, but still tends asymptotically to one, it implies that there is no definite fixed upper limit to human life span, but the probability of survival at high ages is so small that it can be ignored. This is implied
by the Gompertz, Weibull and Heligman and Pollard models (Thatcher, 1999b). The logistic function applied to the force of mortality implies that the probability of death increases with age and tends asymptotically to a limit of less than 1. This means that there is no definite, fixed upper limit to life and no age when the probability of further survival is negligible (Thatcher, 1999b).

**Highest attained ages**

Increases in the highest duration of life recorded in any calendar year, referred to by Robine and Paccaud (2005) as the maximal age, accompanied declines in mortality at very elderly ages and rising modal ages at death. For example, in Sweden, no one survived past age 105 in the period between 1860 and 1889, but the maximum attained life span gradually rose thereafter, to reach age 112 in 1994 (Vaupel, 2001). Consistent with the acceleration in the pace of older-age mortality decline in the second half of the 20th century, researchers found that the pace of increase in the maximal ages at death also accelerated from the 1950s in some countries, and from the 1970s in others (Robine & Paccaud, 2005; Robine & Saito, 2003; Wilmoth et al., 2000). In Switzerland the average maximal age increased from around 102 between 1880 and 1920, to 104 between 1920 and 1960, to around 107 between 1961 and 2001, reaching 110 in 2001 (Robine & Paccaud, 2005). According to Thatcher (1999b), in England and Wales two people reached the highest verified attained age of 111 in the period 1968-72, compared to 17 people reaching age 115 in 1993-97.

### 2.6.3 Conclusion

Many demographers have concluded that improvements in life expectancies that have been observed in low mortality countries such as Japan, France and Switzerland since the 1970s show no signs of slowing and life expectancies do not appear to be approaching a maximum (Oeppen & Vaupel, 2002; Rau et al., 2008; Robine, 2006). The significant improvement in survival at the very high ages since the 1970s and continuing increases in maximal ages at death is contrary to the premise made by Fries that there is a fixed biological limit to the human life span (Vaupel, 1997a). While mortality reductions were accompanied by greater age concentrations of death, or smaller variation in ages at death, it seems that a continuation towards the completion of rectangularization of the survival curve, which is dependent on the existence of a fixed maximum life span, has not recently been evidenced (Kannisto, 2000).
To the best of the author’s knowledge, no study exists of the changes in modal ages at death in Australia and there is no knowledge of the degree of compression or shifting that has taken place. Objective 3 of this thesis is to develop an understanding of the patterns and trends of change in adult mortality in Australia. A detailed analysis of historical trends in death frequency distributions may be used to inform the likely future course of mortality rates and projected very elderly populations.

2.7 Mortality forecasting

The previous section discussed features of the age profile of death rates and death frequency distributions, together with observations of international trends in these features over the last few decades. Objective 4 of this thesis relates to applying insights gained from studying historical patterns and trends to create realistic projections of death rates and very elderly populations. Appropriate projections of Australia’s very elderly population at national and state levels are vital for national and regional planning, policy making and budgeting of social welfare, health and aged care services and housing. Retrospective tests have revealed that very elderly population projections produced by the ABS have turned out to be too low, due to underestimated mortality improvements (Wilson, 2007, 2012a). In order to create more reliable projections, appropriate projection approaches need to be identified. According to Booth and Tickle (2008), mortality forecasting methods can be broadly divided into extrapolation, expectation, and explanation methods. Explanation methods are also referred to in the literature as risk models. In this section, an overview of mortality forecasting methods is given, followed by criteria to be considered when choosing a suitable method. Extrapolation models, which are considered the most appropriate, are then discussed in more detail.

2.7.1 Overview of mortality forecasting methods

Extrapolation models

Extrapolation is the simplest and most widely used method for mortality forecasting. Extrapolation methods are based on an assumed relationship between past and future patterns of mortality (Ediev, 2008; Olshansky, 1988). According to Olshansky (1988: 501) “extrapolation models refer to forecasting methods in which a hypothetical mortality schedule of the future is mathematically derived either from trends in overall or cause-specific mortality rates observed in the past, or expert opinion (that is based on observations
of past mortality) as to how mortality rates may change”. The reliability of extrapolation thus depends on the stability of patterns of mortality-related measures (Wilmoth, 2000).

Separate projections are typically made for males and females, and sometimes also for main causes of death. Measures extrapolated may include life expectancy at birth, rates of change in life expectancy, the level of individual age-specific mortality rates or probability of death, rates of change in age-specific mortality rates or probability of death or parameters of mathematical models used to describe the age profile of mortality rates.

The analysis of mortality by cause of death allows epidemiological knowledge to be incorporated when formulating assumptions about the future (Tabeau, 2001). However, this approach is fraught with problems. For example, disaggregation of mortality rates by cause of death relies on different causes of death being independent, which is seldom the case (Andreev & Vaupel, 2006; Horiuchi & Wilmoth, 1998; Olshansky, 1988). In addition, changes over time in the classification of causes of death, coding practices, and improvements in the way causes are diagnosed present issues in analysing trends over time (Horiuchi & Wilmoth, 1998). At advanced ages the stated cause of death may be less meaningful or inaccurate due to “the frequent coexistence of several life-threatening conditions in people who survive to more advanced ages” (Olshansky 1988: 499). Some researchers argue that this method tends to produce overly pessimistic forecasts and is unnecessarily complex, less transparent and result in internal inconsistencies (Ediev, 2008; McNown & Rogers, 1992; Vaupel, 2012).

**Expectation methods**

A variation on pure extrapolation of past trends is when a target level, to be reached by a target date, is specified for the measure being projected. These are referred to as ‘anticipating methods’ or methods based on ‘expectation’ (Willekens, 1990). The projected measure during the interim period (between the launch date and the target date) is then derived by interpolation. The target is typically set, based on expert opinion, with reference to historical trends and errors made in earlier forecasts, combined with subjective judgement regarding the degree to which past trends will continue (Andreev & Vaupel, 2006; Crimmins, 1984; Olshansky, 1988). In some cases, especially where there is little data, the life table of another
population, or model life tables based on the experience of a number of similar populations, are used as targets (Coale & Guo, 1989; Coale & Kisker, 1986).

Expectation approaches are or have in the past been popular with many official statistical agencies and international organisations such as the United Nations (UN), the World Health Organisation (WHO), the World Bank, the Office for National Statistics in the UK and the US Social Security Administration (Andreev & Vaupel, 2006; Olshansky, 1988; ONS, 2012a; Tabeau, 2001; Willekens, 1990). The target used is either life expectancy, a rate of change in life expectancy, the level of mortality rates or rates of change in mortality rates (Andreev & Vaupel, 2006; McNown & Rogers, 1989; Olshansky, 1988; ONS, 2012a; Tabeau, 2001). Following the determination of the future levels of life expectancy, the age-specific death rates for the projection period are calculated by means of either model life-tables or by a combination of model life-tables and observed age-specific mortality rates (Andreev & Vaupel, 2006; Murphy, 1995; Olshansky, 1988; Tabeau, 2001).

By specifying future target levels of life expectancy, mortality rates or rates of change in mortality rates, a greater degree of subjectivity is introduced in projections. Mortality forecasts based on subjective expert opinion are often biased by opinions about biological upper limits on human life span, and as a result, future improvements in mortality are capped (Olshansky, 1988). Mortality levels projected in this way have often turned out to be overstated, and population numbers underestimated, because mortality improved more than was foreseen or believed possible, particularly at older ages (Keilman, 1997; Lee & Carter, 1992; Murphy, 1995; Olshansky, 1988; Wilson, 2007). The existence of a fixed biological limit on the human life span, and its likely level, continues to be a subject of debate and disagreement among biologists, demographers and actuaries (Fries, 1980; Oeppen & Vaupel, 2002; Olshansky, 1988; Vaupel, 2001).

Population projections provided by the ABS are based on assumed future improvements in life expectancies at birth (ABS, 2013c). For their medium series projections, life expectancies are assumed to continue to increase at past rates for the first five years, and then by gradually decreasing rates over the next 50 years, after which they are assumed to remain constant (ABS, 2013c). Age-specific mortality rates are initially extrapolated assuming a continuation of past patterns of change by age-group, and thereafter are derived from the projected life
expectancies. While also containing characteristics of extrapolation methods, the ABS’ method is considered by the author to be an expectation method.

**Explanation methods or Risk models**

Risk models, also referred to in the literature as epidemiological models, process models or biomedical process-based models, structural models or explanatory models, are models whereby mortality rates are explained by way of disease processes and various causal or risk factors (Andreev & Vaupel, 2006; Booth, 2006; Tabeau, 2001; Willekens, 1990). These factors may include biological and behavioural factors (e.g. smoking habits, diet, obesity, exercise), socio-economic status, environmental factors (e.g. incidence of infectious diseases, advances in medical technology), or genetic factors. Mortality by cause is projected indirectly by extrapolating trends in the risk factors into the future (Andreev & Vaupel, 2006; McNown & Rogers, 1989; Pollard, 1987).

In theory, risk models which quantify anticipated changes in all the variables that affect mortality represent an ideal approach to mortality forecasting. However, their extensive data requirements such as individual-level data on risk factors, the need for a complete understanding of the interactions between various risk factors and mortality at different ages, the difficulty of forecasting many of the factors which influence mortality, and the interdependence between various risk factors, mean that the conceptually simpler extrapolation methods will continue to be used (Bongaarts & Bulatao, 2000; Tabeau, 2001; Vaupel, 2012; Wilmoth, 1998).

**2.7.2 Criteria for choosing a method**

In order for a projection method to be practical and useful, it needs to be simple, make use of methods and assumptions that do not incorporate a high degree of subjectivity and the results produced should be valid and plausible. These requirements are often in conflict and compromises need to be sought. Greater simplicity, for example, from using fewer parameters, may result in compromising the validity of the assumptions and the accuracy of the results. These requirements will be discussed in more detail below.
**Simplicity**

Bongaarts (2005: 23) states, “A good model provides a simple but adequate mathematical description of mortality by age and/or time. The objective is to identify fundamental and persistent patterns in the data and summarize them with as few parameters as possible”. According to Pollard (1987: 56-57), “the formula chosen for mathematical extrapolation should have few parameters and behave in a simple, appropriate and well-understood fashion”. According to Keyfitz (1991), more parameters may provide a better fit to individual life tables, but may produce less regular time series, and therefore lend themselves less readily to extrapolation. Keyfitz (1982: 329) states: “the simpler the curve, the more realistic is likely to be its projection into the future”.

Smith, Tayman, and Swanson (2013) also listed cost of production and timeliness as important criteria to consider in the choice of projection model. This partly relates to the model simplicity and data availability and reliability, and this in turn impacts the timeliness of projections. Another important consideration mentioned by Smith et al. (2013) is the ease with which the model can be applied and the results explained, which is directly related to the simplicity of the model.

**Objectivity**

The greater the degree of discretion or judgement being exercised when applying a method, the greater the level of subjectivity introduced. All methods, including pure extrapolation methods, involve judgement and therefore some degree of subjectivity (Vaupel, 2012). This may relate to, for example, the choice of past period used to estimate the parameters, the measure to forecast (e.g. age-specific death rates, probabilities of death or life expectancy), the degree of disaggregation (e.g. by sex or cause), or the statistical techniques and measures used. Vaupel (2012: 9) states: “Sometimes expert judgment can provide useful additional information but such judgments should be used with great care and caution and only to modestly adjust time series extrapolations of central trends and to widen uncertainties around these trends”.

**Validity**

According to Smith et al. (2013), ‘face validity’ of a projection refers to the use of reliable data, reasonable assumptions, and whether all the factors impacting the results are
incorporated into the method. The model and methods employed should produce reliable, accurate and valid forecasts. One factor impact the validity of the results is whether the data support and justify the assumptions. For example, if it is to be reasonably assumed that future changes in mortality will follow a certain trend, past patterns in mortality rates must reflect this also. The past period used to estimate the parameters should be such as to support the assumed trends in the parameters being projected. For example, a linear relationship can only be assumed if the data support this. Keyfitz (1982: 331) states, “One aims to fit to a succession of tables curves whose parameters move in a clear progression, perhaps in a straight line. If that is true of the past record, then it is likely to be true over the future, and by extrapolating the parameters one can extrapolate the life table”. Keyfitz (1991: 1) argues that the decision regarding “what past period describes the future” is more important than the choice of method.

Similarly, the observed period should be of sufficient length, and appropriate to the projection period, so as to reflect stable long-term trends rather than short term fluctuations (Wilmoth, 1998). According to Vaupel (2012:3), a forecast of n years into the future should be based on data n years in the past, and past periods of 2n or 3n will be even better if “comparable data are available and the time series shows persistence of patterns”.

Similar considerations will apply to the age range over which a mathematical function is fitted (Wilmoth, 1995). The ONS (2012a) found that the addition of another year’s data or extending the age range may produce very different rates of improvement at some ages, referred to as ‘edge effects’, and it is important to do sensitivity testing.

**Plausibility**

A further requirement is that the results should be realistic and plausible, and thus in line with historical trends (Smith et al., 2013). For example, a mortality forecasting method should not produce negative death rates or probabilities of death exceeding one. By modelling a logarithmic or logit transformation of a mortality rate, a floor of zero is applied when modelling a mortality decline. Furthermore, care should be taken to ensure the shape of the mortality age-profile remains plausible when applying different rates of change in mortality for different ages (Ediev, 2008; Keyfitz, 1982). For example, mortality rates should not
decrease with age, and projected death rates should be consistent between males and females (Vaupel, 2012).

The validity of results produced is also a function of the measure being projected. Some researchers are of the view that it is more appropriate to extrapolate age-specific mortality rates than life expectancy (Keyfitz, 1982; Lee & Carter, 1992; Wilmoth, 1998). According to Pollard (1988: 58), “it is important that the life table function or transform chosen to be extrapolated be sensitive to changes in mortality”, and that life expectancy (at age x) “is affected by mortality at all ages beyond x, [but] is sensitive to none”. Lee and Carter (1992: 663) states that this is because a constant linear decline in ages-specific rates will result in a “slowing rate of change in life expectancy”. Because increases in life expectancy were initially driven by mortality improvements at young ages, and more recently by improvements at older ages, increases in life expectancy have slowed down in many countries (Tuljapurkar, 2005; Wilmoth, 1998). Given the very low rate of mortality at younger ages, any further improvements in life expectancy will be the result of improvements at the older ages, and mortality reductions at older ages have less effect on life expectancy.

2.7.3 Extrapolation methods

In light of the criteria discussed in the previous section, extrapolation methods are considered the most appropriate for the purpose of this thesis. The main strength of extrapolation methods is the relative simplicity compared to, for example, explanatory methods. It also provides more objective as well as plausible results compared to expectation methods (Keilman, 1997; Lee & Carter, 1992; Murphy, 1995; Olshansky, 1988; Shaw, 2007). There are two main extrapolation approaches, namely, the direct extrapolation of (transformations of) mortality rates, and the extrapolation of parameters of mathematical functions fitted to the age profile of mortality rates. These methods are discussed in detail below.

Geometric mortality improvement model

If age-specific mortality rates are assumed to change at a constant geometric rate, the probability of death for a specific age (x) in any future year t (\(q_{x,t}\)) can be determined as follows (Pollard, 1987; Willekens, 1990):

\[
q_{x,t} = q_{x,t_0} \gamma_x^t
\]

where
\( q_{x,t} \) is the probability of death at age \( x \), in a future year \( t \), measured from the launch date \((t>t_0)\);

\( q_{x,t_0} \) is the probability of death at age \( x \), in year \( t_0 \), the launch year;

\( \gamma_x \) is the factor reflecting the assumed annual rate of change in the probability of death at age \( x \), based on observed mortality improvements over a historical period. Pollard (1987: 56) refers to this as ‘the improvement factor’. In the case of assumed mortality decline, \( \gamma_x \) will have a value between 0 and 1, and if mortality is assumed to increase this value will be greater than 1.

The base rate \( q_{x,t_0} \) reflects the latest available probability of death at age \( x \). It may be based on the average observed rates over a few years in order to smooth out the effect of random fluctuations, provided there is continuity between the base rate and projected rates. Pollard (1987) suggested that equation 2.33 be applied at selective ages (e.g. 5-yearly), with rates at intermediate ages being derived by interpolation, to ensure the resulting age profile of mortality is smooth. By taking the logarithm on both sides, equation 2.33 can be written as a linear equation:

\[
\ln(q_{x,t}) = \ln(q_{x,t_0}) + t\ln(\gamma_x) \tag{2.34}
\]

Ediev (2008: 5) referred to this method as the “direct (linear) extrapolation of log-mortality rates”, and applied it to the log of central death rates rather than the probabilities of death, i.e.

\[
\ln(m_{x,t}) = \ln(m_{x,t_0}) + t\ln(\gamma_x) \tag{2.35}
\]

where \( m_{x,t} \) is the central death rate at age \( x \), in year \( t \).

**Relational models**

A relational model is one where mortality (or other life table function) in one population is expressed as a function of mortality in another population. This may be a ‘standard’ population, reflecting the combined experience of a number of populations and may also extend to the time dimension by expressing mortality at a future time as a function of mortality at a past date. Projections of mortality rates are produced from the time series of the parameters (Brass, 1974). This can be achieved by fitting curves to historically observed time series of the parameters or some transformation thereof (e.g. logarithmic or logit), and extrapolating them into the future (Keyfitz, 1991).
Relational models were introduced by Kermack, McKendrick, and McKinlay (1934). Brass (1974) developed this further by expressing the logit of the probability of death between birth and age x for a particular population as a linear function of the logit of the probability of death between birth and age x for a standard population.

\[ Y_{x,j} = \alpha_j + \beta_j Y_{x,\text{Std}} \]  

(2.36)

where

\( Y_{x,j} \) is the logit of the probability of death between birth and age x for population j, calculated as \( \%\ln \left( 1 - \frac{l_x}{l_0} \right) \), where \( \frac{l_x}{l_0} \) is the proportion surviving from birth to age x;

\( Y_{x,\text{Std}} \) is the logit of the probability of death between birth and age x for a standard population;

\( \alpha_j \) is the intercept of the logits for population j with the standard table, and thus reflects differences in the level of the mortality curve for the population relative to the standard;

\( \beta_j \) represents the slope of the logits for population j relative to the standard table. By definition, this parameter will impact rates at lower and higher ages in different directions (Himes et al., 1994).

Applying the function to probabilities of death between birth and age x creates complications for deriving death rates at individual ages (Pollard, 1987). Himes et al. (1994) avoided these complications by applying a relational model for deriving central death rates at individual ages between 45 and 99. They constructed a standard based on the combined period data from 1950 to 1985 for 16 low-mortality countries that were considered of acceptable quality. They then expressed the logit of death rates at specific ages (x) for a particular country (j) as a two-parameter linear function of the logit of death rates in the standard. Therefore, death rates for a particular population were derived from equation 2.36 where \( Y_{x,\text{Std}} \) is \( \text{logit}(m_{x,\text{std}}) \) and \( Y_{x,j} \) is \( \text{logit}(m_{x,j}) \), the logistic transformations of the death rates at age x for the standard population and population j respectively, with \( \text{logit}(m_{x,\text{std}}) = \ln\left( \frac{m_{x,\text{std}}}{1-m_{x,\text{std}}} \right) \).

Ewbank, Gómez de León, and Stoto (1983) introduced two additional parameters to the Brass relational model in order to improve the fit at the youngest and oldest ages. The additional parameter at the high ages allows the steepness of death rates above the median age to be manipulated. While this may improve the fit to mortality rates at a particular point in time, it
adds complexity and is likely to be less practical when using the relational model for projection purposes.

**Lee-Carter model**

The Lee-Carter (LC) model (Lee & Carter, 1992) has been widely used for forecasting mortality. It expresses the logarithmic transformation of age-specific death rates at some time in the future as a linear function of a ‘base rate’, as follows:

\[
\ln(m_{x,t}) = a_x + b_x k_t + \varepsilon_{x,t}
\]  

(2.37)

where

\(\ln(m_{x,t})\) is the log-transformation of central death rates for age \(x\), at a future time \(t\);

\(a_x\) is the ‘base rate’, reflecting the average of log of death rates at age \(x\) over the fitting period;

\(k_t\), an index of the level of mortality at time \(t\), drives the overall time trend in mortality across all ages;

\(b_x\) determines the extent to which the log of the central death rate at age \(x\) changes for a given change in \(k_t\);

\(\varepsilon_{x,t}\) is an error term.

Gómez de León (1990) derived essentially the same model through exploratory data analysis of Norwegian mortality data. Lee and Carter (1992) found that, based on United States data over the period 1900-1989, the parameter \(k_t\) decreased roughly linearly at a relatively constant pace, implying a constant geometric rate of change in central death rates over time. Therefore, for forecasting purposes, \(k_t\) was modelled with an ARIMA time series model as a ‘random walk with drift’:

\[
k_t = k_{t-1} + c + u_t
\]  

(2.38)

where \(c\) is the drift term, representing the assumed annual change in \(k_{t-1}\); \(u_t\) is white noise and reflects how well the linear function fits past time series trends of \(k_t\). The white noise and error terms allow probabilistic characteristics to be attached to forecast values (Lee & Miller, 2001).

Separate parameters \((a_x\) and \(b_x\)) are determined for each age or age group \((x)\) and these remain constant over the projection period. Projecting mortality involves projecting only \(k_t\). The assumed linearity of \(k_t\) over time means that mortality at each age changes at a different,
constant exponential rate. The parameter $k_t$ determines the assumed change in the log of mortality rates across the whole age range relative to the ‘base’ rate, and $b_x$ converts this rate of change to an age-specific rate of change. Lee and Carter (1992: 661) state that this method makes use of the “high degree of inter-temporal correlation across the ages, by making all death rates functions of the same time-varying parameter”. This avoids implausible age profiles that may result from separately modelling projected mortality for different ages.

The $b_x$ parameters are derived by applying a singular value decomposition method (least squares solution) to the matrix of values $b_x \times k_t = \ln(m_{x,t}) - a_x$ (Lee & Carter, 1992). According to Mitchell et al. (2013) this is equivalent to performing principal component analysis on the covariance matrix of log mortality rates. A unique solution can only be found if the following constraints are applied: $\Sigma_x b_x = 1$, and $\Sigma_t k_t = 0$. $a_x$ is determined as the average of the observed values $\ln(m_{x,t})$ over a past period (the fitting period) from time $n$ to time $t_0$ ($n < t_0$), calculated as $\frac{1}{t_0-n+1} \sum_{i=n}^{i=t_0} \ln(m_{x,i})$. The values of $k_t$ are further iteratively adjusted for the age distribution of deaths to ensure that the implied number of deaths equal the actual number of deaths for each year. Therefore: $D_t = \sum e^{a_x + b_x \times k_t} N_t$, where $D_t$ is the number of deaths at all ages at time $t$, and $N_t$ is the total population at time $t$.

Over the last two decades the LC model has become more popular as an alternative to expectation methods. A significant literature has developed around the LC method and various extensions and adjustments to it. Only a few examples are mentioned here. Lee and Miller (2001) suggested the use of observed death rates in the launch year to avoid discontinuities between observed mortality rates in the base year and forecast values for the first year. Booth et al. (2002) further refined Lee and Carter’s (1992) method of adjusting $k_t$. Wilmoth (1993) and Booth et al. (2002) also investigated alternative statistical techniques to derive parameters for the LC model. Lee and Carter (1992) applied this model to the total US population, rather than separately for males and females, to avoid forecasting implausible sex differentials. Various extensions of the Lee-Carter model were also proposed by Haberman and Renshaw (2008) and Renshaw and Haberman (2003a, 2003b, 2006, 2008). Li and Lee (2005) proposed an extension to the Lee-Carter method, referred to as coherent forecasts, such that it can be applied simultaneously to multiple populations and avoid divergence of long-term results. This is achieved by using the same $b_x$ and drift term for $k_t$ for all the related populations. This approach is suitable for modelling the different states of Australia or

53
for modelling males and females. Hyndman, Booth, and Yasmeen (2013) proposed a generalization of the Li and Lee (2005) approach called the product-ratio functional forecasting method for coherent forecasting of subpopulations and applied it to mortality data for Australian states. However, the focus was on the entire age range and not specifically on the high ages and an assessment of the accuracy of such methods has not been done.

The implicit assumption that the age-related pattern of mortality change will remain constant (constant $b_x$) over the projection horizon is one of the most criticised aspects of the LC model and is contrary to empirical evidence. Over the course of the previous century, the drivers of mortality decline have changed, with impacts varying by age and over time. Up to around 1950, mortality decline in developed countries occurred mainly among younger age groups while mortality decline after 1950 occurred among the older age groups (Lee, 2000; Lee & Miller, 2001). This may also be the underlying cause for Lee and Carter’s (1992) finding that for the 85+ age group, the model only explained 86% of the variance over time.

Because the same period is used to estimate all the parameters, an appropriate fitting period for the LC model needs to satisfy both the requirement of linearity in the historical pattern of change in $k_t$ across all ages (to derive $c$) and stability over time of the age-related factors $b_x$ (Booth et al., 2002). Lee and Miller (2001), in line with Tuljapurkar et al. (2000), thus suggested using a past period for deriving the parameters starting only from 1950. Booth et al. (2002) found that mortality did not decline at a constant exponential rate from 1950 in Australia. This was contrary to the findings by Tuljapurkar et al. (2000, p.789) for the G7 countries. According to Booth et al. (2002), an optimal fitting period for Australia for deriving $k_t$ starts only in 1968. Data after this date also supported a more stable pattern of $b_x$ over time. Gavrilova and Gavrilov (2011) similarly concluded that the most appropriate fitting period for Sweden started in 1980, given that senescent mortality declined at a stable linear rate only from this date.

Booth and Tickle (2003) applied an adjusted Lee-Carter method to Australian data for 1968-2000 to forecast mortality to 2031 for 5-year age groups up to an aggregate group for ages 85+. Projected life expectancies at birth, absolute and proportional elderly (65+) and very elderly (85+) populations (relative to the total population) were greater from this adjusted Lee-Carter method compared to official ABS projections. Differences in projected very
elderly populations between LC and official projections were found to increase along the forecast horizon from around 11% after 10 years up to 34% after 30 years for females and from 4% to 19% for males (Booth & Tickle, 2003).

Ediev (2008) agreed that the use of constant $b_x$ factors in the LC model over time implies constant correlations between age-specific deviations from the overall linear trend. However, an investigation into the existence of correlations of residuals of the linear model (USA, Japan, France, Russia, Sweden and Austria) over the period 1980-2004 did not provide strong support for the Lee-Carter model. Ediev (2008: 7) states that the Lee-Carter model captures “only the overall trend of the average log-mortality rate, hence neglecting different developments of the trend at different ages”. Ediev (2008) thus suggested the use of the Geometric model with past periods of different lengths being used for determining the geometric rate of decline applicable at different ages. These fitting periods should be such that the linearity requirement is separately met for each age or age group modelled. A constraint is placed on fitting periods used for different ages to ensure the resulting age profile of mortality is plausible.

*Forecasting parameters of fitted mathematical functions*

An alternative to directly forecasting age-specific death rates is to fit a mathematical function to the age profile of death rates and forecast the parameters of the function (McNown & Rogers, 1989; Pollard, 1987). Mathematical functions are fitted and projections made separately to data for males and females and may also be applied separately to mortality rates for different causes of death (McNown & Rogers, 1992). By fitting functions separately for each year over a past period, time series of the parameters are created. The trends in the time series of the parameters may then be extrapolated (Keyfitz, 1982, 1991; McNown & Rogers, 1989, 1992). This may be applied to any of the models discussed in section 2.5.1.

2.8 Conclusion

This literature review chapter addressed knowledge currently available relating to the objectives of this thesis, namely the creation of very elderly population estimates and projections for Australia and insights into mortality patterns and trends and drivers of growth in very elderly populations. The literature reviewed include the methods available for creating accurate very elderly population estimates using death counts, for example the Extinct Cohort, Survivor Ratio and Das Gupta’s methods, and methods of allowing for
declining trends in mortality. The accuracy of these methods will be evaluated for Australia, at both the national and state levels. This will contribute an understanding of the performance of these methods for Australia, as well as an understanding of the suitability of the methods for application at a sub-national level. The accuracy of the methods at a sub-national level has not been assessed in Australia or elsewhere, to the author’s knowledge. A method to allow for internal migration will be proposed, as well as alternative methods for creating sub-national estimates. A new, simpler method which explicitly allows for survival improvement, with no need to rely on official estimates, is also tested. The best methods (at state and national level respectively) will be used to create reliable and detailed estimates of Australia’s very elderly population at the state and national level.

A decomposition method for quantifying the proximate drivers of very elderly population growth was discussed, as well as findings of international studies into drivers of growth. The second objective of this thesis is, based on the reliable estimates created under the first objective, to analyse changes in the demographic characteristics of Australia’s very elderly population and undertake a decomposition of the growth. Specifically, this will improve our understanding of how increases in births, improvements in survival at different ages, and changes in net migration contributed to the growth of Australia’s very elderly population. It will furthermore contribute substantive knowledge relating to temporal changes in the age-sex distribution Australia’s very elderly population Australia.

Features of mortality at high ages were discussed, together with mathematical functions for modelling age profiles of adult death rates, and observed patterns of change in modal ages at death and dispersions of ages at death. In light of this knowledge, temporal changes in Australian mortality experience will be analysed as part of objective 3, adding to the international literature on patterns and trends in adult mortality. Specifically, it will be identified whether death frequency distributions in Australia has been compressed, shifted or expanded, and how Australia compares in this regard with other low-mortality countries. A new method for understanding relationships between changes in death rates and death frequency distributions will also be proposed, thus making a substantive methodological contribution to the literature.

Finally, section 2.6 contained a brief overview of mortality projection methods, criteria to be applied in choosing a suitable method, and detail on extrapolative methods. A number of
these methods are conceptually compared and empirically evaluated as part of objective 4, with the aim of creating reliable very elderly population projections. This will contribute to international knowledge regarding the relative accuracy of different extrapolative mortality forecasting methods for projecting adult death rates and very elderly populations. An appropriate method will furthermore be used to create detailed and accurate very elderly population projections for Australia at a state and national level.
Chapter 3. Assessing the accuracy of population estimation methods

3.1 Introduction

In order to meet the aim of creating accurate population estimates for the very elderly in Australia at a national and state level, appropriate estimation methods need to be identified (objective 1). This was done by retrospectively assessing the accuracy of various nearly-extinct-cohort estimation methods, both at the national and state levels. Estimates by sex and single years of age were compared against numbers derived from the Extinct Cohort method. Errors were measured by the Weighted Mean Absolute Percentage Error. This is the first such assessment with Australian data, and the first assessment that the author is aware of at a sub-national level anywhere in the world.

The methods evaluated here include different variants of the Survivor Ratio (SR) and Das Gupta (DG) methods, described in section 2.3, as well as a new method proposed by the author which explicitly allows for survival improvement. The SR method makes use of a survivor ratio which reflects the proportion of a cohort which survives over a given age range. Variations of this method use different age ranges in deriving this ratio. For both the SR and DG methods, further variants relate to the number of older cohorts averaged when deriving survivor ratios or death ratios. It was also investigated whether results constrained to official estimates are more accurate than those without constraining, and whether it is best to explicitly allow for survival improvement (mortality decline) over time or take it into account implicitly. The accuracy of official very elderly population estimates was also assessed.

The rest of this chapter is organised as follows. The data and methods are described in section 3.2. These include the new proposed approach to directly allow for survival improvement, a proposed approach to allowing for interstate migration as well as a new method for deriving population estimates at state level from national level estimates. The results of retrospectively testing the methods are set out in sections 3.3 (national level) and 3.4 (state level). Section 3.5 concludes with a summary and recommendations.
3.2 Data and methods

3.2.1 Data

Deaths and Estimated Resident Populations

Formal birth registration systems were only in place across the largest part of Australia by the 1860s, and hence deaths data for centenarians can probably only be considered fully reliable from the early 1970s. This is consistent with the ABS’ view that deaths data after 1964 are considered to be more accurate than data for an earlier period (Andreeva, 2012). Deaths data at both national and state level from 1971 to 2012 by sex and single ages to 109 and 110+ were obtained from the ABS. Data for ten earlier years are required when determining survivor ratios over five-year age ranges and for five older cohorts. Deaths from 1960 were downloaded from the Human Mortality Database.

The states included in this study are New South Wales (NSW), Victoria (Vic), Queensland (Qld), Western Australia (WA), and South Australia (SA), and the rest, Northern Territory, Tasmania and Australian Capital Territory (Tas-NT-ACT), are combined due to small numbers. Deaths data at ages above 95 were adjusted for randomisation applied by the ABS, such that death counts at individual ages summed across states reconcile to the national number of deaths, and that at both national level and for each state, the sum of deaths at single ages 100 and older equalled the 100+ totals provided.

Estimated Resident Populations (ERPs), the official population estimates in Australia, were required to apply 85+ and 90+ constraining to nearly-extinct-cohort estimates, and to permit a comparison with Extinct Cohort estimates to be made. They were obtained by state, sex and single year of age from the Australian Bureau of Statistics (ABS, 2013a). Because Extinct Cohort and nearly-extinct-cohort methods were used to create 31 December estimates for each year, the 30 June ERPs were interpolated to 31 December.

Splitting of death counts by age and year into age-period-cohort triangles

Deaths data were provided by single year of age and individual calendar years. In order to apply the Extinct Cohort and nearly-extinct-cohort estimation methods, death counts needed to be further split into cohorts by factors referred to as triangle factors. This allows
aggregation to period-cohort death counts. Triangle factors applied by the HMD are obtained from regression equations based on multiple regression analysis of the data for three countries (Sweden, Japan and France) (Wilmoth et al., 2007). In order to derive triangle factors for the purpose of this research, a new, simpler method was applied, described below.

Triangle factors reflect the proportion of deaths at a specific age and in a certain calendar year that is attributed to a particular cohort. The number of deaths at age x in year t from cohort c can be expressed as the number of births in year t-x, multiplied by their survival to age x, their probability of death between exact age x and exact age x+1, and the proportion of deaths occurring in the earlier triangle of the age-cohort parallelogram. For cohort c and cohort c-1 respectively this is expressed as follows:

\[ D_{x,t}^c = B_c \cdot x p_0^c q_x^c y \]  
\[ D_{x,t}^{c-1} = B_{c-1} \cdot x p_0^{c-1} q_x^{c-1} (1 - y) \]

where

\( D_{x,t}^c \) is the number of deaths from cohort c at age x in year t, where cohort c are those people born in year t-x;

\( B_c \) is the number of births in year t-x;

\( x p_0^c \) is the probability of survival from birth to age x for cohort c;

\( q_x^c \) is the probability of death between exact age x and exact age x+1 for cohort c;

\( y \) is the proportion of deaths occurring in the later triangle of the age-cohort parallelogram.

By assuming deaths are evenly distributed throughout the year between exact age x and exact age x+1, y is taken as 0.5.

The age-period-cohort deaths are illustrated in Figure 3.1. If z is the ratio of deaths in the lower triangle relative to those in the upper triangle, i.e. the number of deaths at age x in year t from cohort c relative to those from cohort c-1:

\[ z = \frac{D_{x,t}^c}{D_{x,t}^{c-1}} = \frac{B_c \cdot x p_0^c q_x^c y}{B_{c-1} \cdot x p_0^{c-1} q_x^{c-1} (1 - y)} \]

then the proportion of deaths in the lower triangle equals \( \frac{z}{1+z} \).
Figure 3.1: Lexis diagram illustrating age-period-cohort deaths

Notes: Each of the triangles represents age-period-cohort deaths. Different shades represent different cohorts. Age-period deaths at age x in year t (D_{x,t}), are attributed to cohorts c (D^c_{x,t}) and c-1 (D^{c-1}_{x,t}).

This relationship was used to split national and state death counts by age and year into deaths by age, year and cohort. Probabilities of survival and death applied at both national and state level are those measured at a national level, while birth numbers are state-specific. Triangle factors varied by sex and state. National level probabilities of survival and death up to 2009 were obtained from the HMD and are thus a function of the triangle factors applied in the HMD. Death and survival probabilities for 2010-2012 were calculated based on the regression equations used in the HMD as set out in the HMD Methods Protocol (Wilmoth et al., 2007).

**Interstate migration data**

Before applying the nearly-extinct-cohort methods at a state scale, it was determined whether an explicit allowance for interstate migration was required. A proposed approach which accounts for interstate migration in the Extinct Cohort and nearly-extinct-cohort methods is...
set out in section 3.2.2. This makes use of ABS census migration data. For each current state of usual residence, the census provides counts of people living at a different address one and five years prior to the census. Interstate migration data were obtained from the 1981, 1986, 1991, 1996, 2001, 2006 and 2011 censuses. Data for interstate migration over five-year periods were used due to their lower volatility. For censuses up to 1996 data were available by single age up to age 98 and in aggregate for ages 99+ and for censuses from 2001 by single age to 99 and in aggregate for ages 100+. Respondents who did not provide their state of usual residence five years ago (‘not stated’) were allocated proportionally between the numbers who did not move, moved from other states or moved from overseas.

Using these data, net interstate migration ratios \( (nm_{x,t}) \) by age over the previous 5 (k) years were calculated for each census date as the number of people who moved in from other states \( (IM_{t-i}) \) minus the number of people who moved away to other states \( (OM_{t-i}) \) during this period, expressed as a percentage of the end-of-period census population aged \( x \) \( (\text{Census}P_{x,t}) \):

\[
nm_{x,t} = \frac{\sum_{i=0}^{k-1} (IM_{t-i} - OM_{t-i})}{\text{Census}P_{x,t}}
\]

By expressing net migration from census data as a percentage of the census population, it is assumed that any age misstatement inherent in census data occurs to a similar extent in migration counts and population counts. Given that both are derived from the same data source this is considered a reasonable assumption. It is acknowledged that the more common population balancing equation assumes movements across borders and that census data is not movement data, but rather reflects transitions only between two points in time and the net effect of movements over a period of time (Rees & Willekens, 1986). However, in this particular instance, it is assumed that net migration from the movement and transition perspectives is approximately the same.

Figure 3.2 shows the five-yearly net interstate migration ratios for each state for ages 85+ for by sex. From these graphs it is clear that net migration at these high ages is small and does not vary significantly between intercensal periods. Ratios also do not vary appreciably by sex. For NSW, net migration ratios were negative indicating that moves out exceeded moves in. More people moved into Qld and Tas-NT-ACT than out, while net migration ratios varied between -0.5% and +0.5% for Vic, SA and WA. An analysis of net five-yearly migration
ratios by state and age for each intercensal period indicates that there are no clear patterns by age and time.

![Net migration ratio, 5 years](image)

**Figure 3.2: 5-yearly net migration ratios at ages 85+ by sex, state and census period**

### 3.2.2 Population estimation methods

The nearly-extinct-cohort methods assessed were the Survivor Ratio and Das Gupta methods described in section 2.3 together with a new technique adapted from the Das Gupta Advanced and Mortality Decline methods, termed here the Survivor Ratio Advanced (SA) method. This new method is described below. In assessing accuracy, the results of these estimation methods were compared against more robust estimates derived from the Extinct Cohort method (see section 2.3). Table 3.1 lists the Survivor Ratio, Das Gupta and Survivor Ratio Advanced variants which were assessed at the national level. Separate calculations were
made for males and females. In addition to testing these methods, the accuracy of the ABS’ Estimated Resident Population (ERP) was also assessed.

Table 3.1: Nearly-extinct-cohort methods tested at the national level

<table>
<thead>
<tr>
<th>Constraint</th>
<th>SR variants</th>
<th>DG variants</th>
<th>SA variants</th>
</tr>
</thead>
<tbody>
<tr>
<td>No constraining</td>
<td>SR(5,5,NC)</td>
<td>DG(1,5,NC)</td>
<td>SA(5,5,NC,10)</td>
</tr>
<tr>
<td></td>
<td>SR(5,3,NC)</td>
<td>DG(1,3,NC)</td>
<td>SA(1,5,NC,10)</td>
</tr>
<tr>
<td>Constrained to total official estimates for ages 85+</td>
<td>SR(5,5,85+)</td>
<td>DG(1,5,85+)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>SR(5,3,85+)</td>
<td>DG(1,3,85+)</td>
<td></td>
</tr>
<tr>
<td>Constrained to total official estimates for ages 90+</td>
<td>SR(5,5,90+)</td>
<td>DG(1,5,90+)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>SR(5,3,90+)</td>
<td>DG(1,3,90+)</td>
<td></td>
</tr>
<tr>
<td>Constrained to total alternative estimates for ages 85+</td>
<td>SR(5,5,ALT 85+)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note. The notation is Method (age range, number of cohorts for averaging survivor ratio or death ratio, official population estimate used as a constraint). In the case of the SA method the last parameter refers to the number of older cohorts over which survivor ratio change is measured.

The Survivor Ratio method is denoted by SR(k, m, constraint), where k refers to the age range over which the survivor ratio is measured and m refers to the number of older cohorts over which the survivor ratio is averaged. ‘Constraint’ indicates whether a constraint was applied and, if so, the age range of the official population estimate. It is either ‘NC’, ‘85+’ or ‘90+’, referring to no constraint, constraining to the official 85+ population estimate, or the official 90+ population estimate. Different variants assessed relate to different values for k, m and constraining age groups.

A further variant of the SR method that was tested is where results are constrained to an alternative 85+ estimate. This is a new variant and has not been used or tested before. The alternative 85+ estimate was derived by deducting subsequent cohort-deaths from cohort-specific ABS estimates (ERPs) at age 84, for all cohorts currently aged 85+. This variant is denoted by SR(k,m,ALT 85+).

In the literature variants of the Das Gupta method are denoted by DG(m,constraint) or DG(m) where no constraint is applied, where m refers to the number of older cohort averaged and ‘constraint’ indicates whether constraining to official estimates for ages 85+ or 90+ is applied. However, a slightly different notation is used here. While Das Gupta’s method
makes use of death ratios between consecutive ages, and the Survivor Ratio method makes use of survivor ratios measured over \( k \) years of age, it can be shown that the DG method produces exactly the same results as the SR method when \( k=1 \). Therefore, while the calculations differ mechanically, method DG\((m)\) gives the same results as SR\((1,m)\), and the DG method can thus be considered a variant of the SR method. The algebraic proof of this is set out in Appendix B. Thus, for consistency, the DG method will be denoted by DG(1,m,constraint).

Thatcher et al. (2002) estimated \( w \) (equation 2.1) as the highest age where there was expected to be only one survivor. This age varied between years and between males and females but did not exceed 110. Hence, Extinct Cohort estimates were produced for cohorts born before 1902. In fact, it was also used to create estimates for cohorts born up to 1906 by ‘completing’ the number of deaths in these very nearly extinct cohorts. There is therefore a small degree of indirect estimation in these Extinct Cohort estimates. However, given the tiny numbers involved, the possible extent of error is very small. The accuracy of the nearly-extinct-cohort methods were thus assessed by retrospectively applying them to create estimates for 31 December each year from 1976 to 1996, and comparing them against the Extinct Cohort estimates.

Retrospective testing of nearly-extinct-cohort methods at a national level for Australia (reported in section 3.3 below) has shown that the number of cohorts used to derive survivor ratios or death ratios make little difference. Therefore, a smaller selection of variants was tested at a state level. Furthermore, the SA method based on a one-year age range produced high and very volatile errors and was not tested for the states, where results are expected to be even more volatile. The methods tested at a state level are shown in Table 3.2.
Table 3.2: Nearly-extinct-cohort methods tested at a state level

<table>
<thead>
<tr>
<th></th>
<th>SR variants</th>
<th>DG variants</th>
<th>SA variants</th>
<th>Proportional variants</th>
</tr>
</thead>
<tbody>
<tr>
<td>No constraining</td>
<td>SR(5,5,NC)</td>
<td>DG(1,5,NC)</td>
<td>SA(5,5,NC,10)</td>
<td>Prop SR NC</td>
</tr>
<tr>
<td>Constrained to total</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>official estimates</td>
<td>SR(5,5,85+)</td>
<td>DG(1,5,85+)</td>
<td></td>
<td>Prop SR 85+</td>
</tr>
<tr>
<td>for ages 85+</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constrained to total</td>
<td>SR(5,5,90+)</td>
<td>DG(1,5,90+)</td>
<td></td>
<td>Prop SR 90+</td>
</tr>
<tr>
<td>official estimates</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>for ages 90+</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note. The notation is Method (age range, number of cohorts for averaging survivor ratio or death ratio, official population estimate used as a constraint). In the case of the SA method the last parameter refers to the number of older cohorts over which survivor ratio change is measured.

**Proportional method**

A new approach proposed by the author, referred to as the Proportional method, was also tested at state level. According to this method, variants of the SR method are applied at the national level, and resulting estimates at single ages apportioned between the states based on the state-level proportions of ERPs in the 85+ age group relative to the national 85+ ERPs. The Proportional method is set out in algebraic form below. The ratio of state-level ERPs at ages 85+ relative to the national level ERP at ages 85+ in year t is:

\[
prop^S_t = \frac{\sum_{x=85}^{100+} ERP^S_{x,t}}{\sum_S \sum_{x=85}^{100+} ERP^S_{x,t}}
\]  

(3.5)

where \( ERP^S_{x,t} \) is the ERP on 31 December of year t for age \( x \) last birthday and state \( S \), and the summation across states equals the total ERP at national level. Following the estimation of national-level estimates using a variant of the SR method, this ratio is applied to the estimates at single ages in year t, to derive state-level estimates at single ages in year t:

\[
p^S_{x,t} = prop^S_t \cdot p^{nat}_{x,t}
\]  

(3.6)

where \( p^{nat}_{x,t} \) is the national population estimate for age \( x \) on 31 December of year t and \( p^S_{x,t} \) is the corresponding estimate for state \( S \). For the purpose of this study, the variants SR(5,5,85+), SR(5,5,90+) or SR(5,5,NC) were applied at a national level and corresponding state-level proportional variants were denoted Prop SR 85+, Prop SR 90+ and Prop SR NC respectively.
**Survivor Ratio Advanced method, allowing explicitly for mortality decline**

The Das Gupta Advanced (DA) method, introduced by Andreev (2004), adjusts death ratios to reflect mortality decline over time prior to their application to derive estimated future cohort deaths. Similar to Coale and Caselli (1990), these adjustments are based on an assumption that mortality decline diminishes linearly with increasing age. The validity of these methods relies on empirical evidence that the decline in mortality rates reduces with increasing age at ages 80 and above. This is not, however, evident in Australian data. Rates of mortality decline increase with age in some years and are generally quite volatile. Overall, there are no clear and consistent patterns of decline with age.

The Mortality Decline method (Andreev, 1999; Thatcher et al., 2002) also allows for mortality decline over time by modelling the temporal decline in age-specific mortality rates by way of a loglinear function fitted to mortality rates observed for a number of older cohorts. A new method, similar in concept to the MD method, but believed to be simpler, is evaluated in this study. Like the MD method, it extrapolates average mortality decline observed in the previous 10 cohorts to estimate the mortality for the youngest unobserved cohort. However, instead of fitting a loglinear curve to mortality rates at a particular age across 10 older cohorts, the average annual improvement in survivor ratios is measured. The average improvement in survivor ratios measured at a specific age over 10 older cohorts is assumed to apply to the youngest cohort. This method is referred to as the Survivor Ratio Advanced (SA) method, and denoted by $SA(k,m,NC,n)$, where $k$ and $m$ are as defined for the SR method. NC indicates that no constraint is being applied to the results, as survival improvement is explicitly allowed for.

According to equation 2.1, the survivor ratio at a particular age for cohort $c$ is calculated as:

$$R^c_x = \frac{P^c_{x,t}}{P^c_{x-k,t-k}}$$  \hspace{1cm} (3.7)

where $P^c_{x,t}$ is the population of the cohort $c$, aged $x$ last birthday on 31 December of year $t$.

The average improvement in the survivor ratios over $n$ cohorts can be written as:

$$r_x = \left( \frac{R^c_x - n}{R^c_{x,t}} \right)^{1/(n-1)} - 1$$  \hspace{1cm} (3.8)
where \( n \) is the number of older cohorts for which changes in survivor ratios are measured. The survivor ratio based on \( m \) older cohorts (equation 2.6) is then increased with this average improvement as follows:

\[
R'_x = \frac{\sum_{j=1}^{m} p_{x,t-j}}{\sum_{j=1}^{m} p_{x-k,t-k-j}} \times (1 + r_x)^{\frac{1}{2(m+1)}}
\]  

\( (3.9) \)

Due to the volatility of survivor ratios at very high ages a combined improvement factor was calculated based on aggregate ratios for ages 103+ in the case of Australia at national level and ages 100+ at state level. When applying this method, care is needed where the rate of improvement is very high to ensure that no survivor ratio is allowed to exceed 1.

**Method to allow for interstate migration**

When applying the Extinct Cohort and nearly-extinct-cohort methods at a sub-national level, allowance may need to be made for interstate migration, if significant. The approach followed here is set out below.

The population for a particular cohort in a state can be estimated from the population at an earlier date by deducting deaths and out-migrants from, and adding in-migrants into that cohort in the interim period:

\[
P_{x,t}^c = P_{x-k,t-k}^c - \sum_{i=0}^{k-1} D_{t-i}^c + \sum_{i=0}^{k-1} (IM_{t-i}^c - OM_{t-i}^c)
\]

\( (3.10) \)

where:

- \( P_{x,t}^c \) is the number of people aged \( x \) last birthday on 31 December of year \( t \)
- \( D_{t}^c \) is the number of cohort deaths during year \( t \)
- \( IM_{t}^c \) and \( OM_{t}^c \) represent in-migration and out-migration respectively (between states) into- or out of the cohort during the time interval.

Net internal (in this case, interstate) migration over the previous \( k \) years, calculated as the number of people who moved in from other states minus the number of people who moved away to other states during this period, expressed as a percentage of the end-population, is:

\[
nm_{x,t} = \frac{\sum_{i=0}^{k-1} (IM_{t-i}^c - OM_{t-i}^c)}{P_{x,t}^c}
\]

\( (3.11) \)

Combining equations 3.10 and 3.11 and rearranging the terms gives:

\[
P_{x,t}^c - nm_{x,t} P_{x,t}^c = P_{x-k,t-k}^c - \sum_{i=0}^{k-1} D_{t-i}^c
\]

\( (3.12) \)
Estimating the population aged \( x \) at time \( t \) by way of the survivor ratio method (equation 2.5), this equation can be written as follows (See Appendix C for more detailed algebraic proof):

\[
P_{x,t}^{c} \times \text{nm}_{x,t} \times P_{x,t}^{c} = \frac{R_{x}}{1 - R_{x}} \sum_{i=0}^{k-1} D_{t-i}^{c}
\]  \hspace{1cm} (3.13)

so that the state-level population aged \( x \) at time \( t \), allowing for internal migration, is:

\[
P_{x,t}^{c} = \left[ \frac{R_{x}}{1 - R_{x}} \sum_{i=0}^{k-1} D_{t-i}^{c} \right] / (1 - \text{nm}_{x,t})
\]  \hspace{1cm} (3.14)

Equation 3.14 was applied to estimate state-level populations at the end of each year for cohorts aged between 90 and 110+ in that year. The cohorts’ populations in earlier years and at younger ages are then obtained by summing deaths as in the Extinct Cohort method. For this purpose, death counts in each cohort-year space were adjusted as follows:

\[
D_{t}^{c} = D_{t}^{c} / (1 - \text{nm}^{'}_{x,t})
\]  \hspace{1cm} (3.15)

where \( \text{nm}^{'}_{x,t} \) is the net interstate migration ratio over a single year, derived from the k-yearly ratios as follows:

\[
\text{nm}^{'}_{x,t} = (1 + \text{nm}_{x,t})^{\frac{1}{k}} - 1
\]  \hspace{1cm} (3.16)

Net migration for a particular state over the previous k years into cohort \( c \), \( \text{nm}_{x,t} \), was measured from census data and expressed as a percentage of the census population aged \( x \) at time \( t \) (see equation 3.4). Before applying net migration ratios, checks were carried out to ensure that net migration across states summed to zero. Furthermore, after splitting state level and national level deaths into age-period-cohort triangles and adjusting state deaths to allow for interstate migration, state data were proportionally adjusted to ensure that deaths in each age-period-cohort triangle, when added across states, added up to national level death counts. Only minor adjustments were required.

The results of testing different estimation methods presented in section 3.4 incorporate adjustments for net interstate migration. Net migration has been included in both the estimates tested and the Extinct Cohort estimates. The impact of this allowance is discussed in section 3.4.3.
3.2.3 Error measures

Error is defined as the population estimate (E) minus the actual population (A), where ‘population estimate’ refers to numbers obtained by applying nearly-extinct-cohort methods and ‘actual population’ describes the populations calculated by the Extinct Cohort method.

In order to compare the relative accuracy of different methods, Thatcher et al. (2002) assigned ranks based on absolute errors for ages 95+ and 100+ in each year and for each country. Andreev (2004) considered total relative errors for ages 90+. In focusing on relative errors, the extent of total deviations above certain ages may be understated if overestimations at some ages are offset by underestimations at others. For this study, absolute errors are measured, which are believed to be a more appropriate indication of accuracy at single ages. Population numbers decrease rapidly with advancing age above 90. In order to measure the accuracy of estimation methods at ages 90+ or 100+, it is therefore appropriate to apply a weight of population size at each individual age to the absolute error measured at that age, when determining the average. This measure of accuracy is referred to as the Weighted Mean Absolute Percentage Error (WMAPE) (Siegel, 2002) and is calculated as follows:

\[
WMAPE_t = \frac{\sum_x |E_x - A_x| \times 100}{\sum_x A_x}
\] (3.17)

WMAPEs are calculated for each year and the summation is over ages 90-94, ages 95-99 and ages 100+ respectively. Lower WMAPEs indicate greater accuracy.

Because the highest age at which ERPs are published is 100+, a more appropriate measure for comparing the accuracy of ERPs and nearly-extinct-cohort methods at these ages is Percentage Error (PE). Percentage Error is calculated as follows:

\[
PE_t = \frac{\sum_x E_x - \sum_x A_x}{\sum_x A_x} \times 100
\] (3.18)

In order to compare the accuracy of ERPs for the final open-ended age group with those of the other methods, Mean Absolute Percentage Error (MAPE) will be used, calculated as:

\[
MAPE_t = \frac{\sum_x |PE_t|}{n}
\] (3.19)

where \(n\) refers to the number of years in the study.

3.3 Accuracy of estimation methods at the national level for Australia

In this section the results of evaluating the nearly-extinct-cohort methods at the national level are discussed. Section 3.3.1 sets out the errors and relative rankings of the different methods.
tested at a national level, on average over the period 1976 to 1996. Section 3.3.2 describes how errors for the different methods changed over the period.

3.3.1 Best method on average over 1976-1996 at the national level

Figures 3.3 and 3.4 show the weighted mean absolute percentage errors (WMAPEs) of each nearly-extinct-cohort estimation method for Australian females and males respectively, at ages 90-94 and ages 95-99. These errors are simple averages of WMAPEs for each year in the period 1976 to 1996. The graphs clearly demonstrate that errors tend to increase with age and are generally higher for males than females. The male populations aged 90-99 were only around a quarter that of females and their numbers decrease rapidly with increasing age. The greater errors for males and at higher ages are partly due to increased volatility resulting from smaller volumes of data.

For both females and males at ages 90-94 the average WMAPE for the ABS ERPs proved relatively low at 2.0% and 3.8% respectively. Not surprisingly therefore, variants where results were constrained to ABS 85+ or 90+ ERPs produced the best estimates, with WMAPEs varying between 2.4% and 4.4% for females and between 3.9% and 6.9% for males. Results were not much different whether the 85+ or the 90+ ERP constraints were applied, but use of these constraints gave more accurate estimates than either explicitly allowing for an improvement in survivor ratios or not constraining.

Furthermore, variations in errors between different numbers of cohorts (m) were small, but errors were generally lower if based on longer age ranges (k). As shown in Appendix B, results from the DG method are equivalent to those from the SR method when k=1. For the same number of cohorts, errors from the DG method were higher than for the SR method, indicating that the use of a 5-year age range yielded more accurate results than a one-year age range.

For females at ages 90-94 and 95-99 and males at ages 90-94, the SA method based on a 5-year age range resulted in lower errors than unconstrained SR methods, as would be expected when mortality rates have been declining. Lower errors were, however, achieved when constraining to official estimates than explicitly allowing for survival improvement. Andreev (2004) found that for the large countries in his study, relative errors for the DA method,
which explicitly allows for mortality decline, were lower than both unconstrained SR and DG variants. However, results were less consistent for smaller populations. In this study it was found that the SA method based on survivor ratios measured at adjacent ages (k=1) gave very poor results, as did the unconstrained DG method, which also used one-year age ranges.

While the accuracy of all the methods tested deteriorated with increasing age, the deterioration was more rapid for ERPs, resulting in a fall in their accuracy relative to nearly-extinct-cohort estimation methods. Average WMAPE for ABS ERPs decreased from 2% for females at ages 90-94 to 7% at ages 95-99, and from 3.8% for males ages 90-94 to 19.6% at ages 95-99. The ranking of ABS ERPs dropped from most accurate for both males and females at ages 90-94, to eighth for females at ages 95-99 and to least accurate for males at ages 95-99. According to Andreev (1999), Hill et al. (2000) and Preston et al. (1999), estimates based on census counts at the high ages become increasingly inaccurate with age due to the steep slope of the mortality curve at these ages, and the increasing tendency for age to be misreported. Furthermore Preston et al. (1999) found that the steeper the decline in population numbers with age, the greater the degree of overstatement of population estimates. The more rapid reduction in the number of males with increasing age compared to females would thus also explain the greater error in ERPs for males.
Figure 3.3: Weighted Mean Absolute Percentage Error of nearly-extinct-cohort methods for females at ages 90-94 and 95-99 at national level, averaged over 1976-1996
Figure 3.4: Weighted Mean Absolute Percentage Error of nearly-extinct-cohort methods for males at ages 90-94 and 95-99 at national level, averaged over 1976-1996

For females at ages 90-94 and 95-99 the SR variant constrained to 90+ ERPs, the variant used in the HMD, produced lower errors on average than the other methods tested. However, for males SR variants constrained to 85+ ERPs produced lower errors. An analysis of ERP errors for males at single ages from 85-99 shows the accuracy of ERPs deteriorates rapidly with increasing age from around age 93. This explains why, for males, constraining to 85+ ERPs resulted in more accurate estimates compared to constraining to 90+ ERPs. The accuracy of ERPs for females also deteriorates from around age 95 but to a much smaller extent than for males. For males aged 95-99 the unconstrained SR variant produced the lowest average WMAPEs (7.4%) over the period, followed by the SA(5,5,NC,10) method (7.8%), which explicitly allows for mortality decline.

Figure 3.5 shows the average WMAPEs for the methods tested over the period 1976 to 1996 for females (left) and males (right) at ages 100+. ERPs are not included because only an aggregate ERP is available for ages 100+, which means WMAPE cannot be determined. Percentage errors (PEs) for ERPs at ages 100+ are discussed later.
Figure 3.5: Weighted Mean Absolute Percentage Error of nearly-extinct-cohort methods at ages 100+ by sex, at national level, averaged over 1976-1996

Note: ABS ERPs are not included because only aggregate ERPs are available for ages 100+, which means WMAPEs cannot be determined.

For female centenarians SR variants constrained to ABS 90+ or 85+ ERPs produced the lowest errors. Unconstrained SR variants were the most accurate for male centenarians. However, for both males and females, the differences in errors between constrained and unconstrained SR variants were small. For ages 100+, SR methods where survivor ratios were measured over 5-year age ranges gave the best results. This was the case whether they were based on the experience of 3 or 5 cohorts, and irrespective of whether estimates were constrained or not. The DG variants and the SA method based on one-year age ranges proved to be the least accurate.

Table 3.3 shows the Mean Absolute Percentage Error (MAPE) for the methods tested for males and females aged 100+. Consistent with WMAPEs, SR(5,5,85+) generated the lowest MAPE for females at 9.2% and unconstrained SR variants generated the lowest MAPE for
males at 12.8%. The accuracy of ABS ERPs for ages 100+ was relatively poor for both females and males with MAPEs of 16.1% and 34.4% respectively.

Table 3.3: Mean Absolute Percentage Error of nearly-extinct-cohort methods at ages 100+ by sex, averaged over 1976-1996

<table>
<thead>
<tr>
<th>Method</th>
<th>Females</th>
<th>Males</th>
</tr>
</thead>
<tbody>
<tr>
<td>SR (5,5,85+)</td>
<td>9.2%</td>
<td>15.4%</td>
</tr>
<tr>
<td>SR (5,5,90+)</td>
<td>9.3%</td>
<td>15.9%</td>
</tr>
<tr>
<td>SR (5,3,90+)</td>
<td>9.5%</td>
<td>14.5%</td>
</tr>
<tr>
<td>SR (5,3,85+)</td>
<td>9.7%</td>
<td>14.7%</td>
</tr>
<tr>
<td>SR (5,3,NC)</td>
<td>10.0%</td>
<td>12.8%</td>
</tr>
<tr>
<td>SR (5,5,NC)</td>
<td>10.3%</td>
<td>12.8%</td>
</tr>
<tr>
<td>DG (1,5,90+)</td>
<td>12.2%</td>
<td>22.5%</td>
</tr>
<tr>
<td>SA (5,5,NC,10)</td>
<td>12.6%</td>
<td>14.6%</td>
</tr>
<tr>
<td>DG (1,5,NC)</td>
<td>12.8%</td>
<td>20.6%</td>
</tr>
<tr>
<td>DG (1,5,85+)</td>
<td>13.0%</td>
<td>22.9%</td>
</tr>
<tr>
<td>DG (1,3,90+)</td>
<td>13.5%</td>
<td>29.1%</td>
</tr>
<tr>
<td>DG (1,3,NC)</td>
<td>14.5%</td>
<td>27.1%</td>
</tr>
<tr>
<td>SA (1,5,NC,10)</td>
<td>14.9%</td>
<td>36.7%</td>
</tr>
<tr>
<td><strong>ABS ERP</strong></td>
<td><strong>16.1%</strong></td>
<td><strong>34.4%</strong></td>
</tr>
<tr>
<td>DG (1,3,85+)</td>
<td>19.3%</td>
<td>30.1%</td>
</tr>
</tbody>
</table>

Note: MAPEs for the most accurate method for each state have been highlighted in bold and MAPEs for ABS ERPs are highlighted in bold italics.

In summary, for females the SR variants constrained to 90+ ERP performed best on average at ages 90+, but errors were only marginally (0.3%) higher for the SR variants constrained to 85+ ERP. For males, the SR variants constrained to 85+ ERP performed best on average for all ages 90+, and the difference in error compared to SR variants constrained to 90+ ERP was higher than for females (1.8%). While the variants constrained to 90+ ERP consistently produced the lowest errors within all age groups above 90 for females, this was not the case for males. Unconstrained SR variants produced lower errors for age groups 95-99 and 100+. The next section will consider the performance of the different methods in single years over the time period.
3.3.2 Changes over the period 1976-1996 at the national level

**Females aged 90-99**

Figure 3.6 shows the WMAPEs for Australian females at ages 90-94 and ages 95-99 for different methods in each year from 1976 to 1996. Results for SR(5,3,90+) and SR(5,3,85+) are not shown because they are very similar to SR(5,5,90+) and SR(5,5,85+) respectively. At ages 90-94, the ABS ERPs and SR variants constrained to ABS ERPs for ages 90+ or 85+ produced the lowest errors on average over the whole period, and also consistently throughout the period, with only a few exceptions. Unconstrained variants and the SA method generated high and volatile errors, especially from 1976 to 1981. However, the SA method was the most accurate from 1986 to 1988 and 1996 at ages 90-94 and from 1982 to 1984 and 1995 at ages 95-99.

At ages 95-99, WMAPEs for all methods are slightly higher and more volatile, due to smaller numbers. At these higher ages the volatility of WMAPEs for ERPs increased more than the other methods, with errors increasing from 3% in 1977 to 13.7% in 1985, followed by a decrease to 2.8% in 1991, and an increase to 13.5% in 1996.
Figure 3.6: Weighted Mean Absolute Percentage Error of nearly-extinct-cohort methods for females at ages 90-94 and 95-99 in each year from 1976-1996

**Females aged 100+**

Percentage errors (PEs) for the different methods for females aged 100+ in each year from 1976 to 1996 are shown in Figure 3.7. From the graph it is clear that ABS ERPs at ages 100+
exhibit significantly more volatile error patterns compared to methods based on deaths. For females PEs for ERPs vary between extremes of -46% in 1984 and 26% in 1988, indicating underestimates in some years and overestimates in others. Until 1981 the majority of PEs were negative, indicating that most methods underestimated the total number of females for ages 100+. Thereafter, nearly-extinct-cohort methods mostly overstated the numbers.

On average over the study period, the SR method with survivor ratios measured over 5-year age ranges resulted in the lowest average WMAPEs (see Figure 3.5) and MAPEs (see Table 3.3), with only marginal differences between variants which applied different ERP constraints. Figure 3.7 shows that up to 1983 variants of the DG and SR methods constrained to 90+ and 85+ ERPs were the most accurate for estimating totals for ages 100+. Thereafter, the most accurate methods were SR(5,5,ALT 85+) and SR(5,5,NC). Both these methods were among the three most accurate in 11 of the 21 years studied, more than any of the other methods. They were particularly accurate from 1984.

Figure 3.7: Percentage Error of nearly-extinct-cohort methods for females at ages 100+ in each year from 1976-1996
**Males aged 90-99**

Figure 3.8 compares WMAPEs for different estimation methods in each year from 1976 to 1996 for males at ages 90-94 and 95-99. Results for SR(5,3,90+) and SR(5,3,85+) are very similar to SR(5,5,90+) and SR(5,5,85+) respectively and are not shown.

Whereas for females at ages 90-94, the ERPs and variants constrained to 85+ and 90+ ERPs performed consistently well throughout the study period, this was not the case for males. ABS ERPs and SR variants constrained to ERPs produced errors below 5% in the years 1976 to 1983, but their accuracy deteriorated thereafter. Errors for ABS ERPs gradually increased from 1.1% in 1978 to 8.9% in 1994. From 1984, errors for variants constrained to 90+ ERPs varied between 5% and 10% but exceeded 10% from 1994 and were among the highest of all the methods considered. Following a similar deterioration to 1989, the SR variant constrained to 85+ ERPs improved from 1990 to produce errors below 5% in 1993 to 1995.

As was the case for females at ages 90-94, unconstrained variants and the SA method generated high and volatile errors for males. Errors for DG variants also exceeded 5% in most years and were very volatile over the period. The SR(5,5,ALT 85+) method also performed poorly initially, with errors exceeding 10%. However, from 1984 this method produced gradually improving estimates and from 1989 to 1995 it produced the lowest errors overall for males at ages 90-94.

At ages 95-99 WMAPEs for ERPs were significantly more volatile compared to the other methods, and the errors for males were also significantly higher than for females. Errors for ERPs increased from the lowest level of 8.2% in 1981 to 30.7% in 1985, after which it decreased to 10.4% in 1991 and increased to 41.4% in 1996. From 1989, the three methods that consistently produced the most more accurate estimates for males at ages 95-99 were SR(5,5,NC), SA(5,5,NC,10) and SR(5,5,ALT 85+).
Figure 3.8: Weighted Mean Absolute Percentage Error of nearly-extinct-cohort methods for males at ages 90-94 and 95-99 in each year from 1976-1996
**Males aged 100+**

PEs in each year for males at ages 100+ are shown in Figure 3.9. PEs for ERPs varied between extremes of -18% and 99%, but ERPs were mostly overestimated. From 1982 to 1990 the SR variant SR(5,5,85+) produced the most accurate estimates with PEs varying between -4% and 5.6%, followed by the SR(5,5,90+) variant. However, from 1991 to 1994 both PEs and WMAPEs (not shown) for these methods exceeded 20%. During the last six years, the SR(5,5,NC) variant was the most accurate in producing aggregated 100+ estimates.

![Figure 3.9: Percentage Error of nearly-extinct-cohort methods for males at ages 100+ in each year from 1976-1996](image)

In summary, for both males and females, ERPs proved very accurate at ages 90-94 but become increasingly unreliable with advancing age, varying between significant overstatement and understatement for females from ages 100 and for males from age 95. While SR variants constrained to 90+ ERPs and 85+ ERPs produce the most accurate results at ages 90+ on average over the study period for females and males respectively, a study of the trends in errors over time indicate that from 1991 the SR variant constrained to the alternative 85+ estimate (ALT 85+) is the most accurate method for both males and females at ages 90-99, and the unconstrained SR variant is the most accurate for ages 100+. It is expected that the accuracy of both the new methods proposed in this chapter, namely the SA
method and SR method constrained to ALT 85+ will continue to improve. Increasing volumes as well as more reliable numbers at the very high ages will allow more reliable measurement of death rate decline. Furthermore, continued improvement in vital statistics and official estimates will facilitate more accurate ALT 85+ estimates.

3.4 Accuracy of estimation methods at the state level for Australia

In this section the results of retrospective testing of nearly-extinct-cohort estimation methods at state level are presented. Section 3.4.1 sets out the errors and relative rankings of the methods tested on average over the period 1981 to 1996. Section 3.4.2 describes trends in the errors for the different methods over the period. Net migration is included in these results and the impact of allowing for net migration is discussed in section 3.4.3.

3.4.1 Best method on average over 1981-1996 at the state level

Females

Figure 3.10 shows WMAPEs for each method tested for females aged 90-94 and 95-99 by state. These are average WMAPEs over the period 1981 to 1996. Consistent with the findings at a national level, unconstrained variants and variants of the DG method produced the least accurate estimates. More accurate estimates were obtained when constraining results for either the SR or Proportional methods to 90+ or 85+ ERPs than explicitly allowing for survivor ratio improvement as with the SA(5,5,NC,10) method.

At ages 90-94 the ABS ERPs proved more accurate than nearly-extinct-cohort estimates for all states, with WMAPEs varying from 1.9% (Vic) to 3.8% (WA). At ages 95-99 ABS ERPs were less accurate with WMAPEs of between 7.6% (NSW) and 15.0% (Tas-NT-ACT). The accuracy of ERPs deteriorated more with increasing age compared to the nearly-extinct-cohort estimates so that their relative ranking dropped. Across the states at ages 95-99 errors for the most accurate nearly-extinct-cohort estimates were on average 2.8% lower than for ERPs.

While no single method consistently performed best for all the states, at ages 90-94 the SR variants and Proportional methods constrained to 90+ and 85+ ERPs were consistently among the top four methods for all states. This was also true at ages 95-99 with the exception of WA. In contrast with the other states, the Proportional variants performed relatively poorly.
for WA, and the top four methods were the three SR variants and the SA method. The relative ranking of the SR variants and Proportional methods constrained to 90+ and 85+ ERPs varied between the states but their accuracy generally differed only marginally. At ages 90-94 the errors produced by the best four methods were generally below 5% for all states except Tas-NT-ACT. At ages 95-99 the average errors were below 5% for NSW and Qld, between 5% and 10% for Vic, SA and WA and between 10% and 15% for Tas-NT-ACT. On average for ages 90-99, the variant SR(5,5,90+) was the most accurate for NSW and Vic. In line with national level findings, this was also the best performing method across the states with an average error of 3.8%, followed by Prop SR 90+ with 4.0%, and Prop SR 85+ with 4.2%.

The accuracy of ABS ERPs for ages 100+ was generally fairly low, as can be seen in Table 3.4, which shows MAPEs for each method and state for females aged 100+. MAPEs for ERPs varied between 14.3% (SA) and 27.8% (WA), compared to MAPEs produced by the best method in each state of between 6.2% (NSW) and 22.5% (WA). On average over the study period, the Prop SR NC method produced the most accurate estimates for females in aggregate at ages 100+ for NSW, Vic and Qld. The Prop SR 90+ and Prop SR 85+ methods produced marginally higher MAPEs for these states and the most accurate estimates for SA and Tas-NT-ACT. For WA, the best performing methods were the unconstrained SR and Proportional methods and SR(5,5,90+).
Table 3.4: Mean Absolute Percentage Error of nearly-extinct-cohort methods for females at ages 100+, by state

<table>
<thead>
<tr>
<th>Method</th>
<th>NSW</th>
<th>Vic</th>
<th>Qld</th>
<th>SA</th>
<th>WA</th>
<th>Tas-NT- ACT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prop SR NC</td>
<td>6.2%</td>
<td>10.1%</td>
<td>5.7%</td>
<td>8.7%</td>
<td>23.4%</td>
<td>18.3%</td>
</tr>
<tr>
<td>Prop SR 90+</td>
<td>6.8%</td>
<td>11.0%</td>
<td>8.0%</td>
<td>7.5%</td>
<td>27.3%</td>
<td>14.0%</td>
</tr>
<tr>
<td>SR (5,5,NC)</td>
<td>6.8%</td>
<td>14.7%</td>
<td>9.3%</td>
<td>14.6%</td>
<td>22.5%</td>
<td>39.9%</td>
</tr>
<tr>
<td>Prop SR 85+</td>
<td>6.9%</td>
<td>11.5%</td>
<td>8.6%</td>
<td>7.8%</td>
<td>27.6%</td>
<td>13.9%</td>
</tr>
<tr>
<td>SR (5,5,90+)</td>
<td>8.6%</td>
<td>12.7%</td>
<td>13.5%</td>
<td>15.2%</td>
<td>23.6%</td>
<td>34.1%</td>
</tr>
<tr>
<td>SR (5,5,85+)</td>
<td>9.4%</td>
<td>13.0%</td>
<td>14.6%</td>
<td>16.5%</td>
<td>24.0%</td>
<td>36.1%</td>
</tr>
<tr>
<td>SA (5,5,NC,10)</td>
<td>13.0%</td>
<td>17.2%</td>
<td>27.5%</td>
<td>33.2%</td>
<td>29.8%</td>
<td>60.3%</td>
</tr>
<tr>
<td>DG(1,5,90+)</td>
<td>13.2%</td>
<td>14.8%</td>
<td>27.4%</td>
<td>28.1%</td>
<td>41.5%</td>
<td>55.4%</td>
</tr>
<tr>
<td>DG (1,5,85+)</td>
<td>14.9%</td>
<td>16.6%</td>
<td>30.6%</td>
<td>30.1%</td>
<td>43.3%</td>
<td>58.8%</td>
</tr>
<tr>
<td>DG (1,5,NC)</td>
<td>15.1%</td>
<td>17.7%</td>
<td>33.0%</td>
<td>28.2%</td>
<td>43.5%</td>
<td>62.6%</td>
</tr>
<tr>
<td>ABS ERP</td>
<td><strong>16.5%</strong></td>
<td><strong>16.5%</strong></td>
<td><strong>18.6%</strong></td>
<td><strong>14.3%</strong></td>
<td><strong>27.8%</strong></td>
<td><strong>27.2%</strong></td>
</tr>
</tbody>
</table>

Note: MAPEs for the most accurate method for each state have been highlighted in bold and MAPEs for ABS ERPs are highlighted in bold italics.

WMAPEs cannot be determined for ABS ERPs because only aggregate 100+ ERPs are published. However, WMAPEs for the proportional and SR methods are shown in Figure 3.12 for comparative purposes. With the exception of WA, the Proportional variants produced more accurate estimates than the SR variants. WMAPEs were around 11-12% for NSW and Qld and around 15% for Vic and SA. For WA and Tas-NT-ACT, errors were significantly higher, at 30%.

**Males**

Figure 3.11 shows WMAPEs for each method for males aged 90-94 and 95-99 by state. Consistent with the results for females, the SR variants performed better than the DG variants and the variants constrained to 85+ or 90+ ERPs performed better than unconstrained variants. Constraining to ERPs also generally produced more accurate estimates than explicitly allowing for survival improvement. Similar to females, ABS ERPs for males at ages 90-94 were the most accurate estimates in NSW, Vic, SA and WA and the second most accurate in Qld and Tas-NT-ACT. WMAPEs for ERPs varied from 4.6% (NSW) to 7.6% (Tas-NT-ACT), higher than for females. However, ERPs for ages 95-99 were significantly
less accurate in both absolute terms and relative to the nearly-extinct-cohort estimates, with WMAPEs varying between 19.4% for SA and 32.9% for Tas-NT-ACT.

As was the case for females, no single method consistently produced the lowest errors for all states for all age groups. However, at ages 90-94 the SR(5,5,85+) and the Prop SR 85+ methods were generally the best performers. At ages 95-99 the Prop SR NC method was the most accurate for NSW, Vic, WA and Tas-NT-ACT and the Prop SR 85+ method was among the top three in Vic, Qld, SA and Tas-NT-ACT. On average for ages 90-99 the Prop SR 85+ variant was the most accurate for NSW, Vic, Qld and Tas-NT-ACT. This was also the best performing method across all states with an average error of 6.9%, followed by SR(5,5,85+) with 7.3%.

As shown in Table 3.5, MAPEs for ERPs for males aged 100+ were high, varying from 27.4% (SA) to 71.4% (WA). For NSW and Qld ERPs for ages 100+ were less accurate than all the nearly-extinct-cohort estimates with MAPEs of 53.2% and 47.3% respectively. The Prop SR NC method produced the most accurate estimates in aggregate at ages 100+ for NSW, Vic, Qld, WA and Tas-NT-ACT.
Table 3.5: Mean Absolute Percentage Error of nearly-extinct-cohort methods for males at ages 100+, by state

<table>
<thead>
<tr>
<th>Method</th>
<th>NSW</th>
<th>Vic</th>
<th>Qld</th>
<th>SA</th>
<th>WA</th>
<th>Tas-NT- ACT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prop SR NC</td>
<td>15.4%</td>
<td>11.1%</td>
<td>14.6%</td>
<td>20.1%</td>
<td>13.9%</td>
<td>31.7%</td>
</tr>
<tr>
<td>SR (5,5,NC)</td>
<td>16.0%</td>
<td>24.1%</td>
<td>22.6%</td>
<td>30.3%</td>
<td>53.1%</td>
<td>44.9%</td>
</tr>
<tr>
<td>SR (5,5,85+)</td>
<td>19.3%</td>
<td>27.7%</td>
<td>25.4%</td>
<td>33.0%</td>
<td>55.1%</td>
<td>45.4%</td>
</tr>
<tr>
<td>SR (5,5,90+)</td>
<td>20.4%</td>
<td>27.5%</td>
<td>26.1%</td>
<td>34.0%</td>
<td>52.8%</td>
<td>44.6%</td>
</tr>
<tr>
<td>SA (5,5,NC,10)</td>
<td>21.4%</td>
<td>32.6%</td>
<td>38.1%</td>
<td>36.5%</td>
<td>132.1%</td>
<td>57.3%</td>
</tr>
<tr>
<td>Prop SR 85+</td>
<td>24.4%</td>
<td>12.5%</td>
<td>22.4%</td>
<td>17.8%</td>
<td>15.6%</td>
<td>33.0%</td>
</tr>
<tr>
<td>Prop SR 90+</td>
<td>27.1%</td>
<td>13.6%</td>
<td>24.7%</td>
<td>16.7%</td>
<td>16.4%</td>
<td>32.1%</td>
</tr>
<tr>
<td>DG (1,5,NC)</td>
<td>32.9%</td>
<td>33.7%</td>
<td>34.8%</td>
<td>89.1%</td>
<td>59.5%</td>
<td>76.9%</td>
</tr>
<tr>
<td>DG (1,5,85+)</td>
<td>33.2%</td>
<td>42.8%</td>
<td>35.5%</td>
<td>85.2%</td>
<td>55.5%</td>
<td>72.2%</td>
</tr>
<tr>
<td>DG (1,5,90+)</td>
<td>34.9%</td>
<td>43.3%</td>
<td>35.7%</td>
<td>80.9%</td>
<td>50.9%</td>
<td>74.4%</td>
</tr>
<tr>
<td>ABS ERP</td>
<td>53.2%</td>
<td>39.7%</td>
<td>47.3%</td>
<td>27.4%</td>
<td>71.4%</td>
<td>55.3%</td>
</tr>
</tbody>
</table>

Note: MAPEs for the most accurate method for each state have been highlighted in bold and MAPEs for ABS ERPs are highlighted in bold italics.

As shown in Figure 3.12 variants of the proportional methods generally produced the most accurate estimates for males at single ages at 100 and above, although WMAPEs were high, varying between 26.0% (VIC) and 47.4% (WA). These errors are significantly higher than for females, probably as a result of the smaller numbers and greater volatility in death numbers.

To summarise, ERPs for all the states proved very accurate at ages 90-94 for both males and females, but their accuracy decreased rapidly with increasing age, and significantly more accurate estimates were obtained for ages 95+ using nearly-extinct-cohort estimation methods. Across the states, variants of both the SR and Proportional methods with either 85+ or 90+ ERPs constraints produced reasonably accurate estimates for both sexes at ages 90-99. At ages 100+ the unconstrained Proportional method performed the best, closely followed by the contrained Proportional variants. The better performance of unconstrained methods at ages 100+ is due to the smaller degree of survival improvement at these ages. By constraining, an indirect allowance is made for survival improvement, which, over the study period, was appropriate at ages below 100, but less so at ages 100+.
Figure 3.10: Weighted Mean Absolute Percentage Error of nearly-extinct-cohort methods for females at ages 90-94 and 95-99, by state, averaged over 1981-1996
Figure 3.11: Weighted Mean Absolute Percentage Error of nearly-extinct-cohort methods for males at ages 90-94 and 95-99, by state, averaged over 1981-1996
Figure 3.12: Weighted Mean Absolute Percentage Error of nearly-extinct-cohort methods at ages 100+ by sex and state
3.4.2 Changes over the period 1981-1996 at state level

Females aged 90-99

Figure 3.13 shows the WMAPEs in each year from 1981 to 1996 for females aged 90-94 and 95-99 in NSW, Vic and Qld. Figure 3.14 shows the same information for SA, WA and Tas-NT-ACT. Only WMAPEs for ERPs and the SR variants and Proportional methods constrained to 85+ and 90+ ERPs are shown as these were the best-performing methods on average over the period. Errors for the other methods were volatile and at times very high and are not shown.

WMAPEs for ERPs for females aged 90-94 were consistently below 5% throughout the period 1981 to 1996 for all states. The nearly-extinct-cohort estimation methods also performed well, producing errors below 5% in most years for NSW, Vic and Qld, and below 10% for SA, WA and Tas-NT-ACT. The exceptions were the Proportional variants, which generated errors above 10% for WA from 1986 to 1988, and SR(5,5,85+), which produced errors in excess of 10% for Tas-NT-ACT from 1990 to 1994.

At ages 95-99 ERPs and nearly-extinct-cohort estimates were less accurate on average and errors were more volatile from year to year. WMAPEs for ERPs at ages 95-99 in the larger states of NSW, Vic and Qld were more often higher than those from the SR and Proportional variants, resulting in higher average errors. The Proportional methods fared particularly well in Qld. WMAPEs for the Proportional methods showed an increasing trend in NSW, however, with WMAPEs for Prop SR 85+ increasing from 1.5% in 1985 to 8.0% in 1995, but a decreasing trend in Vic, from 14.9% in 1983 to 2.1% in 1994. In WA, the Proportional methods produced significantly higher errors than ERPs from 1989 to 1994 and the SR variants constrained to either 85+ or 90+ ERPs were the most accurate.
Figure 3.13: Weighted Mean Absolute Percentage Error of selected nearly-extinct-cohort methods for females at ages 90-94 and 95-99 for NSW, Vic and Qld
Figure 3.14: Weighted Mean Absolute Percentage Error of selected nearly-extinct-cohort methods for females at ages 90-94 and 95-99 for SA, WA and Tas-NT-ACT
**Males aged 90-99**

Figure 3.15 shows the WMAPEs in each year from 1981 to 1996 for males in NSW, Vic and Qld aged 90-94 and 95-99 for ERPs and the SR variants and proportional methods constrained to 85+ and 90+ ERPs. Figure 3.16 shows the same information for SA, WA and Tas-NT-ACT. At ages 90-94 ERPs were generally the most accurate with errors below 10% for all states, although from 1990 its relative ranking dropped in most states. During most of the last few years the Prop SR 85+ and SR(5,5,85+) methods produced more accurate estimates compared to ERPs in all states.

At ages 95-99, WMAPEs for ERPs were significantly higher and more volatile than for the other methods. The accuracy of ERPs for males at ages 95-99 also varied significantly more than for females and errors were much higher. Following a period of steady increase from around 1990-1992, WMAPEs for ERPs in most states reached their highest levels in 1995 or 1996, with 45.8% in NSW, 38.8% in Vic, 37.4% in Qld, 30.8% in SA, 55.2% in WA and 38.2% in Tas-NT-ACT. Differences in errors between ERPs and the other methods were large while differences between SR and Proportional variants were relatively small.
Figure 3.15: Weighted Mean Absolute Percentage Error of selected nearly-extinct-cohort methods for males at ages 90-94 and 95-99 for NSW, Vic and Qld
Figure 3.16: Weighted Mean Absolute Percentage Error of selected nearly-extinct-cohort methods for males at ages 90-94 and 95-99 for SA, WA and Tas-NT-ACT
Ages 100+

Figures 3.17 and 3.18 show Percentage errors (PEs) for females and males aged 100+ in NSW and WA. Trends for Vic, Qld and SA were similar to NSW and are not shown.

Figure 3.17: Percentage Error of selected nearly-extinct-cohort methods for females at ages 100+ for NSW and WA

Figure 3.18: Percentage Error of selected nearly-extinct-cohort methods for males at ages 100+ for NSW and WA
It is clear from these graphs that ABS ERPs at ages 100+ exhibit more volatile error patterns compared to nearly-extinct-cohort estimates. ERPs for females were either significantly underestimated or significantly overestimated, while for males they were mostly overstated and to a greater degree. Much more accurate estimates could be obtained using either the SR or proportional variants.

3.4.3 Impact of allowing for migration

The previous two sections discussed the results of population estimation methods explicitly incorporating interstate migration. This section compares some of the results of the tests where no allowance was made for interstate migration and population estimates are based on deaths only. Table 3.6 compares the MAPEs for a number of estimation methods for males at ages 100+ with and without adjustments for interstate migration.

It is clear that the results of accuracy tests differ very little whether interstate migration is allowed for or not. As was clear from the Figure 3.2 the net migration ratios are small. Although the small size of the errors is partly a function of both the Extinct Cohort and nearly-extinct-cohort estimates consistently including or excluding migration, the difference in population estimates at single ages with or without migration never exceeds 0.3% at any age for any state. This suggests that while it would be theoretically more accurate to allow for interstate migration, it is probably not worth the effort. This is consistent with the findings of Kannisto (1990).
Table 3.6: Mean Absolute Percentage Error of nearly-extinct-cohort methods for males at ages 100+, allowing for interstate migration, not allowing for interstate migration (middle) and the difference (bottom)

### A: WMAPE for methods based on deaths and interstate migration

<table>
<thead>
<tr>
<th>Method</th>
<th>NSW</th>
<th>Vic</th>
<th>Qld</th>
<th>SA</th>
<th>WA</th>
<th>Tas-NT-ACT</th>
</tr>
</thead>
<tbody>
<tr>
<td>SR (5,5,NC)</td>
<td>16.0%</td>
<td>24.1%</td>
<td>22.6%</td>
<td>30.3%</td>
<td>53.1%</td>
<td>44.9%</td>
</tr>
<tr>
<td>SR (5,5,90+)</td>
<td>20.4%</td>
<td>27.5%</td>
<td>26.1%</td>
<td>34.0%</td>
<td>52.8%</td>
<td>44.6%</td>
</tr>
<tr>
<td>SR (5,5,85+)</td>
<td>19.3%</td>
<td>27.7%</td>
<td>25.4%</td>
<td>33.0%</td>
<td>55.1%</td>
<td>45.4%</td>
</tr>
<tr>
<td>DG (1,5,NC)</td>
<td>32.9%</td>
<td>33.7%</td>
<td>34.8%</td>
<td>89.1%</td>
<td>59.5%</td>
<td>76.9%</td>
</tr>
<tr>
<td>DG (1,5,90+)</td>
<td>34.9%</td>
<td>43.3%</td>
<td>35.7%</td>
<td>80.9%</td>
<td>50.9%</td>
<td>74.4%</td>
</tr>
<tr>
<td>DG (1,5,85+)</td>
<td>33.2%</td>
<td>42.8%</td>
<td>35.5%</td>
<td>85.2%</td>
<td>55.5%</td>
<td>72.2%</td>
</tr>
</tbody>
</table>

### B: WMAPE for methods based on deaths only

<table>
<thead>
<tr>
<th>Method</th>
<th>NSW</th>
<th>Vic</th>
<th>Qld</th>
<th>SA</th>
<th>WA</th>
<th>Tas-NT-ACT</th>
</tr>
</thead>
<tbody>
<tr>
<td>SR (5,5,NC)</td>
<td>16.1%</td>
<td>24.2%</td>
<td>22.6%</td>
<td>30.2%</td>
<td>53.1%</td>
<td>44.8%</td>
</tr>
<tr>
<td>SR (5,5,90+)</td>
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<td>26.2%</td>
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<td>44.4%</td>
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<tr>
<td>DG (1,5,NC)</td>
<td>32.9%</td>
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<td>34.8%</td>
<td>89.1%</td>
<td>59.6%</td>
<td>76.8%</td>
</tr>
<tr>
<td>DG (1,5,90+)</td>
<td>34.8%</td>
<td>43.3%</td>
<td>34.5%</td>
<td>80.7%</td>
<td>50.8%</td>
<td>74.1%</td>
</tr>
<tr>
<td>DG (1,5,85+)</td>
<td>33.1%</td>
<td>42.8%</td>
<td>35.2%</td>
<td>85.1%</td>
<td>55.4%</td>
<td>72.1%</td>
</tr>
</tbody>
</table>

### A-B: difference in errors with and without interstate migration

<table>
<thead>
<tr>
<th>Method</th>
<th>NSW</th>
<th>Vic</th>
<th>Qld</th>
<th>SA</th>
<th>WA</th>
<th>Tas-NT-ACT</th>
</tr>
</thead>
<tbody>
<tr>
<td>SR (5,5,NC)</td>
<td>-0.1%</td>
<td>-0.1%</td>
<td>0.0%</td>
<td>0.1%</td>
<td>0.0%</td>
<td>0.1%</td>
</tr>
<tr>
<td>SR (5,5,90+)</td>
<td>0.0%</td>
<td>0.0%</td>
<td>-0.1%</td>
<td>0.1%</td>
<td>0.5%</td>
<td>0.2%</td>
</tr>
<tr>
<td>SR (5,5,85+)</td>
<td>0.0%</td>
<td>-0.1%</td>
<td>-0.1%</td>
<td>0.0%</td>
<td>0.5%</td>
<td>0.2%</td>
</tr>
<tr>
<td>DG (1,5,NC)</td>
<td>-0.1%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>-0.1%</td>
<td>0.1%</td>
</tr>
<tr>
<td>DG (1,5,90+)</td>
<td>0.0%</td>
<td>0.0%</td>
<td>1.2%</td>
<td>0.2%</td>
<td>0.1%</td>
<td>0.3%</td>
</tr>
<tr>
<td>DG (1,5,85+)</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.3%</td>
<td>0.1%</td>
<td>0.0%</td>
<td>0.1%</td>
</tr>
</tbody>
</table>
3.5 Conclusion

This chapter assessed the accuracy of a number of estimation methods for creating population estimates at ages 85 and above based on deaths data. Previous studies have evaluated such indirect estimation methods for a number of European countries, Japan and the US, but no evaluations had been undertaken for Australia. The results of one of these studies (Thatcher et al., 2002) were used to determine the appropriate method for estimating very elderly population numbers in the Human Mortality Database. While there are indications that official estimates for the very elderly in Australia are too high, no research had previously been undertaken to quantify the extent of this overestimation, and it was not known whether the methods used in the HMD, which constrain very elderly population estimates to official estimates for ages 90+, would be the most accurate for Australia.

This was also the first time that the author is aware of, that such methods have been evaluated at a sub-national level. This is thus a substantive contribution to the literature, providing insights into the suitability of such methods for application at a sub-national level, as well as the relative accuracy of different variants and official estimates. A method was proposed for allowing for internal migration when applying nearly-extinct-cohort methods at the sub-national level. It is proposed that allowance for interstate migration be incorporated in the estimation process if net interstate migration ratios exceed levels of around 5%. A new, proportional, method was also proposed for deriving sub-national estimates from national-level estimates and was found to perform well for Australia. A new method that explicitly allows for survival improvement was also assessed, referred to as the Survivor Ratio Advanced method.

In this chapter, the method which Thatcher et al. (2002) found to be the most accurate was also shown to produce the most accurate population estimates for females aged 90+ in Australia. However, for males aged 90-95 the SR method constrained to 85+ ERPs proved more accurate, while for Australian males aged 95+, an unconstrained SR variant was most accurate. For females in age ranges 90-94, 95-99 and 100+, average errors for the SR variant constrained to 85+ ERPs were only marginally higher than for the SR variant constrained to 90+ ERP. Similarly, for males at ages 95-99 and 100+, average errors for the SR variant constrained to 85+ ERPs also proved only marginally higher than for the unconstrained SR variant. However, for males aged 90-94, differences in the relative accuracy of SR methods
constrained to 85+ ERPs and 90+ ERPs were larger. This is largely due to the rapid
deterioration in accuracy of ERPs for males at single ages from around age 93. As a result, on
average across sex and all age ranges, the SR variant constrained to 85+ ERPs proved the
most accurate for Australia at a national level.

With regards to the values of k (age range) and m (number of cohorts), all the results
indicated that the accuracy of variants changed little whether their inputs were averaged over
3 or 5 cohorts, but that estimates were definitely more accurate when based on 5-year age
ranges, as used in the SR variants, compared to one-year age ranges, as is implicit in the DG
variants. The approach of adjusting results by constraining them to official 85+ or 90+ ERPs
was also found to produce more accurate estimates than explicitly allowing for improvement
in survival.

This study also assessed how the various estimation methods performed over time. It was
found that variants of both the SR and DG methods where no constraint was applied were
prone to large variations in errors compared to estimates constrained to either 85+ or 90+
ERPs, or an alternative estimate for ages 85+. While SR variants constrained to 90+ ERPs
and 85+ ERPs produce the most accurate results on average over the study period for females
and males respectively, a study of the trends in errors over time indicate that from 1991 the
SR variant constrained to the alternative 85+ estimate, SR(5,5,ALT 85+), is the most accurate
method for both males and females at ages 90-99. It is recommended that this study be
updated in a few years’ time and if this trend continues, the SR variant based on this
alternative constraint be used to estimate very elderly populations in Australia.

It was more difficult to estimate populations at higher ages, as seen in the higher errors and
greater volatility in errors at ages 100+ compared to ages 95-99, and the higher errors at ages
95-99 compared to ages 90-94. For Australian females aged 90-94, SR and DG variants
where results were constrained to ABS ERPs were reasonably accurate with errors at 5% or
below throughout the study period. For females at ages 95-99 and males at ages 90-94,
WMAPEs varied to up to 10%, and up to 15% for females at ages 100+ and males at ages 95-99.
For males at ages 100+ WMAPEs were as high as 30% in some years.

The performance of official population estimates produced by the ABS deteriorated with
increasing age. For both males and females at ages 95+ the accuracy of ERPs varied
significantly from year to year. For females they alternated between substantial over- and
underestimation, while for males ERPs were generally too high. On average over the study
period, the accuracy of ABS ERPs for ages 100+ was poor relative to the methods evaluated,
with MAPEs of 16% and 34% for females and males respectively.

This research also explored the appropriateness of applying the nearly-extinct-cohort methods
of estimation at a sub-national level, which has not been done before in Australia or
internationally. It has shown that the nearly-extinct-cohort methods can be successfully
applied at a sub-national level to produce reasonably accurate estimates by single years of age
at the highest ages. These methods can be used to produce significantly more accurate
estimates for ages 95 and above compared to official estimates. In addition, the availability of
such estimates at single ages above 100 will facilitate calculation of more accurate and
detailed mortality rates and better projections at these high ages. Accurate estimates at a sub-
national level are important because this is typically where planning and budgeting for
services relating to very elderly care takes place. While nearly-extinct-cohort methods were
applied at a state level in Australia, the methods and principles should be applicable in other
contexts and for other sub-national geographies, subject to the availability of sufficient
volumes of data. The methods appear to become less effective for smaller populations due to
the increased volatility of survivor and death ratios.

The results of applying the methods at a state level were consistent with those at the national
level. For example, DG variants also produced less accurate results than SR variants and
variants constrained to 90+ or 85+ ERPs were generally more accurate than explicitly
allowing for mortality decline. It was furthermore found that the level of interstate migration
at ages 85+ is small and allowing for this does not significantly improve the estimates.

As was the case at the national level, state-level ERPs proved very accurate at ages 90-94 but
their accuracy deteriorated more rapidly with increasing age than nearly-extinct-cohort
methods. For both sexes at ages 95 and above the accuracy of ERPs varied significantly from
year to year and more accurate estimates could be derived by applying nearly-extinct-cohort
methods to state-level death counts, or by apportioning estimates obtained by applying
nearly-extinct-cohort methods at the national level to states. Across the states and for both
males and females, both the average level of WMAPEs of different methods and their
variation over time were consistent with those at the national level. While no single method
proved to be the most accurate for all states and age ranges, reasonably accurate estimates for most states, at most ages 90 and above and for both sexes could be produced by either the Prop SR 85+ or the SR(5,5,85+) methods.

This study has contributed to our understanding of the performance of various nearly-extinct-cohort methods for estimating very elderly populations in Australia, both at national and state levels. It has confirmed that the SR(5,5,90+) method used in the HMD works well for Australian females aged 90 and above but that slightly more accurate estimates may be obtained for Australian males, and on average across males and females, by constraining results from the Survivor Ratio method to ERPs for ages 85+ rather than 90+. Given that SR(5,5,90+) and SR(5,5,85+) give very similar errors for Australian females, for ease of application it is recommended that SR(5,5,85+) be used to create very elderly population estimates for both sexes for Australia at a national level. Similarly, based on the results of this study, the simpler Prop SR 85+ method is recommended to be used to allocate the resulting national level age distributions to the state scale. These methods were used to create very elderly population estimates for Australia used in the rest of this thesis.

In closing, it is acknowledged that the results of this study are dependent on the study period used and that consideration of a different period may yield different average results. For example, mortality improvements during the last two to three decades may result in better performance of methods that allow for mortality improvements, and worse performance for methods that do not. However, the general ranking of methods and hence the conclusions drawn, are not expected to be significantly different.
Chapter 4. Understanding the growth of Australia's very elderly population

4.1 Introduction

The second objective of this thesis is to understand how the age- and sex composition of Australia’s very elderly population has changed over time and to identify the demographic drivers of the growth in numbers. Specifically, this involves undertaking a decomposition to determine the extent to which growth in Australia’s very elderly population over the last three decades has been due to increases in birth cohort sizes, declines in mortality and changes in international migration. This enhances our understanding of Australia’s experience in the international context and will allow better planning for the future.

The results of retrospectively testing a number of nearly-extinct-cohort methods for creating population estimates at ages 85 and above based on deaths data, both at the national and state levels, are set out in chapter 3. Estimates by sex and single years of age were compared against numbers derived from the Extinct Cohort method. Based on the results from these analyses, the Extinct Cohort and the Survivor Ratio methods were used to derive estimates from deaths at a national and state level by sex for ages 85-110. Total national-level SR estimates were constrained to 85+ ERPs. State-level estimates were derived by applying state-specific proportions of 85+ ERPs to national estimates.

These newly created estimates are analysed in this chapter. The analysis of growth of very elderly numbers nationally and at the state level, including changes in the age and sex composition of this population segment is set out in section 4.2. The results of the decomposition of growth in nonagenarian and centenarian numbers into the contributions from births, survival and net migration changes are set out in section 4.3. This includes a comparison of the states. Analyses are presented for New South Wales (NSW), Victoria (Vic), Queensland (Qld), South Australia (SA), Western Australia (WA), and combined for the last state and two territories, Tasmania, Northern Territory and Australian Capital Territories (Tas-NT-ACT).
4.2 Changes in Australia’s very elderly population

*Total growth in very elderly numbers at state and national level*

The significant growth in Australia’s very elderly population since 1981 is clear from Figure 4.1. Total numbers increased from 105,000 or 0.7% of the total population on 31 December 1981 to 430,000 or 1.9% of the total population on 31 December 2012. Growth rates in the number of males accelerated from the mid-1980s to the mid-1990s. Before 1985, the number of females increased at an average annual rate of around 4.6% and males at around 2.3%. The rate of increase for males overtook that of females in 1984. Over the last ten years, the very elderly male population has grown at an average 6.3% per year compared to 4.1% for females. This growth pattern was similar in the states. Very elderly numbers in WA grew the fastest over the period from 1981 to 2012 with an average growth rate of 5.7% per year, followed by Qld and Tas-NT-ACT, with average growth rates of 5.1% and 5.0%. Growth rates in NSW, Vic and SA of 4.6% were below the national average of 4.8% per year.
Figure 4.1: Growth in Australia’s male and female very elderly (ages 85+) populations from 1981-2012

Source: Author’s population estimates
Changing age composition of very elderly population

Accompanying the growth in the size of the very elderly population is an ageing of this group itself. The change in composition of the very elderly population by age groups is shown in Table 4.1. From 1981 the proportion aged 85-89 consistently decreased from 70.4% to 65.4% in 2012, with corresponding increases in the proportions aged 90-99 and 100 and older. From 1981 to 2012 the numbers aged 85-89 increased at an average annual rate of 4.4% compared to 5.2% and 6.4% respectively in age groups 90-99 and 100+.

Table 4.1: Change in age distribution (%) within the very elderly age group, 1981-2012

<table>
<thead>
<tr>
<th>31 December</th>
<th>85-89</th>
<th>90-99</th>
<th>100+</th>
<th>85+</th>
</tr>
</thead>
<tbody>
<tr>
<td>1981</td>
<td>70.4</td>
<td>29.1</td>
<td>0.47</td>
<td>100</td>
</tr>
<tr>
<td>1991</td>
<td>70.1</td>
<td>29.2</td>
<td>0.70</td>
<td>100</td>
</tr>
<tr>
<td>2001</td>
<td>69.4</td>
<td>30.0</td>
<td>0.63</td>
<td>100</td>
</tr>
<tr>
<td>2011</td>
<td>66.4</td>
<td>32.9</td>
<td>0.73</td>
<td>100</td>
</tr>
<tr>
<td>2012</td>
<td>65.4</td>
<td>33.8</td>
<td>0.79</td>
<td>100</td>
</tr>
</tbody>
</table>

Source: Author’s population estimates

Numbers and growth at single ages

The numbers of males and females at individual ages 85+ in 1981, 1996 and 2012 are shown in Figure 4.2. Figure 4.3 shows the increases from 1981 to 1996 and from 1996 to 2012. This comparison of population pyramids illustrates significant increases in the numbers at all the high ages, for both males and females. A number of observations can be made about Figures 4.2 and 4.3. For example, in common with other countries, the very elderly population is more heavily weighted towards females at all ages. Proportional growth in numbers generally increased with age for both males and females. Furthermore, proportional growth in the numbers of males up to age 103 over the period 1996 to 2012 significantly exceeded that over the period 1981 to 1996. Growth in the numbers of females above age 100 was higher in the period from 1981 to 1996 compared to 1996 to 2012.
Changing sex composition

Due to the different rates of growth in very elderly male and female numbers, the sex composition has changed over time. Figure 4.4 shows the ratios of the numbers of males to females in age groups 85-89, 90-99 and 100+. The lower sex ratios at higher ages reflect the greater longevity of women. The gap between the numbers of females and males gradually increased for birth cohorts from 1872 to 1895, resulting in a decreasing sex ratio. This was followed by a reversal of the trend which is still continuing. The reversal of the decreasing trend at all age groups for cohorts born after 1895 indicates that the cause for this was strongly cohort-related, rather than period-related. This pattern of first a decrease and then an increase in the sex ratio among the very elderly has also been observed in other developed countries and is ascribed to factors such as differences and changes in relative smoking prevalence among men and women (Thatcher, 1999a; Tomassini, 2005; Trovato, 2005).
Smoking is a significant death risk factor. According to Bongaarts (2014), the age-standardised all-cause death rate attributable to smoking for males in low mortality countries declined from the mid-1980s, after increasing substantially from the 1950s. In contrast, the smoking-related death rate for females increased from 1955 to 2010, although it was only around half the level for males at 2010. This pattern was also observed in Australia. Peto et al. (2006) showed that female death rates attributed to smoking, in particular lung cancer deaths, increased since the 1970s despite an overall decline in death rates.

Figure 4.4: Ratio of males to females at ages 85-89, 90-99 and 100+ for Australian cohorts born from 1872-1922
Source: Author’s population estimates

The recent reduction in the difference between sex ratios for the 85-89 and 90-99 age groups indicates a narrowing of the gender gap in survival to these higher ages. A greater proportion of men relative to women now survive into their nineties than was the case in earlier years. Figure 4.4 also shows that, after following a decreasing trend, the sex ratio among Australian centenarians born after around 1893 increased from a level of 0.12 to around 0.2. This is equivalent to a ratio of 5.0 females for each male and is lower than the average of 6.0 observed in Europe and Japan in 2006, and specifically, lower than that in France (7.1), England and Wales (7.5) and Japan (5.9) (Robine & Saito, 2009a), suggesting better relative survival for Australian males.
Centenarians

Despite increasing sex ratios the number of male centenarians is still significantly lower than the number of female centenarians. Table 4.2 shows the estimated number of male and female centenarians from 1981 to 2012, the ratio of males to females and the total number of centenarians per million of the total population.

<table>
<thead>
<tr>
<th>31 December</th>
<th>Males</th>
<th>Females</th>
<th>Male: Female ratio</th>
<th>Centenarians per million of total population</th>
</tr>
</thead>
<tbody>
<tr>
<td>1981</td>
<td>76</td>
<td>422</td>
<td>0.18</td>
<td>33.2</td>
</tr>
<tr>
<td>1991</td>
<td>157</td>
<td>942</td>
<td>0.17</td>
<td>63.2</td>
</tr>
<tr>
<td>2001</td>
<td>239</td>
<td>1,389</td>
<td>0.17</td>
<td>84.0</td>
</tr>
<tr>
<td>2011</td>
<td>468</td>
<td>2,550</td>
<td>0.18</td>
<td>134.0</td>
</tr>
<tr>
<td>2012</td>
<td>538</td>
<td>2,850</td>
<td>0.19</td>
<td>147.8</td>
</tr>
</tbody>
</table>

Source: Author’s population estimates

Over the three decades from 1981 to 2012 centenarian numbers in Australia increased at an average rate of 6.4% per year, compared to the average growth rate of the total population of 1.4% per year. The number of centenarians in Australia has roughly doubled every ten years since the 1970s. This is in line with many other developed countries, although in some countries the exponential increase started as early as the 1950s (Kannisto et al., 1994; Robine & Saito, 2009a; Thatcher, 1999a; Vaupel, 2001; Vaupel & Jeune, 1995). Growth rates in the number of female centenarians increased rapidly from around 1976 and males from 1999, following a period of almost no growth in male numbers from 1990. From 1996 to 2006 the number of Australian males increased 2.1 times and females 1.6 times, similar to the average in Europe of 2.0 for both males and females, but lower than the levels of 3.0 and 4.5 for males and females respectively in Japan (Robine & Saito, 2009a).

The number of centenarians increased significantly from a low of 14.5 per million of the total population in 1967 to 147.8 in 2012. In 2004 the number of centenarians per million of the total population in Australia was 93, lower than the average of 131 for 32 developed
countries, and especially low compared to countries such as France (178), Japan (169), the US (154), England and Wales (138), Sweden (130) and Canada (122) (Rau et al., 2008).

In Australia, the number of male centenarians as well as total centenarians per million of the total population decreased between 1951 and 1971, a period during which many other countries started to experience exponential growth in centenarian numbers. This tendency for centenarian numbers to decrease is consistent with that in some other countries and reflects the implementation of formal birth and death registration systems (Jeune & Skytthe, 2001; Robine, 2011; Robine & Saito, 2009a). Registrar’s offices in the various states and territories took over responsibility for birth and death registrations between 1838 and 1870, resulting in the increasing reliability of centenarian statistics in the decades up to around 1970. The number of centenarians per million of the total population in Australia on 1 January 1960 of 17.2 per the HMD also seems implausibly high compared to the number in other low-mortality countries, for example, 4.1 for Denmark, 8.1 for France and 5.4 for the Netherlands (Kannisto, 1994; Rau et al., 2008), suggesting problems with data accuracy before the 1970s.

**Centenarians in the Australian states**

At 31 December 2012, 35% of Australian centenarians were in NSW, 26% in Vic, 17% in Qld, 9% in each of SA and WA and 4% in Tas-NT-ACT. Growth rates in centenarian numbers varied slightly between the states so that the proportions of centenarians in NSW, Vic and SA decreased while proportions in Qld and WA increased over the period from 1981 to 2012.

Table 4.3 shows male and female centenarians per million of the total male and female populations in 1981 and 2012 by state. In 1981 there were only 33.2 centenarians per million of the total Australian population (Table 4.2). By 2012, this number has increased to 147.8. In 2012 the proportions of centenarians relative to total populations were the highest in SA and NSW, with 191.5 and 160.8 respectively, compared to only 115.2 in Tas-NT-ACT. Because these are proportions, they are also impacted by demographic factors such as births and migration at all ages. Low birth rates as well as low net interstate and overseas migration for SA contributed to the relatively higher proportion of centenarians compared to the other states. In comparison, Qld’s high net migration over the last few decades and relatively
higher birth rates (ABS, 2008) contributed to the comparatively low proportion of centenarians.

Table 4.3: Centenarians per million of the total population in 1981 and 2012, by sex and state

<table>
<thead>
<tr>
<th></th>
<th>NSW</th>
<th>Vic</th>
<th>Qld</th>
<th>SA</th>
<th>WA</th>
<th>Tas-NT-ACT</th>
<th>AUS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1981</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Males</td>
<td>8.8</td>
<td>11.2</td>
<td>13.3</td>
<td>11.5</td>
<td>8.7</td>
<td>5.1</td>
<td>10.2</td>
</tr>
<tr>
<td>Females</td>
<td>55.4</td>
<td>55.6</td>
<td>57.1</td>
<td>63.8</td>
<td>54.1</td>
<td>52.0</td>
<td>56.2</td>
</tr>
<tr>
<td>Persons</td>
<td>32.2</td>
<td>33.6</td>
<td>35.1</td>
<td>37.8</td>
<td>31.1</td>
<td>28.4</td>
<td>33.2</td>
</tr>
</tbody>
</table>

|        |      |      |      |      |      |            |     |
| 2012   |      |      |      |      |      |            |     |
| Males  | 50.9 | 49.7 | 41.1 | 60.3 | 37.3 | 36.7       | 47.1|
| Females| 269.1| 257.7| 210.9| 320.4| 201.6| 195.0      | 247.5|
| Persons| 160.8| 154.8| 126.2| 191.5| 118.6| 115.2      | 147.8|

Source: Author’s estimates

Up to around 1999, female centenarian numbers in NSW, Vic, Qld and SA increased faster than male numbers, so that by the end of the century there were only between 12 and 14 males for every 100 females. From the start of the 21st century, however, this trend was reversed and by 2012 the ratio was up to 19. During the 1990s the ratios of males to females in WA and the combined Tas-NT-ACT were significantly higher than in the larger states, but by 2012 the ratios were very similar in all the states.

Not only are more people reaching the milestone of a 100th birthday, but more are also living longer beyond age 100. Figure 4.5 shows the estimated number of males and females at single ages 100 and older in 1981, 1996 and 2012 in each state. In 1981 there were an estimated 26 semi-supercentenarians (ages 105+), 24 females and 2 males. By 1996 the number of semi-supercentenarians had increased to 63 females and 7 males, a total of 70. By 2012 this number more than doubled again to 150, or 135 females and 15 males. Similar pictures emerge for all the states, albeit at different scales, with higher numbers at all ages 100+.
Figure 4.5: Estimated numbers by single age in each state in 1981, 1996 and 2012
Source: Author’s estimates
4.3 Demographic drivers of very elderly population growth

The extent to which the growth in the numbers of nonagenarians and centenarians in Australia over the last four decades were due to increases in births, survival and net migration is presented in this section. Variation of survival improvement between cohorts, ages and between males and females in the different Australian states are also shown. Section 4.3.1 sets out the decomposition method used and section 4.3.2 sets out the result of this analysis.

4.3.1 Decomposition method

The decomposition method described in section 2.4.1 was used to split total growth in numbers into growth in births, survival improvements and net migration increases. For this purpose, the very elderly population was divided into two groups, namely ages 90-99 (nonagenarians) and 100+ (centenarians) with the decomposition explaining the demographic drivers of their growth from 1981 to 2012. An initial year of 1981 had to be chosen because of data constraints relating to the earliest cohorts for which cohort life tables could be constructed. The increase in the numbers of males and females aged 90-99 and 100+ between two points in time (time t-h and time t) was decomposed into the factors:

- births;
- survival between birth and age 65;
- survival between ages 65 and 85;
- survival between ages 85 and 90 or 85 and 100;
- survival beyond ages 90 or 100;
- the impact of net migration (including survival of migrants) on these cohorts.

Ages 65 and 85 have been chosen as these are typically used as thresholds for entering the ‘elderly’ and ‘very elderly’ population groups respectively.

By expanding equation 2.9, the number of people aged between x and x+9 on 31 December of year t can be expressed as:

\[
\sum_{i=0}^{9} P_{x+i,t} = \sum_{i=0}^{9} B_{t-x-i} 65 p_{65}^{c-i} 20 p_{65}^{c-i} x-85 p_{85}^{c-i} p_{x}^{c-i} + \text{net migration}
\]

where \( c = t - x \), the birth year of the youngest cohort, and x is 90 or 100.

The net migration ratio was derived as a residual:

\[
x+i m_{c-i} = \frac{\sum_{i=0}^{9} P_{x+i,t}}{\sum_{i=0}^{9} B_{t-x-i} 65 p_{65}^{c-i} 20 p_{65}^{c-i} x-85 p_{85}^{c-i} p_{x}^{c-i}}
\]
Similarly, the number of people aged between \( x \) and \( x+9 \), \( h \) years earlier, i.e. on 31 December of year \( t-h \) is:
\[
\sum_{i=0}^{9} P_{x+i,t-h} = \sum_{i=0}^{9} B_{t-h-x-i} g_{t-h-x-i} p_{x}^{c-x-h-i} q_{x}^{c-x-h-i} p_{x}^{c-x-h-i} x_{x+i}^{c-x-h-i} p_{x}^{c-x-h-i} (4.3)
\]

The increase factor in numbers aged \( x \) to \( x+9 \) from 31 December year \( t-h \) to 31 December year \( t \) is \( \frac{(4.1)}{(4.3)} \), so that the total factor increase can be written as the product of factors relating to increases in birth numbers, survival ratios and migration ratios as follows:
\[
\frac{\sum_{i=0}^{9} P_{x+i,t}}{\sum_{i=0}^{9} P_{x+i,t-h}} = \frac{\sum_{i=0}^{9} B_{t-x-i}}{\sum_{i=0}^{9} B_{t-h-x-i}} x_{x}^{c-x-h-i} x_{x+h}^{c-x-h-i} x_{x+i}^{c-x-h-i} (4.4)
\]

Using age-cohort deaths data, age-cohort death rates were calculated in line with HMD methods as follows (Wilmoth et al., 2007):
\[
m_{x,t}^{c} = \frac{D_{x,t}^{L} + D_{x,t+1}^{U}}{P_{x,t}^{L} + P_{x,t+1}^{U}} (4.5)
\]

where \( m_{x,t}^{c} \) is the central death rate at age \( x \) in year \( t \) for cohort \( c \), and the superscripts \( L \) and \( U \) refer to the lower and upper age-period triangles respectively. \( P_{x,t}^{L} \) is the population aged \( x \) last birthday at 31 December of year \( t \). Cohort death rates were then converted into cohort probabilities of dying:
\[
q_{x,t}^{c} = \frac{m_{x,t}^{c}}{1 + (1 - a_{x,t}^{c}) m_{x,t}^{c}} (4.6)
\]

with
\[
a_{x,t}^{c} = \frac{D_{x,t}^{L} + D_{x,t+1}^{U}}{D_{x,t}^{L} + D_{x,t+1}^{U}} (4.7)
\]

The life table population is then derived from:
\[
l_{x+1} = l_{x}(1 - q_{x}) (4.8)
\]

with \( l_{0} \) set to 100,000.

National cohort life tables were constructed using data from the Human Mortality Database, supplemented by the estimates derived from deaths data (chapter 3), and by historical life tables published by the ABS (ABS, 2008). State-level cohort life tables were constructed using the state-level deaths and population estimates derived by the author using the methods described in chapter 4. National-level life table values were used to supplement early lifetime histories for the states where these could not be constructed from available data. Complete
national-level cohort life tables could be constructed for cohorts born from 1872. Cohort life tables for states were constructed from age 85 for 1886 cohorts and progressively earlier ages for younger cohorts. State-level cohort survival histories at younger ages thus partly reflect national averages.

4.3.2 Demographic drivers of very elderly population growth

Nonagenarians

Between 1981 and 2012 the number of females aged 90-99 increased by a factor of 4.4, from 23,285 to 101,924. Nonagenarian males increased by a factor of 6.0, from 7,295 to 43,609. These increases were due to higher birth numbers for the 1913-1922 cohorts compared to the 1882-1891 cohorts, higher survival probabilities experienced by the younger group of cohorts, and due to the group born during 1913-1922 being supplemented to a greater extent by international migration. Out of 100,000 males born in 1882-1891, 3,000 survived to age 90, compared to 10,500 per 100,000 males born in 1913-1922. While the probability of survival was higher for females, the improvement has been smaller, from 8,600 to 24,000 per 100,000.

The factor increases in births, cohort survival between birth and age 65, age 65 to 85, age 85 to 90 and beyond age 90, and net migration, contributing to the growth in nonagenarian population from 1981 to 2012 are shown in Figure 4.6. The total increase in the numbers aged 90-99 from 31 December 1981 to 31 December 2012 is the product of the factor increases in cohort birth numbers, survival ratios and net migration ratios shown in Figure 4.6.

The increases in birth numbers, survival between ages 85 and 90, and survival between birth and age 65 contributed roughly similar amounts to the increase in numbers aged 90-99 over the 31 year period. Survival beyond age 90 also improved and the impact of migration increased, but to a smaller extent than the other components. By far the most significant factor driving the increasing numbers of nonagenarians has been an improvement in survival between ages 65 and 85, especially for males. The greater total increase in the number of male nonagenarians compared to females is due to greater improvements in survival above age 65.
Figure 4.6: Factor increases in births, survival and net migration, explaining growth in population aged 90-99 from 1981-2012, by sex
Source: ABS (birth numbers) and author’s calculations

The rate of improvement in male survival only started to exceed that of females for cohorts born after 1908. Birth numbers for both males and females started to increase from around 1903. Males born during the period 1913-1922 experienced significantly greater improvements in their survival probabilities between ages 65 and 85 compared to older cohorts. In comparison, the average rate of improvement in survival between ages 65 and 85 for female cohorts born during 1913-1922 was lower than for older cohorts and significantly lower than for males.

**Centenarians**

The demographic drivers of the growth of centenarian numbers exhibit a number of differences compared with those affecting nonagenarians. Between 1981 and 2012, the number of male centenarians (100+) in Australia increased by a factor of 7.0, with numbers increasing from 76 to 538, while the number of female centenarians increased by a factor of 6.8 from 421 to 2,850. Centenarians in 1981 were born during 1872-1881, and centenarians in 2012 were born during 1903-1912. Figure 5.7 compares the demographic components of the increases between these two groups.
Figure 4.7: Factor increases in births, survival and net migration, explaining growth in population aged 100+ from 1981-2012, by sex
Source: ABS (birth numbers) and author’s calculations

There were almost 60% more births during the years 1903-1912 compared to the period 1872-1881, and the younger group had a 30% (females) or 29% (males) greater probability of surviving to age 65. Survival from age 65 to 85 improved 46% for males and 61% for females. However, the greatest component of growth in the number of centenarians was improved survival beyond age 85, with factor increases of 2.3 and 2.5 for males and females respectively. The probability of surviving beyond age 100 increased by only 4% for males and decreased by 13% for females, while net migration decreased slightly.

The significant improvements in survival to the very high ages warrants further investigation. Specifically, it would be interesting to understand trends across cohorts. Figure 4.8 shows how survival probabilities from birth to age 90 and then further to age 100 have improved for cohorts born from 1872 to 1912. For both males and females the probabilities of survival improved gradually and consistently, with each younger cohort experiencing greater survival, both to age 90 and from age 90 to age 100.
Males born in 1872 had a 0.025 probability of surviving to age 90 in 1962, and then, once attaining age 90, a 0.023 probability of surviving a further ten years to age 100 in 1972. For males born in 1912 the probability of surviving to age 90 in 2002 had increased more than three-fold to 0.077, and those attaining age 90 had a 0.045 probability of reaching age 100. Between the 1872 cohorts and the 1912 cohorts the probabilities of survival from birth to age 100 increased 6.1-fold for males and 8.3-fold for females.

**International comparisons**

In order to understand the Australian experience in the international context, it is useful to consider how these findings compare with those for other countries. Thatcher (1999a) quantified the contribution of different demographic components to increasing numbers of centenarians in England and Wales from 1951 to 1996, comparing the experience of the 1850 cohort with that of the 1895 cohort. Dini and Goldring (2008) updated this analysis to 2001 and also compared the experience split between two periods, from 1951 to 1976 and from 1976 to 2001. Table 4.4 shows the results for England and Wales over the period 1976 to 2001 and those for Australia over the same period.
Table 4.4: Factor increases in births, survival and net migration, explaining growth in population aged 100+ from 1976-2001, comparing Australia and England and Wales

<table>
<thead>
<tr>
<th></th>
<th>Birth numbers 1875 to 1900</th>
<th>Survival from birth to age 80</th>
<th>Survival from age 80 to 100</th>
<th>Ratio of survival beyond 100</th>
<th>Migration / other reasons</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Australia</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Males</td>
<td>1.5</td>
<td>1.3</td>
<td>2.4</td>
<td>0.8</td>
<td>0.8</td>
<td>3.3</td>
</tr>
<tr>
<td>Females</td>
<td>1.5</td>
<td>1.5</td>
<td>2.7</td>
<td>1.2</td>
<td>0.9</td>
<td>6.7</td>
</tr>
<tr>
<td><strong>England and Wales</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Males</td>
<td>1.1</td>
<td>1.2</td>
<td>2.8</td>
<td>1.1</td>
<td>0.7</td>
<td>2.8</td>
</tr>
<tr>
<td>Females</td>
<td>1.1</td>
<td>1.5</td>
<td>2.4</td>
<td>1.1</td>
<td>0.9</td>
<td>3.9</td>
</tr>
</tbody>
</table>

Source: Author's calculations and Dini and Goldring (2008)

This comparison shows that the number of centenarians increased more in Australia than in England and Wales, mainly due to a greater increase in birth numbers from 1875 to 1900. Australian males experienced a greater improvement in survival ratios up to age 80 than males in England and Wales, but a lower improvement beyond age 80. Australian females experienced greater improvements in survival ratios from age 80 to 100 and similar improvements at other ages. Vaupel and Jeune (1995) applied a similar decomposition method to explain the increase in numbers of people attaining age 100 from 1970 to 1989 in Denmark, Norway and Sweden, finding that improved survival from age 80 to age 100 accounted for two-thirds of the total rates of increase.

Improvement in survival, especially at very elderly ages, was one of the major contributors to centenarian growth over the last few decades. Robine and Saito (2009b) compared the centenarian rate for a large number of European countries in 2006. The centenarian rate is defined as the ratio of the number of people aged 100 in a particular calendar year per 10,000 of people who were aged 60, forty calendar years earlier, denoted by CR[60] (Robine & Paccaud, 2005). Centenarian rates varied significantly between countries and the sexes, from 156.5 for females and 27.9 for males in France, to 16.7 for females and 4.8 for males in Bulgaria (Robine & Saito, 2009b). Centenarian rates of 177.2 for Australian females in 2006 exceeded the highest measured in Europe (France, 156.5) whilst the rate for Australian males
of 41.4 was exceeded only by that in Lithuania, 46.8. Centenarian rates in Japan were significantly higher, at 259.4 for females and 49.3 for males (Robine & Saito, 2009a).

**Increasing centenarian numbers at a state level**

Table 4.5 shows estimated numbers of male and female centenarians in each state in 1981 and 2012 and the factor by which the numbers increased. Across Australia centenarian numbers increased almost 7-fold. In NSW, SA, WA and Tas-NT-ACT males increased more than females, albeit from smaller bases. Female centenarians in Qld increased more than in the other states and male centenarians less. Centenarian numbers in WA increased more than in any other state.

**Table 4.5: Growth in numbers of centenarians from 1981-2012, by sex and state**

<table>
<thead>
<tr>
<th></th>
<th>Males</th>
<th></th>
<th></th>
<th>Females</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number aged 100+ in 1981</td>
<td>Number aged 100+ in 2012</td>
<td>Factor increase</td>
<td>Number aged 100+ in 1981</td>
<td>Number aged 100+ in 2012</td>
<td>Factor increase</td>
</tr>
<tr>
<td>NSW</td>
<td>23</td>
<td>186</td>
<td>8.1</td>
<td>145</td>
<td>997</td>
<td>6.9</td>
</tr>
<tr>
<td>Vic</td>
<td>22</td>
<td>140</td>
<td>6.4</td>
<td>110</td>
<td>740</td>
<td>6.7</td>
</tr>
<tr>
<td>Qld</td>
<td>16</td>
<td>95</td>
<td>5.9</td>
<td>68</td>
<td>488</td>
<td>7.2</td>
</tr>
<tr>
<td>SA</td>
<td>8</td>
<td>50</td>
<td>6.6</td>
<td>42</td>
<td>269</td>
<td>6.5</td>
</tr>
<tr>
<td>WA</td>
<td>6</td>
<td>47</td>
<td>8.1</td>
<td>35</td>
<td>247</td>
<td>7.0</td>
</tr>
<tr>
<td>Tas-NT-ACT</td>
<td>2</td>
<td>21</td>
<td>10.4</td>
<td>20</td>
<td>109</td>
<td>5.4</td>
</tr>
<tr>
<td>Australia</td>
<td>76</td>
<td>538</td>
<td>7.0</td>
<td>421</td>
<td>2,849</td>
<td>6.8</td>
</tr>
</tbody>
</table>

Source: Author’s calculations

Figure 4.9 shows the extent to which growth in births, survival and net migration increased from 1981 to 2012 for males and females in each state. The product of the factor increases in births, survival and net migration shown in Figure 4.9 make up the total factor increases in Table 4.5. ‘Births’ represents the growth in birth numbers from 1872-1881 to 1903-1912. On average across Australia, births increased 60%, but this varied significantly between the states. In Vic and SA births increased by less than 20%, in Tas-NT-ACT combined by 60%, in NSW by 80%, while births in Qld more than doubled. WA, however, experienced an 8.6-fold increase in birth numbers, from 9,000 male and female births in 1872-1881 to 77,000
births in 1903-1912. The 1903-1912 cohorts were born after the discovery of gold in WA in the 1890s, which precipitated a significant influx of people from overseas as well as the other states (Rowland, 1979). The discovery of gold in WA followed the much earlier gold rushes of the 1850s and 1860s in NSW, Vic and Qld and these states were experiencing depression and high unemployment when gold was discovered in WA, which further encouraged the flow of people to WA (West, 2010). Earlier gold rushes in NSW, Vic and Qld caused similar influxes of people and spikes in birth rates. The patterns observed are thus a function of the period analysed. If data were available from an earlier date, it is expected that similar patterns would have been observed in the other states following their respective gold rushes.

Similar to Qld’s experience in the 1860s, WA’s net interstate and overseas migration rate spiked in the 1890s to a level never to be reached again (ABS, 2008). As a result, the WA cohorts born in 1903-1912 were supplemented to a significantly smaller extent by net migration compared to the 1872-1881 cohorts. This is clear from the low factor increases for net migration in WA for both males and females in Figure 4.9. Changes in net interstate and overseas migration varied in the other states. On average across Australia, the impact of net overseas migration on centenarian populations in 1981 and 2012 was very similar, with factor increases of 0.97 and 0.95 for males and females respectively, indicating a slightly smaller contribution to the 2012 centenarian population. Net migration for the states, which reflects not only overseas but also interstate migration, supplemented the 2012 centenarian populations in NSW and Qld to a smaller extent than the 1981 centenarian populations, and to a larger extent in Vic, SA and Tas-NT-ACT. These factor changes were similar for males and females.
Figure 4.9: Factor increases in births, survival and net migration, explaining growth in population aged 100+ from 1981-2012, by sex and state
Source: Author’s calculations

Improvements in survival from age 85 to 100 had a significant impact on the growth in centenarian numbers from 1981 to 2012 in all the states, with survival rates more than doubling for both males and females. With the exception of WA, where births increased significantly, this was the most significant driver of growth. Survival from age 85 to 100 improved the most for males in WA and the least for females in Tas-NT-ACT. Survival
beyond age 100 improved for males in NSW, Vic, SA and the Tas-NT-ACT, but decreased for males in Qld and WA and for females in all the states. Survival improvement at the very elderly ages (85+) exceeded those at elderly (65+) and younger ages. Cohorts born in 1903-1912 had a 30% greater chance to survive to age 65 compared to the 1872-1881 cohorts, and a 50% (males) or 60% (females) higher chance to survive to age 85.

4.4 Conclusion

A detailed analysis of the growth in numbers and changing composition by age and sex of the very elderly in Australia, based on estimates derived from death data rather than census data, has not previously been completed for Australia. Until now, very little was known about the growth and demographic drivers of very elderly numbers, especially centenarian numbers, at a state level in Australia. In this chapter the exponential growth in very elderly numbers in Australia since the 1980s was illustrated, both in absolute terms and relative to the total population. This contributes to our understanding of the trends in very elderly numbers and composition by age and sex at both a national level and state level in Australia, and how this compares with other low-mortality countries. During the last three decades the composition of the very elderly age group has changed to be more heavily weighted towards older people. Growth rates in the numbers of very elderly females exceeded that of males until 1985, resulting in a gradually decreasing sex ratio. However, from the mid-1980s, growth rates in the number of very elderly males accelerated and exceeded that of females, resulting in an increasing trend in sex ratios which is still continuing. In all the states very elderly numbers increased over the three decades showing similar trends, although growth rates were slightly higher in WA and Qld, and lower in SA, Vic and NSW. SA has the highest proportion of centenarians while Tas-NT-ACT has the lowest.

This chapter also made new contributions to our understanding of the contribution of different demographic components to this growth. It was found that, consistent with other low-mortality countries, mortality decline at the very elderly ages (85-100) has been the main driver of increasing numbers of very elderly people in Australia. Survival improvement above age 65 contributed more to male nonagenarian growth compared to females. However, the contribution of survival improvement at elderly and very ages to growth in centenarian numbers was higher for females than for males. This greater contribution from survival improvement at ages 85-100 was seen across the states, except in WA, where by far the
The largest driver of growing centenarian numbers was the significant increase in births following the influx of people into this state due to the gold rush in the 1890s. This was, however, partly offset by lower net migration rates in later years. The number of centenarians increased most in WA from 1981-2012, by a factor of 7.1, followed by Qld And NSW with factor increases of 7.0 each. Survival improvement at younger ages also contributed to higher centenarian numbers in all the states, but not to the same extent as at the very high ages. Survival probabilities have been improving for a number of decades and show no signs of reaching a plateau. Increasing sizes of birth cohorts also played a substantial role, although to a varying extent in the different states. Contrary to expectations, net international migration has not contributed much to increasing numbers of nonagenarians and centenarians because migration ratios did not increase significantly between the cohorts considered. However, the selective effect of migration may have contributed to relatively high survival rates. Survival rates for very elderly males improved significantly in the last 15 years, exceeding those for females. If this trend, which is strongly cohort-driven, continues, we can expect rapidly increasing numbers of males reaching very old ages. This has implications for health and care needs. While a smaller proportion of very elderly men currently suffer from severe and profound disabilities, further research is needed into whether the improved longevity of men has been accompanied by a deteriorating ratio of those in good health.

Continued growth in births, especially after World War II, and a continuation of past trends of survival improvement mean that centenarian numbers are likely to continue increasing rapidly in coming decades. Given the high prevalence of disability at the very high age groups and the lack of any recent declines in age-specific prevalence rates (AIHW, 2000), the centenarian population is especially important for health and aged care service planning. The availability of accurate centenarian estimates at a state level should allow the relevant authorities to provide for the needs of this population group.
Chapter 5. Analysis of adult mortality patterns and trends in Australia

5.1 Introduction

Understanding past patterns and trends of mortality change is essential for forecasting likely future changes. Analyses of mortality changes in Australia typically use Australian Bureau of Statistics (ABS) data and focus on changes in life expectancy at birth or causes of death. Furthermore, studies of temporal changes in age-specific death rates generally consider only aggregated rates for ages 75+ or 85+ (AIHW, 2006). ABS death rates for very elderly ages are, however, problematic. The analyses presented in Chapter 3 revealed that ABS estimates at ages 95+ are too high, and a comparison of ABS death rates with those based on estimates derived from death counts confirm that ABS death rates for these ages are too low. Life table values created using population estimates derived from death counts, as described in earlier chapters, allow more detailed and accurate analyses of temporal changes in mortality patterns at the highest ages for Australia.

Mortality changes in many developed countries have been studied extensively. Some studies focussed on temporal changes in age-specific death rates or probabilities of death (Kannisto, 1994, 1996; Kannisto et al., 1994; Rau et al., 2008), while others considered changes in certain features of death frequency distributions or survival curves (Cheung & Robine, 2007; Kannisto, 2000, 2001; Robine, 2001). Analysing the age profile of death rates allows an understanding of how the risk of death varies between ages and how improvements in death rates vary between ages and over time. Death frequency distributions reflect how total life table deaths in any year are distributed in terms of age at death or life span. Features of death frequency distributions typically studied include the modal age at death (M), which is the single year age interval at which the highest concentration of deaths occur (Kannisto, 2000), and mortality compression. Compression measures describe the variability of life spans or how concentrated or spread out deaths are across ages (Kannisto, 2000). Empirical studies have found similar trends in many developed countries. Declines in death rates at younger ages during the first half of the twentieth century have caused death frequencies to become more concentrated around the modal age. When death rates at older ages started to decline in the second half of the twentieth century this resulted in both increasing modal ages and death frequencies becoming more compressed. More recently, the pace of compression and modal age increase started slowing down or, in some countries, stopped. It is not, however, known whether the experience in Australia has been similar.
Both death rates and death frequencies are life table functions, derived from the same data. One would thus expect relationships between their respective patterns of change. Such relationships have not been studied in depth, however. Mortality studies typically cover either changes in death rates or death frequency distributions. Only a few studies considered interactions between these (Canudas-Romo, 2008, 2010; Thatcher et al., 2010; Wilmoth & Horiuchi, 1999), and the impact of changes in death rates at specific ages on various features of the death frequency distribution and vice versa is not well understood.

In the first instance, this chapter seeks to develop insights into the historical development of Australian mortality patterns, with a focus on elderly and very elderly ages. This covers changes in both the age profile of death rates and death frequency distributions. The chapter then presents a new method for linking changes in the level and slope of the age profile of adult death rates with changes in the shape of the death frequency distribution.

The rest of this chapter is organised as follows. In section 5.2 the measures used to describe each of the death frequency distribution and the age profile of death rates are explained. A new decomposition method is proposed for analysing how changes in the parameters of a function describing the age profile of death rates contributed to changes in the features of death frequency distributions. Section 5.3 sets out historical changes in Australian age-specific death rates and death frequency distributions, followed by the results of the decomposition.

5.2 Data and methods

The analyses in this chapter make use of the period life table functions, central death rates by age \(m_x\) and death frequencies by age \(d_x\) for Australia over the period from 1921 to 2012. Unsmoothed period life tables from 1921 to 1970 were obtained from the Human Mortality Database. Life tables from 1971 were created from deaths and the author’s population estimates derived from death counts for ages 85-110 (see Chapter 3) and Estimated Resident Populations (ERPs) for ages below 85 obtained from the ABS (ABS, 2013a).

In earlier years the concentration of deaths in infancy exceeded that at any other age. The modal age studied here refers specifically to the modal age at death at later ages. Compression measures include the concentration of deaths either at the modal age, denoted
or above the modal age or within certain age ranges (Kannisto, 2000). These include, for example, the interquartile range (IQR), defined as the age span of the middle 50% of deaths across all ages (Wilmoth & Horiuchi, 1999); the standard deviation of age at death above the modal age, denoted as SD(M+) (Kannisto, 2000), and the shortest age interval in which a given proportion of deaths (e.g. 25% or 50%) takes place, denoted as C25 or C50 (Kannisto, 2000). Kannisto (2000) showed that SD(M+) and life expectancy at the mode (e_M) are strongly correlated so that this can also be used as a compression measure.

Because life table deaths across all life spans add up to 100,000, all the compression measures are impacted by death frequencies across the age range. Lower d_x at younger ages must cause higher d_x at older ages. Measures such as the IQR, favoured by Wilmoth and Horiuchi (1999), and the C measures suggested by Kannisto (2000) work well for describing how spread out or concentrated life spans are across all ages. However, given the focus of this thesis on mortality at very elderly ages, the measures d_M and SD(M+) are considered most appropriate. The concentration of deaths at the modal age, d_M, is regarded as suitable because it measures compression directly rather than indirectly, as the width of an age range within which a given proportion of deaths take place. SD(M+) reflects the dispersion of life spans beyond the modal age only and was calculated as follows:

$$SD(M+) = \sqrt{\frac{\sum_{M}^M(x-M)^2d_x}{\sum_{M}M^2d_x}}$$ (5.1)

Death frequencies from unsmoothed life tables may require smoothing before a unique modal age can be identified. Various approaches to smoothing have been applied in the literature, including fitting a normal curve (Cheung & Robine, 2007; Kannisto, 2000) or a second order polynomial to death frequencies at ages around the modal age (Cheung, Robine, & Caselli, 2008; Kannisto, 2001). A second order polynomial fitted to Australian death frequencies (d_x) at ages [M-10:M+10] did not yield a very good fit (R^2 of 0.9193) and seemed to underestimate modal ages. Given that death rates and death frequencies are derived from the same data, an alternative approach is to smooth the death rates and then derive death frequency measures from these (Canudas-Romo, 2008; Thatcher et al., 2010). This is the approach taken for the purpose of this analysis.

A number of parametric functions have been found to accurately describe observed adult death rates (section 2.5.1), of which the Gompertz function is one of the most well-known
According to the Gompertz function, adult death rates increase exponentially with increasing age. A number of researchers have, however, found that the rate of increase in death rates decelerates with increasing age at very high ages and that a logistic function describes this pattern more accurately (Horiuchi & Wilmoth, 1998; Thatcher et al., 1998; Vaupel, 1997b, 2010). A simplified, two-parameter logistic function, referred to as the ‘Kannisto model’ has been found appropriate for modelling mortality at very elderly ages (Bongaarts, 2005; Thatcher, 1999b; Thatcher et al., 1998). In this investigation, Gompertz’s exponential and Kannisto’s logistic functions were fitted to unsmoothed death rates at ages 40-101 for Australian males and females in the years 1921-2012. Death rates above age 101 were too volatile to be included. Both Ordinary Least Squares (OLS) and Weighted Ordinary Least Squares (WOLS) fitting methods were used.

According to the Gompertz function, death rates increase exponentially with age:

$$ m_x = \alpha e^{\beta x} \quad (5.2) $$

By taking the natural logarithm on both sides, this can be transformed to a linear function:

$$ \ln(m_x) = \ln(\alpha) + \beta x \quad (5.3) $$

where $\alpha$ is the initial level of mortality and $\beta$, the slope of curve, is the rate of increase in mortality with age, or the rate of senescence (Curtsinger et al., 2005).

According to Kannisto’s logistic function, death rates increase at a decreasing rate with increasing age:

$$ m_x = \frac{\alpha e^{\beta x}}{1 + \alpha e^{\beta x}} \quad (5.4) $$

By taking the logit on both sides, this can be transformed into a linear function:

$$ \text{logit}(m_x) = \ln(\alpha) + \beta x \quad (5.5) $$

All four combinations of the two functions and the two fitting methods produced $R^2$ of over 0.99 for ages 40+ and $R^2$ of over 0.97 for ages 80+. The Gompertz function fitted by the OLS method produced the best fit followed by the Gompertz function fitted by WOLS. The logistic function fitted by OLS produced the worst fit. Which model provides the better fit at ages exceeding 101 could not be reliably tested due to the volatility of these observed death rates.
According to Canudas-Romo (2008), the modal age at death in year $t$ can be derived from the parameters of the Gompertz function as follows:

$$M_t = \frac{\ln(\beta_t) - \ln(\alpha_t)}{\beta_t}$$

(5.6)

Where $\alpha_t$ and $\beta_t$ are the level and slope parameters of equation 5.2 at time $t$. The concentration of deaths at the modal age can be derived as (Canudas-Romo, 2008):

$$d(M_t) = \beta_t e^{\left(\frac{\alpha_t}{\beta_t} - 1\right)}$$

(5.7)

In this study, equations 5.6 and 5.7 were used to perform decompositions to determine how changes in each of the modal age and concentration of deaths at the modal age may be attributed to changes in the level ($\alpha$) and slope ($\beta$) of the Gompertz function. Using equation 5.6, the change in modal age between two dates (times $t$ and $t+n$) can be written as:

$$M_{t+n} - M_t = \frac{\ln(\beta_{t+n}) - \ln(\alpha_{t+n})}{\beta_{t+n}} - \frac{\ln(\beta_t) - \ln(\alpha_t)}{\beta_t}$$

(5.8)

By adding and deducting a term using $\alpha_{t+n}$ and $\beta_t$ this becomes

$$M_{t+n} - M_t = \frac{\ln(\beta_{t+n}) - \ln(\alpha_{t+n})}{\beta_{t+n}} - \frac{\ln(\beta_t) - \ln(\alpha_t)}{\beta_t} + \frac{\ln(\beta_t) - \ln(\alpha_{t+n})}{\beta_t} - \frac{\ln(\beta_t) - \ln(\alpha_t)}{\beta_t}$$

(5.9)

The change in the modal age from time $t$ to $t+n$ is thus expressed as the sum of:

- the change in the modal age due to a change in the level of the age profile of death rates or $\frac{\ln(\beta_{t+n}) - \ln(\alpha_{t+n})}{\beta_{t+n}} - \frac{\ln(\beta_t) - \ln(\alpha_t)}{\beta_t}$, plus

- the change in modal age due to a change in the slope of the curve describing the age profile of death rates, or $\frac{\ln(\beta_t) - \ln(\alpha_{t+n})}{\beta_t} - \frac{\ln(\beta_t) - \ln(\alpha_t)}{\beta_t}$

Similarly, using equation 5.7 the change in concentration of deaths at the modal age can be written as:

$$d(M_{t+n}) - d(M_t) = \beta_{t+n} e^{\left(\frac{\alpha_{t+n}}{\beta_t} - 1\right)} - \beta_t e^{\left(\frac{\alpha_t}{\beta_t} - 1\right)}$$

(5.10)

By adding and deducting a term using $\alpha_{t+n}$ and $\beta_t$ this becomes

$$d(M_{t+n}) - d(M_t) = \beta_{t+n} e^{\left(\frac{\alpha_{t+n}}{\beta_t} - 1\right)} - \beta_t e^{\left(\frac{\alpha_t}{\beta_t} - 1\right)} + \beta_t e^{\left(\frac{\alpha_{t+n}}{\beta_t} - 1\right)} - \beta_t e^{\left(\frac{\alpha_t}{\beta_t} - 1\right)}$$

(5.11)

so that the change in the concentration at the modal age from time $t$ to time $t+n$ is expressed as the sum of:

- the change in concentration at the modal age due to change in the level of the age profile of death rates or $\beta_{t+n} e^{\left(\frac{\alpha_{t+n}}{\beta_t} - 1\right)} - \beta_t e^{\left(\frac{\alpha_t}{\beta_t} - 1\right)}$, plus

- the change in concentration at the modal age due to the change in the slope of the curve describing the age profile of death rates, or $\beta_{t+n} e^{\left(\frac{\alpha_{t+n}}{\beta_t} - 1\right)} - \beta_t e^{\left(\frac{\alpha_t}{\beta_t} - 1\right)}$
5.3 Trends and patterns in adult mortality in Australia

5.3.1 Declines in death rates by age

Similar to the experience in other developed countries, death rates for males and females in Australia reduced significantly over the last 90 years. Figure 5.1 shows how Australia’s death rates by age group at 10-yearly intervals to 2012 compared to death rates in 1922. From 1922 to 2012 death rates for ages 0-14 declined more than at any other age group, to 5% (males) and 6% (females) of their respective levels in 1922. Although the extent to which death rates declined decreased with increasing age, by 2012 death rates for females up to age 79 and for males up to age 69 were all at less than 30% of their 1922 levels. With the exception of ages 0-14 and 100+, death rates for females decreased more than those for males. Decreasing trends also started earlier for females. Deaths rates for females aged 80-89 started to decline from around the 1960s and for males from around the 1980s. By 2012 these death rates had declined to 44% (females) and 54% (males) of their respective levels in 1922.

The initial increase in death rates for ages 90-99 and 100+ is more likely an indication of an underestimation of the 1922 rates rather than a real increase. Historically, churches kept records of births, deaths and marriages in Australia and the responsibility for these registrations were taken over by registrar’s offices in 1838 in Tasmania, 1841 in Western Australia, 1842 in South Australia, 1853 in Victoria, 1856 in New South Wales and Queensland, and 1870 in the Northern Territory. People aged 90+ in 1922 were born before 1833, which was before the formalisation of birth registration systems, and it is therefore likely that ages on death certificates were less reliable. Death rates for ages 100+ are thus likely to be more reliable after the 1950s. A comparison of average death rates for ages 90-99 in Australia and France, a country with similar mortality levels and good quality vital records, suggests Australian rates were understated until the late 1950s.
Figure 5.1: Death rates in 1922-2012 relative to 1922, by sex and age range
Source: Author’s own calculations using HMD data and own estimates

The declines in Australia’s death rates for ages 80+ are consistent with findings of Robine (2006) and Wilmoth (2000) showing that in many developed countries the pace of mortality decline above age 80 accelerated from the 1950s, and especially from the 1970s. It was found that on average in European countries and Japan, the pace of mortality decline increased from about 1% per year in the 1950s to more than 2% per year in the 1990s (Kannisto et al., 1994; Vaupel, 2001). While mortality declines at ages 80+ in Australia also accelerated for females
after the 1950s and for males after the 1970s, they only declined at an average rate of around 1%.

5.3.2 Life expectancy and the modal age at death

Period life expectancy at birth is one of the most commonly quoted measures of the longevity, quality of life, health, and level of development of a nation. Period life expectancy at birth reflects the average length of life of a hypothetical cohort assumed to experience the death rates observed across all ages in a particular period (Bongaarts & Feeney, 2002). Figure 5.2 shows how changes in Australian life expectancy at birth from 1922 to 2012 compared with changes in the modal age at death, for females and males respectively. Apart from a period of little change during the 1960s, both females’ and males’ life expectancy increased consistently from 1922 to 2012. From the 1920s to 1950s increases in life expectancy were driven by declines in death rates at ages 0-44, as shown in Figure 5.1. Up to around 1970 modal ages at death increased very slowly for females but were almost unchanged for males. Modal ages started increasing only when death rates at ages above the modal age declined (Canudas-Romo, 2010). The decline in death rates at ages 80-89 from the 1950s for females and from the 1970s for males seen in Figure 5.1 is thus consistent with the timing of modal age increases shown in Figure 5.2. These death rate declines also contributed to continued increases in life expectancies. From 1972 to 2012, death rates at ages 0-14 for both males and females continued to exhibit greater declines than any other age group, followed by death rates at ages 45-79, thus driving further increases in life expectancies.

The gap between life expectancy and the modal age at death gradually decreased over the period from over 13 years in 1922 to 5.1 and 5.6 for females and males respectively in 2012. From 1944 to 1980 the gender differences in life expectancy and modal ages widened to 7.2 years and 5.6 years respectively, after which they narrowed to 4.1 and 3.6 by 2012.
Figure 5.2: Changes in modal age at death and life expectancy at birth from 1922-2012
Source: Author’s calculations using HMD data and own estimates

Because it is a summary measure which reflects death rates across all ages, changes in life expectancy over time reveal little about the relative contribution of survival improvements at different ages, or of the changing dispersion of ages at death. A study of other features and measures of mortality is essential for developing greater insight into underlying patterns of change. Changes in the age profile of death rates and death frequency distributions are discussed in more detail in the following sections.

5.3.3 Age profiles of death rates and death frequency distributions

Figure 5.3 shows the age profiles of death rates in 1922, 1952, 1982 and 2012 for females and males respectively (top graphs), as well as the corresponding death frequency distributions (bottom graphs). The age profiles of death rates show the substantial increase in the risk of death with increasing age from around age 50. The underestimation of death rates at the highest ages in 1922 and 1952 mentioned earlier is also apparent in these graphs. The temporal declines in death rates at elderly and very elderly ages after the 1950s for females and after the 1970s for males are also clear, with the whole profile appearing to move to the right.
Consistent with the patterns of decline in death rates over time, the proportions of deaths occurring at younger ages also fell over time, causing progressively greater proportions of deaths to occur at older ages. In 1922 (adult) modal ages were 78.2 for females and 74.7 for males, and 2.9% (females) and 2.8% (males) of deaths occurred at these ages. Because mortality declines from the 1920s to the 1950s occurred at ages below the modal age and declines in death rates at ages beyond the modal age were small, the modal age itself increased very little for females and almost not at all for males. The proportions of deaths around the modal age thus increased. This increase in the height of the death frequency curve with little change in the modal age from 1922 to 1952 is clear from Figure 5.3 (bottom). After 1952 the peak of the female death frequency distribution started moving towards the right (increase in modal age) while its height continued to increase. This started later and was more drastic for males. By 2012 modal ages had increased to 89.6 for females and to 86.0 for males, and the proportions of deaths at these ages were 3.9% (females) and 3.7% (males). While in 1982 only 18% of females and 6% of males lived to age 90 or beyond, by 2012 these proportions had increased to 38% and 24% respectively. This therefore also suggests an expansion of the tail of the death frequency distribution.
Figure 5.3: Age profile of death rates and death frequency distributions in 1922, 1952, 1982 and 2012, by sex
Source: Author’s calculations using HMD data and own estimates
5.3.4 Features of death frequency distributions

The death frequency distributions in Figure 5.3 show how modal ages at death and concentrations of deaths at the modal age evolved over time. In this section trends in modal ages and compression measures across the period from 1921 to 2012 are presented in more detail. Figures 5.4 and 5.5 show how changes in the modal ages at death were accompanied by changes in the concentration of deaths around the modal age and changes in the standard deviation of life spans exceeding the modal age, denoted SD(M+). For this purpose, concentration reflects the proportion of deaths occurring in the age range from M-5 to M+5. For both males and females increases in modal ages, increases in the concentration of deaths around the modal age, and decreases in SD(M+) were all strongly correlated (correlations of 96% or more). Trends in Kannisto’s (2000) C measures (C10, C25 and C50) are very similar and are thus not shown.

The shortest age interval in which a 90% of deaths occurs (C90) reduced from 73.9 to 34.9 for females and from 81.8 to 38.9 for males. This is consistent with Kannisto’s findings for low-mortality countries. Therefore, in Australia, as in other low-mortality countries, mortality is very far from being ‘fully compressed’ and rectangularity of the survival curve completed. According to Kannisto (2000), this state would be reached once C90 reduced to one year.

Three broad periods of change can be observed: from the 1920s to the early 1950s, from the early 1950s to the late 1970s and from the late 1970s to 2012. During the first period, modal ages increased only very slowly, from 78.2 to 80.2 (females) and from 74.3 to 76.2 (males). During this time the concentration of deaths around the modal ages (from M-5 to M+5) increased from around 24% to 29% (females) and from 27% to 34% (males) and SD(M+) decreased from 9.2 to 7.6 years (females) and from 9.5 to 8.3 years (males). This reflects the relatively larger declines in death rates below modal ages compared to those above. Between the 1950s and the 1970s both male and female modal ages, concentrations and SD(M+) all remained relatively unchanged, consistent with the lack of decline in death rates at most age groups shown in Figure 5.1. From the 1970s modal ages and concentrations around modal ages consistently increased while SD(M+) decreased. The death frequency distribution (Figure 5.3) thus moved to the right but also increased in height, so that its slope beyond the peak became steeper. Increasing concentrations around modal ages and decreasing SD(M+)
are consistent with mortality declines decreasing with increasing age, as seen in Figure 5.1. Mortality declines at lower ages cause greater numbers of survivors living to older ages, but when death rates at older ages do not decline to the same extent to which survivors to those ages increase, death frequencies at the older ages increase. When the life span does not expand to the same extent at which the modal age increases, the variability of life spans above the modal age decreases, or deaths between the modal age and the maximal age become more compressed.

Figure 5.4: Changes in modal age and concentration of deaths around the modal age from 1921-2012, by sex
Source: Author’s calculations using HMD data and own estimates
The greater concentration of deaths across all possible life spans is also reflected in declines in the IQR. In Australia, 50% of female deaths in 1921 occurred in the 27 years between ages 52 and 79. In 2012, 50% of deaths occurred in the 13 year span between ages 78 and 91. For males the IQR changed from 29 years (ages 47-76) in 1921 to 14 years (ages 74-88) in 2012.

5.3.5 International comparisons

The levels and trends in modal ages and compression measures in Australia are similar to those in other developed countries where modal ages for females also started to increase from around the 1950s and for males from the 1970s. Modal ages for females in low mortality countries were between 88 and 92 in 2007 and for males between 83 and 87 (Robine, 2011). Modal ages in Australia were typically lower than in Japan, France, Italy and Switzerland in 1990-95 but higher than in many other developed countries (Kannisto, 2001).

The pattern of increasing concentrations of life spans (death frequency distribution becoming more compressed) accompanying increases in modal ages was also fairly typical in other low mortality countries (Kannisto, 2000; Robine, 2011). However, in some countries, compression slowed down recently or even stopped completely as continued increases in modal ages were no longer accompanied by increasing concentrations of deaths. The pace of
compression for females in, for example, France, Italy and Switzerland slowed from around the 1960s and from the 1990s compression appears to have stopped, while modal ages continued to increase (Cheung et al., 2009; Kannisto, 2000; Ouellette & Bourbeau, 2011; Robine, 2001). Modal ages in Japan appear to have been stagnant since the early 2000s (Ouellette & Bourbeau, 2011; Robine, 2011). In countries such as Sweden, decreases in SD(M+) continued to accompany increasing modal ages (Cheung et al., 2009). While the pace of increase in concentrations of death around the modal age at death slowed in Australia during the 1950s-60s (and 1970s for males), it accelerated in the last three decades to 2012. This is contrary to the experience in some developed countries where the compression of mortality slowed or ceased. Changes in modal ages in Australia were also more strongly correlated with compression measures such as SD(M+), concentration around the mode and the C measures (correlations of 95% or more) compared to what was found to be the case for other low mortality countries (Kannisto, 2001).

Compression occurs when death rates at higher ages decline less than death rates at lower ages (Thatcher et al., 2010). One could therefore expect that death rates at very high ages declined more in countries where compression stopped compared to countries where it continued. Figure 5.6 shows the extent to which average death rates for ages 90-99 decreased between 1981 and 2011 for a selected number of countries. This confirms that for both males and females these death rates declined to a lesser extent in both Sweden and Australia, where compression continued, compared to countries where it stopped.
5.3.6 Changes in the level and slope of the age profile of death rates

In this section features of the Gompertz curve used to describe the age profile of death rates are explored. The \( \alpha \) parameter in Gompertz’s function fitted to the age profiles of adult death rates represents the level of mortality and affects death rates at all adult ages. The \( \beta \) parameter represents the slope of the curve or the rate at which death rates increase with age. Figure 5.7 illustrates how these parameters changed over time. Decreases in the level of mortality and increases in the slope appear to be mirror images. Following a period of little change during the 1960s and 1970s, the level of mortality decreased and the slope increased during the 1980s and 1990s. The rates of change of both parameters appear to have slowed down in recent years. The slower pace of increase in the slope parameter in recent decades is due to greater mortality declines at older ages. From the 1990s the relative differences between mortality declines at ages below and above modal ages were much smaller than before, causing the slope of death curves to increase at a much slower pace.

Figure 5.6: Ratio of death rates for ages 90-99 in 2011 compared to 1981 by sex, comparison of countries
Source: Author’s calculations using HMD data and own estimates
Figure 5.7: Changes in level (\(\alpha\)) and slope (\(\beta\)) of death rates according to Gompertz’s exponential function from 1921-2012, by sex

Source: Author’s calculations using HMD data and own estimates

Bongaarts (2005) fitted a logistic function to the observed death rates for ages 25-109 for 14 low mortality countries in each year from 1950 to 2000. He found that the slope parameters for these countries were all between 0.10 and 0.12 on average over the period, were slightly lower for males, and in all cases showed almost no variation between 1950 and 2000. The upward trend in the slope parameter for Australia seen in Figure 5.7, especially from the 1970s, therefore seems contrary to these findings. The rate of increase in Australia’s slope parameter started slowing only from around the mid-1990s. These different findings cannot be ascribed to the different functions being used to describe the age profile of death rates, as the trends of the respective \(\beta\) parameters from fitting the Gompertz and logistic functions to Australian data are very similar. Trends in the slope parameter do diverge from the mid-1990s for different statistical methods, however. When using the Ordinary Least Squares (OLS) method, the pace of increase of \(\beta\) starts slowing down earlier compared to when the Weighted Ordinary Least Squares (WOLS) method is used. This applies regardless of whether the exponential or logistic function is fitted. However, when the slope of the age profile for Australian death rates is graphed using a similar scale on the y-axis than that used by Bongaarts, the increase also seems insignificant. While the slope parameters in Australia
were initially at lower levels than in other low-mortality countries as shown by Bongaarts (2005), it is possible that they have caught up and might remain at similar average levels. This issue is discussed further in Chapter 6.

5.4 Relationship between death frequency distributions and age profile of death rates

The correspondence of changes in the age profile of death rates and the death frequency distribution was investigated by applying the decomposition method described in section 5.2. Figure 5.8 shows the extent to which changes in modal ages were caused by changes in the level and slope respectively of the fitted Gompertz functions. Results were smoothed over ten years.

Figure 5.8: Changes in modal age attributed to changes in the level (\( \alpha \)) and slope (\( \beta \)) of the fitted Gompertz model, by sex

Throughout the period, declines in the level of the age profile of death rates drove increases in modal ages, albeit at varying rates. In contrast, increases in the slope of death rates caused decreases in modal ages. Increases in modal ages due to level of death rate changes are almost mirror images of decreases due to slope changes. In other words, if only the level of mortality changed with no changes in the slope, modal ages would have increased to a much
greater extent. The increasing slope of the death curve thus counteracted the impact of falling levels of mortality on the modal age. However, modal age decreases due to slope changes were lower than modal increases due to level changes, so that the net effect was increases in the modal age. The higher rate of increase in modal ages from the 1970s to 2012 compared to earlier periods is also clear from this graph. Before the 1980s, the modal age for females changed more than for males, while the modal age for males increased more after the 1980s. Modal ages for both males and females increased fairly consistently throughout the last few decades, despite the counteracting impacts of changes in each of the level and slope of death rates.

Figure 5.9 shows the extent to which changes in concentration of deaths at the modal age were caused by changes in the level and slope of the fitted Gompertz function. The graph confirms that greater concentrations of deaths around the modal age are directly caused by a steeper slope (higher $\beta$) of the age profile of death rates. A change in $\alpha$, the level of mortality, has no impact on the concentration of deaths around the modal age, and increases in the concentration of deaths at the mode is completely determined by increases in the slope of the age profile of death rates. Changes in concentration were positive throughout most of the period, which means compression occurred, but the rate of increase in concentrations has been decreasing since the 1990s. This confirms that the rate of increase in the slope of the age profile of death rates is slowing.
Figure 5.9: Changes in concentration of deaths at the modal age attributed to changes in the level ($\alpha$) and slope ($\beta$) of the fitted Gompertz model, by sex

5.5 Conclusion

An understanding of historical patterns of change in mortality patterns is an important prerequisite for accurate projections. The creation of reliable estimates and death rates in earlier chapters allowed detailed and reliable analyses of mortality patterns and trends in Australia. This chapter has shown how death rates for males and females in Australia at all ages declined significantly over the last nine decades, although the extent of declines decreased with age. Broadly three periods with clearly different patterns of mortality change can be identified. The period analysed broadly coincide with the latter part of the second stage and the third stage of Omran’s epidemiological transition model, from ‘the age of receding pandemics’ to the ‘age of degenerative and man-made diseases’ (Omran, 1971), and the fourth stage proposed by Valin and Meslé (2004), the ‘cardiovascular revolution’:

- The first period identified in this analysis was from the 1920s to the 1950s during which death rates of children and young adults declined. These declines are likely to be a continuation of experience starting a few decades earlier. These drove increases in life expectancy at birth and greater concentrations of deaths at older ages, while
modal ages remained unchanged. During this period the death frequency distribution was compressed into shorter age spans.

- The second period was from the 1950s to the 1970s. During this period, declines in death rates at the younger ages slowed down while death rates for females at high adult and elderly ages (45+) started declining. Death rates for males changed little. Life expectancy at birth, modal age and the concentration of deaths at the modal age changed little.

- From the 1970s death rates at elderly ages (60+) declined rapidly for both males and females, driving increases in both life expectancy at birth and modal ages. Concentrations of deaths at the modal age also increased and standard deviations of life spans beyond the modal age decreased due to death rates at younger ages declining more than at older ages. Increases in modal ages were thus accompanied by the greater compression of deaths into shorter age spans.

Contrary to the experience in some developed countries where the compression of mortality slowed or ceased in recent decades, it has accelerated in Australia. This appears to be the result of lower declines in death rates at ages 90-99 for both males and females in Australia over the last two decades. The slow-down in the pace of increase in the slope of the age profile of death rates seems to be a more recent trend in Australia, suggesting a lag with some other low-mortality countries.

A new method was used to quantify the extent to which changes in the modal age and concentration of deaths at the modal age were attributed to changes in each of the level and slope of the two-parameter exponential (Gompertz) function describing the age profile of death rates. This enables the relationship between changes in the age profile of death rates and changes in the death frequency distribution to be better understood. This decomposition showed that improvements in the level of mortality drove increases in modal ages, but this was countered to a large extent by increases in the slope of the age profile of mortality. Changes in the concentration of deaths at the mode are determined only by changes in the slope of the mortality curve. It was also found that while improvements in the level of mortality was consistently accompanied by increases in the slope of mortality, the pace of increase in the slope has been slowing since the 1990s and seems to have stagnated since the
start of the 21st century. This can be ascribed to greater declines in death rates at very elderly ages in recent decades.

In line with Vaupel (1997a), Wilmoth (1998) and Oeppen and Vaupel (2002), the continuing observed shifts in Australia’s death frequency distribution to higher ages is contrary to the hypothesis that the human life span is subject to a maximum. While modal age increases due to declines in the level of mortality has been countered by increases in the slope, the overall effect was a steady increase and an expansion of the tail of the death frequency distribution.

The viability of using the proven relationships between death rates and death frequency distributions to forecast mortality is explored in the next chapter. In addition, the regular patterns of death rate decline observed over the last few decades support the use of extrapolation methods to forecast mortality. The application of the insights gained in this chapter to projecting the future is discussed in chapter 6.
Chapter 6. Mortality forecasting methods

6.1 Introduction

The final objective of this thesis is to create reliable very elderly population projections for Australia for the next 30 years. Population projections form an essential input to planning for future services, housing and finance needs. Changing adult mortality is by far the biggest factor impacting elderly and very elderly population projections over the next 30 years, so that the first step towards meeting this objective is finding appropriate mortality forecasting methods.

As discussed in Chapter 2, approaches to projecting mortality include risk models, expectation methods and extrapolation methods. Different methods vary in terms of their complexity, subjectivity and validity of the results. The stable trends in historical mortality patterns in Australia over recent decades, as discussed in Chapter 5, support the use of relatively simple and objective extrapolation models. The Lee-Carter method is a popular extrapolation method and a number of variants and extensions have been developed. However, the accuracy of the Lee-Carter or any other extrapolation method in forecasting death rates and population numbers, especially at the very elderly ages, is not well understood, and it is not known how the performance of the Lee-Carter method compares to that of other extrapolation methods. Specifically, no investigations have been carried out to determine whether mortality could be projected more accurately using a simpler extrapolation approach such as the Geometric method, or whether the extrapolation of fitted mathematical models could be more appropriate.

The purpose of this chapter is two-fold. The first is to compare the direct extrapolation methods conceptually and in terms of the criteria discussed in section 2.6.2. The second is to present the results of retrospectively testing extrapolative mortality forecasting methods for accuracy when applied to Australian adult mortality data. Mortality was projected from 1992 and 2002 to 2012 and compared with actual observed death rates calculated by the author based on population estimates derived in earlier chapters. The methods tested include the Lee-Carter method (Lee & Carter, 1992) and one of its variants known to produce more accurate death rates across the age range for Australia, referred to as the Booth-Maindonald-Smith (BMS) variant (Booth et al., 2002). The performance of the original Lee-Carter method and the BMS variant was compared with that of other direct extrapolation methods such as
the Geometric method, Ediev’s adjusted version of the Geometric method (Ediev, 2008) and a Relational model (Brass, 1974; Himes et al., 1994), as well as with indirect methods. Indirect methods include the extrapolation of parameters of models fitted to the age profile of death rates, and a new method involving the extrapolation of the modal age and concentration, features of the death frequency distribution. These methods are described in section 6.2. Section 6.3 presents a conceptual comparison and section 6.4 the results of the empirical assessment of accuracy.

6.2 Data and methods

6.2.1 Forecasting methods tested

The mortality forecasting methods that were tested are set out below. All the approaches tested allow the measure projected to be expressed as a linear function of either age or time when a logistic or logarithmic transformation is applied. This simplifies the statistical fitting process in that linear regression techniques can be applied (Ordinary Least Squares, Weighted Least Squares or Maximum Likelihood procedures). These transformations also ensure that a floor of zero applies when projecting declines in mortality rates. With the exception of the Lee-Carter methods, parameters for all the methods were fitted using the Ordinary Least Squares method, using the package ‘R’ (R, 2014). Using the Weighted Least Squares method did not significantly alter the results. All functions were fitted to observed death rates in Australia for ages 50-100, separately for males and females. The observed death rates are those derived using the estimates created under the first objective. Death rates for ages beyond 100 were too volatile to be reliably modelled. The method applied for producing forecasted death rates at ages 101+ is explained in section 7.2.2.

Direct extrapolation methods

Geometric method

The Geometric method (Pollard, 1987) is a direct extrapolation method whereby death rates at specified ages are assumed to change at constant geometric rates. The central death rate at time t for age x is modelled as:

\[ m_{x,t} = e^{a_x t + b_x t} \]  \hspace{1cm} (6.1)

Transforming the death rate by taking a logarithm on both sides of the equation, a linear function can be modelled as follows:
\[ \ln(m_{x,t}) = a_x + b_x t \]  
(6.2)

where \( a_x \) and \( b_x \) are age-specific parameters that reflect the base level and geometric rate of change in death rates over time respectively. The same fitting period was used to derive \( a_x \) and \( b_x \) for all ages and these parameters were held constant throughout the projection period.

**Ediev’s variant of the Geometric method**

When declines in death rates at different ages do not show consistent patterns of change over time, it may not be appropriate to use the same fitting period for all ages when applying the Geometric method. A variant of the Geometric method was also tested whereby fitting periods of different lengths were used to derive \( a_x \) and \( b_x \) for different ages.

Ediev (2008) suggested a process for determining the appropriate fitting period for each age by fitting the linear equation \( \ln(m_{x,t}) = a_x + b_x t \) over progressively longer periods and then choosing the period that produced the optimal fit according to a goodness-of-fit criterion. Optimal fitting periods for each age were such that the linearity requirement was met separately for each age. Ediev (2008) suggested the use of fitting periods starting at least 20 years before the jump-off date. However, in this study a minimum of 10 years was used to ensure consistency with fitting periods used for other methods (see section 6.2.2). For example, for a jump-off date of 2002, linear equations were fitted by age over the periods 1992-2002, 1991-2002, 1990-2002 and so on until 1921-2002. The optimal fitting period for each age was the longest period producing a standard deviation of less than 2. The standard deviation for the fitting period starting at time \( t-d \) is calculated as 5-year moving averages of

\[ \varphi_{x,t-d} = \frac{|y_{x,t-d} - \hat{y}_{x,t-d}|}{\hat{\sigma}_{x,t-d}}, \]  
where \( \hat{\sigma}_{x,t-d} = \sqrt{\frac{\sum (y_{x,t} - \hat{y}_{x,t})^2}{n}} \), with \( y_{x,t} = \ln(m_{x,t}) \) observed, and \( \hat{y}_{x,t} = \ln(\hat{m}_{x,t}) \) fitted (Ediev, 2008). This method is referred to as Ediev’s original method.

Due to the volatility of the standard deviations and the resulting challenge to identify unique optimal fitting periods, an alternative goodness-of-fit measure was tested in this study to determine optimal fitting periods by age, namely \( R^2 \). This measure represents the proportion of variance explained by a model (Wilmoth, 1993). Specifically, the better the transform of death rates over a particular fitting period supports the linearity assumption, the higher \( R^2 \). Therefore, for each age, the optimal fitting period was the one yielding the highest \( R^2 \). This method is referred to as the Ediev \( R^2 \) method.
Ediev (2008) suggested a further procedure to correct the estimated slopes of the age profile of death rates in order to avoid implausible age profiles of death rates that may result from the independent forecasting of death rates for different ages. Forecasted death rates were not found to produce irregular patterns with increasing age so that it was not deemed necessary to apply this procedure.

**Original Lee-Carter method**

The original Lee-Carter method (Lee & Carter, 1992) consists of three parts, namely the mortality model, the fitting method and the forecasting method (Booth et al., 2002). Lee and Carter (1992) included the error ($\varepsilon_{x,t}$) and white noise ($u_t$) factors in their model allowing for random variation for the purpose of stochastic modelling and hence the production of probabilistic values (equation 2.37). Given the focus of this thesis on best estimate projections, only the deterministic application of the Lee-Carter method was tested, with central death rates and their logarithmic transformation respectively modelled as:

$$m_{x,t} = e^{a_x + b_x k_t} \quad (6.3)$$

$$\ln(m_{x,t}) = a_x + b_x k_t \quad (6.4)$$

where $a_x$ reflects the average level of log death rates at age $x$ across the fitting period; $k_t$ is the rate of change in mortality in year $t$; and $b_x$ converts the rate of change in year $t$ to an age-specific rate of change. Singular value decomposition is used to estimate $k_t$, and $b_x$ and $k_t$ is adjusted by refitting to total observed deaths. $k_t$, $a_x$ and $b_x$ for all ages are based on the same fitting period and $a_x$ and $b_x$ remain constant throughout the projection period. Lee and Carter employed as the forecasting method the random walk with drift which allows stochastic results and hence confidence intervals to be produced. The Lee-Carter model was implemented using the function “lca” in the package “Demography” in “R” (Hyndman, Booth, Tickle, & Maindonald, 2014). For the purpose of this study, this model will be referred to as the original Lee-Carter model.

**Lee-Carter BMS variant**

The Lee-Carter method will provide valid results if correlations between rates of mortality decline at different ages are stable over time. In order to meet this requirement of consistency, Lee and Miller (2001) and Tuljapurkar et al. (2000) proposed using fitting periods starting from 1950 for industrialised countries. Given indications that Australian
mortality experience does not meet the consistency assumptions underlying the Lee-Carter model, Booth et al. (2002) developed a variant of this model, the Booth-Maindonald-Smith (BMS) variant. Where-as the original Lee-Carter method uses as a fitting period the maximum period for which data is available, the main modification of the BMS method involves the derivation of a fitting period that is considered optimal. An optimal fitting period is one which best supports the assumption of a linear $k_t$, or a constant geometric rate of change in death rates, according to certain goodness-of-fit criteria (Booth et al., 2006; Booth et al., 2002). As for the original Lee-Carter method, the same fitting period is used to derive $a_x$, $b_x$ and $k_t$, but $k_t$ is adjusted to fit the death frequency distribution rather than total observed deaths as for the original Lee-Carter method. Jump-off rates are also affected by this adjustment (Booth et al., 2006; Booth et al., 2002). The BMS variant of the Lee-Carter method was modelled using the “bms” function in the package “Demography” in “R” (Hyndman et al., 2014).

Other variants and extensions of the Lee-Carter method differ, for example, in terms of the method of adjustment of $k_t$, adjustment of the jump-off rates, fitting period used, or the use of smoothed death rates (Booth et al., 2006; Hyndman & Shahid, 2007; Lee & Miller, 2001). An evaluation of the accuracy of death rates forecasted by the original Lee-Carter method and four variants or extensions indicated that the variants and extensions produced more accurate death rates compared to the original Lee-Carter method but the variation in accuracy between the variants and extensions themselves was not statistically significant (Booth et al., 2006). The BMS variant, using fitting periods from 1968 for males and 1970 for females in the case of Australia, was among the most accurate when considering the average errors over all ten countries, males and females, forecast period and age (Booth et al. (2006). Only this variant was therefore tested in this study.

**Relational model**

The relational models introduced by Kermack et al. (1934) and further developed by Brass (1974) and later Himes et al. (1994) make use of death rates from a standard population. Death rates at specific ages for a particular country are derived assuming a linear relationship between the standard population and country-specific death rates. In this thesis, the two-parameter relational model used by Himes et al. (1994) will be applied as a forecasting tool, with the standard representing death rates at a base date and the intercept and slope
representing changes over time. The population dimension is thus replaced with a time dimension. The central death rate is modelled as:

\[ m_{x,t} = \frac{e^{\alpha_t e^{\beta_t a_x}}}{1 + e^{\alpha_t e^{\beta_t a_x}}} \]  

(6.5)

and the logit transformation of death rate can be modelled as a linear equation:

\[ \logit(m_{x,t}) = \alpha_t + \beta_t a_x \]  

(6.6)

where \( a_x \) is the logit of the base mortality level at age \( x \) measured at the start of the fitting period. \( \alpha_t \) allows for a shift in the level of transformed age-specific death rates over time and \( \beta_t \) allows for a change in the slope of the transformed age-specific death rates over time. A linear function was first fitted to the transformed death rates in each year relative to the transformed death rates in the base year to obtain \( \alpha_t \) and \( \beta_t \) by way of Ordinary Least Squares. By inspection these parameters displayed a roughly linear trend. The second step involved fitting linear functions on \( \alpha_t \) and \( \beta_t \) over time so that these parameters can be extrapolated for projection purposes.

**Extrapolation of parametric models**

Indirect forecasting methods were also tested, whereby mathematical functions were fitted to the age profile of death rates and the time-series of the model parameters extrapolated (McNown & Rogers, 1989; Pollard, 1987). Many researchers have found that death rates increase at an exponential rate over most of the adult ages but increase at a decelerating rate at very high ages (Gavrilov & Gavrilova, 1991; Horiuchi & Coale, 1990; Horiuchi & Wilmoth, 1998; Vaupel, 1997a; Vaupel et al., 1998). Functions found to fit the age profile of adult mortality well are Gompertz’s function which models death rates that increase exponentially with age (Gompertz (1825) and a two-parameter logistic function referred to as Kannisto’s logistic function (Bongaarts, 2005; Thatcher, 1999b; Thatcher et al., 1998).

Analyses to identify the most appropriate function for modelling the age profile of death rates are impacted by changes in the quality of data, changes in death rates themselves, and the availability of adequate volumes of data to enable statistically significant conclusions to be drawn. For example, a deceleration in the rate of increase in death rates at very high ages may be a true reflection of experience, or may be due to age misstatement inherent in population estimates underlying the death rates. Previous research indicated that death rates at very high ages were overestimated by the Gompertz’s function and to a lesser extent by the Weibull function and that a logistic function better reflects the decelerating rate of increase at these
ages (Horiuchi & Coale, 1990; Horiuchi & Wilmoth, 1998; Robine & Vaupel, 2002; Thatcher et al., 1998; Weibull, 1951). More recently, Gavrilova and Gavrilov (2012) found that the Gompertz’s law was a better fit than the logistic function of US deaths data at very high ages. For the purpose of this study, the forecasting accuracy of fitting and extrapolating the parameters of Gompertz’s, Kannisto’s logistic and Weibull’s functions were tested. However, these functions were only fitted to ages 50-100, given the extreme volatility of death rates at higher ages. It is thus not possible to draw conclusions about the best-fitting functions at the very high ages. These functions are described below.

**Gompertz’s law**

According to Gompertz’s law of mortality, the central death rate at age x and time t can be expressed as an exponential function of age:

\[ m_{x,t} = \alpha_t e^{\beta_t x} \]  \hspace{1cm} (6.7)

The logarithmic transformation of the death rate at time t for age x is a linear function of age:

\[ \log(m_{x,t}) = \log(\alpha_t) + \beta_t x \]  \hspace{1cm} (6.8)

where \( \alpha_t \) and \( \beta_t \) respectively reflect the base level and slope, or rate of increase with age, of death rates at time t. The first step involves fitting a linear curve to the logarithm of death rates in each year relative to age to obtain \( \alpha_t \) and \( \beta_t \) for that year using simple linear regression (OLS). The second step involves fitting linear functions to each of \( \alpha_t \) and \( \beta_t \) over time so that these parameters can be extrapolated for projection purposes.

**Kannisto’s Logistic function**

According to Kannisto’s two-parameter logistic function, the central death rate at age x and time t can be expressed as:

\[ m_{x,t} = \frac{\alpha_t e^{\beta_t x}}{1 + \alpha_t e^{\beta_t x}} \]  \hspace{1cm} (6.9)

The logistic transformation of the death rate at time t for age x is a linear function of age:

\[ \logit(m_{x,t}) = \log(\alpha_t) + \beta_t x \]  \hspace{1cm} (6.10)

where \( \alpha_t \) and \( \beta_t \) respectively reflect the level and slope of death rates at time t. The first step involves fitting a linear curve to the transformed death rates in each year relative to age to obtain \( \alpha_t \) and \( \beta_t \) for that year. The second step involves fitting linear functions to each of \( \alpha_t \) and \( \beta_t \) over time so that these parameters can be extrapolated.
Weibull

According to the Weibull function central death rates at time \( t \) and age \( x \) are proportional to a power of age (Thatcher et al., 1998):

\[
m_x = \alpha_t x^{\beta_t}
\]  

(6.11)

The logarithmic transformation of death rates is a linear function:

\[
\ln(m_{x,t}) = \ln(\alpha_t) + \beta_t \ln(x)
\]  

(6.12)

where \( \ln(\alpha_t) \) and \( \beta_t \) respectively reflect the level and slope of log-transformed age profile of death rates at time \( t \). The first step involves fitting a linear function to the log of death rates in each year relative to the log of age to obtain \( \alpha_t \) and \( \beta_t \) using simple linear regression (OLS). Given that these parameters exhibited roughly linear trends over the periods considered, the second step involves fitting linear functions to each of \( \alpha_t \) and \( \beta_t \) over time so that they can be extrapolated for projection purposes.

Logistic Constant Beta, Gompertz Constant Beta and Relational Constant Beta methods

Some researchers found that a number of developed countries entered a ‘shifting’ mortality scenario whereby the age profile of death rates has gradually shifted towards higher ages without changing shape (Canudas-Romo, 2008; Edwards & Tuljapurkar, 2005). This is also seen in increases in the modal age at death with concentration measures remaining constant. Based on his findings that the slope parameter changed little over the period 1950-2000 for a number of countries analysed, Bongaarts (2005) proposed using Kannisto’s two-parameter logistic model to forecast mortality but with a constant slope (\( \beta_t \)) parameter, referred to as the shifting logistic model. The forecast accuracy of this method for Australia was also tested in this study. The level parameter (\( \alpha_t \)) was extrapolated over the projection period as for Kannisto’s logistic function described above, while the slope parameter, \( \beta_t \), was held constant at its value at the jump-off date. Similar to the shifting logistic model, a shifting Gompertz function and a shifting Relational model were also tested, whereby the level parameter was extrapolated as described while the slope parameter was held constant at its value at the jump-off date. A detailed discussion of the Relational Constant Beta model can be found in Ediev (2014).

Model based on projected modal age and concentration

A new indirect forecasting model was also tested, making use of two features of death frequency distributions, namely the modal age, or age where the highest proportion of deaths...
occur, and the concentration of deaths at the modal age. This model is introduced for the first time here and will be referred to as the Modal Age Projection Model.

According to Canudas-Romo (2008), the modal age at death in each year can be derived from the parameters of Gompertz’s function as follows:

\[ M_t = \frac{\ln(\beta_t) - \ln(\alpha_t)}{\beta_t} \]  \hspace{1cm} (6.13)

and the concentration of deaths at the modal age can be derived as:

\[ d(M_t) = \beta_t e^{\left(\frac{\alpha_t}{\beta_t} - 1\right)} \]  \hspace{1cm} (6.14)

where \( \alpha_t \) and \( \beta_t \) are the level and slope parameters of the Gompertz’s law at time \( t \).

Simple linear regression models were fitted to the modal age and concentration of deaths at the modal age using Ordinary Least Squares and used to extrapolate modal ages and concentrations over the projection period. From these extrapolated modal ages and concentrations, the Gompertz parameters \( \beta_t \) and \( \alpha_t \) were derived for each year of the projection period.

### 6.2.2 Fitting and projection periods

In order to derive reasonable conclusions about the most accurate forecasting methods, methods were evaluated for different combinations of fitting and projection periods. The combinations of fitting and projection periods used for testing the Geometric, Ediev, Relational, Relational Constant Beta, Gompertz, Gompertz’s Constant Beta, Kannisto’s Logistic, Logistic Constant Beta, and Modal Age Projection methods are shown in Figure 6.1 below.

Two jump-off dates were used, namely 1992 and 2002. For the Geometric, Relational, Gompertz, Logistic and Modal Age Projection models fitting periods started from 1972, 1982 and 1992. The year 1972 was considered a suitable start date based on the analysis presented in Chapter 5, which showed that both male and female death rates for ages 45-89 showed consistent trends of decline from this date. Different fitting and projection periods are considered because if results are consistent across different periods this will allow more general conclusions to be drawn.
Figure 6.1: Fitting and projection periods for the retrospective testing of mortality forecasting methods

Note: The Lee-Carter and Ediev methods are based on different fitting periods, using data from 1921

The original Lee-Carter method uses data for the longest period possible so that data from 1921 to the respective jump-off dates (1992 and 2002) were used. The BMS variant of the Lee-Carter method uses data from 1921 and applies a procedure to find the optimal fitting periods. For comparison purposes the original Lee-Carter method was also applied to data starting from 1972 and was found to produce average errors differing only marginally from that of the Lee-Carter BMS method. This indicates that the procedure incorporated in the Lee-Carter BMS variant to find optimal fitting periods accounts for most of the difference in results between the original Lee-Carter and the Lee-Carter BMS methods. Ediev’s Geometric method was applied to data from 1921 to the respective jump-off dates to find an optimal fitting period for each age.

All the models were applied separately for males and females. Unsmoothed death rates from 1921 to 1970 were obtained from the Human Mortality Database ("Human mortality database," 2013). Unsmoothed death rates from 1971 to 2012 are the author’s estimates, derived using the Survivor Ratio method with total estimates for ages 85+ constrained to ABS’ Estimated Resident Population numbers (ABS 2013), as described in Chapter 3.
6.2.3 Projected population

In addition to considering the accuracy of death rates, the methods were also compared in terms of their accuracy in projecting very elderly populations. Projected population numbers were calculated by way of a cohort-component model as follows (Rees, 1990):

$\hat{P}_{x+1,t+1} = \hat{P}_{x,t} - 0.5m^c_t(\hat{P}_{x+1,t+1} + \hat{P}_{x,t}) \quad (6.15)$

Solving for $\hat{P}_{x+1,t+1}$ gives:

$\hat{P}_{x+1,t+1} = \frac{(1-0.5m^c_t)}{(1+0.5m^c_t)} \hat{P}_{x,t} \quad (6.16)$

where $m^c_t$ is the death rate for cohort c in year t, calculated as $0.5(m_{x,t} + m_{x+1,t})$, and $m_{x,t}$ is the period death rate for age x at time t, smoothed using cubic splines.

6.2.4 Measures of accuracy

Weighted Mean Absolute Percentage Errors (WMAPE) of projected death rates and populations were used to measure the accuracy of the mortality forecast methods (Siegel, 2002). Lower WMAPEs indicate greater accuracy. Because death rates increase rapidly with age, it is appropriate to apply a weight to the absolute error measured at each age when determining the average. Life table deaths (or death frequencies) were used as weights. The Weighted Mean Absolute Percentage Error of projected death rates was calculated as follows:

$WMAPE_t = \frac{\sum_x |\hat{m}_{x,t} - m_{x,t}|}{\sum_x D_{x,t}} \times 100\% \quad (6.17)$

where $\hat{m}_{x,t}$ is the projected central death rate for age x in year t and $m_{x,t}$ is the actual observed death rate. Both are unsmoothed rates. $D_{x,t}$ is life table deaths at age x in year t. Results are presented where the summation are over ages 50-100. WMAPEs were also analysed over ages 80-100 and findings are mostly in line with those at ages 50-100 so that detailed results are not shown.

The Weighted Mean Absolute Percentage Error of projected population numbers was calculated as:

$WMAPE_t = \frac{\sum_x |\hat{P}_{x,t} - P_{x,t}|}{\sum_x P_{x,t}} \times 100\% \quad (6.18)$

where $\hat{P}_{x,t}$ is the projected population aged x at time t and $P_{x,t}$ is the actual observed population.
Percentage Error (PE) was also calculated in order to investigate whether the various methods over- or underestimate population numbers. Percentage Error is calculated as follows:

\[ PE_t = \left( \frac{\sum x P_{x,t} - \sum x P_{x,t}^*}{\sum x P_{x,t}} \right) \times 100\% \] (6.19)

Results are presented for summations over ages 80-89 and ages 90-99. At these ages, net migration was low enough to be ignored (see Figure 3.1). Mean Absolute Percentage Error, the average of absolute percentage errors over the projection period, was also studied, calculated as follows:

\[ MAPE_t = \frac{\sum |PE_t|}{n} \] (6.20)

### 6.3 Conceptual comparison of direct extrapolation models

In this section, a comparison is made of three direct extrapolation methods, namely the Lee-Carter, Relational and Geometric methods. These methods are compared conceptually and in terms of their practical application and the criteria discussed in section 2.6.2.

#### Transformation of the death rate projected

One difference between the methods relates to the transformation of death rates modelled, namely the logarithmic or logistic transformation. The Relational model is applied to the logistic transformation of death rates, while the LC and Geometric methods use a logarithmic transformation. This would result in differences if increases in mortality were projected. A ceiling is applied to \( m_{x,t} \) with the logit transformation in the form of an asymptote (of 1) being approached with increasing forecasting period, whereas the log transformation allows increases in \( m_{x,t} \) without a limit. However, when modelling mortality declines, which is more likely to be the case given past experience, both logarithmic and logistic transformations will result in the forecasted mortality rate \( m_{x,t} \) or \( q_{x,t} \) approaching a floor of zero over time, albeit at different speeds.

#### Probabilistic or deterministic results

The original LC model contains error terms, which facilitates the production of confidence intervals. Probabilistic values can, however, be produced with any model for which the parameters are determined via statistical fitting, with the quality of the fit (errors) affecting predictive intervals (Lee & Miller, 2001). Any of the other extrapolation models could thus
be converted to stochastic models by adding error terms, and adopting a suitable distribution function for the errors. Probabilistic results are not the focus of this research, however, and thus only best-estimate deterministic results are produced.

**Fitting periods**

The deterministic application of the Lee-Carter method projects mortality as
\[ \ln(m_{xt}) = a_x + b_x k_t \text{ with } k_t = c + k_{t-1}. \]
This means, if \( k_0 = 0 \) (i.e. starting at launch values), \( k_t = ct \), so that the Lee-Carter model simplifies to a Geometric method: \( \ln(m_{xt}) = a_x + b_x ct \). The deterministic LC and Geometric methods differ, however, in terms of the fitting periods used for estimating the parameters. Under the LC method the same fitting period is used to estimate the parameters for all ages \((a_x, b_x, \text{ and } c)\), whereas different fitting periods may be used to derive \( a_x \) and \( b_x \) for each age in the Geometric method, subject to the resulting age profile of mortality rates remaining plausible.

**Age- and time-related factors**

The Lee-Carter method has “two age standards plus a trend vector” while the Brass relational model has “one age standard plus two trend vectors” (Gomez de Leon 1990: 31). Age standard in this case refers to the parameter that is a function of age, for example \( a_x \), while the trend vector refers to the parameter which is a function of time. The Relational model thus allows for both the level \( (\alpha_t) \) and the slope of the age profile of mortality \( (\beta_t) \) to vary over time. Similar results are obtained with the Relational and LC models if a ‘sensible’ set of \( b_x \) factors (LC) is used and \( \beta_t \) (in the Relational model) is projected to remain constant over time. ‘Sensible’ \( b_x \)s decrease gradually with increasing age such that the age profile of death rates is not distorted over time.

Based on the results of factor analysis, Gavrilova and Gavrilov (2011) suggested that a minimum of two time-dependent factors are required to describe observed mortality declines, suggesting the Relational model is more appropriate. Furthermore, according to Brass (1974: 128), “the [relational] approach is simpler than projecting parameters separately for each age group, treating each specific death rate as a separate independent measurement, when they are clearly highly dependent.” Which approach is more appropriate depends on the degree of correlation of rates of change in mortality between age groups. The application of
the Relational model is similar to fitting Kannisto’s logistic model to the age profile of death rates and extrapolating its parameters.

The consideration of an appropriate fitting period that supports the assumed linearity of the measure being extrapolated is central to all extrapolation models. However, the requirement to find a fitting period for the LC model that supports both the linearity of time series values of \( k_t \) across all ages and a stable pattern of \( b_x \) over time is more restrictive than that underlying the Relational model. Under the Relational model different fitting periods may be applied to \( \alpha_t \) and \( \beta_t \), such that they separately support linearity of each of the parameters or some transformation thereof. Instead of linear functions, polynomials may also be used to extrapolate these parameters. However, care will be needed to ensure projected results are valid.

**Objectivity**

Supporters of the LC method claim that the method involves minimum subjectivity because forecasts are based entirely on past trends and the same fitting period is used to derive parameters for all ages. It has been claimed that for this reason it provides better results than those from expectation methods (Booth 2003: 196). However, this feature has also been criticised because rates of change in mortality at different ages have varied significantly over time. For this reason different past periods may be appropriate for projection of rates at different ages. The LC method may therefore not be optimal for producing accurate estimates at very high ages. Indeed, Lee and Carter (1992) found that it produced the least accurate results at ages 85+. Under the Geometric model, different past periods may be used to determine base values and the assumed rates of change for each age or age group, resulting in greater flexibility, but also more subjectivity. Appropriate fitting periods will need to be chosen for each age based on the stability of observed trends. Pollard (1987: 56) states: “a lot is left to the discretion of the forecaster”. Ediev (2008) suggested a procedure for finding optimal fitting periods for each age which removes some of the subjectivity.

**Parsimony**

Supporters of the LC model claims that there is only one parameter being forecast, namely \( k_t \). However, separate values of \( a_x \) and \( b_x \) need to be set for each age or age group modelled. Table 6.1 shows the number of parameters required for some of the extrapolation models to
be tested. This is based on 30-year fitting and projection periods, and single ages 50-100. Of
the models considered, projecting the parameters of a two-parameter logistic model involves
the least number of modelled parameters, while the LC model requires the most.

Table 6.1: Comparison of number of parameters for different extrapolation methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Parameters to be estimated</th>
<th>Number of parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometric</td>
<td>$a_x$, $b_x$</td>
<td>51</td>
</tr>
<tr>
<td>Log-Linear</td>
<td>$a_x$</td>
<td>51</td>
</tr>
<tr>
<td>Relational</td>
<td>$a_x$, $\alpha_t$, $\beta_t$</td>
<td>102</td>
</tr>
<tr>
<td>Relational</td>
<td>$a_x$, $\alpha_t$, $\beta_t$</td>
<td>115</td>
</tr>
<tr>
<td>Lee-Carter</td>
<td>$a_x$, $b_x$, $k_t$</td>
<td>51</td>
</tr>
<tr>
<td>Kannisto’s logistic</td>
<td>$\alpha_t$, $\beta_t$</td>
<td>64</td>
</tr>
</tbody>
</table>

6.4 Empirical evaluation of extrapolative mortality forecasting methods

6.4.1 Accuracy of death rates on average

Figure 6.2 shows average WMAPEs of projected death rates for the different methods by
jump-off date and fitting period for females. Equivalent results for males are presented in
Figure 6.3. The WMAPEs reflect the accuracy of death rates over the age range 50-100 and
are averaged over the first 10 years of the projection horizon. WMAPEs averaged over ages
80-100 produced similar rankings of the methods in terms of accuracy and WMAPEs were
generally marginally smaller than for ages 50-100. In most cases the degree of accuracy and
relative ranking of methods are similar or vary only slightly for different combinations of
jump-off date and fitting period. This is particularly true for females. Specifically, the
accuracy of methods does not consistently increase with increasing length of fitting period. In
some cases longer fitting periods result in greater errors. This reflects changes in underlying
patterns of change in death rates over time, discussed in more detail later.
Figure 6.2: Weighted Mean Absolute Percentage Error of death rates for ages 50-100 for different projection methods and fitting periods, projected from jump-off dates 1992 (left) and 2002 (right), averaged over first 10 years of projection period, Females

Note: The Lee-Carter and Ediev methods are based on different fitting periods, using data from 1921
Source: Author’s calculations using HMD data and own estimates
Figure 6.3: Weighted Mean Absolute Percentage Error of death rates for ages 50-100 for different projection methods and fitting periods, projected from jump-off dates 1992 (left) and 2002 (right), averaged over first 10 years of projection period, Males

Note: The Lee-Carter and Ediev methods are based on different fitting periods, using data from 1921

Source: Author’s calculations using HMD data and own estimates
The ranking of methods varied slightly between males and females. In the case of females, the methods can be divided into three broad groups in terms of relative accuracy. The same grouping of methods was observed for both jump-off dates and all fitting periods. The four methods performing best were the two variants of the Ediev method, the Lee-Carter BMS variant and the Geometric method. The accuracy of these methods was very similar, with average WMAPEs varying between 3.9% and 5.3% for the 2002 jump-off date and between 4.4% and 4.9% for the 1992 jump-off date. The use of the $R^2$ goodness-of-fit measure to determine an optimal fitting period by age seems to be slightly more successful compared to the standard deviation measure suggested by Ediev (2008). The ‘middle’ group in terms of accuracy consists of the Relational, Gompertz, original Lee-Carter, Relational Constant Beta, Modal Age Projection and Logistic methods. For fitting periods from 1972, average WMAPEs for these methods varied between 7.6% and 9.6% for the 1992 jump-off date and between 8.8% and 13.6% for the 2002 jump-off date. When averaged over the whole projection horizon of 20 years from the 1992 jump-off date, WMAPEs for this group varied between 9.4% and 13.2%. The least accurate methods for females were the Logistic Constant Beta, Weibull and Gompertz Constant Beta methods.

In the case of males, results were slightly less consistent between jump-off dates. In addition to the Ediev’s, Lee-Carter BMS and Geometric methods, the Gompertz’s and Modal Age Projection methods were among the most accurate for both jump-off dates. Errors were marginally higher than for females, varying between 5.4% and 5.9% for the best four methods for the 1992 jump-off date and between 6.4% and 8.1% for the 2002 jump-off date, for fitting periods from 1972. Similar to females, the least accurate methods were the Logistic Constant Beta, Gompertz’s Constant Beta, and Weibull methods, as well as the original Lee-Carter and Relational Constant Beta methods.

Average WMAPEs of death rates from the Geometric method for females over 2003-2012 were very similar whether the fitting period starts from 1972, 1982 or 1992. This is ascribed to consistent rates of decline in death rates throughout the 40-year period from 1972. Rates of decline in male death rates did not show such consistent patterns of change, with the average rate of decline in death rates for nonagenarians (ages 90-99) slowing between 1992 and 2002, while the pace of decline in death rates for males at ages 60-89 accelerated. As a result, for males, most methods produced more accurate forecasted death rates when using more recent and shorter fitting periods.
Comparison of Geometric, Ediev and Lee-Carter methods

This consistency in death rate reductions, both over time and between age groups, is also what drives the similarity in results between the Geometric, Lee-Carter BMS, and Ediev’s Geometric methods. Both the Lee-Carter BMS and Ediev’s methods model mortality according to the Geometric method and the methods differ in terms of fitting periods and statistical fitting procedures. For both males and females average WMAPEs for the Lee-Carter BMS method were between 50% and 70% lower than for the original Lee-Carter method, indicating that recent fitting periods, starting from around the 1970s, better support the assumption of constant rates of decline in death rates across the 50-100 age range. These findings are consistent with those of Booth et al. (2006). WMAPEs for the original Lee-Carter method applied to data starting from 1972 instead of 1921 differ only very marginally from that of the Lee-Carter BMS method. Application of the Lee-Carter BMS variant to Australian data at all ages by Booth et al. (2006) found optimal fitting periods starting from 1968 for males and from 1970 for females.

Based on simulated life tables, Hansen (2013) found the Lee-Carter BMS method (referred to as LC-V) produced more accurate death rate forecasts than the original Lee-Carter, Geometric and Brass relational methods. However, with the exception of the Lee-Carter BMS method, all the methods tested in Hansen’s investigation were based on simulated death rates over a 100-year fitting period. Hansen found the accuracy of the Geometric and original Lee-Carter methods to be very similar. Similar to the author’s findings, the procedure for determining an appropriate fitting period used by the Lee-Carter BMS model improved its accuracy. It is likely that the Geometric method would have produced results similar in accuracy to the Lee-Carter BMS method in Hansen’s study had similar fitting periods been used. In this context it could be argued, in line with Keyfitz (1991), that the choice of fitting period is more important than the selected method.

Ediev’s method differs from the Geometric method in that fitting periods vary by age and sex. For females, the application of Ediev’s procedure resulted in fitting periods starting from the 1960s for ages up to 84 and from the mid-1950s for ages 85+. For males fitting periods were shorter, starting from the 1970s for ages up to 56, from the mid- to late 1980s for ages up to around 80, and from the 1950s for very high ages. Using different fitting periods for
each age instead of the same period across all ages also did not significantly improve the accuracy of projected death rates.

The small differences in WMAPEs between the Geometric, Ediev and Lee-Carter BMS methods suggests that little was gained from the added complexity of the Lee-Carter statistical fitting procedures and either the Lee-Carter BMS or Ediev’s processes of finding optimal fitting periods. The Geometric method fitted using the Ordinary Least Squares method using data for the same period for all ages and both sexes, is simpler both in concept and practice than the Lee-Carter and Ediev methods. However, this does not automatically extend to all countries, periods and jump-off dates, as the success of the Geometric method is dependent on rates of decline in death rates being stable across both the fitting and projection period as well as on stable relationships between rates of decline at different ages. In particular, the validity of forecasted death rates across the age range is subject to geometric rates of change being equal or decreasing with age. Higher rates of decline at higher ages compared to lower ages may result in lower projected death rates at higher ages. Over the fitting and projection periods used in this study, these conditions were met and this ensured the success of the Geometric method. The added processes inherent in the Lee-Carter BMS and Ediev’s Geometric methods will improve the performance of the Geometric method in circumstances when these conditions are not automatically met. While these processes are markedly different, they seem to achieve very similar results.

**Gompertz, Logistic and Weibull methods**

The indirect forecasting of death rates by extrapolating the parameters of the Gompertz function proved very accurate for males from both jump-off dates, but this method was less accurate for females. The logistic function produced quite accurate forecasts for males for the 1992 jump-off date but less so for the 2002 jump-off date and for females. The Weibull function produced some of the least accurate forecasts for both males and females.

The accuracy of methods for projecting death rates indirectly via parametric models depends both on how well the mathematical function fits the age profile of death rates and on whether the patterns and rates of change in the parameters over the fitting period are linear and representative of changes over the projection period. In the case of females, Gompertz’s function was found to fit death rates across ages 50-100 better than the Logistic and Weibull
functions, with an average $R^2$ over the period 1972-2002 of 0.997 compared to 0.995 and 0.986 for the Logistic and Weibull functions respectively. The Gompertz function was a better fit than the Logistic function for female death rates throughout the period 1972 to 2012. For males, average $R^2$ values over 1972-2002 for the Gompertz, Logistic and Weibull functions were 0.995, 0.997 and 0.994 respectively. The Logistic function was a better fit for male death rates until around 1999 but its goodness-of-fit decreased thereafter while that of the Gompertz function increased to exceed that of the Logistic function. During the projection periods the Gompertz function fitted both male and female death rates at ages 50-100 marginally better than the Logistic function. This fact seems to drive the greater accuracy of the Gompertz model in forecasting death rates. The improving fit of the Gompertz model for males from 2000 also resulted in greater accuracy for forecasting death rates from the 2002 jump-off date compared to the 1992 jump-off date. For both males and females the average $R^2$ goodness-of-fit of the Gompertz model was marginally higher for the 1992-2002 fitting period compared to 1982-2002, and both were higher than for a 1972-2002 fitting period, resulting in greater accuracy of forecasted death rates for more recent albeit shorter fitting periods.

The success of the parametric functions in forecasting death rates also depends on how well the straight lines fitted to level and slope parameters over time (the fitting periods) describe changes over the projection period. An investigation into changes in the parameters of the fitted Gompertz and Logistic functions reveals that their rates of change over the fitting periods were not representative of changes over the projection periods. The paces of decrease in the level parameter and increase in the beta parameters started slowing down from around 2002, after gradually increasing from 1972. The alpha and beta parameters of the Gompertz and Logistic functions showed similar patterns of change. The application of a linear function, equivalent to modelling constant rates of change in these parameters measured over fitting periods up to 2002, was thus not representative of trends after 2002. Extrapolating the rates of change in these parameters measured over the fitting period thus resulted in under-projected alpha values and over-projected beta values. The impact of forecasting a level parameter that is too low and a slope parameter that is too high is counteracting, but to different degrees at different ages. Misestimation of the level parameter impacts death rates equally at all ages while the effect of a misestimated slope parameter increases with age.
The combined impact of trends in the parameters, goodness-of-fit and shifting weights across the age range drive the relative accuracy of the parametric methods for different jump-off dates and fitting periods. The goodness-of-fit of the functions seems to have a greater impact than the accuracy in projecting the parameters. The net effect of how well these functions described death rates across the adult age range and how accurately the parameters were projected was an overstatement of male death rates at most ages, and in the case of females an overstatement at ages 60-85 and an understatement at ages 85+. While this applied to both the Gompertz and the Logistic functions, the Logistic function resulted in greater overstatement of death rates than the Gompertz function at ages 70-85 and greater understatement of death rates for ages 85+, while the Gompertz function overstated death rates for ages 100+. Both the Gompertz and Logistic functions seem to fit death rates above age 80 better than below and the Gompertz function produced smaller WMAPEs above age 80 than the Logistic function. In summary, the better fit of the Gompertz function to male death rates compared to female death rates over both the projection horizons seemed to be the main reason behind its superior performance in forecasting death rates for males.

Given the faster pace of change in the slope parameter over the fitting period compared to the projection period after 2002, one might expect that death rates may be projected more accurately with a constant slope parameter. This was not the case, however, because even though the rate of increase in the beta parameter slowed from around 2000, it was still above 0%. The average rate of change in the Gompertz slope parameter for females during the fitting periods was around 0.5% per year, compared to 0.3% per year over the projection period, and the 0% inherent in the Constant Beta models. As a result, the Gompertz Constant Beta and Logistic Constant Beta models were among the least accurate for both males and females. It is possible that these methods may produce more accurate forecasts for countries where a shift in the age profile of death rates have been observed in recent decades. For example, in France, Italy and Switzerland the pace of compression has slowed substantially or stopped while modal ages continued to increase (Cheung et al., 2009; Kannisto, 2000; Ouellette & Bourbeau, 2011; Robine, 2001). Also, given that Australia seems to be lagging behind in these trends, it is possible that these methods may produce more accurate forecasts in the future.
**Relational Model**

The format of the Relational model is very similar to that of the Logistic model. An analysis of the percentage errors by age confirms that the pattern and size of errors are also very similar for these two models. Both models overestimated death rates between ages 64 and 94 for males and between ages 65 and 86 for females and underestimated death rates at other ages. In both cases the slope parameter continued to trend upwards, so that the Constant Beta models were less accurate. This is in contrast to Ediev’s (2014) findings that the shifting Brass model produced improved forecasts at the older ages compared to the normal Brass model. It is, however, possible that Australia lags other countries and may reach a shifting scenario at a later stage.

**Modal Age Projection Model**

The Modal Age Projection model was in the ‘middle’ group in terms of accuracy for females but performed fairly well for males, especially from the 2002 jump-off date. This is the least ‘direct’ approach of the methods considered and its accuracy is impacted by a number of factors, including the stability of rates of change in modal age and concentration across fitting and projection periods, the appropriateness of the approximation to derive the Gompertz parameters from these and the goodness-of-fit of the Gompertz function to the age profile of adult death rates. Its poor performance for females seems to be mainly the result of the approximation used to derive the level and slope parameters from the projected modal ages and concentrations, which resulted in overstated $\alpha_t$ and understated $\beta_t$. A factor countering the impact of this misestimation was the assumed continuation of trends in the concentration of deaths at the modal age throughout the projection period, which did not occur. While modal ages for both males and females increased at fairly consistent rates from 1972 to 2012, the rates of increase in the concentrations of deaths at the modal age slowed down considerably to almost zero from the mid to late 1990s. As a result, concentrations forecasted from the 2002 jump-off date were too high, resulting in the derived mortality level ($\alpha_t$) and slope ($\beta_t$) parameters derived being respectively too low and too high. The net impact of the misestimation of these parameters from the modal age and concentrations and the erroneous forecasting of past trends was too high $\alpha_t$ and too low $\beta_t$ being projected. The resulting death rates were consequently too high between ages 60-85 and too low beyond age 85.
Errors over the projection horizons

Figure 6.4 shows WMAPEs averaged over the whole 20-year projection period for the 1992 jump-off date, separately for females and males. For females the three broad groups of methods in terms of accuracy are similar to those for 10-year averages, with only minor differences in the ranking of methods within each group. The 20-year averages for the Geometric, Ediev and Lee-Carter BMS methods were only 1-1.5 percentage points higher than the 10-year averages. For males, differences between 10 and 20-year averages were greater, and there was also slightly more variation in the ranking of methods. The Modal Age Projection and Gompertz methods produced the lowest WMAPEs on average over the 20 years for males.

The standard errors of the WMAPEs over the projection horizons for the most accurate methods were very similar. In the case of females, the standard error of WMAPE was the lowest for the Ediev $R^2$ (1.3%), followed by the Geometric method (fitting period 1972-1992) (1.5%) and the Lee-Carter BMS method (1.6%). For males and the fitting period 1972-1992 the standard error of the WMAPEs for the Modal Age Projection method was 1.3% and for the Gompertz method 1.8%. The Lee-Carter BMS, Geometric and Ediev $R^2$ methods followed with 4.0%, 4.4% and 4.7% respectively.
Figure 6.4: Weighted Mean Absolute Percentage Error of death rates for ages 50-100 for different projection methods and fitting periods, projected from jump-off date 1992, averaged over the whole projection period 1993-2012, by sex.

Source: Author’s calculations using HMD data and own estimates.
6.4.2 Accuracy of death rates over the projection period

Figure 6.5 shows WMAPEs for ages 50-100 in each year of the projection period for jump-off dates 1992 and 2002 for a selection of methods. Results for the Gompertz, Logistic, Geometric, Relational and Modal Age Projection methods are based on fitting periods 1972-1992 and 1972-2002 respectively. For females the results were fairly consistent between jump-off dates. WMAPEs for the Lee-Carter BMS, Ediev’s and Geometric methods from 2002 were similar throughout the projection periods. WMAPEs for these methods from the 1992 jump-off date increased gradually over the course of the 20-year projection period but did not exceed 10%. Results for the 1992 jump-off date for males started to deteriorate more from 2005 when declines in death rates at ages 70-90, where the highest proportion of deaths occur, started to accelerate. Volatility between years resulted in relatively low observed death rates in 2005-2007 at ages around 85 compared to subsequent years. The Relational, Gompertz, Modal Age Projection and Logistic methods were less accurate, with WMAPEs of around 3-6 percentage points higher throughout the projection periods. Ediev’s method showed the least variation in errors over the projection periods for both jump-off dates.

In the case of males, the Gompertz and Modal Age Projection methods produced the most accurate forecasts of death rates throughout most of the projection periods for both jump-off dates. These methods also show the least variation over the projection periods. For the 1992 jump-off date death rates forecasted by the Geometric, Ediev’s and Lee-Carter BMS methods were very accurate in the first five years from 1993 to 1997 with WMAPEs of around 4%. However, thereafter WMAPEs for these methods increased over the 20 year projection period to over 15%. WMAPEs of death rates forecasted by the Gompertz and Modal Age Projection methods remained below 10% throughout both projection periods. The Relational and Logistic methods performed better from the 1992 jump-off date than from 2002.
Figure 6.5: Weighted Mean Absolute Percentage Error of death rates for ages 50-100 for the most accurate methods fitted over the periods 1972-1992 (left) and 1972-2002 (right), projected to 2012, by sex

Note: The Lee-Carter and Ediev methods are based on different fitting periods, using data from 1921
Source: Author’s calculations using HMD data and own estimates
6.4.3 Accuracy of projected populations on average

The relative forecast accuracy of death rates across ages 50-100 for the different methods do not necessarily extend to very elderly population projections. WMAPEs for death rates were weighted by life table deaths and the highest weights are around the modal age at death. WMAPEs for populations presented in this section were weighted by projected population numbers which decrease with increasing age. In addition, WMAPEs of death rates reflect their accuracy at particular points in time only while projected populations reflect the cumulative effect of errors in forecasted death rates for a particular cohort up to the various dates over the projection horizon. In this section the accuracy of projected very elderly populations for various mortality forecasting methods is presented.

Figure 6.6 shows a comparison of WMAPEs of projected population numbers for a selection of methods for ages 85-100 by sex and jump-off date. WMAPEs are averaged over the first 10 years of the projection periods. Fitting periods start from 1921 for the Lee-Carter BMS and Ediev’s methods and from 1972 for the other methods. No single method consistently produced the most accurate population forecasts, but most methods produced small errors, and differences between the best-performing methods were small. The ranking of methods was similar when averaging WMAPEs over the whole projection period for the 1992 jump-off date and are not shown.

For females, the Geometric, Ediev and Lee-Carter BMS methods were the most accurate for projected very elderly populations regardless of jump-off date. This was in line with the forecast accuracy of death rates. For males, the ranking of methods differed more between projected death rates and very elderly populations. For example, the Gompertz and Modal Age Projection methods were the most accurate in forecasting male death rates from both jump-off dates. It was also very accurate for projecting very elderly male populations from 2002. However, WMAPEs for very elderly populations projected from 1992 by these methods were more than double those of the Geometric, Ediev, Lee-Carter BMS and Relational methods. For both males and females the Gompertz Constant Beta method was the least accurate of the methods shown. Overall, across sex and jump-off dates, the Ediev $R^2$ method was the most accurate with an average WMAPE of 2.3%, closely followed by the Geometric and the Lee-Carter BMS and methods with average WMAPEs of 2.4%.
Figure 6.6: Weighted Mean Absolute Percentage Error of projected very elderly populations, averaged over the first 10 years of projection periods, comparison of methods by sex and jump-off date, fitting periods from 1972
Source: Author’s calculations using HMD data and own estimates

6.4.4 Accuracy of projected population numbers over projection period

Figure 6.7 shows percentage errors (PE) over the projection period from the 2002 jump-off date for ages 80-89 and 90-99 for females and males. Table 1 shows the projected population at 31 December 2012 for each of the methods tested, for males and females aged 80-89 and 90-99, compared to actual population numbers. These projections are based on a fitting
Table 6.2: Projected population at 31 December 2012 by various methods for males and females aged 80-89 and 90-99 compared to actual population numbers, based on fitting period 1972-2012.

<table>
<thead>
<tr>
<th></th>
<th>Females</th>
<th></th>
<th>Males</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ages 80-89</td>
<td>Ages 90-99</td>
<td>Ages 80-89</td>
<td>Ages 90-99</td>
</tr>
<tr>
<td>Actual</td>
<td>425,987</td>
<td>101,924</td>
<td>300,792</td>
<td>43,609</td>
</tr>
<tr>
<td>Geometric</td>
<td>420,956</td>
<td>100,453</td>
<td>287,316</td>
<td>39,388</td>
</tr>
<tr>
<td>Ediev $R^2$</td>
<td>420,671</td>
<td>101,503</td>
<td>292,519</td>
<td>39,779</td>
</tr>
<tr>
<td>Lee Carter BMS</td>
<td>419,563</td>
<td>97,700</td>
<td>290,696</td>
<td>40,176</td>
</tr>
<tr>
<td>Modal Age and Concentration</td>
<td>417,423</td>
<td>117,028</td>
<td>301,903</td>
<td>47,822</td>
</tr>
<tr>
<td>Logistic Constant Beta</td>
<td>434,623</td>
<td>129,300</td>
<td>331,938</td>
<td>60,763</td>
</tr>
<tr>
<td>Ediev original</td>
<td>416,155</td>
<td>97,662</td>
<td>283,175</td>
<td>38,614</td>
</tr>
<tr>
<td>Gompertz</td>
<td>412,926</td>
<td>109,154</td>
<td>293,306</td>
<td>42,499</td>
</tr>
<tr>
<td>Lee Carter original</td>
<td>406,860</td>
<td>94,145</td>
<td>275,557</td>
<td>38,402</td>
</tr>
<tr>
<td>Relational</td>
<td>406,116</td>
<td>102,568</td>
<td>282,855</td>
<td>40,012</td>
</tr>
<tr>
<td>Logistic</td>
<td>402,049</td>
<td>105,180</td>
<td>278,814</td>
<td>39,763</td>
</tr>
<tr>
<td>Gompertz Constant Beta</td>
<td>450,471</td>
<td>141,128</td>
<td>351,721</td>
<td>70,086</td>
</tr>
<tr>
<td>Relational Constant Beta</td>
<td>396,830</td>
<td>99,226</td>
<td>268,779</td>
<td>37,619</td>
</tr>
<tr>
<td>Weibull</td>
<td>394,283</td>
<td>113,102</td>
<td>273,454</td>
<td>43,070</td>
</tr>
</tbody>
</table>

Source: Author’s calculations

For both sexes projected populations aged 80-89 were very accurate for most methods shown throughout the projection period with PEs falling within the range -5% to 5%. This means that for most methods projected numbers at 31 December 2012 for ages 80-89 were understated by up to 20,000. The least accurate method for females aged 80-89 was the Weibull method with PEs decreasing to -7.4% by 2012, or an understatement of over 31,000 females. For both females and males the Logistic method underprojected populations due to the overprojection of death rates at ages 60 to around 85. With the exception of the Logistic Constant Beta and Ediev’s method for males, projections for ages 80-89 from most of the methods were slightly too low. The constant slope parameter resulted in death rates projected by the Logistic Constant Beta, Gompertz Constant Beta and Relational Constant Beta methods that were too low, resulting in projected populations being too high. The increasing
difference between the actual and projected slope parameter and the cumulative effect over time resulted in rapidly increasing PEs.

Errors at ages 90-99 were higher and more variable. For females, PEs for the Geometric, Ediev, Lee-Carter BMS and Relational models remained low throughout the projection period and were mostly within the range -5% to 5%. For these methods, 2012 projected female numbers aged 90-99 differed from actual populations by no more than 4,300. Projected populations from the Logistic and Gompertz methods were too high by between 2% and 9% due to underprojection of death rates as discussed in section 6.4.1. For males aged 90-99 the Gompertz method was the most accurate, only slightly overstating projected populations for most of the projection period. PEs for the Geometric, Ediev $R^2$ and Lee-Carter BMS methods increased over time to around -8%, with populations progressively being underprojected. The Gompertz Constant Beta and Logistic Constant Beta methods were the least accurate, significantly overprojecting both female and male populations.
Figure 6.7: Percentage Error for projected populations aged 80-89 and 90-99, comparison of methods over the projection period from a 2002 jump-off date, fitting period 1972-2002, by sex
Source: Author’s own calculations using HMD data and own estimates
6.5 Conclusion

This chapter presented the results of forecast accuracy tests for a number of mortality extrapolation methods. Methods tested include some older methods not regularly used for forecasting such as the extrapolation of parameters of models fitted to the age profile of death rates, a relational model and a geometric model. A variant of the Geometric method was also tested whereby assumed rates of decline in death rates at different ages are based on fitting periods of varying lengths (Ediev, 2008) as well as the popular Lee-Carter method (Lee & Carter, 1992) and one of its variants known as the Lee-Carter BMS method (Booth et al., 2002).

The results have shown that the direct extrapolation of age-specific death rates assuming constant geometric rates of decline has been very successful in projecting both death rates and very elderly populations for Australia over the periods considered. This was consistently the case for both the 1992 and 2002 jump-off dates and throughout the 10 and 20-year projection horizons studied. The success of this method, especially for females, is underpinned by consistent rates of decline in death rates across adult ages from 1972 to 2012. Historical rates of decline in male death rates were less regular over time and between adult ages so that the Geometric method was slightly less successful for males, especially for longer fitting periods and beyond a 10-year projection horizon.

The Ediev and Lee-Carter methods also project mortality assuming constant geometric rates of change by age and sex and can be considered variants of the Geometric method. Differences between these methods relate to the fitting periods or methods employed for estimating the parameters. The Lee-Carter method and its variants use a single fitting period to estimate all age-specific parameters, similar to the Geometric method tested in this study. The original Lee-Carter method uses a fitting period starting from the earliest date data are available, in this case 1921, while the Lee-Carter BMS variant identifies a fitting period that best supports the assumption of constant rates of decline in death rates across the age range. Ediev’s method allows for varying trends in rates of decline in death rates between ages by applying different fitting periods for each age. By using the same fitting period across ages an assumption underlying the Geometric and Lee-Carter methods is that of constant relationships between rates of decline in death rates between ages.
This analysis has shown that differences in death rates and very elderly populations projected by the Geometric, Ediev and Lee-Carter BMS methods were small. It could be argued that little was gained in terms of accuracy from employing the additional procedures aimed at finding optimal fitting periods. However, this was only the case because of consistent rates of decline in Australian adult death rates since 1972 and consistent relationships in the reduction of rates between adult ages. If this were not the case, it is likely that these additional procedures would have significantly improved the results. The impact of applying a more appropriate fitting period was clearly seen from the significant improvement in accuracy between the original Lee-Carter method and the Lee-Carter BMS variant. This underlines the importance of using suitable fitting periods when applying extrapolation methods.

Forecasting death rates by the extrapolation of the parameters of functions fitted to the age profile of death rates or by extrapolating certain features of death frequency distributions proved to be successful for males but less so for females. In particular, application of the Gompertz function resulted in more accurate death rates and very elderly populations than any of the other methods tested. Such methods are more complex than those in which age-specific death rates are directly extrapolated and as a result, more conditions need to be met to ensure their accuracy. For example, the function needs to be both a good fit for death rates across the age range and the extrapolation of its parameters should reflect trends over the projection period. Continuing improvements in the fit of the Gompertz function to male adult death rates contributed greatly to the accuracy of projected death rates despite the slowing of trends in its parameters since around 2000. Results for the Relational model were quite similar to that for the Logistic model, resulting from their similar format. The Gompertz, Logistic and Relational methods where the slope parameter was held constant produced poor forecasts, because the slope parameter continued to increase in Australia over the period from around 2000, albeit at a slower pace than before.

The new method introduced in this paper, the Modal Age Projection model, produced quite accurate population estimates at ages 80-89, but overstated populations at ages 90-99. Very elderly populations could be more accurately projected by the direct extrapolation methods. This method may be improved by using non-linear functions to extrapolate trends in modal age and the concentration of deaths. For example, the slow-down in rates of increase in concentrations of deaths at the modal age may be better modelled with a logistic function.
Forecasting different rates of decline for different ages, as with the Geometric method and variants such as the Lee-Carter and Ediev methods, may result in the age profile of death rates becoming implausible over time (Girosi & King, 2007; Ediev, 2008). While this has not been found to be the case in this study, a procedure to prevent such implausible patterns from emerging has been suggested by Ediev (2008). This issue does not arise with the Relational model or the extrapolation of parametric functions because the forecast parameters vary only with the time dimension and are thus independent of age. If the rates of decline in death rates did not decrease with increasing age, the indirect methods studied in this paper are likely to have outperformed the direct extrapolation methods.

In conclusion, the simplest method, the Geometric method, consistently produced accurate, valid and plausible results when based on fitting periods that reflect regular and consistent patterns of decline in death rates across adult ages. Subject to the conditions of linearity being met, assumed rates of decline can be derived using simple linear regression techniques, reducing the need for specialist statistical knowledge. Consistently continuing trends in declines of adult death rates in Australia justify the use of the Geometric method for projecting very elderly populations over the next few decades. This method performed very well for Australian females over all periods studied and is considered suitable for future projections. While the Gompertz method was more accurate for Australian males, the simpler Geometric method also produced reasonably accurate results. If it is desirable to use the same method for both male and female projections, the Geometric method can be used for both. In countries where adult death rate declines have not been decreasing at stable rates over a number of decades, either the Lee-Carter BMS or Ediev’s Geometric methods may be applied to ensure assumed future rates of decline are based on appropriate fitting periods. These methods also performed consistently well over the periods studied, and the added procedures for finding appropriate fitting periods seem to be equally effective.
Chapter 7. Very elderly population projections for Australia

7.1 Introduction

Retrospective assessments of Australian Bureau of Statistics (ABS) population projections revealed that projected numbers for the 85+ age group were generally too low (Wilson, 2012a). According to Wilson (2007), high errors at older ages could largely be attributed to inaccurate mortality rate forecasts. The historical under-projection of very elderly populations stems from the over-projection of death rates, or, more specifically, under-projecting the extent to which death rates would decline in future. The final objective of this thesis is to create reliable very elderly population projections for Australia. Based on the results presented in chapter 6, it was found that both death rates and very elderly populations could be accurately projected with the Geometric, Ediev and Lee-Carter BMS methods. Differences between these methods were shown to be small. For males, extrapolating a Gompertz function fitted to the age profile of adult death rates also produced good results. In this chapter, the Geometric method, found to be both accurate and simple, is applied to create projected adult death rates for Australia. These are input into a cohort-component model to create very elderly population projections for the next 30 years at both national and state levels.

This chapter is organised as follows. Section 7.2 describes the cohort-component population projection model and the Geometric method used to project death rates. The assumptions relating to declines in death rates, the modelling of death rates at very high ages and migration are also described. Projected very elderly numbers for Australia at a national and state level are presented in section 7.3. This includes an analysis of future changes in the very elderly population’s age and sex composition and a decomposition of growth in nonagenarian and centenarian numbers into their demographic drivers. A comparison with ABS projections is given in section 7.4 and section 7.5 presents a conclusion.

7.2 Data and methods

7.2.1 Cohort-component model

A cohort-component model was used to create population projections by sex and single years of age (50-110) from the jump-off date 31 December 2012. Separate calculations were performed at a national level for Australia and the following states: New South Wales
The model was built in Excel based on the following formulae (Rees, 1990):

\[ P_{x+1,t+1} = P_{x,t} - 0.5m_c^c (P_{x+1,t+1} + P_{x,t}) + N_{x,t} \]  

(7.1)

where

\( P_{x,t} \) is the population aged x last birthday on 31 December of year \( t \),

\( N_{x,t} \) is net interstate and international migration flows at age x in year \( t \), and

\( m_c^c \) is the central death rate for cohort \( c \) in year \( t \), calculated as \( 0.5(m_{x,t} + m_{x+1,t}) \), with \( c = t - x \) and where \( m_{x,t} \) is the projected period death rate for age x in year \( t \), smoothed using cubic splines.

Rearranging the terms, this equation can be written as:

\[ P_{x+1,t+1} + 0.5m_c^c P_{x+1,t+1} = P_{x,t} - 0.5m_c^c P_{x,t} + N_{x,t} \]  

(7.2)

Solving for \( P_{x+1,t+1} \) gives:

\[ P_{x+1,t+1} = \frac{(1-0.5m_c^c)P_{x,t}+N_{x,t}}{(1+0.5m_c^c)} \]  

(7.3)

The derivation of projected death rates and net migration flows are discussed below. Given the focus of this study on the very elderly (ages 85+), and the projection horizon of 30 years, only death rates and migration at ages 50 and above are considered. Cohorts are modelled through their lifetimes, so that people aged 55 at the 2012 jump-off date are aged 85 in 2042.

### 7.2.2 Projected death rates

**Method**

The retrospective testing results presented in chapter 6 indicated that no single method consistently produced the most accurate population projections for males and females, different fitting periods and projection horizons. However, the Geometric method (Pollard, 1987), Ediev’s adjusted Geometric method (Ediev, 2008) and the BMS variant of the Lee-Carter method (Booth et al., 2002) all produced reasonably accurate results and differences between them were small. The Ediev and Lee-Carter BMS methods can both be considered variants of the Geometric method but with some of the subjectivity involved in the choice of
fitting period removed. Set procedures are applied to determine an appropriate fitting period based on goodness of fit criteria, either by single age or across an age range (Booth et al., 2002; Ediev, 2008). The regularity in the patterns of decline in adult death rates in Australia over the last three decades rendered the additional complexity introduced unnecessary, however, as these methods did not produce significantly more accurate results than the Geometric method. Therefore, due to both its accuracy and simplicity, the Geometric method was chosen for the death rate projections presented in this chapter.

The Geometric method extrapolates death rates over the projection horizon assuming constant rates of decline. Assumed future rates of decline for each sex and age are based on declines observed over the fitting period. The central death rate at time t for age x is modelled using equation 6.2. National level death rates were projected for single ages 50-100 from the jump-off date of 31 December 2012. Due to low volumes, death rates at ages above 100 were very volatile and could not be reliably extrapolated using the Geometric method. The approaches for projecting death rates at ages 100+ as well as state-level adjustments are described later. Historical death rates make use of the author’s population estimates derived from death counts based on the Extinct Cohort and Survivor Ratio methods (Chapter 3). At ages 95+, these estimates have been shown to be more accurate and plausible than official estimates.

**Fitting period**

Underlying the Geometric method is the assumption that death rates will decline exponentially over the projection horizon at similar rates to those observed over the fitting period. The validity of this assumption depends, in part, on the stability of rates of decline over the fitting period, as well as on consistent relationships between rates of decline at consecutive ages and between males and females. Given universal historical patterns, it is reasonable to expect male death rates to be higher than female death rates at each age, and to expect death rates for both males and females to increase with increasing age. If rates of decline at higher ages exceed those at younger ages and these are extrapolated over a sufficiently long period, the age profile of death rates may become distorted. There were no such distortions in the projected death rates in this study.
Figure 7.1 shows average annual rates of decline in death rates by age and sex over fitting periods starting from 1972, 1982, 1992 and 2002 to 31 December 2012. For females, death rates at ages 55-80 declined at a similar pace of around 2.4% per year. At ages 80+ death rates declined at decreasing rates with increasing age. Average rates of decline for females at ages 50-100 varied little between the fitting periods considered. Above age 100, rates of decline varied to a much greater extent between fitting periods, with increases from 1992 compared to declines from 1972, 1982 and 2002. There was also greater variation in the rates of decline between ages and fitting periods for males compared to females. Rates of decline in male death rates gradually increased with age from 50 to 70 and decreased with increasing age thereafter. Male death rates for ages 60-80 have declined more during 1992-2012 compared to earlier periods. Since 2002, death rates for ages 80+ have also declined more than in earlier periods. Across all fitting periods male death rates at ages 60-95 declined more than female rates.

It is desirable to reflect the greater declines in death rates for ages 85+ in the most recent period in the projections, as these are a more likely indication of future experience than earlier observed trends. However, due to the greater volatility of death rate changes at single ages, a 10-year projection period was considered too short. On the other hand, fitting periods starting from 1982 are more likely to underestimate future death rate declines if most recent trends continue. A fitting period of 1992-2012 was therefore chosen for both males and females. On average across both age ranges 50-100 and 80-100, the goodness-of-fit of fitted death rates was also greater over 1992-2012 than over the period 1982-2012.
Figure 7.1: Average annual rates of decline in death rates by age over fitting periods starting from 1972, 1982, 1992 and 2002, by sex

Source: Author’s estimates

Death rates at ages 100+

The volatility of observed death rates at ages 100+ required a different projection approach. Several approaches for smoothing projected death rates for ages 50-100 and extrapolating to
ages 101-110 were tested. Fitting cubic splines to projected death rates for ages 50-100 worked well for smoothing, but extrapolation to ages 101-110 resulted in rates that seemed too high. The Gompertz (1825) and Kannisto’s logistic functions (Thatcher et al., 1998) fitted to projected unsmoothed rates across ages 50-100 both resulted in fitted death rates that were too high around ages 60-85 and too low at ages 85-100. Centenarian death rates were too high under the Gompertz function and too low under the Logistic function. Both the Gompertz and Logistic functions were also fitted to projected rates in each year across the age range 90-100. For both males and females, death rates for ages 101-110 extrapolated from the Logistic function fitted to ages 90-100 in the jump-off year seemed to be most consistent with average observed rates over the last ten years. These rates also agreed more closely with those from the International Longevity Database (ILD). Gampe (2010) and Robine and Vaupel (2002) found that, based on data in the ILD, the probability of death after age 110 is roughly flat at around 0.5, which approximates to a central death rate of 0.7. Fitting a Logistic function to projected rates for ages 90-100 in each year over the projection horizon, however, resulted in death rates for ages 101-110 increasing over time, which did not seem plausible. Therefore, jump-off year rates at age 110 were kept constant over the projection horizon and rates for ages 101-109 were derived by interpolation.

To summarise, geometrically projected death rates for ages 50-100 in each year of the projection horizon were smoothed over age by way of cubic splines. Death rates in 2012 for ages 101-110 were derived by fitting a logistic curve over ages 90-100 and extrapolating to age 110. Death rates for age 110 were held constant over the projection horizon at fitted 2012 levels and rates for ages 101-109 in each projection year derived by interpolation. Separate calculations were performed for males and females. Figure 7.2 shows the resulting projected death rates for males and females at ages 50-110 in 2022, 2032 and 2042, as well as actual observed rates in 1982, 1992, 2002 and 2012. Projected death rates for ages 101-110 improve only slightly over the projection horizon.
Figure 7.2: Actual death rates for ages 50-109 in 1982, 1992, 2002 and 2012 and projected death rates in 2022, 2032 and 2042, by sex

Source: Author’s estimates and projections

**Variation in death rates by state**

The level of observed death rates varied between the different Australian states. Ratios of female death rates at each age from 50-100 by state relative to the national average death rates are shown in Figure 7.3. Death rates were averaged over 2003-2012 and the ratios
smoothed. Female death rates in the combined Tas-NT-ACT were more than 20% above the national average while those in Qld were more than 5% higher; death rates in WA, SA and Vic were below the average. The largest differences between state death rates and the national average were observed across ages 55-75. Similar patterns were observed for males. State-level death rates were derived by applying these ratios to projected national death rates, and the ratios were assumed to remain constant throughout the projection period.

Figure 7.3: Ratios of female death rates at ages 50-100 in 2003-2012 relative to the national average, by state
Source: Author’s estimates

7.2.3 Migration Assumptions

Projections at a national level allow for international migration only, while projections at the state level allow for both interstate and international migration. In order to ensure that total net migration flows across the states equal international migration flows, or that total interstate migration is zero, migration flows rather than rates were used. These flows were held constant over the projection horizon. Net interstate migration flows at each age and sex were derived from 2011 Census data, calculated as interstate and overseas moves into the state minus interstate and international moves out of the state (more detail in section 3.2.1). Data on interstate moves and immigration over the previous five years by single age from
50-99 and ages 100+ were extracted from the ABS Census website using Tablebuilder. Emigration from each state was estimated from the ratios of arrivals to departures for ages 65+ in 2006-2011 obtained from the ABS. Annual net migration flows by state and age at census date for females are shown in Figure 7.4. Data for males are similar.

![Figure 7.4: Annual net migration flows for females at ages 50-100, by state](image)

Source: ABS 2011 Census data and ABS

Conceptually, the use of directional in- and out-migration rates rather than constant net migration flows would have been more appropriate. However, this would have significantly increased the complexity of the projection model, and at the ages considered, migration has very little impact on the projections. For example, the projected very elderly population in 2042 for Qld, which were most affected, was only 1.7% higher than if zero net migration had been assumed.

7.3 Projected very elderly population

7.3.1 National projections

Figure 7.5 shows the projected growth of Australia’s very elderly population to 2042. Actual populations from 1982 are also shown for comparison purposes. The very elderly population of Australia is projected to continue growing rapidly over the next 30 years, from 430,000 in
2012 to almost 1.5 million in 2042. As a percentage of the total Australian population, it is expected to increase from 1.8% in 2012 to 4.2% in 2042. This reflects an average annual growth rate of 4.2%. Of particular interest is the accelerated growth from around the 2030s when the baby boomers enter this age group. The number of very elderly males is projected to grow at a faster rate than that of females, with an average annual growth rate of 4.9% compared to 3.7% for females. This compares to average growth rates of 5.8% per year for males and 4.3% for females from 1982 to 2012. The faster growth of males is the result of greater projected mortality declines, especially at elderly and very elderly ages. This is discussed in more detail in section 7.3.3.

Figure 7.5: Growth in Australia’s very elderly population (ages 85+) from 1982-2012 (actual) and to 2042 (projected), by sex
Source: Author’s estimates and projections

Figure 7.6 shows the numbers of males and females at ages 85 to 105+ in 1982, 2012 and projected numbers in 2042. Figure 7.7 shows total growth over 1982-2012 and over 2012-2042 at each age. Growth in male numbers during 2012-2042 increased with age and exceeded those of females at all ages. Males aged 93+ are expected to increase more during 2012-2042 than in 1982-2012. In the case of females growth at all ages are expected to be lower during 2012-2042 than during 1982-2012 and do not consistently increase with age as
was the case during 1982-2012. People aged 85+ in 1982 were born in 1873-1897; those aged 85+ in 2012 were born in 1903-1927; and those aged 85+ in 2042 were born in 1933-1957. Births in 1933-1957 for both males and females were higher than in 1903-1927, but increased to a smaller extent than during the previous 30 years. For females, cohort survival at adult and elderly ages for the 1933-1957 cohorts improved to a smaller extent compared to the 1903-1927 cohorts, than was the case for the 1903-1927 cohorts compared to the 1873-1897 cohorts. In contrast, survival improvement for the more recent male cohorts (1933-1957 compared to 1903-1927 cohorts) exceeded that of the older cohorts (1903-1927 compared to 1873-1897 cohorts). These factors resulted in greater increases in very elderly males during 2012-2042 compared to 1982-2012, but smaller increases for females. The drivers of growth are discussed in more detail in section 7.3.3.

![Figure 7.6: Males and females at single ages 85-105+ in 1982 (actual), 2012 (actual) and 2042 (projected)](image)
Source: Author’s estimates and projections

![Figure 7.7: Percentage increase in males and females at single ages 85-105+ from 1982-2012 and 2012-2042](image)
The number of centenarians is projected to grow from 3,388 in 2012 to 15,614 in 2042. Centenarian males are projected to increase at an average rate of 7.3% and females at 4.7% per year compared to 6.4% from 1982 to 2012 for both males and females. Semi-super centenarians (ages 105+) are projected to increase from 15 males and 135 females in 2012 to 148 males and 573 females in 2042. The number of centenarians per million of the total population is projected to increase from 147.8 in 2012 to 448.1 in 2042.

The increasing growth rates with age shown in Figure 7.7 will result in a further ageing of the very elderly population. Table 7.1 shows future expected changes in the proportional split of the total very elderly population in the age ranges 85-89, 90-99 and 100+. From the table it is clear that the trend of the ageing of the very elderly group itself observed over the last 30 years (as shown in Table 4.1) is expected to continue over the next 30 years. The proportion aged 85-89 is projected to decrease from 65.4% in 2012 to 58.8%, while the proportion of nonagenarians is expected to increase from 33.8% to 40.2% and centenarians from 0.79% to 1.06%.

<table>
<thead>
<tr>
<th>31 December</th>
<th>Age Group</th>
<th>85-89</th>
<th>90-99</th>
<th>100+</th>
<th>85+</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>65.4</td>
<td>33.8</td>
<td>0.79</td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>2022</td>
<td>61.5</td>
<td>37.4</td>
<td>1.09</td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>2032</td>
<td>64.8</td>
<td>34.2</td>
<td>0.98</td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>2042</td>
<td>58.8</td>
<td>40.2</td>
<td>1.07</td>
<td></td>
<td>100</td>
</tr>
</tbody>
</table>

Source: Author’s estimates and projections

The faster expected growth of very elderly males will result in increasing sex ratios, as shown in Figure 7.8. The increasing values of sex ratios at ages 85-89, 90-99 and 100+ that started with cohorts born from around 1897 are projected to continue for cohorts born up to around the end of the 1930s. By the time cohorts born in the 1940s are aged 85+, there is expected to be 0.8 males for every female, compared to only 0.6 for cohorts born in the 1920s.
Figure 7.8: Ratio of males to females at ages 85-89, 90-99 and 100+ for Australian cohorts born from 1882-1952
Source: Author’s population estimates and projections

7.3.2 Projections by state

Very elderly populations in all the states are expected to grow significantly over the next three decades, albeit at different rates. Table 7.2 shows projected very elderly populations by sex and state in 2022, 2032 and 2042, compared to actual populations in 2012. Very elderly populations in WA, Tas-NT-ACT, and Qld are projected to grow the most over the 30 years, at average annual rates of 4.9%, 4.8% and 4.6% respectively. The least growth is expected in SA (3.5%), NSW (3.9%) and Vic (4.0%).
Table 7.2: Projected very elderly populations by sex and state

<table>
<thead>
<tr>
<th></th>
<th>2012</th>
<th>2022</th>
<th>2032</th>
<th>2042</th>
</tr>
</thead>
<tbody>
<tr>
<td>Females</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NSW</td>
<td>97,330</td>
<td>118,285</td>
<td>177,841</td>
<td>262,934</td>
</tr>
<tr>
<td>Vic</td>
<td>72,273</td>
<td>90,467</td>
<td>137,228</td>
<td>206,005</td>
</tr>
<tr>
<td>Qld</td>
<td>47,610</td>
<td>61,828</td>
<td>105,054</td>
<td>161,683</td>
</tr>
<tr>
<td>SA</td>
<td>26,251</td>
<td>30,140</td>
<td>44,719</td>
<td>64,800</td>
</tr>
<tr>
<td>WA</td>
<td>24,130</td>
<td>32,964</td>
<td>53,296</td>
<td>86,723</td>
</tr>
<tr>
<td>Tas-N-T-ACT</td>
<td>10,665</td>
<td>13,278</td>
<td>22,520</td>
<td>36,414</td>
</tr>
<tr>
<td>Australia</td>
<td>278,258</td>
<td>346,963</td>
<td>540,658</td>
<td>818,558</td>
</tr>
<tr>
<td>Males</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NSW</td>
<td>52,457</td>
<td>76,697</td>
<td>133,822</td>
<td>205,081</td>
</tr>
<tr>
<td>Vic</td>
<td>39,463</td>
<td>59,049</td>
<td>102,031</td>
<td>159,241</td>
</tr>
<tr>
<td>Qld</td>
<td>26,686</td>
<td>40,879</td>
<td>79,713</td>
<td>125,406</td>
</tr>
<tr>
<td>SA</td>
<td>14,011</td>
<td>19,214</td>
<td>32,576</td>
<td>49,754</td>
</tr>
<tr>
<td>WA</td>
<td>13,179</td>
<td>21,395</td>
<td>40,666</td>
<td>69,149</td>
</tr>
<tr>
<td>Tas-N-T-ACT</td>
<td>5,890</td>
<td>9,122</td>
<td>18,168</td>
<td>30,374</td>
</tr>
<tr>
<td>Australia</td>
<td>151,685</td>
<td>226,356</td>
<td>406,975</td>
<td>639,004</td>
</tr>
</tbody>
</table>

Source: Author’s estimates and projections

Centenarian numbers in all Australian states are projected to grow rapidly, but at varying rates. Table 7.3 shows projected numbers of centenarians in each state in 2022, 2032 and 2042. Male and female centenarian numbers are projected to grow the most in WA at average rates of 8.2% and 5.2%, compared to Tas-N-T-ACT where numbers will grow the least at 6.9% and 4.2% respectively.
Table 7.3: Projected number of centenarians in 2022, 2032 and 2042, by sex and state

<table>
<thead>
<tr>
<th></th>
<th>2012</th>
<th>2022</th>
<th>2032</th>
<th>2042</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Females</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NSW</td>
<td>997</td>
<td>1,708</td>
<td>2,358</td>
<td>3,635</td>
</tr>
<tr>
<td>Vic</td>
<td>740</td>
<td>1,316</td>
<td>1,866</td>
<td>2,895</td>
</tr>
<tr>
<td>Qld</td>
<td>488</td>
<td>888</td>
<td>1,282</td>
<td>2,225</td>
</tr>
<tr>
<td>SA</td>
<td>269</td>
<td>452</td>
<td>596</td>
<td>889</td>
</tr>
<tr>
<td>WA</td>
<td>247</td>
<td>445</td>
<td>685</td>
<td>1,149</td>
</tr>
<tr>
<td>Tas-NT-ACT</td>
<td>109</td>
<td>160</td>
<td>218</td>
<td>377</td>
</tr>
<tr>
<td><strong>Australia</strong></td>
<td>2,850</td>
<td>4,968</td>
<td>7,005</td>
<td>11,171</td>
</tr>
<tr>
<td><strong>Males</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NSW</td>
<td>186</td>
<td>431</td>
<td>771</td>
<td>1,451</td>
</tr>
<tr>
<td>Vic</td>
<td>140</td>
<td>338</td>
<td>621</td>
<td>1,166</td>
</tr>
<tr>
<td>Qld</td>
<td>95</td>
<td>203</td>
<td>371</td>
<td>794</td>
</tr>
<tr>
<td>SA</td>
<td>50</td>
<td>124</td>
<td>209</td>
<td>376</td>
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<tr>
<td>WA</td>
<td>47</td>
<td>124</td>
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<tr>
<td>Tas-NT-ACT</td>
<td>21</td>
<td>40</td>
<td>73</td>
<td>157</td>
</tr>
<tr>
<td><strong>Australia</strong></td>
<td>538</td>
<td>1,259</td>
<td>2,285</td>
<td>4,443</td>
</tr>
</tbody>
</table>

Source: Author’s estimates and projections

7.3.3 Decomposition of growth

What demographic factors will drive these substantial increases in Australia’s very elderly population? Figures 7.9 and 7.10 show how projected growth in nonagenarians (ages 90-99) and centenarians (ages 100+) from 2012 to 2042 is driven by increases in births, net migration and projected survival improvements. The improvement in survival is measured from birth to age 65, age 65 to 85, and 85 to 90 and 100 respectively, as well as beyond ages 90 and 100 respectively. Figure 7.9 compares the experience of 1913-1922 and 1943-1952 cohorts, while Figure 7.10 compares 1903-1912 and 1933-1942 cohorts. The decomposition method used is that described in section 4.3.1.

Male nonagenarians are expected to increase 5.6 times and females 3.3 times over the period 2012-2042. This compares to increases of 6.0 times and 4.4 times for males and females respectively between 1981 and 2012 (see section 4.3.2). This growth is attributed to a 33%
(males and females) increase in births from 1913-1922 to 1943-1952, a 124% (females) and 281% (males) projected improvement in the survival of these cohorts at all ages and an increase in net migration of 12% (females) and 11% (males). It is expected that around 35% of males and 47% of females born in 1943-1952 will survive to age 90, compared to 11% and 24% respectively of males and females born in 1913-1922. Consistent with experience over the previous 30 years, male survival from age 65 to 85 is expected to improve the most.

![Factor increases in births, survival and net migration, explaining growth in population aged 90-99 from 2012-2042](image)

**Figure 7.9:** Factor increases in births, survival and net migration, explaining growth in population aged 90-99 from 2012-2042

Source: Author’s calculations

Survival at elderly and very elderly ages is projected to improve substantially for the 1933-1942 cohorts compared to the 1903-1912 cohorts, as is clear from Figure 7.10. These improvements will significantly drive increasing centenarian numbers from 2012 to 2042, especially males. Increases in survival at younger ages, births and net migration are also expected to positively contribute to increasing centenarian numbers. Survival beyond age 100 is projected to change very little, and the slight decrease shown in Figure 7.10 is largely due to smoothing of projected rates.
Figure 7.10: Factor increases in births, survival and net migration, explaining growth in population aged 100+ from 2012-2042
Source: Author’s calculations

7.4 Comparison with ABS projections

How do these projections of the very elderly compare with those of the ABS? The ABS produces a large number of projections reflecting different combinations of assumptions for mortality, fertility and migration. Series B is widely regarded as the principal projection series. Mortality assumptions underlying the Series B projection from a 2012 jump-off year involve assumed rates of change in age-specific death rates up to 2025-26, based on experience during 1996-2011 (ABS, 2013c). Thereafter, death rates are scaled to reflect the projected ‘medium scenario’ life expectancy at birth. Under the medium scenario, life expectancy at birth is assumed to increase at historical rates only until 2015-16 and at declining rates thereafter.

Figure 7.11 shows a comparison of the author’s projections with ABS Series B projections for very elderly populations from 2012 to 2042, by sex. Actual populations from 1982 are also shown. The author’s and the ABS’ projections were very similar in the first 13 years of the projection horizon but diverge thereafter. From around 2025, the author’s projections
exceed the ABS’ projections with the percentage difference for both males and females increasing over the projection horizon. By 2042 the author’s projected very elderly male and female populations are respectively 19% and 9% higher than those of the ABS, or 13% higher in total. Compared to actual numbers in 2011, ABS projected populations aged 85+ from a 1992 jump-off date (ABS, 1993b) turned out to be too low by 18% for males and 10% for females.

![Figure 7.11: Comparison of author’s and ABS’ Series B projected population aged 85+, actual from 1982 and projected from 2012](image)

Source: Author’s estimates and ABS (2013c)

There are a number of factors contributing to the differences between the ABS’ and the author’s projected very elderly populations. These relate mainly to death rates in the jump-off year and death rate declines projected over the projection horizon. The difference in projected death rate declines accounts for most of the difference in projected very elderly populations. From 2025-26 the ABS’ Series B projection reflects mortality rates derived from projected life expectancy at birth which were assumed to increase at declining rates from 2015-16. The mortality improvements implied by this projected life expectancy at birth are thus lower than when past rates of decline by age are assumed to continue. For example, the author’s death rates for females at age 85 were projected to decline by 41% over the projection horizon.
2012-2042 compared to the ABS’ 25% projected decline to 2042. Life expectancy at age 55, the age of the youngest cohort at the jump-off date, was projected by the author to increase by 13% and 20% for females and males respectively, compared to the ABS’ 8% and 12%. Ignoring the effect of different jump-off death rates, the author’s higher assumed rates of mortality decline resulted in projected very elderly populations that were around 22% (males) and 10% (females) higher in 2042 compared to the ABS projections.

This use of different jump-off rates had the opposite effect on projected populations and thus countered some of the impact of the different assumed death rate declines. The age profile of death rates in the jump-off year used by the ABS start to plateau at around age 85, so that death rates from age 85 are lower than those used by the author, with the differences increasing with age. The ABS’ underestimated death rates stem largely from the use of overstated population estimates at the high ages which determine the denominators in the death rate calculations, as discussed in chapter 3. For example, at 31 December 2012, death rates used by the author for age 95 were 26% higher than the ABS’ death rates for females and 43% higher for males. At the jump-off date, ABS life expectancy at age 85 was 3% higher than the author’s for females and 7% higher for males. These jump-off rates would have resulted in the projected very elderly populations being 5% lower than that of the ABS in the case of males and 2% in the case of females, if the author's projected death rate declines were assumed. The balance of the difference in projected populations is mainly due to different assumptions for net overseas migration. Up to age 80 the ABS assumed much larger net overseas migration flows than the author, and negative flows from around age 85.

Figure 7.12 shows a comparison of the author’s projections with ABS Series B projections for centenarians from 2012 to 2042, by sex. Whereas the ABS projected lower very elderly populations compared to the author’s, the ABS Series B projected numbers of centenarians were substantially higher. By 31 December 2042, the ABS projected 8,437 and 15,821 centenarian males and females respectively, compared to the author’s projections of 4,443 and 11,171. ABS projections of male centenarians were thus 90% higher and females 40% higher, or 55% in total. Alternatively, the author’s projected number of centenarians was 35% lower than official projections. In light of historical growth and the drivers of this growth presented in Figure 7.10, the ABS’ projections seem unrealistically high. For both males and females the percentage differences between the author’s and ABS’ projected centenarians increase until 2026 and decrease thereafter. Until 2025-26, age-specific death rates
underlying the Series B projection reflect historical patterns and rates of decline, after which rates were scaled to reflect a projected life expectancy at birth which is modelled to increase at declining rates from 2015-16. This resulted in a slight narrowing of the gap between the author’s and ABS’ projections.

As with the very elderly population, differences between the ABS’ and the author’s projected centenarians can be ascribed to differences in both jump-off populations and death rates and projected death rate declines. Early in the projection horizon, differences in jump-off centenarian population estimates contributed most, especially for males. The ABS’ estimated numbers of centenarians at 31 December 2012 are 711 males and 2,908 females, compared to the author’s estimates of 538 and 2,850 respectively. These differences are due to different population estimation methods. The author’s estimates were derived from death counts based on the Extinct Cohort and Survivor Ratio methods (discussed in detail in Chapter 3), while the ABS’ estimated resident populations were derived from census counts. The effect of this difference gradually reduces over the projection horizon as these cohorts become extinct. These population estimates, however, also resulted in lower death rates at the high ages,
which caused most of the overstatement in the projected number of centenarians. In the jump-off year the ABS’ death rates at age 100 were 40% and 31% lower than the author’s for males and females respectively. Despite assuming lower rates of decline, the ABS’ projected death rates at the high ages (90 and older) were lower than the author’s throughout the projection period. Together with higher assumed net migration flows at ages up to around 85, the lower death rates at ages 90 and older means that more people are projected by the ABS to live to age 100. The ABS uses single-age death rates to age 100 and assumes a probability of death of one at age 101, equivalent to a death rate of approximately 0.67. In comparison, the author used assumed death rates for single ages up to 109 and around 0.76 and 0.71 at age 110+ for males and females respectively in the jump-off year. The ABS’ higher assumed death rate at age 101 resulted in a lower life expectancy at age 100, which counters some of the impact of higher assumed survival to age 100.

The ABS only produced projections for centenarians from 2004. Until then, projections were provided for single ages until 84, and in aggregate for ages 85+. Figure 7.13 shows ABS (Series B) projected numbers of centenarians from 2004, 2006 and 2012 to 2020, compared to actual numbers in 2004-2012. The author’s projections from 2012-2020 are also shown for comparison purposes. The graph clearly demonstrates the extent of the problem. The ABS’ projected centenarian populations were reduced at each subsequent projection date but continued to be too high.
Figure 7.13: ABS’ Series B projected population aged 100+ from 2004, 2006 and 2012, compared with actual numbers from 2004-2012 and the author’s projections from 2012-2020

Source: Author’s estimates and projections and ABS (2005, 2008b, 2013c)

The Australian Government Treasury’s 2015 Intergenerational Report considered the fiscal implications of expected changes in Australia’s population structure under various policy scenarios (Treasury 2015). The Treasury made use of ABS estimates for the 2014 jump-off
year and their own projection assumptions (Treasury, 2015). Projected mortality rates are derived from assumed future changes in life expectancy at birth by sex, which are higher than the ABS’ Series B assumptions. Despite using higher life expectancy assumptions, the Treasury’s projected very elderly population in 2035 is still around 10% lower than the author’s projections, while projected centenarians are 25% higher. These differences seem to stem from different mortality forecast methods and assumptions in the case of very elderly population numbers, and different jump-off population estimates in the case of centenarians.

7.5 Conclusion

The aim of this chapter was to present reliable projections of Australia’s very elderly population at a state and national level, by sex and for single ages 85-110+ from 2012 to 2042. This chapter also contributes a detailed understanding of the accuracy of official very elderly population projections. Accurate and detailed very elderly projections at the national and state level are vital for the effective planning of this population group’s extensive service needs. Official population projections are currently provided only in aggregate for ages 100+ (national) and 85+ (state-level). Furthermore, the ABS’ 1992 very elderly population projections to 2011 turned out to be too low by 18% for males and 10% for females. Various local and international studies suggested that the tendency for official projections of very elderly populations to be too low was due to the use of expectation methods in forecasting mortality, and in particular overly conservative targets (Lee & Carter, 1992; Murphy, 1995; Olshansky, 1988; Wilson, 2007, 2012). In addition, in earlier chapters, large errors in official census-based estimates for populations aged 90+ were identified. Erroneous population estimates result in unreliable death rates, which, when forecasted, impact population projections. The projections presented in this chapter are based on mortality forecasted using the Geometric method and on very elderly population estimates and death rates derived only from death counts.

The Geometric method was chosen for its simplicity and accuracy in projecting Australian adult death rates and very elderly populations. While Ediev’s adjusted Geometric method (Ediev, 2008) and the Lee-Carter BMS method (Booth et al., 2002) also produced accurate projections for Australia, the continuing consistency in adult death rate trends observed over recent decades justified the use of the simpler Geometric method. While judgement is involved in the choice of an appropriate fitting period, the Geometric method assumes a continuation of assumed rates of mortality decline and applies no subjective limit on the
extent of future mortality decline. This resulted in greater assumed mortality improvements than those in ABS projections.

Australia’s total very elderly population is projected to increase from 430,000 in 2012 to almost 1.5 million in 2042. While very elderly populations in all the Australian states are expected to grow rapidly, the least growth is expected in South Australia and the most in Western Australia. Centenarians are projected to increase from 3,388 to over 15,600 over the same period. The Australian population will continue to age with the proportion of very elderly increasing from 1.9% of the total population to 4.2%. Growth is expected to accelerate from the 2030s when the baby boomers enter this age group.

Past ABS projections of Australia’s very elderly populations have turned out to be too low compared to actual population numbers, while their projected centenarian numbers were significantly overstated. Similarly, from a 2012 jump-off date, the very elderly population projections presented here are 19% (males) and 9% (females) higher by 2042 than the ABS’ projections. ABS projections of male centenarians were 90% higher and females 40% higher than the author’s, or 55% higher in total. These differences are ascribed to different jump-off population estimates and death rates as well as to different projection approaches. The limits applied by the ABS to the extent death rates are assumed to decline in future resulted in their under-projection of very elderly populations. The ABS’ underestimation of death rates at ages 90 and older in the jump-off and earlier years, resulting from overestimating these population numbers from census counts, resulted in their significant over-projection of centenarian numbers.

The projections presented in this chapter indicated that Australia’s very elderly population will itself age as more people continue to live to higher ages, resulting in increasing proportions of nonagenarians and centenarians. Increases in birth numbers over the first half of the twentieth century, improvements in cohort survival and increases in net migration are all expected to contribute to increases in very elderly numbers in the coming decades. Projected improvements in survival from age 65 to 85 and from age 85 to 100 especially are expected to drive rapidly growing numbers of nonagenarians and centenarians over the next 30 years. Cognitive, physical and psychological functional limitations increase with age, with resulting increases in the need for care (ABS, 2010). If the incidence of chronic conditions and disability do not decrease as a proportion, increasing very elderly numbers will result in
greater demands for aged care (Crimmins et al., 1994; Crimmins, 2004). Greater projected improvement in male survival means that very elderly male numbers will grow faster than females, resulting in increasing sex ratios. This could have implications for the type of health care required, the availability of carers and possibly living arrangements.

In closing, it is acknowledged that the projections presented in this chapter are based on the assumption that rates of change in death rates observed over the last two decades will continue in the next three decades. While the Geometric method produced accurate results for projection horizons ending in 2012, there is no guarantee that this will continue to be the case. All projections are subject to limitations relating to the quality of data used and subjective judgment exercised. However, given the use of high quality very elderly population estimates and death rates, and mortality forecasting methods shown to produce accurate projections, it is believed that these projections are an improvement on projections currently available for ages 90+.

The low volumes and hence high volatility of data at ages 100+ allowed only a very approximate approach to projecting death rates for these ages. Increasing population numbers living to beyond age 100 in future decades will enable more accurate modelling of death rates and projections for these ages.
Chapter 8. Conclusion

8.1 Background

The substantial growth in very elderly populations worldwide requires a greater focus on planning for the significant service needs of this unique age group, and thus a need for reliable data. The aim of this thesis was to create accurate estimates and projections for Australia’s very elderly population at a state and national level. This aim was met by retrospectively evaluating the accuracy of very elderly population estimation methods and extrapolative mortality forecasting methods. Mortality forecasting methods were also conceptually compared and evaluated. The insights gained were used to create very elderly estimates and life tables from 1971 to 2012 and projections to 2042 by sex and state at single ages 85-110+. These estimates, life tables and projections were furthermore analysed to improve our understanding of the changes in the age and sex distribution, and drivers of growth, of the very elderly as well as trends in death rates and death frequencies. Four objectives were addressed in this thesis. Section 8.2 presents a summary of the findings for each objective. The main contributions of this study are summarised in section 8.3. This is followed by a discussion of the limitations of this research in section 8.4 and suggested further research in section 8.5.

8.2 Summary of findings

8.2.1 Objective 1: To identify appropriate very elderly population estimation methods

In order to meet the aim of creating accurate estimates for Australia’s very elderly population, appropriate estimation methods needed to be identified. This was the first objective of this thesis. The official estimates provided by the Australian Bureau of Statistics, Estimated Resident Population (ERPs), are derived from census figures. Previous research based on data for a number of low-mortality countries indicated that census-based estimates for high ages are problematic and more accurate estimates can be derived from death counts. The Extinct Cohort method involves no approximations and can be used to derive population numbers from death counts for cohorts for which all the members have died. A number of methods, collectively referred to as nearly-extinct-cohort estimation methods, are available for estimating cohorts which are not fully extinct. In this study, the relative accuracy of various nearly-extinct-cohort estimation methods for creating population estimates at ages 85 and above based on deaths data were assessed. The methods were retrospectively applied to
extinct cohorts and the results compared against those obtained from applying the Extinct Cohort method. The methods assessed here for Australia include some of the methods tested in earlier studies for other countries, including that used by the Human Mortality Database (HMD), as well as a new approach to allow for mortality decline. This includes the Survivor ratio and Das Gupta methods, and the new Survivor Ratio Advanced method. A number of variants of these methods were considered reflecting different age ranges over which the survivor ratio is measured, the number of older cohorts over which the survivor ratio or death ratio is averaged, and the application of different constraints to the resulting estimates. The accuracy of the ABS’ official estimates, estimated resident populations (ERPs), was also assessed. This is the first such evaluation for Australia and it was the first time these methods were evaluated at a sub-national level.

This study found that the accuracy of official estimates provided by the ABS for ages 95+ was poor and deteriorated rapidly with increasing age, with errors varying greatly from year to year. The ABS substantially overestimated the number of male centenarians, while female centenarians were significantly overestimated in some years and underestimated in others. Methods based on death counts proved to be more accurate than census-based estimates and produced more plausible estimates. It was found that the method used in the HMD, the Survivor Ratio method with total estimates for ages 90+ constrained to the ABS’ ERPs, works well for Australian females. However, more accurate estimates may be obtained for Australian males and on average across males and females, by constraining results from the Survivor Ratio method to ERPs for ages 85+ rather than 90+.

This study furthermore showed that these methods can be successfully applied at a sub-national level, and the impact of internal migration was small. Accurate state-level estimates could also be derived by apportioning national estimates, obtained by applying nearly-extinct-cohort estimation methods, between states. Given similar findings for national and sub-national application, the methods and principles should be applicable in other contexts and for other sub-national geographies, subject to the availability of sufficient volumes of data.

Based on these findings, very elderly population estimates for 1971-2012 were created for Australia based on the Extinct Cohort and the Survivor Ratio methods applied to national level deaths, with total estimates constrained to 85+ ERPs. State-level estimates at single ages
were derived from national estimates by applying state-specific proportions of 85+ ERPs. Life table values were created from these estimates.

8.2.2 Objective 2: To understand changes in the age- and sex distribution of Australia’s very elderly population and the demographic drivers of growth

An analysis of the very elderly estimates created under the first objective has improved our understanding of the changing demographic characteristics of Australia’s very elderly population and the demographic drivers of growth. Australia’s very elderly population has grown substantially over the last three decades, from 105,000 or 0.7% of the total population at 31 December 1981 to 430,000 or 1.9% of the total population at 31 December 2012. Growth rates increased with age, resulting in the composition of the very elderly age group to be more heavily weighted towards older people. The gap between the numbers of females and males gradually increased for birth cohorts from 1872 to 1895, resulting in a decreasing sex ratio. From the mid-1980s (cohorts born after 1895), growth of the number of very elderly males accelerated and exceeded that of females resulting in increasing sex ratios.

While the growth of centenarians in Australia is similar to that in European countries, it is well below that in Japan. Centenarians comprise a much lower proportion of the total population in Australia than in many other low-mortality countries. Growth in very elderly numbers showed similar trends in all the Australian states, although growth rates were slightly higher in WA and Qld, and lower in SA, Vic and NSW. SA has the highest proportion of centenarians while Tas-NT-ACT has the lowest.

Nonagenarians increased by a factor of 4.4 for females and 6.0 for males from 1981 to 2012. Centenarians increased by factors of 6.8 and 7.0 for females and males respectively. Increasing birth cohort sizes and improvements in survival at all ages played important roles in the growth of nonagenarian and centenarian populations in Australia. However, by far the most significant factor driving the increasing numbers of nonagenarians has been an improvement in survival between ages 65 and 85, especially for males. Improvement in survival between ages 85-100 was the most important driver of growth in centenarian numbers. This was consistent with findings in other low-mortality countries. Because net migration did not increase greatly between the cohorts considered, this factor has not contributed much to the growth of these populations, although the selective effect of migration may have contributed to higher survival rates.
From 1981 to 2012 the number of centenarians in WA increased by a factor of 7.1, followed by Qld And NSW with factor increases of 7.0 each. In WA, the huge increase in births following the influx of people during the gold rush in the 1890s was the largest driver of growing centenarian numbers from 1981 to 2012. This was, however, partly offset by lower net migration rates in later years. The gold rush occurred later in WA than in the other states and similar patterns of high net migration and birth rates would, however, have been seen in those states if the period of investigation started earlier. In the other states, consistent with national-level results, improvements in survival from age 85-100 were by far the most important drivers of growth.

### 8.2.3 Objective 3: To understand changes in adult mortality

The third objective was to understand the changing patterns and trends in Australian mortality, with a view of developing insights into likely future changes. For this purpose, data from the Human Mortality Database from 1921 were combined with life table data from 1971 developed as part of the first objective. This analysis, discussed in chapter 5, included a study of changes in the age profile of death rates, death frequency distributions, and the interactions between them. It was found that death rates for males and females in Australia at all ages (with the exception of ages 100+) declined substantially over the last nine decades, albeit to a decreasing extent with increasing age. Three broad periods were identified showing different patterns of change namely:

- the 1920s to the 1950s, a period of decline in death rates of children and young adults driving increases in life expectancy at birth, a greater compression of life spans (or the death frequency distribution) and unchanged modal ages;
- the 1950s to the 1970s, characterised by a slowing down of declines in death rates at the younger ages and an acceleration of death rate declines at ages 45+, with very little change in life expectancy at birth, modal ages and the concentration of deaths at the modal age;
- after the 1970s death rates at elderly ages (60+) declined rapidly, driving increases in both life expectancy at birth and modal ages, accompanied by the greater compression of deaths into shorter age spans.
Based on a new decomposition method it was found that declines in the level of the age profile of death rates were behind increases in modal ages, but this was mostly countered by increases in the slope. Changes in the slope of the mortality age-profile drove changes in the concentration of deaths at the mode. However, due to greater declines in death rates at very elderly ages in recent decades, the pace of increase in the slope has been slowing since the 1990s and seems to have stagnated since the start of the 21st century. Continuing shifts in Australia’s death frequency distribution to higher ages is contrary to the hypothesis that the human life span is subject to a maximum. While modal age increases due to falls in the level of mortality has been countered by increases in the slope of the age profile of death rates, the overall effect was a steady increase in modal ages and the expansion of the tail of the death frequency distribution. An understanding of the patterns and trends of mortality change provides valuable insights for forecasting.

8.2.4 Objective 4: To identify appropriate methods for forecasting adult mortality and create very elderly population projections

In order to meet the aim of creating accurate projections for Australia’s very elderly population, appropriate mortality forecasting methods needed to be identified. A number of direct and indirect mortality extrapolation methods were evaluated in terms of their accuracy in projecting adult death rates and very elderly populations for Australia over 10 and 20 years ending in 2012. Direct methods include the Geometric method and a variant referred to as the Ediev method, the Lee-Carter method and one of its variants, the Lee-Cater BMS method, and a Relational model. Indirect methods include the extrapolation of the parameters of models fitted to the age profile of death rates and a new proposed method involving the extrapolation of two features of death frequency distributions namely the modal age and concentration of deaths at the modal age. It was found that both death rates and very elderly populations could be accurately projected with the Geometric, Ediev and Lee-Carter BMS methods, or by fitting and extrapolating a Gompertz function to the age profile of death rates. While no single method consistently produced the most accurate projections for males and females, different fitting periods and projection horizons, differences between the Geometric, Ediev and Lee-Carter BMS methods were small.

The Ediev and Lee-Carter BMS methods can both be considered variants of the Geometric method but with some of the subjectivity involved in the choice of fitting period removed.
Other differences relate to statistical procedures. For both the Ediev and Lee-Carter methods, set procedures are applied to determine an appropriate fitting period based on goodness of fit criteria, either by single age or across an age range. The regularity in the patterns of decline in adult death rates in Australia over the last three decades rendered the additional complexity introduced with these procedures unnecessary, however, as these methods did not produce much more accurate results than the Geometric method. Therefore, due to its accuracy and simplicity and based on continuing regularity in adult death rate trends in Australia, death rates by age and sex were extrapolated using the Geometric method. These rates were input into a cohort-component population projection model to create projections to 2042 by single age 85-110+, sex and state.

It is expected that very elderly populations in all the Australian states will continue their substantial growth over the next three decades, increasing to almost 1.5 million or 4.2% of the total population in 2042. Growth is expected to accelerate from the 2030s when the baby boomers enter this age group. The very elderly population is expected to continue its ageing trend resulting in increasing proportions of nonagenarians and centenarians. Of the total very elderly population, the proportion aged 85-89 is projected to decrease from 65.4% in 2012 to 58.8%, while the proportion of nonagenarians is expected to increase from 33.8% to 40.2% and centenarians from 0.79% to 1.06%. Sex ratios are also expected to continue to increase due to greater projected improvements in male survival. By the time cohorts born in the 1940s are aged 85+, there is expected to be 0.8 males for every female, compared to only 0.6 for cohorts born in the 1920s. Very elderly populations in WA, Tas-NT-ACT, and Qld are projected to grow the most over the 30 years, at average annual rates of 4.9%, 4.8% and 4.6% respectively. The least growth is expected in SA (3.5%), NSW (3.9%) and Vic (4.0%).

8.3 Contributions

This thesis has made contributions both to knowledge, in particular for Australia, and methodological contributions, which are applicable internationally. The following contributions were made to substantive knowledge:

1. More plausible and detailed estimates of Australia’s very elderly population at a state and national level, by sex and for single ages 85-110+ were created based on death data and methods found to be reliable (Chapter 3). Based on evaluations of the accuracy of very elderly population estimation methods for Australia, it was found
that more accurate estimates can be created than are currently available from the ABS or the HMD. The ABS derives very elderly estimates from census counts and at single ages only to 99. These were found to be increasingly inaccurate with age from 95, especially for males. It was found that the Survivor Ratio method with total estimates constrained to 85+ ERPs produced more accurate very elderly estimates for Australia across males and females, compared to when a 90+ ERP constraint is applied in line with the HMD. In addition, it was shown that the nearly-extinct-cohort estimation methods can be successfully applied at a sub-national level to produce more accurate estimates for ages 95 and above compared to official estimates. The simpler approach of deriving state-level estimates by applying state-specific proportions of 85+ ERPs to national level estimates also proved quite accurate, and is the recommended method. More accurate and detailed very elderly population estimates and mortality rates formed an important base for accurate projections.

2. Similarly, more plausible and detailed projections of Australia’s very elderly population were created for the 30 years to 2042, based on the Geometric method, found to be simple and reliable (Chapters 6 and 7). Projections were created at a state and national level, by sex and for single ages 85-110+. These projections are both more detailed and different from those currently available. Official national and state projections are currently provided in aggregate only for ages 100+ and 85+ respectively. Historical projections of Australia’s very elderly population, based on a projected life expectancy at birth that is assumed to increase at declining rates, have been found to be too low, although projected centenarian numbers were significantly overstated. In this study, very elderly population projections were created based on methods shown to produce accurate projections, underpinned by continuing regular patterns of death rate declines. The availability of accurate and detailed very elderly population projections allows more effective planning and policy decisions related to the housing, infrastructure, services and care needs of the very elderly. These data also allow better management of longevity risk.

3. This thesis contributes a detailed understanding of the accuracy of official very elderly population estimates (Chapter 3) and projections (Chapter 7) for Australia, and highlights a need for the ABS to revisit some of the methods used. While there existed some indications that official estimates for the very elderly in Australia, derived from
census counts, are too high, no research had previously been undertaken to quantify the extent of this overestimation. This study has shown that ERPs are reasonably accurate up to age 99 for females and 94 for males but their accuracy deteriorate rapidly with increasing age from ages 100+ (females) and 95+ (males). The accuracy of official estimates also varies significantly from year to year. For females they alternated between substantial over- and underestimation, while for males ERPs were generally too high. These estimates also resulted in death rates at the high ages that are too low.

It was found that official projections 20 years ago have underestimated Australia’s very elderly population in 2012 by 18% for males and 10% for females. Similarly, the author’s 85+ population projections from 2012 are 19% (males) and 9% (females) higher by 2042 compared to the ABS’ projections. The author’s projected 85+ male and female populations are 639,003 and 818,559 respectively at the end of 2042 compared to the ABS’ 538,932 and 750,113. In contrast, past ABS projections have greatly overestimated centenarian populations. Projected from 2012, the ABS’ projected centenarian population in 2042 were 90% (males) and 40% (females) higher than the author’s projections. The ABS projected 8,437 and 15,821 centenarian males and females respectively by 2042 compared to the author’s 4,443 and 11,171. Differences between the ABS’ and the author’s projected very elderly populations stem mainly from the different assumed death rate declines. In particular, the ABS assumes declining rates of improvement in life expectancy at birth, whereas the author assumes a continuation of past trends of death rate decline by age throughout the projection period. The ABS’ overstatement of projected centenarian numbers is largely due to too-low death rates at ages 90+, resulting from over-estimated population numbers.

4. This thesis provides insights into how the age-sex composition of Australia’s very elderly population has changed over the last three decades and is expected to change over the next three decades (Chapters 4 and 7). It was found that the very elderly population itself has aged over the last three decades as more people continue to live to higher ages, resulting in greater proportions of nonagenarians and centenarians. This trend is expected to continue over the next three decades. Furthermore, due to greater survival improvements for males from the mid-1980s, sex ratios among the
very elderly have been increasing, and will likely continue to increase. This could have implications for the type of health care required, the availability of carers and possibly living arrangements. Projected improvements in survival from age 65 to 85 and especially from age 85 to 100 are expected to drive rapidly growing numbers of nonagenarians and centenarians over the next 30 years. If disability incidence rates do not reduce accordingly, higher numbers at these ages will have implications for care and residential needs.

5. This study provides an understanding of how increases in births, improvements in survival at various ages and changes in net migration contributed, and are expected to contribute, to the substantial past and future growth of very elderly populations (Chapters 4 and 7). Increasing birth cohorts and improvements in survival at all ages contributed to the growth of nonagenarian and centenarian populations in Australia over the last three decades. However, consistent with findings in other low-mortality countries, improvements in survival beyond age 65 and especially beyond age 85 were by far the most important drivers of growth. Survival improvements at elderly and very elderly ages are expected to continue driving substantial increases in the numbers of nonagenarians and centenarians in the next three decades. When the baby boomers enter this age group from the 2030s, growth is expected to accelerate, with implications for services catering for their health and care needs.

6. This study contributes insight into changes in the age profile of death rates and death frequency distributions in Australia over the last nine decades from 1921 to 2012 (Chapter 5). It was found that death rates for males and females in Australia at all ages declined significantly, with the extent of declines decreasing with age. Broadly three distinct periods of mortality change were identified. Similar to many other low-mortality countries, Australian male and female death rates at elderly ages declined rapidly since the 1970s, resulting in increased life expectancy at birth and modal ages. Declines in the level of mortality were consistently accompanied by increases in the slope of mortality, so that deaths were compressed into shorter age spans. However, greater declines in death rates at very elderly ages in recent decades meant that the pace of increase in the slope has been slowing since the 1990s and seems to have stagnated since the start of the 21st century. Continuing decreases in the level of the age profile of death rates, with little change in the slope, implies a shift of the profile
to higher ages, and thus improved survival to higher and higher ages. Australia thus seems to be moving out of a compression phase into a shifting phase. The observed shift of the death frequency distribution to higher ages is contrary to the hypothesis that the human life span is subject to a maximum.

The findings presented here are not just important for Australia, but have wider applications in the global context. For example, the findings on estimation methods have implications for countries which rely upon census data for population estimates, and findings on forecasting approaches are relevant where targeted approaches for mortality projection are used. Insights gained on mortality change patterns are relevant to other low-mortality countries. The following methodological contributions were made.

7. This thesis contributed to the identification of appropriate methods for estimating very elderly populations at a national and state level as well as an improved understanding of the relative accuracy of different methods by age, sex and state (Chapter 3). It was shown that The Das Gupta (DG) method produces the same results as the Survivor Ratio (SR) method when a one-year age range is used in the calculation of the survivor ratio. The results indicated that estimates were more accurate and less volatile when survivor ratios were based on 5-year age ranges, as used in the SR variants, compared to one-year age ranges, as is implicit in the DG variants. Results changed little whether their inputs were averaged over 3 or 5 cohorts. The approach of adjusting results by constraining them to official 85+ or 90+ ERPs was also found to produce more accurate estimates than explicitly allowing for improvement in survival.

8. Confirmation that nearly-extinct-cohort estimation methods, based on death counts, can be successfully applied at a sub-national level, subject to the availability of adequate volumes of data (Chapter 3). This was the first time that the author is aware of that these methods have been evaluated at a sub-national level. A method was proposed for incorporating internal migration when applying indirect estimation methods at the sub-national level. The level of interstate migration at ages 85+ was small, however, and allowing for this did not significantly improve the estimates. It is proposed that allowance for interstate migration be incorporated in the estimation process if interstate migration ratios exceed levels of around 5%. The results of testing the accuracy of the nearly-extinct-cohort methods at the state level were consistent
with those at the national level. A new, proportional method was also evaluated for deriving sub-national estimates from national-level estimates and was found to perform well.

9. A new method, the Survivor Ratio Advanced (SA) method, was proposed for explicitly allowing for survival improvements in nearly-extinct-cohort estimation methods. This method, which is simpler than other comparable methods currently available, can be used when official estimates are not considered reliable (Chapter 3).

10. A new decomposition method was proposed to quantify the extent to which changes in modal ages and concentrations of deaths can be attributed to changes in each of the level and slope of the age profile of death rates. This enables the relationship between changes in the age profile of death rates and changes in the death frequency distribution to be better understood (Chapter 5). It was shown that increases in modal ages were driven by declines in the level of the age profile of mortality, but this was partly countered by increases in the slope. Increases in the slope of the age profile of mortality determined increases in the concentration of deaths at the modal age at death. This method thus allows the quantification of the extent to which changes in the modal age are due to changes in the level of the age profile of mortality and changes in the slope respectively.

11. A conceptual comparison of direct mortality extrapolation methods was done in Chapter 6. It was shown that both the Ediev and the Lee-Carter methods model mortality according to the Geometric method. Differences between the methods relate to fitting periods, statistical fitting procedures and whether deterministic or probabilistic results are projected. The Lee-Carter BMS variant identifies a single fitting period across the age range which reflects constant rates of decline in death rates, while Ediev’s method applies a different fitting period for each age reflecting separate linear trends. For all the methods tested, a logistic or logarithmic transformation of death rates allowed the use of linear functions and thus simple linear regression techniques to be used.

12. This thesis finally contributed an understanding of the comparable accuracy of both old and newer extrapolative mortality forecasting methods for projecting adult death
rates and very elderly populations. In particular, insights were gained into how the performance of the popular Lee-Carter method compares with older and less well-known methods (Chapter 6). It was found that, due to consistent rates of decline in death rates across adult ages over the last four decades, the direct extrapolation of age-specific death rates assuming constant geometric rates of decline has been very successful in projecting adult death rates and very elderly populations. Differences between the Geometric, Ediev and Lee-Carter BMS methods were small. If death rates continue to decline at consistent rates across the adult age range, the Geometric method can be used as a simpler alternative to the Lee-Carter method. Ediev’s variant of the Geometric method can also be used as an alternative to the Lee-Carter method when rates of decline in death rates show varying trends between ages.

A new method, the Modal Age Projection Model, was also proposed whereby death rates were indirectly forecasted by extrapolating trends in two features of death frequency distributions, namely the modal age and the concentration of deaths at the modal age. This method produced very accurate death rates and very elderly population projections for males throughout the projection periods, but less so for females. The method may be improved by fitting non-linear curves to modal age trends.

8.4 Limitations

Despite these contributions, it is important to point out that the thesis has a number of limitations. The population estimates are based only on death data, and population projections are based on these population estimates and death rates. The validity of the results is therefore subject to the quality of death data used. Historically, churches kept records of births, deaths and marriages and the responsibility for these registrations were taken over by registrar’s offices of the various colonies in 1838 in Tasmania, 1841 in Western Australia, 1842 in South Australia, 1853 in Victoria, 1856 in New South Wales and Queensland, and 1870 in the Northern Territory. Considering that birth registration systems were only in place across the largest part of Australia by the 1860s, deaths data for the very elderly (85+) can probably only be considered reliable from after the Second World War, and for centenarians from the early 1970s. While the introduction of formal vital statistics registration will have the effect of gradually improving death data over time, it is likely that there may still have been some inaccuracies at the very high ages in the periods studied.
Death data supplied by the ABS were subject to adjustments to protect confidentiality in any age-year where there are only one or two deaths. This impacted death data from around age 95. As volumes of deaths at these ages increased over time, fewer adjustments were made, and from the year 2000 this impacted state-level death data at ages above 100 only. Where appropriate, the author applied adjustments to state-level age-year death counts to ensure that when death counts at individual ages were summed across states the totals reconciled to the national number of deaths. Adjustments were applied at both national and state scales to ensure the sum of deaths at single ages 100+ equalled the 100+ totals. This impacted data at the highest ages only and it is not expected to have significantly impacted the conclusions.

The retrospective evaluation of population estimation methods were performed in a particular context (Australian national and state level) and over certain time periods and there is no guarantee that the results will hold in other contexts or eras. For example, mortality declines during the last two to three decades may result in improved performance of methods that allow for survival improvements (implicitly or explicitly), and possibly worse performance for methods that do not.

The decomposition of growth into the demographic drivers makes use of historical birth rates and period life tables published by the ABS. These were used in the construction of cohort life tables relating to younger ages. The validity of the decomposition analyses thus depend on the reliability of these historical births and life tables.

The analyses of mortality trends and patterns make use of data from 1921. However, data relating to the period before 1971 is likely to be less reliable, for reasons stated earlier. The observed increase in death rates at the highest ages (95+) between the 1920s and 1950s suggests that these rates may previously have been underestimated. These rates have been obtained from the Human Mortality Database and were thus based on population estimates derived from death counts, implying an overstatement of these early death counts. The analyses of death rates at very high ages in earlier periods should thus be used with caution.

The results of retrospective tests of different mortality forecasting methods are also dependent on the mortality experience in Australia over the study period, 1972-2012. Differences in experiences in other geographical or historical context will lead to different
results. In particular, by nature, the success of extrapolation methods depends on consistency in trends over the fitting and projection period. Methods based on a specified function fit to the age profile of death rates are subject to the further condition that the function should be an appropriate reflection of the pattern of death rates. Variations in death rate changes between ages may impact the goodness-of-fit of different functions over time.

Finally, the projections are based on the assumption that rates of change in death rates observed over the last two decades will continue in the next three decades. While the Geometric method produced accurate results for projection horizons ending in 2012, there is no guarantee that this will continue to be the case in future periods. The estimates and projections presented are nevertheless regarded a considerable improvement on official estimates currently available for ages 90+.

8.5 Research Outlook

8.5.1 Population estimation, forecasting and death rates at the highest ages

As longevity continues to improve and very elderly populations continue to grow and age, the topic of very elderly estimates, death rates and projections will become increasingly important. Furthermore, as more data become available for ages 100+, the accurate modelling of death rates and projections for these ages will become more viable. Higher volumes and reduced volatility at the very high ages will probably also improve the viability of population estimation methods which explicitly allow for survival improvement, removing a reliance on an additional set of official estimates. The evaluation of estimation methods should be repeated in a few years’ time to confirm this. It may also be instructive to apply some of the methods introduced in this thesis in other contexts. It will, for example, be interesting to know whether an evaluation of the relative accuracy of different mortality forecasting methods yielded similar results when applied to data for other countries. The availability of greater volumes of data at ages 100+ will allow more detailed analysis of trends in maximal ages and death rates as well as a more viable approach to forecasting death rates at these ages. While there are some indications that Australia may be entering a shifting phase, with death frequency distributions moving to the right, this cannot yet be confirmed. It will be useful to update the analyses presented in Chapter 5 in a few years’ time to confirm whether this is the case. When compression has stopped while modal ages continue to increase, there will be firmer evidence against the theory that human life spans are subject to a specified limit.
Whether the age profile of death rates at the very high ages in Australia reflects an exponential or logistic pattern can only be confirmed as sufficient volumes of data become available.

8.5.2 Projections by cause of death

Trends in age-standardised death rates vary significantly by cause of death. For example, the risk of death due to Ischaemic heart diseases decreased by around 50% and 66% for Australian males and females respectively from 1993 to 2013 (ABS, 1993a; 2013b). Similarly, the risk of death from cerebrovascular diseases decreased by 34% for males and 56% for females. In comparison, trends in death rates for cancer showed much less improvement and death rates from mental disorders increased. Projecting overall mortality by separately projecting death rates for different causes is not common practice and has met with various problems. These include, for example, data problems relating to changes over time in coding practices, classification of causes of death, and improvements in the way causes are diagnosed (Andreev & Vaupel, 2006; Bongaarts, 2014; Horiuchi & Wilmoth, 1998). Furthermore, at advanced ages the stated cause of death may be inaccurate due to “the frequent coexistence of several life-threatening conditions in people who survive to more advanced ages” (Olshansky 1988: 499). Further research is needed into how data problems may be overcome, and to understand the trends and relationships between cause-specific death rates by age and how these can be effectively incorporated into projections.

Bongaarts (2014) demonstrated how age-standardised death rates attributable to smoking changed for males and females in 15 low-mortality countries from 1955 to 2010. In particular, male smoking-related death rates peaked in the 1980s and declines in smoking-related death rates in the last three decades were the main contributor to overall mortality decline since. In contrast, smoking-related death rates for females continued to increase, despite a decline in overall mortality, and seem to have peaked around 2010. Researchers have suggested that mortality projections may be improved by adjusting for these underlying trends in smoking-related mortality (Bongaarts, 2014; Janssen et al., 2013). Trends in smoking-related deaths in Australia and the viability of incorporating trends into projections could be investigated with a view to improve projections. Further research should also be carried out into how trends in other major risk factors, such as obesity, can be allowed for to improve mortality projections.
8.5.3 Health status trends

The implications of increasing very elderly numbers for the cost of aged and health care depend on the health and care needs of this population segment. If not accompanied by proportional decreases in the incidence of chronic conditions and disability, growth in very elderly numbers will drive substantial increases in health and aged care needs, with associated rises in costs (Crimmins et al., 1994; Crimmins, 2004). Current research includes, for example, comparisons of summary measures such as life expectancy and health expectancy and trends in disability to determine whether compression or expansion of morbidity, or dynamic equilibrium has occurred (AIHW, 2014; Crimmins & Beltrán-Sánchez, 2011; OECD, 2007; Robine et al., 2009c). Compression of morbidity occurs when increasing life expectancy is accompanied by a shorter period of disability before death, resulting from a delay in the onset of chronic illness and disability (Fries, 1980). Under the dynamic equilibrium theory, less severe disability increases but more severe disability decreases so that the ratio of disability-free life expectancy to total life expectancy remains stable (Manton, 1982). The expansion of morbidity hypothesis is the premise that improved survival is accompanied by worsening health – the result of fewer deaths from previously fatal conditions but with the resulting longer life being spent in poor health or disability (Gruenberg, 2005). This gives an indication of whether the health status of a population segment is improving at a faster rate than survival, or whether the proportion of remaining life years that is disabled is increasing or constant (Fries, 1980; Gruenberg, 2005; Manton, 1982). The relationship between improving survival and the consequences for quality of life is not well understood. Different studies have also lead to varying and often contradictory conclusions, partly due to different definitions of health status employed, changes in definitions over time and lack of data (AIHW, 2000). According to Parker and Thorslund (2007), health trends and the accompanying care needs among the very elderly cannot be adequately understood by considering general morbidity measures such as functional limitations disability only and there is a need to consider all the separate health components.

More detailed research is needed into trends in the health conditions and disability status of Australia’s very elderly cohorts, and the correlations with trends in mortality by cause. These trends in health status can then be extrapolated and combined with projected very elderly numbers to estimate future health and care needs and costs. Greater insights are also needed into the specific health conditions the very elderly suffer from, how these conditions and their
prevalence are changing, and what the implications are for the type of care required. For example, dementia is a leading cause of disability among the elderly resulting in dependency, with an estimated 30% of the population aged 85+ living with dementia (AIHW, 2012). Growth in the very elderly population results in a rise in the number of people living with dementia, with care needs which differ from those of, for example, cancer patients and people suffering from heart disease. While dementia is currently more prevalent among women, it is likely that the prevalence among men will increase in line with their greater survival improvements. How the incidence of various chronic and degenerative conditions among the very elderly is changing will greatly impact both the type and level of future care requirements. Related to this is a need for understanding trends in the living arrangements of the very elderly, to provide greater insights into likely care providers. This will further facilitate more detailed planning of the required services and associated costs.

8.6 Conclusion

The substantial growth in very elderly numbers and proportions means that this population group has attracted much attention from demographers and policymakers in recent decades. This thesis sought to add insights into the most appropriate methods for estimating and projecting very elderly population numbers. Improved and more detailed estimates and projections should facilitate better planning for and financing of their unique needs. This study furthermore sought to improve our understanding of the trends and demographic drivers of the growth of Australia’s very elderly population. The continued growth expected in coming decades, together with its significant social and economic implications, means that very elderly population estimates and projections, and understanding their demographic trends, will continue to be a topic of great importance. There is substantial scope for developing further insights into, for example, the quality of life of the very elderly, and specifically, implications of living longer for health and care needs.
REFERENCES


Human Mortality Database. (2013). University of California, Berkeley (USA), and Max Planck Institute for Demographic Research (Germany). Available at www.mortality.org


Appendix A

Figure 8.1 Map of Australian States and Territories
Appendix B

In this Appendix it is shown algebraically that population estimates derived from the Survivor Ratio method with an age range (k) of one year are equivalent to those estimated from applying Das Gupta’s method, provided the number of cohorts (m) in both are the same.

Survivor Ratio method with k=1:

From equation 2.5, the population aged x at time t can be estimated with the survivor ratio method where k=1 with:

\[ P_{x,t}^c = \frac{R_x}{1 - R_x} \times D_t^c \]  \hspace{1cm} (B1)

Where, according to equation 2.6:

\[ R_x = \frac{\sum_{j=1}^{m} p_{x,t-j}^c}{\sum_{j=1}^{m} (p_{x,t-j}^c + D_{t-j}^c)} \]  \hspace{1cm} (B2)

Applying equation 2.3, \( R_x \) can be re-written as:

\[ R_x = \frac{\sum_{j=1}^{m} p_{x,t-j}^c}{\sum_{j=1}^{m} (p_{x,t-j}^c + D_{t-j}^c)} \]  \hspace{1cm} (B3)

So that \( \frac{R_x}{1 - R_x} \) simplifies to:

\[ \frac{R_x}{1 - R_x} = \frac{\sum_{j=1}^{m} p_{x,t-j}^c}{\sum_{j=1}^{m} D_{t-j}^c} \]  \hspace{1cm} (B4)

And the population aged x at time t can be written as:

\[ P_{x,t}^c = \frac{\sum_{j=1}^{m} p_{x,t-j}^c}{\sum_{j=1}^{m} D_{t-j}^c} \times D_t^c \]  \hspace{1cm} (B5)

Das Gupta’s method

With Das Gupta’s method, the population aged x at time t is estimated with equation 2.1:

\[ P_{x,t} = \sum_{i=1}^{w-x} D_{t+i}^x \]  \hspace{1cm} (B6)

where (according to equation 2.8):
\[ D^c_{t+i} = dr_{x+i} \times D^c_{t+i-1} \quad \text{(B7)} \]

and (equation 2.7):

\[ dr_{x+i} = \frac{\sum_{j=1}^{m} D^c_{t-j+i}}{\sum_{j=1}^{m} D^c_{t-j-1+i}} \quad \text{(B8)} \]

Expanding equation B6:

\[ P^c_{x,t} = D^c_{t+1} + D^c_{t+2} + D^c_{t+3} + D^c_{t+4} + \ldots \quad \text{(B9)} \]

and re-writing it by combining it with equation B7 gives:

\[ P^c_{x,t} = D^c_{t}(dr_{x+1} + dr_{x+1}dr_{x+2} + dr_{x+1}dr_{x+2}dr_{x+3} + dr_{x+1}dr_{x+2}dr_{x+3}dr_{x+4} + \ldots) \quad \text{(B10)} \]

The numerators and denominators in consecutive death ratios (equation 2.7) are the same so that they cancel, simplifying this equation to:

\[ P^c_{x,t} = D^c_{t}\left(\frac{\sum_{j=1}^{m} D^c_{t-j+1}}{\sum_{j=1}^{m} D^c_{t-j}} + \frac{\sum_{j=1}^{m} D^c_{t-j+2}}{\sum_{j=1}^{m} D^c_{t-j}} + \frac{\sum_{j=1}^{m} D^c_{t-j+3}}{\sum_{j=1}^{m} D^c_{t-j}} + \frac{\sum_{j=1}^{m} D^c_{t-j+4}}{\sum_{j=1}^{m} D^c_{t-j}} + \ldots\right) \quad \text{(B11)} \]

which simplifies to:

\[ P^c_{x,t} = \frac{D^c_{t}}{\sum_{j=1}^{m} D^c_{t-j}} \sum_{j=1}^{m} \sum_{i=1}^{w-x} D^c_{t-j+i} \quad \text{(B12)} \]

and given that \( P^c_{x,t-j} = \sum_{i=1}^{w-x} D^c_{t-j+i} \), this equation can be written as:

\[ P^c_{x,t} = \frac{D^c_{t}}{\sum_{j=1}^{m} D^c_{t-j}} \sum_{j=1}^{m} P^c_{x,t-j} = \frac{\sum_{j=1}^{m} P^c_{x,t-j}}{\sum_{j=1}^{m} D^c_{t-j}} \times D^c_{t} \quad \text{(B13)} \]

and this is the same as equation B5. This therefore shows that the survivor ratio with \( k=1 \) is equivalent to Das Gupta’s method for the same m.
Appendix C

In this Appendix it is shown population estimates are derived from deaths and net migration ratios.

From section 2.4.2, we have the following:

\[ R_x = \frac{p_{x,t}}{p_{x-k,t-k}} \quad (2.2) \]

\[ p_{x-k,t-k} = p_{x,t} + \sum_{i=0}^{k-1} d_{t-i}^c \quad (2.3) \]

\[ p_{x,t} = \frac{r_x}{1-r_x} \times \sum_{i=0}^{k-1} d_{t-i}^c \quad (2.5) \]

Where net migration is ignored in all the above.

In section 3.2.2, formulae are derived where allowance is made for net migration in the period from time \( t-k \) to time \( t \):

\[ p_{x,t}^c - nm_{x,t} p_{x,t}^c = p_{x-k,t-k}^c - \sum_{i=0}^{k-1} d_{t-i}^c \quad (3.12) \]

If population numbers where allowance is made for net migration is written as accented notation, equation 3.12 becomes:

\[ p_{x,t}^c - nm_{x,t}^c p_{x,t}^c = p_{x-k,t-k}^c - \sum_{i=0}^{k-1} d_{t-i}^c \quad (C1) \]

We set \( p_{x-k,t-k}^c = p_{x-k,t-k}^c \), as the starting population is not impacted by whether allowance is made for migration in the period from time \( t-k \) to time \( t \).

\[ p_{x,t}^c - nm_{x,t}^c p_{x,t}^c = p_{x-k,t-k}^c - \sum_{i=0}^{k-1} d_{t-i}^c \quad (C2) \]

We therefore have:

The population, not allowing for migration (from equation 2.3):

\[ p_{x,t}^c = p_{x-k,t-k}^c - \sum_{i=0}^{k-1} d_{t-i}^c \]

The population, allowing for migration (from equation C2):

\[ p_{x,t}^c = p_{x-k,t-k}^c - \sum_{i=0}^{k-1} d_{t-i}^c + nm_{x,t}^c p_{x,t}^c \]

Therefore

\[ p_{x,t}^c = p_{x,t}^c + nm_{x,t}^c p_{x,t}^c \quad (C3) \]

So that

\[ p_{x,t}^c = \frac{p_{x,t}^c}{1-nm_{x,t}} \quad (C4) \]
Given that (equation 2.5):

\[ P_{x,t} = \frac{R_x}{1 - R_x} \times \sum_{i=0}^{k-1} D_{t-i}^x \]

It follows that

\[ P'_{x,t}^c = \left( \frac{R_x}{1 - R_x} \times \sum_{i=0}^{k-1} D_{t-i}^c \right) / (1 - nm_{x,t}) \]  

(C5)

Where the survivor ratio \( R_x \) is based on deaths only, i.e. makes no allowance for migration. This is equation 3.14.

This can also be shown by using survivor ratios that does allow for migration.

Define: \( R'_x = \frac{P'_{x,t}^c}{P'_{x-k,t-k}^c} \)

which, using equation C1, can be written as

\[ R'_x = \frac{P'_{x,t}^c}{P'_{x,t}^c(1 - nm_{x,t}) + \sum_{i=0}^{k-1} D_{t-i}^c} \]

Solving for \( P'_{x,t}^c \), gives:

\[ P'_{x,t}^c = \frac{R'_x}{1 - R'_x(1 - nm_{x,t})} \times \sum_{i=0}^{k-1} D_{t-i}^c \]  

(C6)

By using the relationship (see proof below)

\[ R'_x = \frac{R_x}{(1 - nm_{x,t})} \]  

(C7)

It follows that C5 (and equation 3.14) is the same as C6, i.e.

\[ P'_{x,t}^c = \left( \frac{R_x}{1 - R_x} \times \sum_{i=0}^{k-1} D_{t-i}^x \right) / (1 - nm_{x,t}) \]

The proof for this relationship (C7) is shown below:

\[ R'_x = \frac{P'_{x,t}^c}{P'_{x-k,t-k}^c} \]

is the same as (from C4)

\[ R'_x = \frac{P_{x,t}}{P_{x-k,t-k}} \]

And it therefore follows that

\[ R'_x = \frac{R_x}{(1 - nm_{x,t})} \]