Laser feedback interferometry: a tutorial on the self-mixing effect for coherent sensing

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This tutorial presents a guided tour of laser feedback interferometry, from its origin and early development through its implementation to a slew of sensing applications, including displacement, distance, velocity, flow, refractive index, and laser linewidth measurement. Along the way, we provide a step-by-step derivation of the basic rate equations for a laser experiencing optical feedback starting from the standard Lang and Kobayashi model and detail their subsequent reduction in steady state to the excess-phase equation. We construct a simple framework for interferometric sensing applications built around the laser under optical feedback and illustrate how this results in a series of straightforward models for many signals arising in laser feedback interferometry. Finally, we indicate promising directions for future work that harnesses the self-mixing effect for sensing applications. © 2015 Optical Society of America

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1. Introduction

Schawlow and Townes’ 1958 prediction of lasers [1], and their first demonstration in 1960 in ruby [2,3], was quickly followed by the demonstration of laser action in 1961 in helium–neon gas mixture [4] and cesium vapor [5], and in 1962 in gallium–arsenide p–n junctions [6–8]. Soon after the suppression of unwanted resonant laser modes [9,10] was considered by feedback from a third external mirror (“self-mixing”) [11], the first use of the self-mixing effect for laser metrology by King and Steward was reported, in 1963 [12–14]. This tutorial is intended as a guide to the reader interested in the basic theory of the self-mixing effect and its practical use for laser feedback interferometry (LFI) and metrology. A great deal of literature on the subject has accumulated over the past half-century, which we will survey only lightly during the course of this tutorial.

The self-mixing effect—the mixing of the intracavity electromagnetic wave with an emitted electromagnetic wave reinjected into the laser cavity after interaction in the external cavity—is a remarkably universal phenomenon, occurring in lasers regardless of type. Among others, the effect has been reported in gas lasers [12], in-plane semiconductor diode lasers [15–17], vertical-cavity surface-emitting lasers (VCSELs) [18], mid-infrared [19] and terahertz quantum cascade lasers (THz QCLs) [20], interband cascade lasers [21], fiber [22–24] and fiber ring lasers [25,26], solid-state lasers [27–29], microring lasers [30–32], and quantum dot lasers [33].

Using a photosensitive detector to collect radiation emitted from one mirror, King and Steward demonstrated displacement sensing using radiation reinjected into the laser through the second mirror after reflection from an external mirror. They noted the effect was observable when as little as 0.1% of emitted radiation was reflected from an external mirror situated up to 10 m from the laser operating in continuous wave mode. Their patent application [14] discusses the potential for their device in measuring physical parameters that are capable of altering an optical path length. These parameters include physical length and velocity measurement, measuring changes in the propagation medium, such as by pressure change or change in composition of a medium interposed in the external cavity, and the use of both the visible and infrared emission of...
the He–Ne laser separately or concurrently for coarse and fine measurement. Almost immediately after King and Steward’s demonstration, LFI was employed in 1963 to measure plasma density \([34,35]\) and the refractive index of plasma \([36]\) (see \([37]\) for an early discussion), and in 1968 used for laser Doppler velocimetry \([38]\).

Since its first demonstration \([12,34]\), the research on the laser feedback effect for metrology has been carried out under many names: induced-modulation \([39]\), back-scatter-modulation \([40]\), self-mixing \([18]\), (optical/external) feedback \([41]\), self-coupling \([16,17]\), and autodyning \([42]\), to name a few. Early work employed photodetection to acquire signals from LFI \([12,15,16,34,36–38]\)—LFI signals. It was later pointed out that monitoring the voltage across the laser terminals was an alternative means of acquiring LFI signals \([17,43,44]\), removing the need for an additional detector—a substantial benefit when appropriate detectors of the laser radiation are cumbersome, expensive, or unavailable. Techniques for the acquisition of LFI signals in quadrature appeared as early as 1967 \([45]\). However, it was noted in 1978 that quadrature signal acquisition could be achieved simply in lasers supporting two resonant modes with orthogonal polarization \([39,46]\). The 1980 work of Lang and Kobayashi \([47]\) presented the core model for a semiconductor laser experiencing optical feedback (though Spencer and Lamb gave essentially the same model in 1972 \([48]\)). This model captures the essence of laser dynamics under feedback and has remained the foundation for phenomenological models of these systems to the present day. Today, the theoretical investigation and practical demonstration of schemes for laser feedback interferometry continues apace \([24,49–59]\).

The remainder of this tutorial is structured as follows. Section 2 discusses the basic architecture and operating regimes of LFI. Section 3 introduces the model of Lang and Kobayashi \([47,48]\) which well describes the dynamics of lasers experiencing optical feedback, which we follow with its reduction (under suitable conditions) to temporal steady state in Section 4. In Section 5 we explore the practicalities of realizing LFI systems based on diode lasers, the principles of which may be adapted to other laser types. In Sections 6 through 9, we discuss the metrology implications of the models, and give concrete examples of LFI schemes for a number of applications. We conclude in Section 10 with a brief summary and our thoughts on promising future directions.

2. Laser Feedback Interferometers: Composition and Operating Regimes

All LFI systems operate according to the same basic principle: light is emitted from a laser, is transmitted to an external target from which it is partially reflected, and transmitted back to the laser where a portion of it re-enters the laser cavity; there the reinjected light interacts (mixes) with the resonant modes of the laser. Due to the self-coherent nature of laser feedback interferometers, they are inherently highly sensitive, suppressing most radiation entering the laser cavity that is not their own.

Through its double transmission through the external cavity and reflection from the external target, the reinjected light is imprinted with information about each. By mixing in the laser cavity, the reinjected light perturbs the intracavity electric field, transferring this information from outside the laser cavity, which then becomes measurable through the resulting perturbations to the operating
parameters of the laser, such as a change in gain leading to variations in optical power, lasing frequency, and laser terminal voltage. The changes in optical power are often monitored using a photodetector (PD). Alternately, the variations in the laser terminal voltage can be monitored directly. See Section 5 for a more detailed discussion of practical considerations.

The simple architecture of LFI can be elegantly captured by a three-mirror laser model (see Fig. 1). Laser light recirculates in a cavity of physical length $L_{\text{in}}$ and effective refractive index $n_{\text{in}}$ between two mirrors $M_1$ and $M_2$ with reflectivities $R_1$ and $R_2$, respectively. Light is emitted through $M_2$, is transmitted through an external cavity of physical length $L_{\text{ext}}$ and effective refractive index $n_{\text{ext}}$ to an external mirror $M_3$ with reflectivity $R$, from which it is reflected, retransmitted, and reinjected into the laser cavity through $M_2$. Note that complicated and compound external targets can be modeled as an effective third mirror $M_3$. The validity of such a model for an effective third mirror should be greatest for lasers with a single longitudinal mode and a single transverse mode interacting with transversely uniform compound external cavities.

When single reflection from $M_3$ is considered (due to a diffusive target or weak optical feedback), the rate $\tilde{\kappa}$ at which reinjected light is coupled into the laser cavity (coupling rate) is related to cavity parameters through $[60–62]$

$$\tilde{\kappa} = \frac{\kappa}{\tau_{\text{in}}} = \varepsilon (1 - R_2) \sqrt{\frac{R}{R_2} \frac{1}{\tau_{\text{in}}}}.$$  

(1)

where $\kappa$ is a measure of the coupling strength between the laser and external cavities, $|\varepsilon| \leq 1$ accounts for possible loss on reinjection (for example, due to mode mismatch), the intracavity round-trip time is denoted by $\tau_{\text{in}} = 2n_{\text{in}}L_{\text{in}}/c$, and $c$ denotes the light velocity in vacuum. In steady state, multiple reflections in the external cavity can be considered simply by modifying the coupling rate $\tilde{\kappa}$ (see, for example, $[63–65]$). A related and equally important quantity for characterizing feedback is the (dimensionless) feedback level—Acket’s characteristic parameter $[57,66]$—$C$:

$$C = \frac{\kappa}{\tau_{\text{in}}} \sqrt{1 + \alpha^2}.$$  

(2)

Figure 1

Three-mirror model of LFI. The laser is represented as the “internal” cavity with length $L_{\text{in}}$, refractive index $n_{\text{in}}$, and round-trip propagation time $\tau_{\text{in}}$. Light leaves the internal cavity through the partially transmissive mirror $M_2$ and traverses the “external” cavity of length $L_{\text{ext}}$, refractive index $n_{\text{ext}}$, and round-trip propagation time $\tau_{\text{ext}}$. A portion of this light re-enters the laser through $M_2$ and mixes with the field inside the laser cavity, affecting the operating state of the laser.
where $\tau_{\text{ext}}$ is the external-cavity round-trip time and $\alpha$ is Henry’s linewidth enhancement factor [67].

For fixed internal and external round-trip times $\tau_{\text{in}}$ and $\tau_{\text{ext}}$, the coupling strength $\kappa$ (and the related $\tilde{\kappa}$ and $C$) can be used to qualitatively categorize the regimes in which a laser feedback interferometer operates (see also Fig. 2). As $\kappa$ increases from zero, the laser operates in five qualitatively different regimes [68,69]:

I. The first regime is characterized by weak optical feedback ($C \leq 1$) with a single emission frequency and a narrowing or broadening of the emission line depending on the phase of the feedback.

II. The second regime is characterized by moderate optical feedback ($C > 1$), which results in multiple emission frequencies and apparent splitting of the emission line due to rapid mode hopping. The laser under optical feedback remains dependent on the phase of the feedback.

III. The third regime is characterized by strong optical feedback ($C \gg 1$), which results in a return to single emission frequency under feedback. The laser under optical feedback remains dependent on the phase of the feedback.

IV. The fourth regime is characterized by chaotic dynamics with islands of stability [69] and broadening of the emission line, a state often referred to as “coherence collapse.” This regime occurs when the rate $\tilde{\kappa}$—at which the optical feedback is coupled into the laser cavity—is comparable with its relaxation oscillation frequency. The laser under optical feedback remains partially dependent on the phase of the feedback.

V. The fifth regime is characterized by a return to stability, when the laser–target system effectively operates as an optically pumped (long) external cavity laser. This regime occurs when the feedback coupling rate $\tilde{\kappa}$ is much greater than the laser relaxation oscillation frequency. The laser under optical feedback is independent of the phase of the feedback.

When $L_{\text{ext}}$ is greater than the coherence length of the laser, feedback phase loses identifiability; the laser operates independent of feedback phase, but still depends on the feedback amplitude (intensity-based). LFI deals with regimes I–III and consequently we restrict our attention to these in this tutorial. For discussions of regimes IV–V, the interested reader is referred to [69–73].

In the sections that follow, we examine in detail the idea of stimulus and response in LFI. The stimulus for each interferometric application differs; however, each induces changes in the laser operating parameters, which are directly observable in the response signal.
3. Dynamics under Feedback: The Lang–Kobayashi Model

In this section we step through the derivation of the model for a diode laser under optical feedback. This process is characterized by the prudent use of appropriate approximations. There are two conventions for dealing with the quantities of photons and carriers in the subsequent equations. One is to use photon and carrier densities, and the other to use photon and carrier numbers. We choose to proceed using photon and carrier densities (after [70, 74, 75]), but we shall present the complete set of equations for both conventions at the end of the derivation. As there are many symbols and constants used in this section and throughout the paper, Table 1 provides a concise summary to aid the reader.

When multiple reflections in the external cavity can be neglected, the plane wave and slowly varying envelope approximations lead to the standard Lang–Kobayashi model for a diode laser under feedback [47, 48, 62, 76]:

\[
\frac{d}{dt} (E(t)e^{j\omega t}) = \left\{ j\omega_m + \frac{1}{2} \left( \Gamma G - \frac{1}{\tau_p} \right) \right\} E(t)e^{j\omega t} \times E(t)e^{j\omega t} + \tilde{\kappa} E(t - \tau_{ext})e^{j\omega(t - \tau_{ext})}, \tag{3}
\]

where \(E(t)\) is the scaled, slowly varying complex envelope of the electric field, rapidly oscillating according to \(e^{j\omega t}\). Equation (3) is implicitly coupled through the gain term \(G\) with one for carrier density, \(N\):

\[
\frac{dN(t)}{dt} = \frac{\eta i I(t)}{qV} - \frac{N(t)}{\tau_n} - GS(t), \tag{4}
\]

and together these two equations describe the laser dynamics under optical feedback. This is our starting point, and as insightful as Eq. (3) is, one must do some work to arrive at a set of equations that may be directly applied to laser modeling problems.

Figure 3 graphically depicts the rate-equation model, showing the interrelationships between the carrier and photon populations (we shall transition from

Operating regimes of LFI, after [68, 69]. Region I, weak feedback; region II, moderate feedback; region III, strong feedback; region IV, chaos with islands of stability; region V, external cavity. We restrict ourselves to regions I–III, where the system operates interferometrically, and does not exhibit chaotic behavior.
the electric field rate-equation to a photon rate-equation below). The model is
deceptively simple—the blue arrows represent number of particles (photons or
carriers) flowing per unit time. Starting from the top, carriers are generated by
the driving current, with injection efficiency \( \eta_i \). Carriers are lost at a rate
governed by the carrier lifetime \( \tau_n = (R_{nr} + R_{sp})^{-1} \) by radiative and nonradiative
means such as lattice vibrations, Auger recombination, and carrier leakage [74].
Following the usual derivation, we do not consider the spontaneous emission in
the electric field (or photon) equation (though it is implicitly present in the car-
rier equation through \( \tau_n \)), as its main effect is to introduce noise, but it may be
simply added to the rate equations if desired; a small portion of spontaneous
emission will couple into the dominant lasing mode, governed by the factor
\( \beta_{sp} \) [74]. The arrow representing gain by stimulated emission assumes a positive
value for the gain \( G \), but note that below transparency the gain will be negative.
The blue arrow leading away from the photon reservoir represents the decrease
in photon number due to the intrinsic scattering out of mode losses, and the large
orange arrow exiting the reservoir represents optical emission of the lasing
mode. These two mechanisms of photon loss (intrinsic scattering out of mode

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Comment</th>
<th>Units</th>
</tr>
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<tbody>
<tr>
<td>( j )</td>
<td>Imaginary unit, ( \sqrt{-1} )</td>
<td>—</td>
</tr>
<tr>
<td>( c )</td>
<td>Speed of light in vacuum</td>
<td>m/s</td>
</tr>
<tr>
<td>( q )</td>
<td>Charge of an electron</td>
<td>C</td>
</tr>
<tr>
<td>( t )</td>
<td>Time</td>
<td>s</td>
</tr>
<tr>
<td>( E )</td>
<td>Slowly varying (complex) envelope of the electric field, ( E =</td>
<td>E</td>
</tr>
<tr>
<td>( \phi )</td>
<td>Phase of ( E )</td>
<td>rad</td>
</tr>
<tr>
<td>( S )</td>
<td>Photon density in the laser cavity</td>
<td>m(^{-3})</td>
</tr>
<tr>
<td>( N )</td>
<td>Carrier density in the laser cavity</td>
<td>m(^{-3})</td>
</tr>
<tr>
<td>( N_{th} )</td>
<td>Carrier density in the laser cavity at threshold</td>
<td>m(^{-3})</td>
</tr>
<tr>
<td>( N_{tr} )</td>
<td>Carrier density in the laser cavity at transparency</td>
<td>m(^{-3})</td>
</tr>
<tr>
<td>( n_{in} )</td>
<td>Laser cavity (effective) refractive index</td>
<td>—</td>
</tr>
<tr>
<td>( n_s )</td>
<td>Laser cavity group refractive index</td>
<td>—</td>
</tr>
<tr>
<td>( n_{in} )</td>
<td>Laser cavity effective refractive index at threshold</td>
<td>—</td>
</tr>
<tr>
<td>( L_{in} )</td>
<td>Laser cavity length</td>
<td>m</td>
</tr>
<tr>
<td>( m )</td>
<td>Mode index (integer)</td>
<td>—</td>
</tr>
<tr>
<td>( \omega )</td>
<td>Laser mode angular frequency</td>
<td>rad/s</td>
</tr>
<tr>
<td>( \omega_{in} )</td>
<td>Cavity resonance angular frequency, ( \omega_{in} = m\pi c/(n_\delta L_{in}) )</td>
<td>rad/s</td>
</tr>
<tr>
<td>( \omega_{in} )</td>
<td>Laser mode angular frequency in the absence of optical feedback at threshold, ( \omega_{in} = m\pi c/(n_\delta L_{in}) )</td>
<td>rad/s</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Laser emission wavelength in vacuum</td>
<td>m</td>
</tr>
<tr>
<td>( k )</td>
<td>Laser emission wavenumber in vacuum, ( k = 2\pi/\lambda )</td>
<td>m(^{-1})</td>
</tr>
<tr>
<td>( v_g )</td>
<td>Laser cavity group velocity, ( v_g = c/n_\delta )</td>
<td>m/s</td>
</tr>
<tr>
<td>( G )</td>
<td>Gain in the laser cavity—a function of ( N ) and ( S )</td>
<td>s(^{-1})</td>
</tr>
<tr>
<td>( g )</td>
<td>Material gain per unit length, ( g = G/v_g )</td>
<td>m(^{-1})</td>
</tr>
<tr>
<td>( V )</td>
<td>Cavity volume</td>
<td>m(^3)</td>
</tr>
<tr>
<td>( V_p )</td>
<td>Effective cavity volume occupied by photons, ( V_p = V/\Gamma )</td>
<td>m(^3)</td>
</tr>
<tr>
<td>( \Gamma )</td>
<td>Optical confinement factor, ( \Gamma = V/V_p )</td>
<td>—</td>
</tr>
<tr>
<td>( I )</td>
<td>Laser driving current</td>
<td>A</td>
</tr>
<tr>
<td>( \eta_i )</td>
<td>Current injection efficiency</td>
<td>—</td>
</tr>
<tr>
<td>( \tau_p )</td>
<td>Photon lifetime in laser cavity</td>
<td>s</td>
</tr>
<tr>
<td>( \tau_{in} )</td>
<td>Laser internal cavity round-trip time, ( \tau_{in} = 2L_{in}/v_g )</td>
<td>s</td>
</tr>
<tr>
<td>( \tau_{ext} )</td>
<td>External cavity round-trip time</td>
<td>s</td>
</tr>
<tr>
<td>( a )</td>
<td>Differential gain</td>
<td>m(^2)</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>Reinjection loss factor</td>
<td>—</td>
</tr>
<tr>
<td>( \epsilon_{G} )</td>
<td>Gain compression factor</td>
<td>m(^3)</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>Feedback coupling coefficient</td>
<td>—</td>
</tr>
<tr>
<td>( \tilde{\kappa} )</td>
<td>Feedback coupling rate, ( \tilde{\kappa} = \kappa/\tau_{in} )</td>
<td>s(^{-1})</td>
</tr>
<tr>
<td>( C )</td>
<td>Feedback level/parameter</td>
<td>—</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Linewidth enhancement factor</td>
<td>—</td>
</tr>
</tbody>
</table>
loss and emission through mirrors) are lumped together in the photon lifetime term $\tau_p$ in (2). A portion of the emitted wave, represented by the large orange arrow, traverses the external cavity by reflection from the target and is subsequently reinjected into the laser cavity; that is, into the photon reservoir in Fig. 3. The reinjected field is time shifted by the cavity round-trip time $\tau_{ext}$, and system stimuli of the order of this time scale will cause complex behavior [70]. For simplicity, the phase shift on reflection from the target is implicitly included in $\tau_{ext}$, as it is a constant for most applications—see Sections 7.A and 7.D for exceptions.

Now we turn to the derivation at hand, and at the outset, we note that the optical confinement factor $\Gamma = V/V_p$, a consequence of the photon and carrier populations occupying different volumes, appears in Eq. (3) but not in Eq. (4). This asymmetry is due to our use of density equations—were we using number equations the gain terms would be symmetric [74]. Furthermore, the gain is written in its general form, simply as $G$, as its precise functional form may be chosen as needed. For instance, it may be written as [74]

$$G = v_g a g \ln(N/N_w),$$

(5)

where the expression in the denominator models gain compression. However, two common simplifications applied to Eq. (5) are to ignore gain compression, and to linearize the logarithm, yielding

$$G = v_g a (N - N_w).$$

(6)

By keeping the gain term general we leave the choice regarding its functional form to the reader without losing any generality in what follows.
Performing the differentiation of the product on the left-hand side of Eq. (3), and dividing both sides by $e^{i\omega t}$, one obtains

$$\frac{dE(t)}{dt} = \left\{ j(\omega_m - \omega) + \frac{1}{2} \left( \Gamma G - \frac{1}{\tau_p} \right) \right\} E(t) + \kappa E(t - \tau_{\text{ext}}) e^{-i\omega t_{\text{ext}}}.$$  \hspace{1cm} (7)

It is customary to convert the complex rate equation [Eq. (7)] into two real rate equations, one for the photon density, $S$, in the laser cavity, and one for the phase of the electric field, $\phi$. To arrive at the rate equation for photon density, note from Table 1 that we scale $E$ so that

$$S(t) = |E(t)|^2 = E(t)E^*(t),$$ \hspace{1cm} (8)

where $\ast$ denotes complex conjugation. Differentiating Eq. (8) with respect to time gives

$$\frac{dS(t)}{dt} = E(t) \frac{dE^*(t)}{dt} + E^*(t) \frac{dE(t)}{dt},$$ \hspace{1cm} (9)

which may be directly applied to Eq. (7) to obtain

$$\frac{dS(t)}{dt} = \left( \Gamma G - \frac{1}{\tau_p} \right) S(t) + 2\kappa \sqrt{S(t)S(t - \tau_{\text{ext}})}$$

$$\times \cos[\omega \tau_{\text{ext}} + \phi(t) - \phi(t - \tau_{\text{ext}})].$$ \hspace{1cm} (10)

In order to obtain the rate equation for the phase, $\phi(t)$, of the slowly varying envelope of the electric field, note that

$$\phi(t) = \arctan \left( \frac{\text{Im}(E(t))}{\text{Re}(E(t))} \right).$$ \hspace{1cm} (11)

Differentiating Eq. (11) yields

$$\frac{d\phi(t)}{dt} = \frac{1}{(\text{Re}(E(t)))^2 + (\text{Im}(E(t)))^2}$$

$$\times \left[ \text{Im} \left( \frac{dE(t)}{dt} \right) \text{Re}(E(t)) - \text{Re} \left( \frac{dE(t)}{dt} \right) \text{Im}(E(t)) \right].$$ \hspace{1cm} (12)

Noting that for any two complex numbers $x$ and $y$, $\text{Im}(x)\text{Re}(y) - \text{Re}(x)\text{Im}(y) = \text{Im}(y^*x)$ and making use of Eq. (8), one obtains

$$\frac{d\phi(t)}{dt} = \frac{1}{S(t)} \left[ \text{Im} \left( E^*(t) \frac{dE(t)}{dt} \right) \right].$$ \hspace{1cm} (13)

which may be directly applied to Eq. (7) to yield

$$\frac{d\phi(t)}{dt} = (\omega_m - \omega) - \kappa \left( \frac{S(t - \tau_{\text{ext}})}{S(t)} \right)^{1/2} \sin[\omega \tau_{\text{ext}} + \phi(t) - \phi(t - \tau_{\text{ext}})].$$ \hspace{1cm} (14)

Here we make the assumption that $\omega \approx \omega_{\text{th}}$, which is constant. To simplify the expression $(\omega_m - \omega_{\text{th}})$ we will need to linearize both the laser refractive index, $n_{\text{th}}$, and gain, $G$, with respect to carrier density, $N$. We start by linearizing the refractive index about threshold:
\[ n_{\text{in}} = n_{\text{th}} + (N - N_{\text{th}}) \frac{\partial n_{\text{in}}}{\partial N}. \] (15)

Using this expression for \( n_{\text{in}} \), and noting that \( \omega_{\text{th}} = m \pi c / (n_{\text{th}} L_{\text{in}}) \), we may proceed with

\[
\omega_m - \omega_{\text{th}} = \frac{m \pi c}{L_{\text{in}}} \left( \frac{1}{n_{\text{in}}} - \frac{1}{n_{\text{th}}} \right)
= \frac{m \pi c}{L_{\text{in}} n_{\text{th}} n_{\text{in}}} (n_{\text{th}} - n_{\text{in}})
= \frac{m \pi c}{L_{\text{in}} n_{\text{th}} n_{\text{in}}} \left( - (N - N_{\text{th}}) \frac{\partial n}{\partial N} \right)
= \frac{\omega_m}{n_{\text{th}}} \left( - (N - N_{\text{th}}) \frac{\partial n}{\partial N} \right).
\] (16)

Now we will linearize gain for the purposes of further simplifying Eq. (16), but we will still keep the gain general in the context of Eq. (7). Linearizing the gain about threshold and noting that at threshold the gain is equal to the cavity loss, \( 1/\tau_p \), gives

\[
\Gamma G = \Gamma G_{\text{th}} + \Gamma (N - N_{\text{th}}) \frac{\partial G}{\partial N} = \frac{1}{\tau_p} + \Gamma (N - N_{\text{th}}) \frac{\partial G}{\partial N},
\] (17)

which may be applied to Eq. (16) to yield

\[
\omega_m - \omega_{\text{th}} = - \frac{\omega_m}{\tau_p} \left( \frac{\Gamma G - 1}{\tau_p} \right) \frac{\partial n_{\text{in}}}{\partial n} \left( \Gamma \frac{\partial G}{\partial N} \right).
\] (18)

Here we make use of the linewidth enhancement factor, defined as the ratio of the change in the real part of the laser refractive index to the change in the imaginary part of the laser refractive index \([62,67,75]\), to give

\[
\omega_m - \omega_{\text{th}} = \frac{1}{2} \alpha \left( \Gamma G - 1 \right).
\] (19)

In most instances \( \alpha \) is conveniently treated as a constant that is determined experimentally; however, in reality \( \alpha \) is a function of, among other things, both optical feedback level and carrier density \([77]\).

Substituting Eq. (19) into Eq. (14) (and using \( \omega \approx \omega_{\text{th}} \)) yields

\[
\frac{d\varphi(t)}{dt} = \frac{1}{2} \alpha \left( \Gamma G - 1 \right) \frac{S(t - \tau_{\text{ext}})}{S(t)} \times \sin[\omega_{\text{th}} \tau_{\text{ext}} + \varphi(t) - \varphi(t - \tau_{\text{ext}})].
\] (20)

Equations (10), (20), and (4) form the standard set of rate equations expressed in terms of densities. To convert from density equations to number equations, one must keep in mind that the volumes occupied by the photon and carrier populations differ, and are related by the optical confinement factor, \( \Gamma \). We write the photon number as \( \tilde{S} = V_p S \) and the carrier number as \( \tilde{N} = VN \).
Now, for clarity of exposition, we will write the full set of rate equations below in terms of both densities and numbers. For densities they are

\[
\begin{align*}
\frac{dS(t)}{dt} &= \left(\Gamma G - \frac{1}{\tau_p}\right) S(t) + 2\kappa \sqrt{S(t)} S(t - \tau_{\text{ext}}) \cos[\omega_{\text{th}} \tau_{\text{ext}} + \varphi(t) - \varphi(t - \tau_{\text{ext}})] \\
\frac{d\varphi(t)}{dt} &= \frac{1}{2} \alpha \left(\Gamma G - \frac{1}{\tau_p}\right) - \kappa \left(\frac{S(t - \tau_{\text{ext}})}{S(t)}\right)^{1/2} \sin[\omega_{\text{th}} \tau_{\text{ext}} + \varphi(t) - \varphi(t - \tau_{\text{ext}})] \\
\frac{dN(t)}{dt} &= \frac{n_i(t) - N(t)}{q \tau_n} GS(t).
\end{align*}
\] (21)

For numbers they are

\[
\begin{align*}
\frac{d\tilde{S}(t)}{dt} &= \left(\Gamma G - \frac{1}{\tau_p}\right) \tilde{S}(t) + 2\kappa \sqrt{\tilde{S}(t)} \tilde{S}(t - \tau_{\text{ext}}) \cos[\omega_{\text{th}} \tau_{\text{ext}} + \varphi(t) - \varphi(t - \tau_{\text{ext}})] \\
\frac{d\tilde{\varphi}(t)}{dt} &= \frac{1}{2} \alpha \left(\Gamma G - \frac{1}{\tau_p}\right) - \kappa \left(\frac{\tilde{S}(t - \tau_{\text{ext}})}{\tilde{S}(t)}\right)^{1/2} \sin[\omega_{\text{th}} \tau_{\text{ext}} + \varphi(t) - \varphi(t - \tau_{\text{ext}})] \\
\frac{d\tilde{N}(t)}{dt} &= \frac{n_i(t) - \tilde{N}(t)}{q} \Gamma \tilde{G} \tilde{S}(t).
\end{align*}
\] (22)

Regardless of whether the density or number form is used, the phase of the electric field is unaffected. Therefore, we keep its symbol, \(\varphi\), the same in both Eq. (21) and Eq. (22). The only difference in the two sets of equations is the presence or absence of the optical confinement factor in the carrier equation.

This basic time-dependent model is remarkably appropriate for a wide range of lasers. After so many approximations and assumptions in its derivation, one might wonder how well this model mimics reality, yet it predicts a great deal of complex behavior that has been observed in practice, including path-dependent behavior under moderate or strong feedback, as well as the multi-stability of the laser in these cases. Due to its wide applicability, the Lang and Kobayashi model provides important insight into the behavior of lasers experiencing feedback over small time scales, including turn-on delay and the exhibition of relaxation oscillations.

A. Signal Observation: Modeling Output Power and Terminal Voltage Fluctuations

As noted in Section 1, the LFI signal is observed by monitoring the fluctuations in either the laser output power (through the use of a PD), or its terminal voltage. The signals obtained from these two methods are directly proportional to one another, and, for low feedback levels, they are both (somewhat counterintuitively) proportional to the square root of the optical feedback power [78]. However, the PD and voltage signal strengths depend differently on injection current and temperature, leading to radically different biasing strategies for optimal sensor sensitivity in each case [79].

Linking optical output power variation to the rate-equation model can be as simple as noting that photon density is proportional to output optical power. To arrive at a specific formula, one may assume that the rate of stimulated emission
is approximately equal to the inverse of the photon lifetime above threshold, and that spontaneous emission is negligible to obtain [75]

\[ P_{\text{Total}}(t) = \hbar \omega v g \alpha_m V_p S(t) = \hbar \omega v g \frac{1}{2L_{\text{in}}} \ln \left( \frac{1}{R_1 R_2} \right) V_p S(t), \quad (23) \]

where \( P_{\text{Total}} \) is the total optical power emitted from both laser facets, \( \hbar \) is the reduced Planck’s constant, and \( \alpha_m \) is the effective mirror loss.

On the other hand, the laser terminal voltage \( V_{\text{Terminal}} \) for a laser diode may be modeled as [78,80]

\[ V_{\text{Terminal}}(t) = \frac{2k_B T}{q} \ln \left( \frac{N(t)}{N_i} \right), \quad (24) \]

where \( k_B \) is Boltzmann’s constant and \( N_i \) is the intrinsic carrier density of the active region.

Both Eq. (23) and Eq. (24) assume a bulk active region and simple Fabry–Perot cavity structure. More complex laser structures may require a different approach to accurately model the interferometric signal. For example, the voltage of a quantum-well laser may be modeled by taking into account the different carrier concentrations in the quantum-well and barrier regions [81] (see also [82–86] for further details). As a second example, a distributed feedback (DFB) laser has a slightly different expression for the effective mirror loss, \( \alpha_m \), thus altering Eq. (23) [75]. However, both Eq. (23) and Eq. (24) are good starting points for most system configurations.

4. Steady-State Behavior: Excess Phase Equation and Three Mirror Model

A. Excess Phase Equation

The dynamic rate-equation model can be reduced to temporal steady state whenever the photon density \( S \) and the carrier density \( N \) do not change rapidly. This occurs whenever the frequencies of system stimuli (in the form of changing current \( I \), external cavity round-trip time \( \tau_{\text{ext}} \), or feedback coupling rate \( \kappa \)) are slow relative to the natural frequencies of the system—the laser relaxation frequency and the natural resonant frequency of the external cavity. In practice, most continuous-wave LFI systems operate in this quasi-static regime. In steady state, as mentioned in Section 2, multiple reflections in the external cavity can be considered simply by modifying the coupling rate \( \kappa [63–65] \).

Steady-state solutions to the rate equations [Eq. (21)] can be found by substituting \( S(t) = S(t - \tau_{\text{ext}}) = S_s, N(t) = N_s, \varphi(t) = (\omega - \omega_s)t \) (where \( \omega_s \) is essentially \( \omega_{\text{th}} \)), and setting derivatives to zero, yielding [62,76]

\[ N_s = N_{\text{th}} - \frac{2\kappa}{\Gamma v g a \tau_{\text{in}}} \cos(\omega \tau_{\text{ext}}), \quad (25) \]

and

\[ \omega_s - \omega = \frac{\kappa}{\tau_{\text{in}}} \sqrt{1 + \alpha^2} \sin(\omega \tau_{\text{ext}} + \arctan \alpha). \quad (26) \]
By multiplying Eq. (26) by \( \tau_{\text{ext}} \), defining \( \varphi_s = \omega_s \tau_{\text{ext}} \) and \( \varphi_{FB} = \omega \tau_{\text{ext}} \), using the characteristic parameter \( C \) given in Eq. (2), and rearranging, we obtain the **excess phase equation** for solution in \( \varphi_{FB} \):

\[
\varphi_{FB} - \varphi_s + C \sin(\varphi_{FB} + \arctan \alpha) = 0.
\]  

(27)

It is useful to think of the term \( \varphi_s \) as a phase stimulus—symbolically corresponding to the phase accumulated on transmission through the external cavity if the laser were not experiencing optical feedback—and \( \varphi_{FB} \) as a phase response, corresponding to the actual phase accumulated on transmission through the external cavity. The feedback level \( C \) dictates the degree of nonlinear coupling between phase stimulus and response, while the linewidth enhancement factor \( \alpha \) governs the asymmetry of the phase transfer function induced by Eq. (27).

Note that Eq. (27) is a transcendental equation with unique solution when \( C \leq 1 \) (weak feedback) and with multiple solutions when \( C > 1 \) (moderate or strong feedback). Due to the alternating stability of solutions when \( C > 1 \), physical solutions \( \varphi_{FB} \) to Eq. (27) exhibit path dependence (hysteresis) as \( \varphi_s, C, \) or \( \alpha \) varies. This characteristic necessitates that some care be taken in solving Eq. (27) [87]. Figure 4 shows the relationship between the phase stimulus \( \varphi_s \) and the response \( \varphi_{FB} \), including the path dependence of the solution for a harmonic phase stimulus.

The phase response \( \varphi_{FB} \) is not directly observable; rather, the experimentally observable quantity is either the variation in laser power or variation in voltage across the laser terminals. The dependence of these quantities on the phase response \( \varphi_{FB} \) can be found as follows. By considering the carrier rate equation in steady state (and a linear gain function), the steady-state optical power can be determined as [76]

\[
P_s \propto S_s = \frac{1}{G} \left( \frac{\eta I}{qV} - \frac{N_{th}}{\tau_n} \right).
\]  

(28)

Substituting for the gain from the photon rate equation, and substituting for \( N_s \) from Eq. (25) yields

\[
P_s \propto \left( \frac{\eta I}{qV} - \frac{N_{th}}{\tau_n} + \frac{2\kappa}{\Gamma_{\text{ext}} \alpha \tau_n} \cos(\varphi_{FB}) \right) \frac{\Gamma \tau_p}{1 - 2\kappa \tau_p \cos(\varphi_{FB})}.
\]  

(29)

Assuming that \( 2\kappa \tau_p \ll 1 \), and noting that \((1 - x)^{-1} \approx (1 + x)\) for \( x \ll 1 \), gives

\[
P_s \propto \Gamma \tau_p \left( \frac{I}{\eta I qV} - \frac{N_{th}}{\tau_n} \right)(1 + 2\kappa \tau_p \cos(\varphi_{FB})).
\]  

(30)
With a suitable voltage–current model depending on carrier density (or number), the dependence of steady-state voltages on $\phi_{FB}$ may be obtained similarly. For the remainder of this section, we focus exclusively on the ac component of the LFI signal, which is approximated by $\cos(\phi_{FB})$.

Examining the excess phase equation [Eq. (27)], there are effectively only two “inputs”—$C$ and $\phi_s$—that may be varied in order to drive changes to $\phi_{FB}$ ($\alpha$ is usually modeled as a constant). While Eq. (27) appears innocuous, the resulting LFI signal morphology depends crucially on the values of $C$ and $\alpha$, and on magnitude and functional form of the phase stimulus $\phi_s$ [88]. To illustrate this dependence in an intuitive way, we make use of animation [Fig. 5 (Visualization 1, Visualization 2, and Visualization 3)]. In these animations, we show the relationship between the temporal evolution of the phase stimulus and the LFI signal. In addition, we show the effect of $C$ and $\alpha$ on the morphology and strength of the LFI signal, while keeping the stimulus unchanged in all our examples.

Visualization 1 shows the change in the LFI signal morphology as the feedback parameter $C$ varies for a fixed value of $\alpha = 5$. This value of $\alpha$ is representative of the behavior of diode lasers, and these LFI waveforms are most frequently found in the literature [76]. As $C$ increases, fringes in the LFI signal tilt and sharpen, resulting in abrupt vertical features when $C > 1$ (a consequence of jumps between solutions). For moderate to strong feedback, one observes the loss of fringes (associated with loops observed in the transfer function), a reduction of amplitude, and a notable asymmetry between the positive and negative parts of the LFI signal. With further increase in $C$, the transfer function becomes extremely tilted, resulting in the situation where the operating point of the interferometer remains on one (slightly sublinear) fringe for a very large range of input stimuli. As a consequence of the almost linear transfer function in this case, the shape of the LFI signal closely replicates the stimulus.

Visualization 2 shows the change in the LFI signal with varying feedback parameter $C$, when $\alpha$ has been reduced from $\alpha = 5$ to $\alpha = 0.1$, a situation typical for quantum cascade lasers (though carrier and photon lifetimes for these lasers are considerably different to those in diode lasers, which will also affect their
dynamics). All other parameters have been kept unchanged from those in Visualization 1. The main effect of this change in \( \alpha \) is that the shape of the transfer function is symmetric and remains upright while \( C \) increases. This results in the symmetrical shape of fringes observable in the LFI signal. The vertical transitions usually observed for moderate feedback levels in diode lasers (and frequently discussed in the literature) are unobservable. Unlike the more usual situation corresponding to large positive values of \( \alpha \), even for strong feedback, the LFI signal does not replicate the stimulus signal.

In contrast to the previous two media, in Visualization 3 we explicitly explore the effect of \( \alpha \) on the morphology of the LFI signal. We selected a constant value of \( C/\alpha = 1.5 \) (moderate feedback), where asymmetry and hysteresis in the fringe system are easily observable. As we change the value of \( \alpha \) from negative through zero to positive, the transfer function shape changes from leaning to the left, through upright, to leaning to the right.

In Sections 6–9 we will elaborate on how various sensing topologies tie into the excess phase equation through these two inputs.

**B. Three-Mirror Model**

In Section 4.A, Eq. (27) was obtained directly from the Lang–Kobayashi model. The same equation can be obtained in an entirely different way by considering...
only the geometry of the optical system of the laser feedback interferometer (three-mirror model). Referring to Fig. 1, in the presence of an external cavity, the amplitude reflection coefficient, \( r_2 = R_2^{1/2} \), of \( M_2 \) can be replaced by an equivalent (but strongly frequency dependent) version \([75,89,90]\):

\[
\bar{r}_2(\omega) \stackrel{\text{def}}{=} |\bar{r}_2|e^{i\phi} = r_2 + e(1 - R_2)r\exp(-i\omega\tau_{\text{ext}}),
\]

where \( r = R^{1/2} \) is the field amplitude reflection coefficient of the target. Without loss of generality, \( r_2 \) and \( r \) can be considered real and positive. When \( \kappa \ll 1 \) [where \( \kappa \) is as in Eq. (1)], Eq. (31) reduces to

\[
|\bar{r}_2| = r_2[1 + \kappa \cos(\omega\tau_{\text{ext}})],
\]

since \( \text{Re}(\bar{r}_2) \approx |\bar{r}_2| \), and

\[
\theta_r = -\kappa \sin(\omega\tau_{\text{ext}}),
\]

since \( \kappa \sin(\omega\tau_{\text{ext}}) \ll (1 + \kappa \cos(\omega\tau_{\text{ext}})) \) when \( \kappa \ll 1 \), and \( \text{arctan}(x) \approx x \) for small \( x \). As the round-trip phase within the laser cavity must equal an integer multiple of \( 2\pi \), we obtain the phase condition

\[
-2kL_{\text{in}}n_{\text{in}} + \theta_r = -2\pi m, \quad m \in \mathbb{Z},
\]

where \( k \) is the wave number (phase constant) of the cavity resonant mode. Using the effective refractive index \( n_{\text{in}} = ck/\omega \), this phase condition becomes

\[
-2\omega n_{\text{in}}L_{\text{in}}/c + \theta_r = -2\pi m, \quad m \in \mathbb{Z}.
\]

At threshold without feedback (that is, when \( \theta_r = 0 \)), Eq. (35) determines the laser emission frequency \( \omega = \omega_{\text{th}} \). With feedback, both the threshold gain and emission frequency may shift as

\[
\Delta(n_{\text{in}}) = \omega_{\text{th}} \Delta n_{\text{in}} + (\omega - \omega_{\text{th}})n_{\text{in}},
\]

yielding a change \( \Delta \bar{\varphi}_c \) in the compound cavity phase condition Eq. (35) (relative to fixed \( 2\pi m \)) of

\[
\Delta \bar{\varphi}_c = -2L_{\text{in}}(\omega_{\text{th}} \Delta n_{\text{in}} + (\omega - \omega_{\text{th}})n_{\text{in}}) + \theta_r.
\]

By relating the change in effective index \( \Delta n_{\text{in}} \) to its change with carrier density and emission frequency, considering the amplitude condition at threshold of the compound cavity, and rewriting in terms of effective group refractive index \( n_g = n_{\text{in}} + \omega_{\text{th}} \frac{\partial \alpha}{\partial \omega} \) and Henry’s linewidth enhancement factor \( \alpha \), when \( \kappa \ll 1 \) we may rewrite Eq. (37) as \([75]\)

\[
\Delta \bar{\varphi}_c = \frac{2n_g L_{\text{in}}}{c}(\omega - \omega_{\text{th}}) + \kappa(\sin(\omega\tau_{\text{ext}}) + \alpha \cos(\omega\tau_{\text{ext}}))
\]

Considering that \( \Delta \bar{\varphi}_c = 0 \) for the compound cavity’s resonant frequency \( \omega \), by dividing by the cavity round-trip time \( \tau_{\text{in}} \) and rearranging, Eq. (38) reduces to the excess phase equation [Eq. (27)], which was found via entirely different means.

C. Linking Physical Parameters to the Excess Phase Equation

In subsequent sections we explore in detail the many LFI sensing schemes available. The means by which we observe the self-mixing effect—the LFI signal—is
observed via monitoring the variations in the laser optical output power or its terminal voltage. It is then processed in a manner particular to each sensing scheme to extract the quantity being measured. The LFI signal therefore is manifest as a small perturbation of a larger, constant value (the voltage or optical power). As such, while it is possible to measure the laser optical power or the terminal voltage directly [91], it is far more practical to ac couple the signal and observe only the temporal changes in these quantities. When measuring a quantity that inherently induces time-varying perturbations to the laser operating state, the LFI signal is naturally time varying and additional modulation is frequently unnecessary. However, if the quantity being measured does not inherently produce a time-varying LFI signal, a system parameter needs to be modulated externally in order to create the desired time variability. This is typically achieved by modulating the laser current, which in particular induces a change in the laser operating frequency (resulting from temperature-induced changes to the laser cavity, principally its effective refractive index), or by modulating the optical feedback level using mechanical or electro-optical means.

Here we make note that inspection of Eq. (27) identifies \( C \) and \( \phi_s \) as parameters that may be modulated to produce a time-varying LFI signal (whether naturally, for example, by the quantity being measured, or by external modulation). The feedback parameter \( C \) depends on the strength of reflection from the target as well as the external cavity round-trip time. The phase stimulus \( \phi_s \) is affected by changes in the effective optical length of the external cavity—due to changes in the laser frequency, the external cavity length and refractive index, and the phase shift on reflection [92–94]. Therefore these two parameters that one can control are connected, and changing a physical quantity that affects both parameters will cause complex behavior.

Neglecting thermal effects, which can influence many of the parameters in LFI but typically operate over long time scales [95,96], the direct stimuli usually accessible are: (i) current \( I \), (ii) reflectivity of the external target \( R \) (including its phase-shift on reflection \( \theta_R \)), and (iii) optical length of the external cavity (i.e., a combination of the physical length, \( L_{\text{ext}} \), and the corresponding effective complex refractive index, \( \tilde{n}_{\text{ext}} = n_{\text{ext}} - jk_{\text{ext}} \)). Changing current leads to shifting of the solitary laser frequency \( \omega_s \), which can therefore be thought of as an indirect stimulus. In the discussion that follows, we will make reference to known functional forms of the solitary laser frequency with current (for example, linear frequency sweeping). Additional indirect stimuli, such as the change in gain \( G \), laser cavity refractive index \( n_{\text{in}} \), or reflectivity of the cavity mirrors \( R_1 \) and \( R_2 \), will not be considered further in this tutorial.

In examining Eqs. (27) and (30), these stimuli can be grouped into two categories: phase stimuli \( (L_{\text{ext}}, n_{\text{ext}}, k_{\text{ext}}, \theta_R, \omega_s) \) and amplitude stimuli \( (L_{\text{ext}}, n_{\text{ext}}, R, I) \). Note in particular that the physical length of the external cavity \( L_{\text{ext}} \) and its refractive index \( n_{\text{ext}} \) appear in both lists.

5. Implementing Laser Feedback Interferometry with Diode Lasers

Having covered the physical processes underpinning LFI, and explored the modeling methodology used to describe them, we outline the basic architecture for LFI using a diode laser. LFI can be implemented using virtually any laser, as
noted in Section 1. However, the majority of published work and the most compact implementation uses a semiconductor laser, whether it be a diode laser or a quantum cascade laser. The steady-state theory presented in Section 4 holds equally well for all laser types, but the apparatus presented here describes an LFI implementation using a diode laser. While a variety of signals and physical quantities can be acquired by LFI, the topology of the basic LFI system remains virtually the same across the spectrum of applications.

We first present the overall system topology in Fig. 6. As outlined theoretically in Section 3.A, the LFI signal may be observed either by monitoring optical output power, usually via a PD, or via monitoring the laser voltage (which is effectively monitoring the carrier density within the laser active region). If using an external PD to observe the LFI signal, a portion of the laser emission must be guided to the PD, which may be conveniently done via a beam splitter,

![Figure 6](https://example.com/figure6.png)

Schematic diagram of the basic LFI apparatus with (a) an external PD and (b) an internal PD. The laser current is controlled with a laser driver, through which electrical modulation may be applied. In (a), the emitted beam is split, with a portion passing through external optical elements to and from the target and that may be mechanically modulated, and a portion transmitted to an external PD. Interferometric signals are acquired using a data acquisition card connected to a computer (PC) through (i) an external PD together with a trans-impedance amplifier (TIA), followed by a bandpass filter (BPF); and/or (ii) the laser terminal voltage together with a voltage amplifier (Volt. Amp), followed by a BPF. In (b), the system topology is simpler, as the internal PD obviates the need for a beam splitter.
as in Fig. 6(a). Note that, within the collimated path, the order of the beam splitter and mechanical modulator may be swapped from that depicted in Fig. 6(a). However, many commercially packaged laser diodes have an integrated PD for power monitoring. If using this integrated PD or just the laser voltage to observe the LFI signal, then the system topology is simpler, as per Fig. 6(b).

A. Optical Considerations

LFI systems composed of only a laser and a target with no other optical elements have been demonstrated [97,98]; however, the vast majority of systems do incorporate optical elements to guide and shape the light. When designing the optics for an LFI system, it is prudent to begin with the characteristics of the sensing application. For instance, when sensing flow in a channel, the laser beam must have a component in the direction of the flow. If the sensing scheme calls for the target to move over a certain spatial range, the Rayleigh range of the beam must be designed in order to accommodate this movement while maintaining usable signal-to-noise ratio (SNR) [93].

After considering the demands on the optical system imposed by the sensing application, the nature of the target must be considered. Its reflectivity will directly affect system SNR. If specular reflection is dominant, the target will need to be carefully aligned to guide the reflected light back to the laser. If diffuse reflection is dominant, care must be taken to prevent unwanted reflections from coupling back into the laser.

In view of this, it is often useful to incorporate a variable optical attenuator in the beam path in order to control the optical feedback level. It is advisable to angle the attenuator (and any other similar optical components) with respect to the laser beam axis so that the reflection from the attenuator does not couple back into the laser, forming an inadvertent optical cavity.

Consideration must be given to the laser spot size on the target, and, by extension, the sensing volume. The beam may be focused to a very small spot on the target, enabling high-resolution spatial scanning. However, as the focus is made tighter and the spot smaller, the Rayleigh range of the beam will likewise decrease.

B. LFI Signal Retrieval

If one chooses to use a PD to sense the LFI signal, some additional considerations are warranted. The PD may be external to the laser package, or may be integrated within it. When using an integrated PD, an important distinction must be made between in-plane lasers and VCSELS. Figure 7(a) shows the internal structure of a packaged in-plane laser with integrated PD. The emission from the back facet of the laser directly illuminates the PD. However, the photodetection in a packaged VCSEL functions differently, as Fig. 7(b) shows. As the back facet of the VCSEL faces the substrate, no light is emitted from it. Rather, the PD samples the light reflected from the package window, which is emitted from the top facet of the laser. The reflection from the package window may cause undesired effects in an LFI system, in which case the window may need to be removed. Once it has been removed, sensing the LFI signal from a VCSEL using the integrated PD may no longer be possible, and the voltage signal or an external PD may need to be used instead.
If using the PD integrated into the laser package, care must be taken as to whether the configuration is common anode or common cathode. The PD detects power fluctuations in the laser light and generates a proportional photocurrent, which is converted into a voltage using a trans-impedance amplifier (TIA). As such, one needs to take into account the responsivity of the PD if the absolute value or the power/current fluctuation is of interest. The trans-impedance gain

Schematic diagram of (a) an in-plane laser and (b) a VCSEL packaged with a monitoring PD in a transistor outline can. For the in-plane laser, the rectangular aperture results in an elliptical profile of the emitted beam from both the front and rear facets. The PD responds to optical power emitted from the rear facet of the laser. For the VCSEL, the circular aperture results in a circular profile of the emitted beam. The PD responds to optical power backreflected from the glass window.
being used must ensure that the dc part of the current does not get amplified beyond the supply voltage range of the TIA. Another concern when using the PD signal is speed—the bandwidth of the PD must be taken into account when designing the system. A physically large PD, as is typically integrated into VCSEL packages, will have limited BW and may restrict the LFI measurement.

If using the voltage signal, an ac-coupled voltage amplifier needs to be connected to the laser terminals. The input impedance of the amplifier must be large in order to not load the laser and affect its operation. Low-noise laser drivers are available from a number of suppliers, but many of them have a floating ground at their output. This can make it difficult to extract the voltage signal—a common-mode amplifier may simply not work. A differential input amplifier with isolated input and output may need to be used (e.g., the pre-amplifier in some lock-in amplifiers). On the other hand, laser drivers that have a common-mode output may be comparatively more noisy.

Note that the PD and voltage signals are related, but may be phase shifted relative to each other [99–101].

C. Sensors Based on Laser Arrays: Multidimensional Measurement

Simultaneous measurement is advantageous for many applications. One straightforward benefit in applications that acquire multidimensional measurements—such as mapping surface profile or imaging vibration/flow—is that the precise spacing between sensing elements is known. In these instances, a linear array of lasers can reduce acquisition time, potentially reducing one dimension of the measurement completely. Early implementations consisted of discrete lasers with cumbersome circuitry [102], moving presently to monolithically integrated arrays of lasers [103–105]. In the latter case, acquiring PD signals is possible with the additional integration of an array of PDs [106]; however, the acquisition of terminal voltage signals will often be preferred for its relative simplicity. This monolithic integration has the intrinsic advantage that lasers are proximal on the same chip; however, care must be taken to mitigate the effect of optical cross talk [107]. The increased operating temperature of the sensors when operating concurrently has been reported to improve SNR for the sensors (compared to single-channel operation) [105].

Concurrent measurement with multiple lasers opens up applications that are difficult or impossible to achieve with a single-channel LFI system: (i) two integrated sensors for quasi-three-dimensional displacement measurement [108,109], (ii) exploiting the spatial configuration of the laser array to detect and measure an object that passes across the array, or (iii) high-frame-rate imaging using one- or two-dimensional laser arrays (such as the VCSEL arrays developed by Philips [110]).

6. External-Cavity-Related Measurement

A. Large Displacement Measurement (\(\Delta L_{\text{ext}} \gg \lambda/(2n_{\text{ext}})\))

Perhaps the simplest stimulus for LFI is created by changing the length of the external cavity, \(L_{\text{ext}}\), over small distances [23,111–115], and is used for example...
in profilometry [116–118]; Fig. 8 depicts recovered amplitude measured across a vibrating surface. As there is negligible change in feedback strength, $C$ remains constant, and the expression for the phase stimulus is simply

$$\varphi_s(t) = \omega_s \tau_{\text{ext}}(t) = \frac{2\pi c}{\lambda} \frac{2n_{\text{ext}}L_{\text{ext}}(t)}{c} = \frac{4\pi n_{\text{ext}}L_{\text{ext}}(t)}{\lambda}. \quad (39)$$

By changing $L_{\text{ext}}$ by $\lambda/(2n_{\text{ext}})$, which is to say by half a wavelength (taking into account the refractive index of the external cavity), we change $\varphi_s$ by $2\pi$. This gives rise to a periodicity in the observed signal corresponding to half-wavelength displacements [119]—“fringes.” Note that the effect of longitudinal speckle on the interferometric waveform over large distance displacements is manifested in an additional amplitude modulation envelope; see [94, 120, 121]

**Figure 8**

Amplitude in $\mu$m

Measured vibration amplitude of a titanium tweeter membrane at 300, 500, 580, 700, 800, 900, 3200, 3500, 3600, 6600, and 6800 Hz, showing different nodal structures. Reproduced with permission from [118]. Copyright 2014 SPIE.
for details, and [122–124] for the simultaneous recovery of varying \( C \) and for increased accuracy in the recovered displacement stimulus.

Figure 9 shows typical LFI signals resulting from linear and sinusoidal displacement. The LFI signals show periodicity corresponding to half-wavelength displacements. In the weak feedback regime (that is, \( C \leq 1 \)), Eq. (27) has a unique solution and does not exhibit path dependence—and, in particular, there is no loss of fringes [see Figs. 9(c) and 9(d)]. Therefore, by recording the interferometric signal [Eq. (30)], we may easily determine the change in external cavity length relative to \( \lambda/(2n_{\text{ext}}) \) by counting the number of whole and fractional fringes observed [125] or via Fourier transform techniques [126]. If \( \lambda \) is known, we will have reconstructed the displacement, whose accuracy is fundamentally

![Figure 9](image_url)
dependent on the wavelength [127]. Resolution can be improved using multiple reflections on the external target, using, for example, a birefringent crystal or wave plate inserted in the optical path [128–130]. Using two lasers and comparing the LFI signals received from them can also be used to improve resolution [131].

In the moderate and strong feedback regimes (C > 1), Eq. (27) has multiple solutions and exhibits path dependence (see Fig. 4)—and, in particular, may lead to loss of fringes as feedback increases [see Figs. 9(c)–9(h)]. At very high levels of feedback, all fringes are lost, and the stimulus is replicated in the response [see Figs. 9(i) and 9(j)].

As such, the recovery of a stimulus with amplitude $\Delta L_{\text{ext}} \gg \lambda/(2n_{\text{ext}})$ from the measured response (LFI signal) is straightforward only at very high levels of feedback. As discussed in Section 6.B, it is also straightforward to recover a stimulus when $\Delta L_{\text{ext}} \ll \lambda/(2n_{\text{ext}})$, regardless of the level of feedback. In all other cases, recovery of the stimulus can be undertaken, but requires more care; see [132,133] for details.

Tracking the displacement of the target can be applied to many different applications, such as sensing the arterial pulse wave in vivo [56] and two-dimensional tracking of motor shaft displacement [134].

**B. Small Displacement Measurement ($\Delta L_{\text{ext}} \ll \lambda/(2n_{\text{ext}})$)**

When the target displacement is smaller than $\lambda/(2n_{\text{ext}})$, as is the case with small vibrations, the displacement can still be measured [135–139], but the signal morphology tends to be different from that in Section 6.A. Suppose that the length of the external cavity is once again changing as in Eq. (39). For simplicity, consider harmonic variation in external cavity length, $L_{\text{ext}}(t) = L_{\text{ext}}(0) + |\Delta L_{\text{ext}}| \sin(2\pi f_{\text{ext}} t)$, say, where the maximum displacement satisfies $|\Delta L_{\text{ext}}| \ll \lambda/(2n_{\text{ext}})$, varying at frequency $f_{\text{ext}}$. Provided this frequency of vibration $f_{\text{ext}}$ is sufficiently small, we have phase stimulus

$$\phi_s(t) = \omega_s t = \frac{2\pi c 2n_{\text{ext}} L_{\text{ext}}(t)}{\lambda} = \frac{4\pi n_{\text{ext}} L_{\text{ext}}(0)}{\lambda} + \frac{4\pi n_{\text{ext}} |\Delta L_{\text{ext}}| \sin(2\pi f_{\text{ext}} t)}{\lambda}. \quad (40)$$

To obtain good linearity of the stimulus–response transfer function, it is desirable to ensure that the small displacements occur within one $\lambda/(2n_{\text{ext}})$ fringe—and, in particular, at a quadrature point of the transfer function (see Fig. 10). This can be achieved by tuning either the lasing frequency $\omega$ (through the laser bias current, $I$) or the external cavity length $L_{\text{ext}}(0)$.

In this case, the vibration can be measured in an analog fashion directly from the interferometric signal, with scale calibrated to within one $\lambda/(2n_{\text{ext}})$ fringe. Figure 10 (Visualization 4) explores the dependence of the LFI signal on $C$. With increase in $C$, the slope of the transfer function decreases, bringing about a corresponding decrease in sensitivity to the stimulus. However, with the increased tilt of the transfer function, its quasi-linear part extends to a larger range of input phase stimuli, effectively increasing the dynamic range of the sensing scheme.

This approach also works well with nonharmonic stimulus—see Fig. 11 for an example of a surface electrocardiographic signal [140].
C. Measuring Absolute Distance by Linear Frequency Sweeping

A small variation in laser driving current, $\Delta I$, will produce an approximately linear variation in the laser operating frequency, $\Delta \nu$. The empirical frequency modulation coefficient, $\Omega$, relates the two quantities as \( \Delta \omega = 2\pi \Delta \nu = 2\pi \Omega \Delta I \). \[(41)\]

A typical value for $\Omega$ for a semiconductor laser diode is $-3$ GHz/mA; however, the exact value for any particular laser must be determined experimentally. With a fixed external target and a small fractional frequency sweep $[\Delta \omega/\omega_s(0) \ll 1$, so that target reflectivities do not change appreciably over the frequency range due to material dispersion] we have phase stimulus of

![Sinusoidal phase-stimulus–power-response transfer function](image1.png)

![Electrocardiographic phase-stimulus–power-response transfer function](image2.png)
The current is typically modulated using a sawtooth or triangle function to exploit the (approximately) linear relationship between current and frequency. Note that to achieve a linear frequency shift in practice, some distortion (pre-emphasis) of the form of the modulating current is usually required [142–144]. Assuming a sawtooth modulation, let us take one period of the sawtooth to be of the form

\[ \Delta I(t) = At, \]  

where \( A \) is the slope of the sawtooth function. This corresponds to a phase stimulus of

\[ \varphi_s(t) = \tau_{\text{ext}} \omega_s(t) = \tau_{\text{ext}}[\omega_s(0) + 2\pi \Omega \Delta I(t)]. \]  

Note that the form of this phase stimulus is functionally identical to that present in the displacement application discussed earlier. Being a phase stimulus, it will produce periodic modulation at integer multiples of \( 2\pi \). By defining the period of this induced modulation as \( T \), we may write

\[ 2\pi = \tau_{\text{ext}}(2\pi \Omega AT), \]  

which gives the optical length of the external cavity as

\[ n_{\text{ext}} L_{\text{ext}} = \frac{c}{2\Omega AT}. \]  

Hence, by inducing frequency change at a known rate, one can infer the distance to the target \( L_{\text{ext}} \) by fringe counting [125]: \( L_{\text{ext}} = c(N_f \pm 1)/(2n_{\text{ext}} \Delta \omega) \), where \( N_f \) is the number of fringes in the observed interferometric signal. Figure 12 shows a three-dimensional profile reconstructed based on the average frequency variation between the fringes (see also [145]).

A refinement can be achieved by simultaneously modulating the phase by using an electro-optic crystal in the external cavity [146]. These coherent schemes provide high accuracy over a wide range of distances [147]. However, the effect of longitudinal speckle on the interferometric waveform over large distance displacements is manifested in an additional amplitude modulation envelope; see [94,120,121] for details.

While fringe-counting is a straightforward method, Fourier transform methods (in particular, the fast Fourier transform (FFT)) have been shown to provide superior performance in the presence of noise [148–150]. Moreover, a variant of tonal analysis, multiple signal classification, has been shown to provide better performance in the presence of noise than even the FFT [151]. Linear frequency sweeping has also been used in conjunction with synthetic aperture techniques and a THz QCL, allowing contrast to be maintained beyond the diffraction limit [152].

An alternate method for determining absolute distance is to place a chromatic dispersive element in the external cavity, between the laser and the target. In this case, the optical feedback from the focal spot on the target is most strongly coupled back into the laser cavity, providing an LFI signal that corresponds to the distance between the chromatic dispersive element and the target [153].
D. Simultaneous Measurement of Distance and Velocity by Triangular Frequency Sweeping

If the laser current is modulated to produce a periodic triangular frequency sweep [125], then an estimate of the velocity of the external target is possible at the same time as an absolute distance measurement [125]. This is made possible as the increasing linear sweep of frequencies is seen interferometrically as a linear extension of the external cavity, while the decreasing linear frequency sweep is seen as a linear contraction of the external cavity. As such, the number of interferometric fringes on the increasing and decreasing sweeps gives information about the location and change in location of the external target itself.

Figure 13 depicts the two physical stimuli (linear displacement and triangular frequency sweep), which combine into a single phase stimulus. The typical LFI signal (response) in this case is shown in Fig. 13(c), from which fringes on the increasing and decreasing portions of the frequency sweep may be extracted—for example, by numerical differentiation [see Fig. 13(d)]. These fringe counts contain information about the location and change in location of the external target.

E. Measuring Change of Refractive Index in External Cavity

A change in the (real-valued) refractive index of the external cavity is effectively a change in the optical length of the external cavity, and is thus equivalent to
changing the geometrical length of the cavity, as in Sections 6.A and 6.B. An interesting example is the imaging of acoustic fields (see Fig. 14). We may write the phase stimulus as

$$
\varphi_s(t) = \omega_s \tau_{ext}(t) = \frac{2\pi c}{\lambda} \frac{2n_{ext}(t)L_{ext}}{c} = \frac{4\pi n_{ext}(t)L_{ext}}{\lambda}.
$$

(47)

For large phase changes, in a similar way to displacement measurement, changes in external cavity refractive index can be detected simply by fringe counting.
with improved resolution possible by arrangement of a series of external mirrors guiding the laser beam multiple times through a sample placed in the external cavity \[155,156\]. Small phase changes can be detected by automatic calibration at a series of three, four, or five settings of an electro-optic modulator inserted into the external cavity \[157,158\].

Alternately, when the emitted beam is coupled into an external fiber, strain applied to the fiber results in changes in both external cavity length and its refractive index, permitting the integral strain along the fiber to be measured \[159,160\]. Another interesting recent application is the sensing of trace gas in the external cavity \[159,160\].
7. Target-Related Measurement

A. Measuring Material Complex Refractive Index of the Target by Slow Linear Frequency Sweeping: Material Characterization

As in Section 7.D, the optical length of the external cavity $n_{\text{ext}}L_{\text{ext}}$ is held constant, and the temporal variation in complex refractive index of the target is under investigation. To measure the complex refractive index of the target (rather than its temporal change) one can slowly sweep the laser frequency, enabling the extraction of the complex refractive index of the target material. The amplitude and phase of the reflection coefficient of the target are as in Eqs. (53) and (54), but the linear frequency sweep (similar to Section 6.C) modifies the inputs to the excess phase equation as

$$\varphi_s(t) = \omega_s(t)\tau_{\text{ext}} - \theta_R = \frac{2n_{\text{ext}}L_{\text{ext}}\omega_s(0)}{c} + \frac{2n_{\text{ext}}L_{\text{ext}}|\Delta\omega|t}{c} - \theta_R. \quad (48)$$

In Eq. (48), an emitted wave takes time $\tau_{\text{ext}}/2$ to traverse the external cavity, accumulating transmission phase; is reflected from the (stationary) external target, which imparts phase shift on reflection $\theta_R$; and returns to the laser, once again taking time $\tau_{\text{ext}}/2$ to traverse the external cavity, accumulating transmission phase. The offset of the interferometric waveform due to the linear frequency sweep shifts depending on the phase shift on reflection from the target, while the amplitude of the waveform depends on the reflectivity of the target. This pair of target characteristics is related to its refractive index ($n$) and extinction coefficient ($k$), which can therefore be recovered by calibrating the system with respect to known reference materials [54,161]. Even without calibration, this enables the concurrent mapping of amplitude and phase information of the external target, providing two registered images of the target, which could then be used to build up its three-dimensional profile. For example, Fig. 15 shows the imaging of porcine tissue obtained using LFI. However, this type of sensing scheme has to date been demonstrated only with THz QCLs at cryogenic temperatures [54,162,163], and it remains to be seen whether other types of lasers (that may be less stable due, for example, to thermal effects or laser phase noise [164]) are appropriate for this sensing scheme.

B. Imaging by Frequency Shifting Using Acousto-Optic Modulators

An acousto-optic modulator may be used to frequency shift laser emission for imaging and sensing purposes [165]. This has the advantage of not altering the laser operating state, as is the case when employing a current sweep. This technique is often used for imaging as in, for example, biomedical imaging using confocal microscopy [166] or confocal fiber laser imaging of waveguide modes [22], but it can be applied in a wide range of LFI settings. Figure 16 shows some interesting imaging results from these. Acousto-optic techniques can also be used to reduce the effect of parasitic reflections, i.e., from microscope slides [167], and to reach the shot noise limit with mirror scanning [168].
C. Measuring Thickness and Refractive Index of a Transparent Material

An interesting application of LFI is the simultaneous measurement of thickness and refractive index of a transparent material [169]; see Fig. 17. A related approach to measuring the refractive index of a transparent material may be found...
in [170]. This technique uses two concurrently monitored interferometric signals in the weak feedback regime: (1) the interferometric signal at PD2 (resulting from interference of the transmitted and double-reflected beam), and (2) the interferometric signal at PD1. The phase difference between the transmitted and the double-reflected beam is

$$\Delta \varphi_{PD2} = 2knd \cos \theta,$$

(49)

where $n$ is the refractive index of the transparent sample, $d$ is sample thickness, $\theta$ is the angle of refraction at the first sample interface, and $k = 2\pi/\lambda$ is the (free-space) wave number. In addition to depending on $\theta$, the change in phase stimulus at PD2 also depends on the angle $\alpha$ at which the sample is offset from the beam-axis normal, as

$$\Delta \varphi_{PD1} = 2kd(n \cos \theta - \cos \alpha).$$

(50)
The difference between these two,
\[
\Delta \varphi_{PD2} - \Delta \varphi_{PD1} / 0.0136 = 2kd \cos \alpha;
\]

is independent of refractive index \(n\) of the material. Hence, measurement of phase difference dependence in Eq. (51) with \(\alpha\) will yield thickness \(d\). Once \(d\) is known, the dependence of \(\Delta \varphi_{PD2}\) in Eq. (49) on refraction angle \(\theta\) permits a crude estimate for \(n\).

### D. Measuring Change in Target Refractive Index

For this application, the external cavity length and its refractive index are constant. An example is the measurement of a fiber Bragg grating under strain [171]. Now consider the target to have a (complex) refractive index \(\tilde{n} = n - jk\), where the negative sign is once again a consequence of the convention to use \(e^{j\omega t}\) in describing the temporal dependence of the wave. Assuming normal incidence from a lossless medium onto an optically thick target (half-space), the target amplitude reflection coefficient is [172]

\[
r = n_{\text{ext}} - \tilde{n} \quad n_{\text{ext}} + \tilde{n} = \frac{n_{\text{ext}} - n + jk}{n_{\text{ext}} + n - jk}.
\]

Taking the convention \(r = |r|e^{j\theta_R}\), the amplitude, \(|r|\), and phase, \(\theta_R\) of the reflection coefficient are

\[
|r| = \sqrt{(n_{\text{ext}} - n)^2 + k^2} \quad \frac{n_{\text{ext}} + n}{(n_{\text{ext}} + n)^2 + k^2}
\]

\[
\theta_R = \text{Arctan}[(2n_{\text{ext}}k), (n_{\text{ext}}^2 - n^2 - k^2)]
\]

where Arctan\([y, x]\) denotes the four-quadrant arctangent function of a vector with Cartesian coordinates \((x, y)\).

Figure 17

Apparatus for the simultaneous measurement of thickness and refractive index of a transparent material (adapted from [169]). The emitted beam is incident to the transparent sample at angle \(\alpha\) and is refracted at angle \(\theta\) to the partial reflector and second photodetector (PD2). A portion of the beam is reflected and subsequently returns to the laser diode, whose optical power is monitored by the first photodetector (PD1). A part of this returning beam undergoes reflection from the internal surface of the sample, resulting in double reflection. This double-reflected beam is observed only at PD2 and has negligible impact at PD1.

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When \( n > n_{\text{ext}} \)—for example, when the beam is incident from air and probing a higher index material—we have \( k \geq 0 \) and \( n > n_{\text{ext}} \), and Eq. (54) will give a \( \theta_R \) in the second quadrant of the complex plane. Using the standard definition of the arctan function, which has range \((-\pi/2, \pi/2)\)—that is the first and fourth quadrants—we may write

\[
\theta_R = \arctan\left(\frac{2n_{\text{ext}} k}{n_{\text{ext}}^2 - n^2 - k^2}\right) + \pi.
\]  

(55)

If the material extinction coefficient \( k \) is zero, then the phase change is simply \( \pi \) as expected. Note that we may have \( n < n_{\text{ext}} \)—for example, when the beam is incident from a fiber probing a lower index lossy material.

Therefore, changes in complex target refractive index will induce changes in both the amplitude and phase of the reflected optical wave. As \( C \) is directly proportional to \(|r|\) [via Eqs. (1) and (2)], changes in \(|r|\) will change \( C \) by a proportional amount. Changes in \( \theta_R \) will change \( \phi_s \) by an equivalent amount. Explicitly, this results in phase stimulus of

\[
\varphi_s(t) = \omega_s \tau_{\text{ext}} - \theta_R(t) = \frac{4\pi n_{\text{ext}} L_{\text{ext}}}{\lambda} - \theta_R(t),
\]  

(56)

and amplitude stimulus of

\[
C(t) = C(0) \frac{|r(t)|}{|r(0)|}.
\]  

(57)

A scheme of this nature was used recently to image free carriers in semiconductors using a THz QCL [173] (see Fig. 18). Another interesting recent application is to measuring thin layers of residues on surfaces [174].

### E. Alignment and Angle Measurement of Remote Target

When the external target is at a fixed distance from the laser, but possibly misaligned (in the angular sense) [175, 176], its effective (complex) amplitude reflection coefficient can be viewed as a function of its two angular deflections (\( \theta \) and \( \vartheta \), say) from the plane transverse to the laser beam, as

\[
r(\theta, \vartheta) = |r(\theta, \vartheta)| e^{j\phi_s},
\]  

(58)

resulting in phase stimulus (explicitly highlighting phase-shift on reflection) of

\[
\varphi_s(t) = \omega_s \tau_{\text{ext}} - \theta_R = \frac{4\pi n_{\text{ext}} L_{\text{ext}}}{\lambda} - \theta_R,
\]  

(59)

and angle-dependent characteristic feedback parameter \( C \), as

\[
C(\theta, \vartheta) = C(0, 0) \frac{|r(\theta, \vartheta)|}{|r(0, 0)|}.
\]  

(60)

When the effective target reflectivity in the direction of the beam axis decreases with departure from perfect alignment, as is the case for many external targets, including for minor mirror misalignment, this angular dependence provides means for target alignment and therefore angle measurement. By creating an
interferometric signal (for example, by simply chopping the laser beam), adjusting the two angles until maximum peak-to-peak signal is achieved will correspond to a well-aligned target. Indeed, this idea has been used to investigate the effect of optical feedback due to Fresnel reflections from a fiber front end in characterizing laser-to-fiber coupling techniques \[177\]. When the target can be displaced, an alternative approach that exploits the stimulus replication phenomenon of the phase-insensitive coherence collapse regime can be found in \[175\]. One can also employ three laser sensors operating with sawtooth frequency stimulus to measure displacement at three points on the target, thereby recovering angular rotation \[176\].

F. Intensity-Based Measurement

While the many applications of LFI discussed previously rely on the coherent (phase-based) information contained in the LFI signals, for some applications it is desirable or unavoidable to forgo this coherent information and instead rely on the intensity-based information contained in the optical feedback. Laser Doppler velocimetry for rough surfaces and fluid flow is one application discussed in

Figure 18

(a) Representative intensity distributions of the infrared pump laser by placing a charge-coupled device (CCD) camera at the sample position. The pattern was computer controlled by a spatial light modulator (SLM) and projected onto the silicon surface. Dark pixels of SLM liquid crystal maintain the polarization of the incident light and define the exposed area. (b) Terahertz imaging in reflection mode of photoexcited electron plasma on semiconductors. The spatial distribution of free carrier charges corresponds to the structured beam profile. Reprinted with permission from Mezzapesa et al., Appl. Phys. Lett. 104, 041112 (2014) \[173\]. Copyright 2014 American Institute of Physics.
more detail in Section 7.G. Another interesting application is the profiling and imaging of surfaces using LFI in a confocal microscope configuration [78, 91, 178, 179] (see Figs. 19, 20, and 21).

In these instances, the laser is operated at a constant bias current, and simply the change in bias current due to optical feedback is recorded. This change can be directly related to the change in surface profile [91].

A further application is as follows. If the phase information is randomized by reflection from a rough surface, or is otherwise discarded, relative measurement of the target’s reflectivity is possible simply though monitoring the peak-to-peak

Figure 19

(a), (b) Two specimen through-focus images of a semiconductor chip taken at 3 μm axial separation with a semiconductor laser confocal microscope. Field of view is 80 μm × 80 μm. Reprinted with permission from [78]. Copyright 1994 Optical Society of America.

Figure 20

Locating defective transistors using an optical-feedback thermographic microscope. (a) Thermal map of photodetector array sample superimposed on reference confocal image showing anomalous “hotspots” (yellow regions), which are possible defect sites. (b) Regions of increasing thermal activity reveal the integrity of the semiconductor architecture where thermal blooming on two regions is apparent. Image area is 180 μm × 180 μm. (c) Homogeneity of the localized quantum efficiency of regions across a silicon photodiode is evaluated at optical spatial resolution. Region 1 represents a substrate; 2, n-type semiconductor; 3, p–n overlay; and 4, bonding pad. (d) Thermal map of the quantum efficiency of the photodiode revealing the nonuniform response on the boundaries and the n region. Image area is 270 μm × 270 μm. Reprinted with permission from [179]. Copyright 2006 Optical Society of America.
LFI signal amplitude. Translating rough surfaces at a known velocity permits their characterization by monitoring the fluctuations in effective reflectivity [182].

**G. Doppler Velocimetry and Flow Imaging**

This sensing scheme—LFI velocimetry—is based on the well-known Doppler effect that dictates that a moving object will impart a frequency shift to light incident upon it. In this sensing scheme, the Doppler beat frequency of the interferometric signal is observed, just as in a standard laser Doppler velocimeter (LDV) [183]. The scheme differs from LDV in that the self-mixing effect will introduce nonlinearity into the signal and the scheme requires no reference arm. At first glance, it may seem that LFI velocimetry is a simple matter—one simply observes the beat frequency induced by the Doppler effect and thereby deduces the velocity of the target. However, a more careful consideration of this sensing
scheme reveals many subtleties and gives a deeper understanding of LFI in general.

1. Principle of Operation

The Doppler shift, $\Delta f$, incurred by light upon scattering by an object with the velocity vector $v$ is [183]

$$\Delta f = \frac{1}{2\pi} (k_s - k_i) \cdot v,$$

(61)

where $k_s$ and $k_i$ are the scattered and incident wavevectors. Assuming direct backscattering reduces Eq. (61) to

$$|\Delta f| = \frac{1}{\pi} k_j \cdot v = \frac{2nv \cos(\theta)}{\lambda}.$$

(62)

where $v = \|v\|$, $\theta$ is the angle between the laser beam axis and the target velocity vector, $n$ is the refractive index of the medium surrounding the target, and $\lambda$ is the laser emission wavelength in vacuum. This simplified equation works well in practice, as the scattered light must return to the laser in order to contribute to the LFI signal (monostatic geometry) [53,103].

It is instructive to consider Eq. (61) in more detail. Figure 22 depicts a ray of light with incident wavevector $k_i$ impinging on an optically flat surface. We see clearly that the vector quantity $k_s - k_i$ is normal to the surface tangent plane. In order to have a non-zero Doppler shift, Eq. (61) demands that the vector dot product $(k_s - k_i) \cdot v$ be non-zero. This means that the velocity vector must be outside the surface tangent plane for a Doppler shift to occur. That is, the surface must be translating in such a way as to change the external cavity length. If the cavity length is not changing (i.e., the velocity vector is in the plane tangent to the surface), no Doppler shift occurs.

**Figure 22**

Visualization of Eq. (61). The vector $(k_s - k_i)$ is normal to the reflection surface. If the velocity vector $v$ is in the plane tangent to the reflection surface, as pictured, there will be zero Doppler shift. A velocity vector outside this plane will give a non-zero Doppler shift, which means the surface is translating in such a way that it is changing the external cavity length. Therefore, the external cavity length must change for a Doppler shift to occur.
To illustrate this, consider the case of a rotating (but otherwise stationary) disk. If the surface of the disk has flatness sufficiently smaller than the laser emission wavelength such that specular reflection is dominant (i.e., it is optically flat), Eq. (61) gives zero Doppler shift regardless of the incident wavevector’s direction, as \( \mathbf{k}_s - \mathbf{k}_i \) will always be normal to the velocity vector. The mathematics here is clear, but the authors also confirmed this experimentally by coupling the reflected light from a disk of this nature back into the laser; no Doppler shift was observed.

Now let us consider the same disk, but with surface roughness significant compared to the laser wavelength. As the disk spins, its roughness modulates the external cavity length. Analyzing the statistics of the reflection process is beyond the scope of this article, but one may intuitively reason that the mean Doppler shift is given simply by the component of the target velocity vector along the laser beam axis, and this is indeed observed in experiment [103]. That is to say, the mean Doppler shift is given by Eq. (62). These two cases (an optically smooth and a rough disk) illustrate the principle derived through solely mathematical considerations above—cavity length must change for a Doppler shift to occur. In fact, LFI Doppler velocimetry is equivalent to displacement measurement (covered in Section 6.A). This equivalence has been noted even for the more general case of laser Doppler and laser speckle velocimetry [184]. LFI displacement counts fringes corresponding to \( \lambda/2 \) displacement, whereas LFI velocimetry considers the frequency of those same fringes (i.e., the Doppler frequency) and thereby infers the target velocity.

Surface roughness will inevitably lead to speckle effects [185]. Just as for displacement measurement, speckle can have a detrimental effect on systems tracking the motion of rough targets, as the “dark speckles” caused by destructive wave interference will attenuate the LFI signal. This negative effect can be ameliorated by implementing a real-time signal tracking algorithm [94, 186], or by physically moving a lens with piezo-actuators [187].

Fully developed speckle will destroy phase coherence—the beam reflected from a rough surface will have uniformly random phase on \( 2\pi \) [185]. This does not, however, stop the beam from accruing a Doppler shift. When approaching LFI velocimetry from the speckle perspective, the autocorrelation function is typically used to analyze the LFI signal [188]. When approaching the problem from the Doppler perspective, the power spectrum is typically used to analyze the signal, which is simply the Fourier transform of the autocorrelation function by the Wiener–Khintchine theorem [185]. Therefore, both approaches are valid and fundamentally linked. We choose to proceed using the Doppler approach, and analyze the signal by considering its power spectrum.

Finally, one may wonder how it is possible to detect a Doppler shift, frequently in the kilohertz range, far below the typical laser diode linewidth (some tens of megahertz). This dilemma was solved in the field of radar Doppler [189, 190]—the emitter (the laser in our case) is considered as a fixed-frequency source with a
certain phase noise (corresponding to the laser linewidth). As the phase of the reinjected electromagnetic field is statistically correlated to some degree with the phase of the laser emission, the frequency noise in the “baseband” is severely attenuated, an effect termed range correlation. This allows the observation of Doppler shifts far below frequencies corresponding to the laser linewidth.

2. Modeling

Modeling the LFI velocimetry signal is done in two parts—the light–target interaction and the laser dynamics. The nature of the target and the optical system will dictate the light–target interaction and the ensuing Doppler-shift distribution that will re-enter the laser cavity and be subject to the laser dynamics.

A body in motion, such as a rough, rotating disk, exhibit a Gaussian peak in the spectrum centered about the Doppler frequency expected from the velocity at the center of the laser spot [103]. Increasing the numerical aperture of the beam incident on the target will broaden the spectrum and vice versa [191].

The Doppler-shift spectrum for fluids is complicated by scattering effects. If the fluid is weakly scattering—i.e., each photon is most likely to be scattered once or not at all—then the spectrum tends to resemble that of a rough body in motion [53]. Figure 23(a) shows representative spectra for a range of flow rates at the same (weak) scattering level using external optics to focus the beam at the center of the flow channel.

If the fluid is strongly scattering, as is often the case for blood flow measurement [192], then the spectrum does not exhibit a clear Doppler peak, but rather a distribution governed by the statistics of the scattering process [52,193]. Models for the Doppler-shift spectrum have been reported, taking into account scattering effects [52,194–196].

To model laser dynamics, an inverse FFT method may be used to transform the Doppler-shift spectrum into a complex time series \( \psi(t) \) (taking care to assign each frequency component uniformly random phase for the case of fully developed speckle). Then we may write the phase and amplitude stimuli as [87]

\[
\varphi_s(t) = \omega_s \tau_{\text{ext}} + \arg(\psi(t))
\]  

Figure 23

Morphology of Doppler flow spectra, adapted from [53,55]. (a) Doppler flow spectra for six flow rates ranging from 0 to 50 \( \mu \)L/min for the same scattering level of 2% wt. % milk diluted in water. (b) Doppler flow spectra for four scattering levels, ranging from 0.2% to 100% wt. % milk diluted in water, for the same maximum fluid velocity of 1.6 mm/s.
\[ C = C_0 |\psi(t)|, \] 

(64)

where \( C_0 \) is the constant, nominal value of \( C \). As the fractional change in the laser operating frequency due to the Doppler effect is very small, \( \omega_s \) may be considered to remain constant. It should also be noted that here \( \tau_{\text{ext}} \) is a constant, nominal value for the external cavity round-trip time.

The nonlinear process of LFI produces harmonics, and, as such, measurement is ideally performed under weak feedback, though monitoring the ratio of power of the peak Doppler frequency to its second harmonic gives a measure of feedback level [197].

3. Spectral Analysis and Velocity Extraction

The spectral analysis of the LFI signal is commonly carried out by application of the FFT; however, it has been shown that multiple signal classification, a frequency estimator using a single \( a \) priori parameter, provides improved performance in the presence of noise and obviates the need for fitting for Gaussian spectral distributions [151,198].

In this case, the average flow can be extracted by finding the frequency at which the signal spectrum intersects the noise floor [199], by considering the first moment of the spectrum [200], or by tracking the frequency at which the spectral power is reduced from its maximum by a fixed amount [55]. The optical system configuration will also necessarily affect the spectrum [191], and may cause the signal spectrum to be non-Gaussian even for weakly scattering fluids [55]. Figure 23(b) shows representative spectra for a range of scattering levels at the same flow rate.

4. Flow Imaging and Other Applications

The above technique can be applied in many interesting situations. In 1984, de Mul et al. [200] measured the velocity of blood flow in a tube and in vivo on human skin using an external laser. Koelink et al. [199] inserted an optical fiber into a glass tube to measure the velocity of polystyrene spheres in water as well as human blood. Özdemir et al. [201] measured velocity of blood flow in vitro in a 5 mm channel and in vivo in human fingertips and wrists using a laser and a lens at a distance from the target. Lim et al. [104] demonstrated an LFI flow sensor using a VCSEL array, imaging fluid velocity in a planar flow channel of 3 mm thickness. Figure 24 shows the experimental and theoretical flow images of the channel. Norgia et al. [52] performed experimental and theoretical studies of sensing the velocity of extracorporeal blood flow in a 9 mm diameter channel. Campagnolo et al. [53] measured the spatial flow profile of Newtonian and non-Newtonian fluids within a 320 \( \mu \)m channel using external optics. Nikolić et al. [55] demonstrated an LFI flowmeter using an optical fiber embedded within a microfluidic structure.

The shape of the Doppler spectrum induced by Brownian motion of a fluid at rest can be used to measure particle size within the fluid, as scatterers of different sizes will have a different spatial scattering profile [202–204].

The measurement of wind speed by LFI is a related application in which light is scattered from particulates in the air, much as light is scattered by particles in fluid as above [205].
8. Laser-Cavity-Related Measurement

A. Laser Linewidth and Linewidth Enhancement Factor (\(\alpha\)) Measurement

The susceptibility of laser linewidth to optical feedback has long been studied [67,206]. Indeed, the measurement of laser linewidth and linewidth enhancement under feedback is a mainstay of LFI [19,20,207–213]. It is important to keep in mind that the linewidth and Henry’s linewidth enhancement factor \(\alpha\) under feedback often depend on the laser biasing condition, as well as the level of optical feedback [213]. The values reported for \(\alpha\) vary a great deal between different device structure and measurement technique—investigation into which factors influence \(\alpha\) is a topic of ongoing investigation [213].

Experimentally determining \(\alpha\) by LFI may be achieved in a number of ways. The phase-stimulus–power-response transfer function (i.e., see Fig. 10) strongly depends on \(C\) and \(\alpha\). In the moderate feedback regime, where multiple solutions exist and the interferogram (the LFI signal) exhibits hysteresis, the hysteresis observed in the LFI signal can be linked to the transfer function, and hence to \(C\) and \(\alpha\) [214]. The advantage of this method is that no high-frequency or optical spectrum measurements are required, and the reported accuracy of determining \(\alpha\) is \(\pm 6.5\%\). Alternatively, direct fitting of the experimental LFI signal to theory can be employed [207], or a multiple-parameter optimization
may be performed to solve for $\alpha$, $C$, and a number of other laser and target parameters \cite{208}—including the simultaneous recovery of the displacement signal \cite{123,124}. If the phase stimulus can be precisely controlled, another approach making use of the interferometric waveform’s functional form can be found in \cite{57}.

The measurement of laser linewidth under feedback is possible in the moderate feedback regime, as the variance of phase fluctuations when $C > 1$ are proportional to the linewidth, and are easily monitored by tracking the phase fluctuations of a particular interferometric fringe \cite{215}.

### B. Relative Intensity and Phase Noise Measurement

A further extension of extracting laser parameters through LFI is the investigation of relative intensity noise (RIN) and the phase noise in the laser under feedback. In both cases, LFI is used to correctly estimate the feedback conditions of the laser under test.

Measuring RIN using LFI is performed in a way similar to that without LFI (i.e., a high-speed PD coupled to a microwave spectrum analyzer)—except in the former case, an additional optical arm with a displacement target is required. RIN is measured with the target static, and then the coupling coefficient is calculated via extraction of the feedback parameters $C$ and $\alpha$ from the movement of the target \cite{216}. This has been demonstrated in both QCLs and laser diodes. Interestingly, by controlling the feedback (with appropriate attenuation) and correctly estimating the coupling coefficient, it is possible to tune the feedback to the laser and decrease the RIN to make the laser more stable \cite{217,218}.

Phase noise is measured in a similar way, but the fast PD and spectrum analyzer are replaced by a Fabry–Perot cavity to convert phase changes into intensity changes, which can be easily measured \cite{219,220}.

### 9. Industrial Acceptance

The earliest patent involving an LFI sensor dates back to 1976 \cite{221}. Since then, there has been a steady stream of patents, with a notable increase in filed patents over the past decade. In particular, Philips, Microsoft, and Covidien have been granted U.S. patents pertaining to optical input devices and movement sensors (including for flow) \cite{222–239}; Martin Marietta (now Lockheed Martin), Boeing, Microsoft, Mitutoyo, Yamatake, and Philips have been granted U.S. patents pertaining to optical ranging and distance measurement \cite{153,221,236,238,240–243}; and Philips has been granted U.S. patents pertaining to atomic frequency detection \cite{244}.

Of all these companies, Philips is most active in developing LFI sensors using VCSELs \cite{108,109,245}. Liess et al. first reported a miniature motion sensor and input device in 2002 \cite{246}. They demonstrated a working 3D navigation prototype, implemented in mobile telephones, for detecting click and scroll movements of the finger with velocities up to 150 mm/s. In 2008, Pruijmboom et al. reported a commercial product based on this technology, the Philips Twin-Eye Laser LFI sensor \cite{108,109}. The sensor is a system-in-package solution with a very small form factor of less than 0.2 cm$^3$. Figure 25 shows the Twin-Eye sensor. A lens is incorporated on top of the package, focusing the light a few millimeters from the lens. This is ideal for optical mouse applications;
however, the optical configuration could be easily modified to suit other types of application.

10. Summary

The study of LFI is multifaceted, encompassing the fields of physics, optics, device design, systems design, simulation, and signal processing. These interferometers have a natural simplicity, both in their abstraction to a three-mirror model and in their experimental manifestations. One very attractive consequence of their autodyne nature is that the laser itself may be used as its detector. This can be achieved in practice by monitoring the variations in the optical output power or the terminal voltage of the laser—the latter offering significant advantage when external detectors of emitted radiation are unattainable. The self-mixing effect can be used for a great range of metrological applications, some of which we have explored in more detail in the body of the text. Much of the art of LFI is in the design and execution of novel external cavities, and the successful modeling and interpretation of the resulting interferometric signals. In the quasi-static regime, in broad strokes, this amounts to devising a clever equivalent second mirror that is strongly dependent in a known manner on the interferometric system parameters.

The earliest demonstrated laser [2] operated in pulsed mode, and, even today, new pulsed lasers typically operate at significantly higher heat-sink temperatures than their continuous-wave counterparts [247, 248]. To tame the self-mixing effect in these cases and push the boundaries of LFI further, the modeling and design of these metrological schemes operating on time scales at which laser dynamics operate is becoming increasingly important [249–251].

Appendix A: Derivation of the Excess Phase Equation

Assuming a linear gain function [Eq. (6)] and noting that, at threshold, the cavity gain is equal to the material losses ($1/\tau_p$), one obtains

$$N_{th} = N_{tr} + \frac{1}{\Gamma v_g d \tau_p}.$$  \hspace{1cm} (A1)

Philips Twin-Eye Laser LFI sensor, adapted from [108]. (a) Sensor with integrated lenses. (b) System-in-package with lenses removed.

Figure 25

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Using this expression in the photon density rate equation [Eq. (21)], and assuming linear gain, leads directly to Eq. (25).

To arrive at Eq. (26), we first substitute the steady-state conditions into the photon and phase rate equations from Eq. (21), yielding

\[
0 = \left( \Gamma G - \frac{1}{\tau_p} \right) + 2\kappa \cos(\omega \tau_{\text{ext}}),
\]

(A2)

\[
\omega - \omega_s = \frac{1}{2} \alpha \left( \Gamma G - \frac{1}{\tau_p} \right) - \kappa \sin(\omega \tau_{\text{ext}}).
\]

(A3)

Combining this pair of equations leads to

\[
\omega - \omega_s = -\alpha \kappa \cos(\omega_s \tau_{\text{ext}}) - \kappa \sin(\omega \tau_{\text{ext}}),
\]

(A4)

which can be simplified via three trigonometric identities, \( A \cos(\omega t) + B \sin(\omega t) = \sqrt{A^2 + B^2} \cos(\omega t - \arctan(B/A)) \), \( \arctan(1/\theta) = \pi/2 - \arctan(\theta) \), and \( \cos(\theta - \pi/2) = \sin(\theta) \), to obtain Eq. (26).

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member. He has been Guest co-Editor for Journal of Optics (June 1998 and November 2002) and Optical Engineering (January 2001) on Distance/Displacement Measurements by Laser Techniques and chaired the International Conference ODIMAP in 1997. With Prof. Marc Lescure, he has edited the Milestone Volume entitled “Selected Papers on Laser Distance Measurements” published by SPIE in 1995. In 2011, he was chairing the special session on self-mixing during the IEEE Sensors 10th Annual Conference. He is now a Senior Member of IEEE and serves as Chairman of the IEEE Instrumentation & Measurement Technical Committee “Laser & Optical Systems” and as an Associate Editor of the IEEE Transactions on Instrumentation & Measurements (1997–2010). He won the Mechatronics Award (Research Category) during the European Mechatronics Meeting in 2010 and the Jean Ebbeni Prize from the CMOI Technical Committee of the French Optical Society in 2011.

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