Analytical Considerations When Measuring Income Mobility

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Examining income mobility un_masks some of the key features of the income distribution that are hidden when we simply look at conventional cross-sectional indicators of poverty and inequality. Studying income mobility is also important because the concept is linked to a number of socio-economic issues like economic insecurity. However, measuring income mobility is not straightforward. In this study, I review the important considerations when measuring income mobility. The statistical analyses highlight that:

1) Income mobility is multi-dimensional. The existing literature offers different perspectives of what income mobility means. The results of my simulation suggests that while there is a general linear relationship among the income mobility indices that I examined, the strength of the correlation depends on the income mobility concept being measured. For instance, indices that do not differentiate between downward and upward income mobility are strongly correlated with each other but exhibit more variability when compared to equalization indices and poverty dynamics.

2) Different income mobility regimes call for different mix of policies. Thus, it is important to gauge the magnitude of bias caused by measurement errors on income mobility estimates. My simulation experiment reveals that both classical and non-classical measurement error can lead to both downward and upward bias in the estimates. Moreover, the characteristics of the measurement error can have offsetting effects on the severity of the bias.

3) Income mobility is not always a desirable outcome as it may reflect income insecurity when it is driven by fluctuations in the transitory component of income. Thus, it is important to decompose income mobility estimates into mobility of its permanent and transitory components.
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Abstract

Defining income mobility is not a trivial matter, where both practical and technical considerations depend on the perspective of the researcher. This study provides a literature review of three important analytical considerations when measuring income mobility. First, I review several definitions of income mobility and how each concept is operationalized using empirical data. The study shows that income mobility estimates based on various definitions do not necessarily agree in levels and trends. Using a simple simulation experiment, I find a general linear relationship among the income mobility indices considered and the strength of the correlation depends on the income mobility concept being measured. Second, the study reviews the impact of both classical and non-classical measurement error on income mobility estimates. Unlike previous studies that suggest that measurement error always inflates income mobility, the simulation experiment reveals that the different features of measurement error can lead to either downward or upward bias. Third, high level of income mobility may also represent economic insecurity when it is driven by fluctuations in the transitory component of income. Thus, it is important to examine mobility of permanent and transitory components of income using different econometric methods.

Keywords: income mobility, measurement error, permanent income
1. Introduction

While conventional measures of income poverty, income inequality and other indicators of economic development are useful for gauging a country’s socio-economic progress, most of these analytical tools only provide static snapshots of a country’s growth performance. However, sole reliance on these static development indicators may mask important features of a country’s development process. For example, while small changes in cross-sectional estimates of income poverty and inequality portray a stagnant income distribution at the macro-level, it may also be characterized by strong offsetting effects between large income gains of some poor individuals and large income losses of others at the micro-level. On the other hand, high income inequality may merit less concern when it is possible for an individual to work his way up the social hierarchy. This emphasizes the importance of examining income mobility patterns in conjunction with cross-sectional indicators of poverty and inequality. In general, income mobility can be linked to a concept of a ladder where the ladder represents the income distribution. Some individuals climb up while others go down. High income mobility rates are traditionally associated with social justice as it allows those who are initially poor to get out of socio-economic dearth. However, high income mobility rates may not always be desirable. At the extreme case, complete reversal of incomes wherein the richest swaps income with the poorest, second richest swaps income with the second poorest and so on, will still give the same cross-sectional estimates of poverty and inequality but at the same time will portray a highly unstable income distribution.

Examination of income mobility trends is important for policy planning as different income mobility regimes call for different mix of socio-economic policies. By measuring income mobility patterns, research can provide important inputs that will enable socio-economic planners to develop more efficient evidenced-based policies. However, examining income mobility patterns is not a trivial matter and various practical and technical considerations need to be taken into account. First, the multi-dimensional feature of income mobility lends itself to a number of ways to measure it using empirical data. As different measures may produce varying trends, it is important to examine more than one dimension of income mobility. At the same time, practicality dictates that examining all dimensions may cause more confusion than good. In other words, measuring mobility in terms of all of its dimensions may just lead to a bewildering array of numbers. Thus, analysis should strike a balance between having to provide a thematic discussion while trying to paint a comprehensive picture of the income mobility process. To be able do this, it is important to
examine the relationship among the different income mobility concepts. Second, incomes reported in household surveys are prone to measurement error. If left unaddressed, the presence of measurement errors may render estimated levels of income mobility biased and thus, lead to misleading conclusions. Third, it is important to distinguish whether the observed income mobility reflects changes in permanent or transitory components of income. Although high income mobility rate tends to taper off the adverse consequences of high income inequality when it is driven by positive changes in permanent income, it may also reflect an unstable income distribution when much of the mobility comes from unexpected fluctuations in transitory income.

The main objective of this study is to provide a literature review of several important analytical considerations when estimating income mobility. While Jenkins (2011) and Fields (2008) provide general discussion of measurement issues that are critical when one examines the income distribution from a longitudinal perspective, here, I provide more detailed discussion of three topics. First, I review the various definitions of income mobility used in the existing literature and shows that estimates based on various definitions do not necessarily agree in levels and trends. Second, I examine the behaviour of the most commonly used income mobility measures in the presence of measurement errors. I find that the different features of the measurement error can have offsetting effects on the severity of the bias on income mobility estimates. Third, I describe a methodology for decomposing income mobility into its two fundamental components: permanent and transitory income. This allows us to differentiate between mobility as a desirable outcome and mobility as risk.

The rest of the paper is outlined as follows. First, I discuss the importance of examining income mobility. Then I review the different conceptualizations of income mobility and how each definition is measured empirically. Using simulated data, I also examine the effect of measurement errors on income mobility estimates. Furthermore, I also differentiate income mobility as a corrective tool for the adverse effects of inequality from income mobility as an indicator of socio-economic insecurity. The last section summarizes the main results.

2. Why it is important to examine income mobility?

From a policy perspective, measuring income mobility is important because different income mobility levels call for different mix of socio-economic policies. In theory, there are several possible income mobility regimes that may be observed for a given society. Zero mobility
and perfect mobility are the two extreme cases. A society is said to have zero mobility if there is a complete persistence of the income distribution and perfect mobility when all individuals are income mobile. In empirical application, the true income mobility regime often falls in between these two extreme cases. When income mobility is low, previous studies suggest that individuals find less merit to work hard as the distribution of socio-economic opportunities is perceived to be inequitable (McNamee and Miller 2013). This is true, in particular, if low levels of income mobility are accompanied by very high poverty rates and high income inequalities. In this context, the underlying economic development process may exhibit prominent poverty traps wherein those who are at the bottom of the income distribution are being systematically marginalized. In other words, without any mobility-enhancing intervention, the poor are likely to remain poor. This represents a significant waste of human resources (ADB 2012, OECD 2008). In such scenario, finding an appropriate redistributive policy (e.g., conditional cash transfer) might be the way to move forward. Second, when low levels of income mobility are accompanied by low to moderate poverty rates with high income inequalities, then much of the rigidity may be occurring at the top of the income hierarchy. In a society where cumulative advantage is persistent, creation of more high quality jobs and provision of trainings to meet the skill-requirement of these jobs might be the way to expand the income mobility prospects of other segments of the population. On the other hand, a modest increase in inequality may not be too problematic (i.e., regarded as inequality of outcome) if it is accompanied with high levels of income mobility and decreasing poverty rates. Up to some extent, this increasing inequality of outcome may be a necessary feature of a progressive economic system that is focused on uplifting the standard of living of the poor. On the other hand, it is also worth pointing out that high levels of income mobility need not always be a desirable outcome. This is particularly true when mobility is mainly driven by large fluctuations in transitory income as this would represent a very unstable economic system (Jarvis and Jenkins 1998). Analogously, low levels of income mobility may also be regarded as a good indicator if it represents a mature economy that has already achieved long-run equilibrium. By measuring and examining income mobility patterns, research can help socio-economic planners identify more efficient evidenced-based policies for these economic development regimes.1 However, as will be elaborated in the next section, income mobility has multiple definitions (Fields 2008 and 2010). Naturally, the

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1 It is often regarded that static analysis of development is more suitable for treating symptoms of social exclusion while a more dynamic analysis allows us to tease out causal relations among different factors and thus plan a more effective intervention of escaping social exclusion.
ideal level of income mobility as well as the appropriate policy interventions depend on how income mobility is conceptualized within the context of a society’s stage of development.

3. Income Mobility Concepts

Over the years, a number of conceptual definitions have been proposed in the income mobility literature. In fact, Fields (2010) identified more than 20 different income mobility measures that are currently being used. While these measures may differ in terms of functional forms and how each individual’s income movements are aggregated to come up with an overall income mobility estimate, Ferreira et al. (2013) argued that the choice of income mobility concept to be examined goes beyond these technical distinctions. For instance, the objective of one’s study interacts powerfully with the different concepts of income mobility such that some concepts are more relevant for specific research questions that in others. This section reviews several definitions of income mobility and how each concept is operationalized using empirical data. In reviewing the different income mobility concepts that are commonly used in the literature, this section adopts the taxonomy used by Fields (2000 and 2008). In particular, income mobility is conceptualized into three broad groups: mobility as movement, mobility as origin independence and mobility as equalizer of long term income.\(^2\) Without loss of generality, the discussions in this section assume that the observation period consists of two time points. For notation purposes, I use \(Y_{it}\) to refer to the income of the \(i^{th}\) individual at \(t^{th}\) time period where \(i = 1, 2, \ldots, N\) at time \(t = 1, 2\). Interested readers may also refer to the works of Fields (2008 & 2010), Fields and Ok (1999), Solon (1999), Maasoumi (1998), Atkinson, Bourguignon and Morrission (1992), Cowell (1985), King (1983) and Shorrocks (1978) for more comprehensive discussion.

The first main perspective views mobility as movements in income. It has four sub-concepts. First, income mobility is gauged in terms of gross movements or what is commonly referred to as “income flux”. Operationally, this approach entails estimating the absolute value of the difference between \(Y_{i1}\) and \(Y_{i2}\). Second, mobility may be measured with respect to net movements in incomes. Since the individual income differences \(Y_{i2} - Y_{i1}\) are estimated instead of taking the absolute value \(|Y_{i2} - Y_{i1}|\), this (sub-) concept implicitly distinguishes...
upward from downward income mobility. Third, mobility as movement may also be measured in terms of changes in relative income or income shares $\frac{Y_{i1}}{\sum Y_{i1}} - \frac{Y_{i2}}{\sum Y_{i2}}$. Fourth, mobility may be measured with respect to positional movement. This concept entails quantifying the extent of re-ranking from $Y_{i1}$ to $Y_{i2}$. Under the positional movement perspective, perfect mobility is encapsulated when each individual’s income destination is a complete reversal of his/her income origin. Unlike the first two (sub-) concepts, mobility based on income shares and mobility based on positional movement examine changes in an individual’s income in relation to the incomes of everyone else in the society. Thus, an individual’s mobility depends not only on whether his/her income changed over time but also on how the change alters his/her income share or income rank (Jenkins 2011).³

Income mobility may also be examined with respect to the extent to which an individual’s income in the past influences his/her current income (Lillard and Willis 1978). Hence, the second main perspective views income mobility with respect to origin or temporal dependence. Empirically, this is gauged in terms of the correlation between $Y_{i1}$ and $Y_{i2}$. Under this perspective, mobility is high when an individual’s income destination is weakly related to one’s income origin. In other words, the basic property underpinning origin independence-based measures is that a more mobile society is one where an individual’s first period-income is less important in predicting his/her income in the succeeding periods (Ferreira et al. 2013).

The third main perspective views mobility as equalizer of long-term incomes. In general, an individual’s income at any given time will differ from his/her average income taken over several successive time periods. Using these longitudinally averaged incomes will smoothen the longitudinal variability in each individual’s income as well as the variability across individuals.⁴ Under this perspective, mobility is characterized in terms of the speed at which inequality is reduced as the observation period is lengthened (Shorrocks 1978 and Jenkins 2011).⁵ In general, mobility is high when the inequality in longitudinally-averaged income is less than the inequality at any particular point in time (Ferreira et al. 2013). The rationale behind introducing this perspective is to evaluate the extent to which long-term incomes are distributed more or less equally over time. It is particularly appealing because it directly

³ Under the positional movement concept, it is not possible for all individuals to be uniformly upwardly (or downwardly) mobile (Jenkins 2011).

⁴ The longitudinally averaged income is usually referred to as the permanent or long-term income.

⁵ The speed of inequality reduction depends on the chosen inequality measure (Schluter and Trede 2003).
links income mobility with inequality. For socio-economic researchers, it is important to examine whether an increase in income mobility can contribute to transitory variations in income so that permanent income inequality would be less than observed income inequality (Jarvis and Jenkins 1998). In other words, high income inequalities in fluid societies might be less problematic because the distribution of lifetime income would be generally even through income mobility (Krugman 1992). Table 1 provides a summary of the different conceptualizations of income mobility and the corresponding indices that measure these concepts. While the list is not exhaustive, these are the most commonly used in empirical studies of income mobility.

Table 1. Income Mobility Indices based on Different Conceptualizations

<table>
<thead>
<tr>
<th>Concept</th>
<th>Index</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Income Movement</strong></td>
<td>Fields-Ok</td>
<td>$\frac{1}{N} \sum_{i=1}^{N} \ln \left( \frac{\text{Income}<em>{it}}{\text{Income}</em>{it-1}} \right)$</td>
</tr>
<tr>
<td></td>
<td>King</td>
<td>$1 - \frac{1}{N} \sum_{i=1}^{N} \left( \frac{\text{Income}<em>{it}}{\text{Income}</em>{it-1}} - 1 \right) \ln \left( \frac{\text{Income}<em>{it}}{\text{Income}</em>{it-1}} \right)$</td>
</tr>
<tr>
<td></td>
<td>Average Rank Jump</td>
<td>$\frac{1}{N} \sum_{i=1}^{N} \left[ \text{rank(\text{Income}<em>{it})} - \text{rank(\text{Income}</em>{it-1})} \right]$</td>
</tr>
<tr>
<td></td>
<td>Poverty Persistence</td>
<td>$\frac{1}{N_p} \sum_{i=1}^{N_p} I(\text{Income}<em>{it} \leq z_i) \text{ where } P \equiv {i: \text{Income}</em>{it-1} \leq z}$</td>
</tr>
<tr>
<td></td>
<td>Poverty Inflow</td>
<td>$\frac{1}{N_C} \sum_{i=1}^{N_C} I(\text{Income}<em>{it} &gt; z_i) \text{ where } C \equiv {i: \text{Income}</em>{it-1} &gt; z}$</td>
</tr>
<tr>
<td>Time dependence</td>
<td>Hart</td>
<td>$1 - \text{Correl}(\ln(\text{Income}<em>{it}), \ln(\text{Income}</em>{it-1}))$</td>
</tr>
<tr>
<td><strong>Mobility as Equalizer of Income</strong></td>
<td>Shorrocks</td>
<td>$1 - \frac{1}{\sum_{i=1}^{N} w_i I(\text{Income}<em>{it})} \sum</em>{i=1}^{N} w_i I(\text{Income}_{it})$</td>
</tr>
<tr>
<td></td>
<td>Fields</td>
<td>$1 - \frac{1}{I(\text{Income}<em>{it-1})} I(\text{Income}</em>{it})$</td>
</tr>
<tr>
<td></td>
<td>Chakravarty, Dutta and Weywark (CDW)</td>
<td>$\frac{I(\text{Income}<em>{agg})}{I(\text{Income}</em>{it-1})} - 1$</td>
</tr>
</tbody>
</table>

Source: Fields (2000 and 2008)

To examine how income mobility trends may vary depending on how mobility is conceptualized, consider the examples of several income mobility scenarios based on a four-individual society that are presented in Table 2. First, we can use the two income vectors provided in Scenario A to illustrate the difference between gross and net income changes, i.e., non-directional and directional income mobility, two of the four sub-concepts under the mobility as movement perspective. In this scenario, the incomes of the two individuals
increased by one unit each while the remaining two noted a unit decline in income. Thus, Scenario A portrays non-zero gross income movements but negligible net income movements. In other words, if we do not consider the direction of the income changes, we can say that there is income mobility in Scenario A. However, when we subtract the negative income differences from the positive income differences, we say that there is no income mobility under Scenario A. Scenario B, on the other hand, can be used to illustrate the difference between directional and non-directional income mobility and mobility based on changes in income share or income ranks. Since the income of each individual doubled in Scenario B, everyone observed positive gross and net income movements. However, since each individual’s share to total income and income rank remained fixed, it also portrays negligible mobility based on the concepts of share and positional movement. Scenario C illustrates the difference between mobility as movement and mobility as origin independence perspectives. The income vectors provided in Scenario C portray a rank-reversal scenario wherein the initially poorest swaps income with the initially richest, the initially second poorest swaps income with the initially second richest, and so on. In this example, the correlation between the two income vectors is -1. Since the initial income vector perfectly predicts the values of the final income vector, we can say that there is negligible mobility based on the origin independence perspective. At the same time, since the observed increases in the income of the initially two poorest individuals offset the observed income declines of the remaining two individuals, we can say that there is no directional income mobility in Scenario C. However, since all individuals observed change in both the actual income levels and income ranks, Scenario C portrays a mobile society in terms of non-directional income mobility and positional movement. Lastly, Scenario D illustrates the difference between mobility based on origin independence and mobility as equalizer of long-term income perspectives. The relatively high yet negative correlation (-0.8) between the two income vectors implies that there is low income mobility based on the origin independence perspective. However, since inequality in average income (4.5, 3, 4.5, 3) is lower compared to either the initial or final income vector, we can say that there is mobility based on the concept of equalizer of long-term income.
Table 2. Examples of Income Mobility Scenarios

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Initial income vector</th>
<th>Final income vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(1, 2, 3, 4)</td>
<td>(2, 3, 2, 3)</td>
</tr>
<tr>
<td>B</td>
<td>(1, 2, 3, 4)</td>
<td>(2, 4, 6, 8)</td>
</tr>
<tr>
<td>C</td>
<td>(1, 2, 3, 4)</td>
<td>(4, 3, 2, 1)</td>
</tr>
<tr>
<td>D</td>
<td>(1, 2, 3, 4)</td>
<td>(8, 4, 6, 2)</td>
</tr>
</tbody>
</table>

In income mobility studies, we often see seemingly conflicting results across different analytical approaches even though in most cases, the same data set was used. In many cases, these “inconsistencies” stem from the fact that they measure different aspects of income mobility (Fields 2008). In general, the differences are not necessarily of limited practical interest because each concept corresponds to inherently distinct notions of what mobility is (Ferreira et al. 2013). Figure 1 illustrates this point. Using simulated data, I compare the relationship among different income mobility indices. While there is a general linear (either positive or negative) relationship among the indices considered, the strength of the correlation depends on the income mobility concept being measured. For instance, indices that do not differentiate between downward and upward income mobility (e.g., Fields-Ok’s, King’s, average rank jump, Hart’s and Shorrocks’ indices) are strongly correlated with each other but exhibit more variability when compared to equalization indices and poverty dynamics. Thus, it is important to explicitly identify which concept is being examined as the conclusions will depend on this choice.

In summary, the broad point that the discussions in this section seek to convey is that to be able to provide a more holistic picture of the underlying income mobility process, it is important to examine income mobility based on different perspectives. This is because there is hardly a single mobility measure that can adequately capture all income mobility concepts at the same time. In many cases, a satisfactory measure of one particular concept may be a poor measure of another. However, calculating too many income mobility indices may also result in a confusing array of numbers. Instead of providing a more comprehensive view of the income mobility process, this may just obscure the big picture. To strike a balance between these two considerations, a possible approach is to focus on a core set of indicators. To reduce the dimension, one may decide to choose one or two indicators among income mobility indicators that are highly correlated with each other while ensuring that most of the dimensions are covered.
4. Impact of Measurement Error and Data Contamination

Although the analytical tools presented earlier are useful for measuring income mobility, the discussions assume that income is measured accurately. However, previous studies suggest that income, particularly those derived from household surveys, are prone to measurement errors (Glewwe 2012, Gottschalk and Huynh 2010; Bound and Kruger 1991; Duncan and Hill 1985). If measurement error is significant, the results of income distributional analysis may produce biased results and thus, lead to misleading conclusions and policy implications. In addition, the effect of measurement errors can vary across different indicators of income mobility. This section examines how estimates of different income mobility indicators behave in the presence of measurement errors. For notation purposes, I use $Y_{it}$ and $Y_{it}^*$ to refer to the observed and true income, respectively, of the $i^{th}$ individual at $t^{th}$ time period where $i = 1, 2, ..., N$ and $t = 1, 2$. I also denote the (natural) logarithm of $Y_{it}$ and $Y_{it}^*$ by $Z_{it}$ and $Z_{it}^*$, respectively. The term $v_{it} = u_i e_{it}$ represent the time-invariant and time-varying measurement errors such that $Z_{it} = Z_{it}^* + \ln(v_{it})$. Hence, $Y_{it} = Y_{it}^* v_{it}$. 
Figure 1. Correlation of Income Mobility Indices

Source: Author’s computations using simulated data and the Stata tools for income mobility analysis developed by Van Kerm (2002).
Note: The simulated data is available upon request from the author.
What are the common sources of measurement errors? The most basic forms of data contamination may arise from randomly misreporting income or data encoding mistakes. Conventionally, these errors are assumed to average out (i.e., zero-mean) and are uncorrelated with the true income. In statistical parlance, this is referred to as classical measurement error. Classical measurement errors contribute to additional noise in observed incomes and findings from previous studies suggest that they lead to biased estimates of cross-sectional poverty and inequality (Jenkins 2011). In poverty estimation, Chesher and Schluter (2002) concluded that the severity of the bias induced by classical measurement error in estimates that use distribution-independent poverty line will depend on the level at which the poverty line is set and the number of individuals with incomes near this poverty threshold. On the other hand, the bias may be more severe when the analysis is anchored on a distribution-dependent poverty line. Furthermore, as can be seen in the derivation below, the variance of the observed income overestimates the variance of the true income in the presence of classical measurement error. Hence, estimates of inequality which can be expressed as functions of the variance of income are also inflated in the presence of classical measurement error (van Praag, Hagenaars and van Eck 1983). Moreover, the derivation below suggests that when the measurement error is correlated with the true income, the direction of the bias for the variance of income depends on the ratio of the covariance between the true income and the measurement error and the variance of the measurement error. In particular, a negative correlation can offset the inequality-increasing effect of the additional variability introduced by the measurement error.

\[ V(Z_{it}) = V(Z_{it}^* + \ln(v_{it})) \]  
\[ V(Z_{it}) = V(Z_{it}^*) + V(\ln(v_{it})) + 2Cov(Z_{it}^*, \ln(v_{it})) \]  
\[ V(Z_{it}) = V(Z_{it}^*) \text{ if } Cov(Z_{it}^*, \ln(v_{it})) = -0.5V(\ln(v_{it})) \]  
\[ V(Z_{it}) > V(Z_{it}^*) \text{ if } Cov(Z_{it}^*, \ln(v_{it})) > -0.5V(\ln(v_{it})) \]  
\[ V(Z_{it}) < V(Z_{it}^*) \text{ if } Cov(Z_{it}^*, \ln(v_{it})) < -0.5V(\ln(v_{it})) \]

While several studies have examined the effect of measurement errors on cross-sectional estimates of poverty and inequality and have proposed adjustment procedures to correct the bias induced by these errors (Ravallion 1994, Chesher and Schluter 2002), not much has been
said about the extent of measurement error bias on income mobility estimates.\(^6\) The few studies that have tackled the issue of measurement error when estimating mobility focus only on the origin independence perspective. These previous studies suggest that measurement errors make income less correlated over time. Consequently, there seems to be more income mobility in the presence of measurement error (Glewwe 2012). However, this conclusion does not necessarily hold when mobility is conceptualized in other ways. For instance, even under classical assumptions, Boheim and Jenkins (2006) noted that the effect of classical measurement error on poverty dynamics is less clear. Furthermore, when the magnitude of the measurement error is severe, some studies have hinted that the observed income mobility would likely be an artefact of the contamination of the income data. For example, Glewwe (2012) find that between 15\% to 42\% of observed mobility in Viet Nam in the 1990s can be considered as upward bias due to measurement errors in Vietnam Living Standards Survey. Krebs, Krishna and Maloney (2013) also find that the effect of measurement errors on income mobility estimates in Mexico to be non-negligible. In such cases, estimates of observed mobility may lead to incorrect inferences for development policies. Thus, it is important to investigate the magnitude of the effect of measurement error on income mobility estimates.

To contribute to this gap in the literature, this study briefly extends the discussions by examining the effect of measurement errors on different indicators of income mobility presented in the previous section.

When income is measured with error, the true historical income profile denoted by \{\(Y_{i1}^*, Y_{i2}^*\}\) is unobserved. In turn, the estimated mobility of observed income \{\(Y_{i1}, Y_{i2}\)\} will reflect the changes in the joint distribution of the true income and measurement errors \((Y_{i1}^*, Y_{i2}^*, u_i, e_{i1}, e_{i2})\). First, suppose mobility is defined with respect to non-directional income movement such that the income mobility is a function of the absolute difference between the (natural) logarithm of income, i.e., \(|Z_{i2} - Z_{i1}|\). In particular, consider the Fields-Ok index. We can see in the derivation below that the magnitude of the bias is bounded above by the total non-directional mobility reflected in the measurement errors.\(^7\) In particular, it is intuitive to think that the size of the bias will depend on the levels and variability of the measurement error.

\[
\frac{1}{N} \sum \left| \ln \left( \frac{Y_{i2}}{Y_{i1}} \right) \right| = \frac{1}{N} \sum \left| \ln \left( \frac{Y_{i2}^p}{Y_{i1}^p} \right) \right| 
\]

\(^6\) Glewwe (2007) also showed that when both income and its measurement errors follow a lognormal distribution, the estimated mean income of the poor and income growth of the poor are biased.

\(^7\) The derivation above is robust in the presence of a time-invariant component \(u_i\) in the measurement error as that term will cancel out.
\[
\frac{1}{N} \sum |Z_{i2} - Z_{i1}| = \frac{1}{N} \sum |Z_{i2}^* + \ln(v_{i2}) - Z_{i1}^* - \ln(v_{i1})| 
\]  
(7)

\[
\frac{1}{N} \sum |Z_{i2} - Z_{i1}| \leq \frac{1}{N} \sum |Z_{i2}^* - Z_{i1}^*| + \frac{1}{N} \sum |\ln(v_{i2}) - \ln(v_{i1})| 
\]  
(8)

\[
\frac{1}{N} \sum |Z_{i2} - Z_{i1}| - \frac{1}{N} \sum |Z_{i2}^* - Z_{i1}^*| \leq \frac{1}{N} \sum |\ln(v_{i2}) - \ln(v_{i1})| 
\]  
(9)

Second, suppose mobility is measured using the Hart’s index which is based on the correlation between the (natural) logarithm of the initial and final income. Under classical measurement error, \(\text{Cov}(Z_{i1}, \ln(v_{i1})) = \text{Cov}(Z_{i1}, \ln(v_{i2})) = \text{Cov}(Z_{i2}, \ln(v_{i1})) = \text{Cov}(Z_{i2}, \ln(v_{i2})) = 0\). In this context, the numerator of \(\rho(Z_{i1}, Z_{i2})\) is the same as the numerator of \(\rho(Z_{i1}^*, Z_{i2}^*)\). However, since the denominator of \(\rho(Z_{i1}, Z_{i2})\) is potentially higher because \(V(\ln(v_{i1}))\) and \(V(\ln(v_{i2}))\) are both non-negative, then \(\rho(Z_{i1}, Z_{i2}) \leq \rho(Z_{i1}^*, Z_{i2}^*)\). In turn, correlation-based indices of mobility for the observed income profile denoted by \(\{Z_{i1}, Z_{i2}\}\) will tend to overestimate the true (correlation-based) income mobility implied by \(\{Z_{i1}^*, Z_{i2}^*\}\).

\[
1 - \rho(Z_{i1}, Z_{i2}) = 1 - \frac{\text{Cov}(Z_{i1}, Z_{i2})}{\sqrt{\text{Var}(Z_{i1})\text{Var}(Z_{i2})}} 
\]  
(10)

\[
1 - \rho(Z_{i1}, Z_{i2}) = 1 - \frac{\text{Cov}(Z_{i1}^*, Z_{i2}^* + \ln(v_{i2}))}{\sqrt{\text{Var}(Z_{i1}^* + \ln(v_{i1}))\text{Var}(Z_{i2}^* + \ln(v_{i2}))}} 
\]  
(11)

\[
1 - \rho(Z_{i1}, Z_{i2}) = 1 - \frac{\text{Cov}(Z_{i1}, Z_{i2}^*) + \text{Cov}(Z_{i1}^*, \ln(v_{i2}^*)) + \text{Cov}(Z_{i2}^*, \ln(v_{i1}^*)) + \text{Cov}(\ln(v_{i1}), \ln(v_{i2}))}{\sqrt{\text{Var}(Z_{i1}) + \text{Var}(\ln(v_{i1})) + 2 \text{Cov}(Z_{i1}^*, \ln(v_{i1}))\text{Var}(Z_{i2}^*) + \text{Var}(\ln(v_{i2})) + 2 \text{Cov}(Z_{i2}^*, \ln(v_{i2}))}} 
\]  
(12)

In empirical application, measurement errors are not always classical in form. In some cases, observed incomes may be systematically flawed. Misunderstanding the reference period (e.g., misreporting weekly as monthly income), under-reporting income to evade tax obligations, over-reporting income to impress interviewers are some examples when this could happen (Krebs, Krishna and Maloney 2013; Glewwe 2007). In this context, it may not be safe to assume that the measurement error is distributed with zero mean. In addition, measurement errors may also be auto-correlated over time wherein some individuals have fixed propensity to misreport their income (Bound, Brown, Duncan and Rodgers 1994 and Pischke 1995). For example, self-employed often confuse personal income from business expenditures due to

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\textsuperscript{8} As pointed out in (12), the denominator of \(\rho(Z_{i1}, Z_{i2})\) is equal to the denominator of \(\rho(Z_{i1}^*, Z_{i2}^*)\) when \(\text{Cov}(Z_{i1}^*, \ln(v_{i1})) = -0.5V(\ln(v_{i1}))\).
lack of adequate accounting records, leading to erroneous income estimates (Deaton 1997; Daniel 2001). Furthermore, measurement errors may also be negatively correlated with true income if high income individuals are likely to report lower income while low income individuals are likely to inflate their reported income (Deaton 1997). In general, the severity of the bias on mobility estimates due to measurement errors will depend on how income mobility is defined and how the measurement errors are distributed. For instance, recall the derivation for the Fields-Ok index. As pointed out earlier, the magnitude of the bias is increasing with the level of the measurement error. On the other hand, in addition to the variability of the measurement errors, the severity of the bias on the Hart’s index depends on two additional factors: the covariance of the measurement error with the true income and the covariance of the measurement error over time.

In this study, we consider different forms of measurement errors. However, even if the form of measurement error is known, providing an analytical derivation to infer the direction of the bias it induces on mobility estimates is not always straightforward for each concept of income mobility. Hence, we turn to a simple simulation experiment to gauge the impact of measurement errors on different income mobility indicators. As pointed out earlier, we assume that the measurement error is linearly additive with the (natural) logarithm of income. Moreover, the measurement error can be decomposed into time-invariant and time-varying components as described below.

(i) \[ Z_{it} = Z_{it}^* + \ln(e_{it}), \quad E(\ln(e_{it})) = \mu_e, \quad Var(\ln(e_{it})) = \sigma_{e_{it}}^2, \quad Corr(Z_{it}^*, \ln(e_{it})) = \rho, \quad Corr(\ln(e_{it}), \ln(e_{i2})) = \delta \]

Here, the measurement error is assumed to be time-varying. When \( \mu_e = 0, \rho = 0, \) and \( \delta = 0, \) the measurement error is a white noise process. The classical measurement error is subsumed in the time-varying component. In particular, setting \( \mu_e, \rho \) and/or \( \delta \) to be non-zero yields different forms of non-classical measurement errors. For instance, when \( \mu_e > 0, \) individuals, on the average, are likely to over-estimate their true income. On the other hand, setting \( \rho > 0 \) portrays the scenario when high income individuals tend to under-estimate (or under-report) their true income while low income people tend to over-estimate their true income. In

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9 Compared to when income is measured in terms of earnings, some studies suggest that the extent of measurement error is less severe when income is measured in terms of household consumption. Nevertheless, consumption data is not free from measurement error. For instance, Scott and Amenuvegbe (1990) find that reported consumption decreases with the length of the recall period.
addition, the measurement errors are correlated over time when $\delta \neq 0$. For this configuration, the simulation experiment entails drawing from the multivariate normal distribution\(^{10}\) such that $(Z_{it}^*, Z_{it2}, \ln(e_{i1}), \ln(e_{i2})) \sim MVN(\mu, \Sigma_u)$ where $\mu' = (\mu_{Z_{it}^*}, \mu_{Z_{it2}}, \mu_e, \mu_e)$ and

$$
\Sigma_u = 
\begin{pmatrix}
\sigma_{Z_{it}^*}^2 & \text{Cov}(Z_{it}^*, Z_{it2}^*) & \rho \sqrt{\sigma_{Z_{it}^*}^2 V(\ln(e_{i1}))} & 0 \\
\text{Cov}(Z_{it}^*, Z_{it2}^*) & \sigma_{Z_{it2}^*}^2 & 0 & \rho \sqrt{\sigma_{Z_{it2}^*}^2 V(\ln(e_{i2}))} \\
\rho \sqrt{\sigma_{Z_{it}^*}^2 V(\ln(e_{i1}))} & 0 & V(\ln(e_{i1})) & \delta \sqrt{V(\ln(e_{i1}))V(\ln(e_{i2}))} \\
0 & \rho \sqrt{\sigma_{Z_{it2}^*}^2 V(\ln(e_{i2}))} & \delta \sqrt{V(\ln(e_{i1}))V(\ln(e_{i2}))} & V(\ln(e_{i2}))
\end{pmatrix}
$$

(ii) $Z_{it} = Z_{it}^* + \ln(u_i) + \ln(e_{it}), \quad E(\ln(e_{it})) = \mu_e, \quad \text{Corr}(Z_{it}, \ln(u_i)) = \rho, \quad \text{Corr}(Z_{it}, \ln(e_{it})) = 0, \quad \text{Corr}(\ln(u_i), \ln(e_{it})) = 0$

Here, the non-classical measurement error is subsumed in both the time-invariant and time-varying components. The time-invariant component $u$, can be perceived as a constant propensity of an individual to under- or over-overestimate their true income. It explicitly forces the measurement error to be correlated over time. For the second configuration, I draw from the multivariate normal distribution such that $(Z_{it}^*, Z_{it2}, \ln(u_i), \ln(e_{i1}), \ln(e_{i2})) \sim MVN(\mu, \Sigma_u)$ where $\mu' = (\mu_{Z_{it}^*}, \mu_{Z_{it2}}, 0, \mu_e, \mu_e)$ and

$$
\Sigma_u = 
\begin{pmatrix}
\sigma_{Z_{it}^*}^2 & \text{Cov}(Z_{it}^*, Z_{it2}^*) & \rho \sqrt{\sigma_{Z_{it}^*}^2 V(\ln(u_i))} & 0 & 0 \\
\text{Cov}(Z_{it}^*, Z_{it2}^*) & \sigma_{Z_{it2}^*}^2 & \rho \sqrt{\sigma_{Z_{it2}^*}^2 V(\ln(u_i))} & 0 & 0 \\
\rho \sqrt{\sigma_{Z_{it}^*}^2 V(\ln(u_i))} & \rho \sqrt{\sigma_{Z_{it2}^*}^2 V(\ln(u_i))} & V(\ln(u_i)) & 0 & 0 \\
0 & 0 & 0 & V(\ln(e_{i1})) & 0 \\
0 & 0 & 0 & 0 & V(\ln(e_{i2}))
\end{pmatrix}
$$

\(^{10}\) Glewwe (2007) noted that the effect of measurement error is robust even if one departs from the normality assumption, particularly when measuring income growth of the poor.

\(^{11}\) If $Z_{it}^*$ is strongly correlated with $Z_{it2}^*$, then the correlation between the error terms $\ln(e_{i1})$ and $\ln(e_{i2})$ may not be totally independent of Corr($Z_{it}^*$, $Z_{it2}^*$). In other words, forcing $\text{Cov}(\ln(e_{i1}), \ln(e_{i2}))$ to be zero may produce non-positive semi-definite $\Sigma_u$. In such cases, we allow $\text{Cov}(\ln(e_{i1}), \ln(e_{i2}))$ to be non-zero.
For the simulated true income data, I set $\mu_{Z_1^*} = 3.9$, $\mu_{Z_2^*} = 4$. This assumes that the growth in mean income is equal to approximately 10.5% ($= \exp(4)/\exp(3.9) - 1$). In addition, the assumed true variances for the (natural) logarithm of income are not unrealistic. In empirical studies, a value of 0.5 for the variance of the (natural) logarithm of income is on the high side for expenditure data but in the low side for income data (Glewwe 2007). Furthermore, the simulated data assumes that $\text{Cov}(Z_1^*, Z_2^*) = 0.9 \sqrt{\sigma_{Z_1^*}^2 \sigma_{Z_2^*}^2} = 0.9 \times 0.5 = 0.45$. In other words, we assume that $\text{Corr}(Z_{i1}^*, Z_{i2}^*) = 0.9$. Overall, these parameters values portray a relatively low income mobility. Table 3 shows the mobility estimates based on the simulated true income.

Table 3. Mobility based on Simulated True Income (Low Income Mobility Scenario)

<table>
<thead>
<tr>
<th>Indicator</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ave movement</td>
<td>0.48</td>
</tr>
<tr>
<td>Immobility</td>
<td>0.57</td>
</tr>
<tr>
<td>Hart</td>
<td>0.10</td>
</tr>
<tr>
<td>Shorrocks</td>
<td>0.05</td>
</tr>
<tr>
<td>King</td>
<td>0.22</td>
</tr>
<tr>
<td>CDW</td>
<td>0.02</td>
</tr>
<tr>
<td>Ave rank jump</td>
<td>10.13</td>
</tr>
<tr>
<td>Fields-Ok</td>
<td>0.26</td>
</tr>
<tr>
<td>Fields</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Source: Author’s computations using simulated data

For the measurement error, I allow different values for $\sigma_e^2$ such that the measurement error inflates the observed variance of the (natural) logarithm of income by 0%, 5%, 10% up to 40%. In addition, we also allow $\mu_e$ to be non-zero to represent instances when the observed incomes are systematically under- or over-estimated. The measurement error is also allowed to be correlated with the true income. We simulated 1000 data sets for each permutation of parameter values.

The estimates in Tables 4 to 6 present the results when the measurement error consists of only time-varying component. The figures presented in Table 4 show that, on the average, classical measurement error leads to overestimation of all income mobility indices and hence, underestimation of immobility measures. Nevertheless, the results also suggest that there could also be substantial variability in the extent of the bias within each mobility indicator.
The severity of the bias depends on the indicator being used. Among the indices considered, the effect of classical measurement error tends to be smallest for measures based on mobility as movement perspective, particularly the average quintile move, immobility ratio and Fields-Ok indices. On the other hand, the effect of classical measurement error is most-severe when using CDW and Fields indices which are both based on the mobility as equalizer of long-term income perspective. Recall that positive values for these indices suggest socially desirable mobility while negative values suggest undesirable mobility. Note that the results of the simulations suggest that it is possible for classical measurement errors to change the sign of the mobility estimates and yield misleading conclusions.\textsuperscript{12} At the same time, the magnitude of the measurement error also depends on by how much the measurement errors increase the variance of the (natural) logarithm of the observed income. For instance, focusing on the average quintile move, our simulations suggest that when measurement error overestimates the variance of the (natural) logarithm of income by a factor of $q = 5\%$, the true income mobility is only 85\% of the observed mobility. The extent of overestimation of the average quintile move index can be as large as 50\% if measurement errors inflate the variance of (natural) logarithmic income by 40\%.

\textsuperscript{12} This could be an artefact of the values we assigned for the parameters of the measurement error. Recall that since CDW and Fields indices are both functions of income inequality as discussed in the previous section, then they are also implicitly related to the variance of income. Since we assume that the variances of the true income is fixed over time (i.e., $V(Z_{i1}^*) = V(Z_{i2}^*)$) and that its temporal correlation is high (i.e., $\text{Corr}(Z_{i1}, Z_{i2}^*) = 0.9$), then we would expect that the true mobility based on CDW and Fields indices will be very small (approaching zero). Consequently, small perturbations in the income data caused by measurement errors may produce significantly different results.
Table 4. Robustness of Income Mobility Indicators to Classical Measurement Error (Mean Ratio of Actual to Observed Mobility)

<table>
<thead>
<tr>
<th>Income Mobility Indicator</th>
<th>( V(\ln(e_{it})) = q * V(Z_{it}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( q=0.05 )</td>
</tr>
<tr>
<td>Ave. quintile move</td>
<td>0.85</td>
</tr>
<tr>
<td>Immobility ratio</td>
<td>1.09</td>
</tr>
<tr>
<td>Hart (correlation)</td>
<td>0.70</td>
</tr>
<tr>
<td>Shorrocks</td>
<td>0.71</td>
</tr>
<tr>
<td>King</td>
<td>0.83</td>
</tr>
<tr>
<td>CDW</td>
<td>0.68</td>
</tr>
<tr>
<td>Ave. rank jump</td>
<td>0.84</td>
</tr>
<tr>
<td>Fields-Ok</td>
<td>0.83</td>
</tr>
<tr>
<td>Fields</td>
<td>0.75</td>
</tr>
</tbody>
</table>

Source: Author’s computations using simulated data.

To examine different forms of non-classical measurement error, I first allow \( \ln(e_{it}) \) to be distributed with non-zero mean. In particular, I set \( \mu_e \) to be proportional to \( \mu_{Z_{it}} \) and assume that the variance of the measurement error does not raise the variance of the (natural) logarithm of the true income, i.e., \( \ln(e_{it}) = \ln(e_t) \). Although not shown here, the results suggest that this type of measurement error has nil effect on mobility estimates. In other words, when a constant value is added to each of the individual’s (natural) logarithm of income, such that \( \mu_e \) is proportional to \( \mu_{Z_{it}} \), the mobility estimates are mostly unaffected. On the other hand, if \( \ln(e_i) \)’s are random, we can see that among the mobility indicators considered in this study, only the Fields-Ok index is clearly affected by measurement errors with non-zero mean.

Table 5. Robustness of Income Mobility Indicators to Classical Measurement Error (Actual - Observed Mobility)

<table>
<thead>
<tr>
<th>Income Mobility Indicator</th>
<th>( \mu_e = q * \mu_{Z_{it}} ) (assuming ( V(e_{it}) = 0.05 * V(Y_{it}^\ast) ))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( q=0 )</td>
</tr>
<tr>
<td>Ave. quintile move</td>
<td>-0.2630</td>
</tr>
<tr>
<td>Immobility ratio</td>
<td>0.1277</td>
</tr>
<tr>
<td>Hart (correlation)</td>
<td>-0.1418</td>
</tr>
<tr>
<td>Shorrocks</td>
<td>-0.0667</td>
</tr>
<tr>
<td>King</td>
<td>-0.1176</td>
</tr>
<tr>
<td>CDW</td>
<td>-0.0355</td>
</tr>
<tr>
<td>Ave. rank jump</td>
<td>-5.4898</td>
</tr>
<tr>
<td>Fields-Ok</td>
<td>-0.1688</td>
</tr>
<tr>
<td>Fields</td>
<td>-0.0310</td>
</tr>
</tbody>
</table>

Source: Author’s computations using simulated data.
Next, the numbers provided in Table 6 show the robustness of the mobility estimates when I allow the measurement error to be correlated with the true income, i.e., $\rho \neq 0$. Here, I find that the (relative) bias is smallest when $\rho = -0.7$. Hence, the severity of bias is not necessarily increasing or decreasing with the correlation between the measurement error and the true income. This is consistent with the derivation presented for the Hart’s index in the first part of this Section. In general, it seems that the severity of the bias for different income mobility indicators will depend on the interaction between the covariance of the measurement error with the true income and the variance of the measurement error.

Table 6. Robustness of Income Mobility Indicators to Measurement Error
that is Correlated with True Income
(Mean Ratio of Actual to Observed Mobility)

<table>
<thead>
<tr>
<th>Income mobility indicator</th>
<th>$\text{Corr}(Z_{it}^*, \ln(e_{it})) = q$</th>
<th>$q = -0.9$</th>
<th>$q = -0.7$</th>
<th>$q = -0.3$</th>
<th>$q = 0$</th>
<th>$q = 0.3$</th>
<th>$q = 0.7$</th>
<th>$q = 0.9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ave. quintile move</td>
<td>1.09</td>
<td>1.00</td>
<td>0.89</td>
<td>0.83</td>
<td>0.78</td>
<td>0.75</td>
<td>0.72</td>
<td></td>
</tr>
<tr>
<td>Immobility ratio</td>
<td>0.96</td>
<td>1.01</td>
<td>1.07</td>
<td>1.11</td>
<td>1.16</td>
<td>1.18</td>
<td>1.21</td>
<td></td>
</tr>
<tr>
<td>Hart</td>
<td>1.16</td>
<td>0.99</td>
<td>0.80</td>
<td>0.70</td>
<td>0.61</td>
<td>0.56</td>
<td>0.52</td>
<td></td>
</tr>
<tr>
<td>Shorrocks</td>
<td>1.16</td>
<td>1.00</td>
<td>0.80</td>
<td>0.70</td>
<td>0.62</td>
<td>0.56</td>
<td>0.52</td>
<td></td>
</tr>
<tr>
<td>King</td>
<td>1.07</td>
<td>1.01</td>
<td>0.89</td>
<td>0.83</td>
<td>0.78</td>
<td>0.74</td>
<td>0.73</td>
<td></td>
</tr>
<tr>
<td>CDW</td>
<td>1.15</td>
<td>1.23</td>
<td>0.84</td>
<td>0.73</td>
<td>0.57</td>
<td>0.45</td>
<td>0.37</td>
<td></td>
</tr>
<tr>
<td>Ave. rank jump</td>
<td>1.07</td>
<td>1.00</td>
<td>0.89</td>
<td>0.83</td>
<td>0.78</td>
<td>0.75</td>
<td>0.72</td>
<td></td>
</tr>
<tr>
<td>Fields-Ok</td>
<td>1.07</td>
<td>1.00</td>
<td>0.89</td>
<td>0.83</td>
<td>0.77</td>
<td>0.73</td>
<td>0.70</td>
<td></td>
</tr>
<tr>
<td>Fields</td>
<td>1.18</td>
<td>1.29</td>
<td>0.86</td>
<td>0.78</td>
<td>0.61</td>
<td>0.50</td>
<td>0.41</td>
<td></td>
</tr>
</tbody>
</table>

Source: Author’s computations using simulated data.
Table 7. Robustness of Income Mobility Indicators to Non-Classical (Zero-Mean) Measurement Error that is Correlated with True Income (Mean Ratio of Actual to Observed Mobility)

<table>
<thead>
<tr>
<th>Income Mobility Indicator</th>
<th>Corr($Z_{it}$, $\ln(u_{it})$) = $q$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(assuming $V(e_{it}) = 0.05 * V(Y_{it}^2)$)</td>
</tr>
<tr>
<td></td>
<td>$q = -0.9$ $q = -0.6$ $q = -0.3$ $q = -0.1$ $q = 0.1$ $q = 0.3$ $q = 0.6$ $q = 0.9$</td>
</tr>
<tr>
<td>Ave. quintile move</td>
<td>0.79 0.84 0.88 0.91 0.93 0.96 1.00 1.03</td>
</tr>
<tr>
<td>Immobility ratio</td>
<td>1.15 1.10 1.07 1.05 1.04 1.02 1.00 0.99</td>
</tr>
<tr>
<td>Hart</td>
<td>0.63 0.70 0.78 0.82 0.88 0.92 1.00 1.07</td>
</tr>
<tr>
<td>Shorrocks</td>
<td>0.63 0.70 0.78 0.83 0.88 0.93 1.00 1.07</td>
</tr>
<tr>
<td>King</td>
<td>0.91 0.91 0.91 0.91 0.91 0.92 0.92 0.92</td>
</tr>
<tr>
<td>CDW</td>
<td>-20.82 -64.24 0.31 0.32 1.08 -0.02 1.80 -0.32</td>
</tr>
<tr>
<td>Ave. rank jump</td>
<td>0.79 0.83 0.88 0.90 0.93 0.96 1.00 1.03</td>
</tr>
<tr>
<td>Fields-Ok</td>
<td>0.96 0.96 0.97 0.97 0.97 0.97 0.97 0.97</td>
</tr>
<tr>
<td>Fields</td>
<td>-9.29 1.93 2.28 3.23 3.28 3.37 1.54 -4.53</td>
</tr>
</tbody>
</table>

Source: Author’s computations using simulated data.

The estimates in Table 7 also depart from the classical measurement error setting by allowing its time-invariant component to be correlated with the true income. Similar to the findings earlier, the impact of measurement errors tend to be smallest for movement-based measures and largest for equalizer-based measures. Interestingly, as the correlation increases positively, some of the mobility estimates (e.g., average quintile move, immobility ratio, Hart’s, Shorrocks, and average rank jump indices) tend to be less biased. Up to some extent, this is consistent with the findings of Glewwe (2007) who concluded that consistent estimation of (correlation-based) mobility indices is easier when the measurement error has the same amount of mobility implied in the joint distribution of income. However, when measurement errors are negatively correlated with the true income, my findings suggest that for most of the indicators, the magnitude of bias becomes more severe as the correlation between the measurement error and the true income increases negatively.

Table 8 provides the results when the measurement error is dominated by either the time-invariant or time-varying component. Interestingly, as the variation of the measurement error that can be attributed to a fixed propensity to commit an error increases, the effect of measurement error on mobility estimates tend to taper-off. This result is quite intuitive especially when we are measuring relative mobility since the term $u_{it}$ will tend to cancel out if we take the ratio of the initial and final income.

To examine the effect of each parameter of the measurement error on income mobility estimates simultaneously, I regress $M_{ratio} = \text{abs} \left( \frac{M(Y_{it1}Y_{it2})}{M(Y_{it1}Y_{it2})} - 1 \right)$ on $V(\ln(u_{it}) + V(e_{it}))$, ...
Corr(Z_{it}^*, \ln(u_i)) \cdot \frac{V(\ln(u_i))}{V(\ln(u_i) + \ln(e_{it}))} \text{ and } \mu_e. \text{ The main point of Table 9 is that, in general, the different parameters of the non-classical measurement error have offsetting effects on the bias in income mobility estimates. In particular, whereas the magnitude of the measurement error bias is an increasing function of both the mean and the variance of the measurement error, the bias decreases when the time-invariant component dominates the measurement error and is positively correlated with the true income. Furthermore, the result suggest that income mobility is not always overestimated in the presence of non-classical measurement errors.}

<table>
<thead>
<tr>
<th>Income Mobility Indicator</th>
<th>( \frac{V(\ln(u_i))}{V(\ln(u_i) + \ln(e_{it}))} = q ) (assuming ( V(e_{it}) = 0.05 \cdot V(Y_{it}) ); Corr(Z_{it}^*, \ln(u_i)) = 0.6)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>q=0.1</td>
</tr>
<tr>
<td>Ave. quintile move</td>
<td>0.81</td>
</tr>
<tr>
<td>Immobility ratio</td>
<td>1.13</td>
</tr>
<tr>
<td>Hart</td>
<td>0.65</td>
</tr>
<tr>
<td>Shorrocks</td>
<td>0.67</td>
</tr>
<tr>
<td>King</td>
<td>0.77</td>
</tr>
<tr>
<td>CDW</td>
<td>0.33</td>
</tr>
<tr>
<td>Ave. rank jump</td>
<td>0.81</td>
</tr>
<tr>
<td>Fields-Ok</td>
<td>0.86</td>
</tr>
<tr>
<td>Fields</td>
<td>-0.21</td>
</tr>
</tbody>
</table>

Source: Author’s computations using simulated data.

<table>
<thead>
<tr>
<th>Income Mobility Indicator</th>
<th>( \frac{V(\ln(u_i))}{V(\ln(u_i) + \ln(e_{it}))} )</th>
<th>( \mu_e )</th>
<th>Corr(Z_{it}^*, \ln(u_i))</th>
<th>( \frac{V(\ln(u_i))}{V(\ln(u_i) + \ln(e_{it}))} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ave. quintile move</td>
<td>0.59</td>
<td>0.01</td>
<td>-0.15</td>
<td>-0.24</td>
</tr>
<tr>
<td>Immobility ratio</td>
<td>0.70</td>
<td>0.01</td>
<td>-0.18</td>
<td>-0.26</td>
</tr>
<tr>
<td>Hart</td>
<td>0.86</td>
<td>0.00</td>
<td>-0.22</td>
<td>-0.35</td>
</tr>
<tr>
<td>Shorrocks</td>
<td>0.84</td>
<td>-0.03</td>
<td>-0.21</td>
<td>-0.33</td>
</tr>
<tr>
<td>King</td>
<td>0.70</td>
<td>0.00</td>
<td>-0.01</td>
<td>-0.29</td>
</tr>
<tr>
<td>CDW</td>
<td>-4.84</td>
<td>10.77</td>
<td>-1.38</td>
<td>2.49</td>
</tr>
<tr>
<td>Ave. rank jump</td>
<td>0.59</td>
<td>0.01</td>
<td>-0.15</td>
<td>-0.24</td>
</tr>
<tr>
<td>Fields-Ok</td>
<td>0.46</td>
<td>0.44</td>
<td>0.00</td>
<td>-0.19</td>
</tr>
<tr>
<td>Fields</td>
<td>29.56</td>
<td>20.72</td>
<td>12.41</td>
<td>-6.54</td>
</tr>
</tbody>
</table>

Source: Author’s computations using simulated data.
Robustness of Findings for Other Income Mobility Regimes

The previous section implicitly assumed that the true income mobility regime was relatively low. In this context, the results presented earlier suggest that, on the average, the true income mobility tends to be overestimated in the presence of either classical or non-classical measurement errors. Moreover, the severity of the bias on mobility estimates increases as the variance of the measurement error increases. On the other hand, the bias tends to taper-off when much of the variability of the measurement error can be attributed to its time-invariant component. In this section, we investigate whether these conclusions also apply for higher income mobility scenarios. To do this, I use the following parameter values for the true income. Table 10 shows the estimated true mobility based from each of the configurations depicted below.

Scenario 1
\[ \mu_{Z_1^*} = 3.9, \mu_{Z_2^*} = 4.2, \sigma_{Z_1^*}^2 = 0.5, \sigma_{Z_2^*}^2 = 0.3, \text{Corr}(Z_1^*, Z_2^*) = 0.6 \]

Scenario 2
\[ \mu_{Z_1^*} = 3.9, \mu_{Z_2^*} = 4.5, \sigma_{Z_1^*}^2 = 0.5, \sigma_{Z_2^*}^2 = 0.2, \text{Corr}(Z_1^*, Z_2^*) = 0.3 \]

Table 10. Mobility based on Simulated True Income
(Medium and High Income Mobility Scenarios)

<table>
<thead>
<tr>
<th>Income Mobility Indicator</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ave. quintile move</td>
<td>0.98</td>
<td>1.31</td>
</tr>
<tr>
<td>Immobility ratio</td>
<td>0.35</td>
<td>0.26</td>
</tr>
<tr>
<td>Hart</td>
<td>0.41</td>
<td>0.70</td>
</tr>
<tr>
<td>Shorrocks</td>
<td>0.21</td>
<td>0.37</td>
</tr>
<tr>
<td>King</td>
<td>0.32</td>
<td>0.34</td>
</tr>
<tr>
<td>CDW</td>
<td>0.13</td>
<td>0.20</td>
</tr>
<tr>
<td>Ave rank jump</td>
<td>20.66</td>
<td>27.48</td>
</tr>
<tr>
<td>Fields-ok</td>
<td>0.53</td>
<td>0.76</td>
</tr>
<tr>
<td>Fields</td>
<td>0.20</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Source: Author’s computations using simulated data.
After replicating the same process implemented in the previous section, I find that on the average, the mobility estimates still tend to be overestimated in the presence of measurement errors although the severity of the (upward) bias seems to be less pronounced for higher income mobility regimes. This is quite intuitive in the sense that as the true income mobility increases, there is less room for overestimation. In addition, it is interesting to note that unlike in the low-mobility scenario wherein our simulation suggests that all mobility indicators were overestimated in the presence of classical measurement error, here, I encountered several instances when the average quintile move and the Fields-Ok indices are underestimated by classical measurement error. This proves that measurement errors do not always inflate income mobility estimates upwards. Overall, the same factors contribute to the severity of the bias on income mobility estimates due to measurement errors. In particular, the variance of the measurement error generally inflates the bias on mobility estimates especially when the error is time-varying. The interaction between the covariance of the time-varying measurement error with true income and the variance of the measurement error also affects the severity and the direction of the bias. Furthermore, bias on income mobility estimates decreases as the time-invariant component dominates the measurement error.

Correcting for Measurement Error

The results presented in the previous section emphasize that the effect of measurement errors on income mobility estimates are not always trivial. Given this information, how do we correct for measurement errors when examining income mobility trends? There are several ways to do so. For instance, if the main income data source is household survey, survey records may be matched with reports by the same respondents from administrative data that are assumed to be error-free. In turn, this supplementary data can be used to derive appropriate income adjustment factors. In particular, some studies use tax data records to correct for the potential bias present in the data on observed income\(^{13}\). On the other hand, some researchers minimize measurement error by restricting the sample to population groups that are less likely to misreport income. For example, Gottschalk and Huynh (2010) proposed excluding those who are self-employed or those whom a large portion of income is imputed from the analysis. Furthermore, others rely on more sophisticated statistical modeling

\(^{13}\) However, this approach may not be relevant for many developing countries where such type of administrative tax data is usually inaccessible if not unavailable. Several validation studies also find that survey data on income contain substantial measurement errors (Jenkins 2011). Moreover, the findings also depart from the classical assumptions regarding measurement errors.
techniques. In particular, when estimating income transition matrices, one can fit latent class Markov models (Worts, McDonough and Sacker 2010 and Breen and Moisio 2004) to gauge the impact of measurement errors on income transition matrices. Given sufficient length of longitudinal data, the main idea behind this approach is to assume that the true transition probabilities are stable over time and this can be estimated from the Markov model. The resulting residuals are considered measurement errors. On the other hand, the use of instrumental variables is an alternative statistical tool that can be used to address the bias caused by measurement error. This is particularly useful when the concept of origin-independence is being used. In particular, consider the dynamic model,

\[ Z_{it}^* = \alpha Z_{it-1}^* + \beta X_{it} + \gamma_i + \epsilon_{it} \]  

(13)

where \( \alpha \) is the income mobility parameter of interest, \( X_{it} \) is a vector of individual characteristics at time \( t \), \( \gamma_i \) is time-invariant unobserved heterogeneity for individual \( i \) and \( \epsilon_{it} \) is a random disturbance term. In a two-period setting, the corresponding model with measurement error is given by

\[ Z_{i2} = \alpha Z_{i1} + \beta X_{it} + \gamma_i + (1 - \alpha) \ln(u_i) + \ln(e_{i2}) - \alpha \ln(e_{i1}) + \epsilon_{i2} \]  

(14)

In this model, \( \alpha \) is the mobility parameter of interest which corresponds to the income elasticity. If this dynamic model is estimated using ordinary least squares (OLS), the estimate for \( \alpha \) is biased due to and measurement error. The idea behind instrumentation is to use a proxy variable that is highly correlated with the outcome of interest but is uncorrelated with the measurement error. For instance, Arellano and Bond (1991) proposed to use the income of lagged two periods as an instrument. However, in this context where there is unobserved individual-level heterogeneity \( \gamma_i \), the Arellano and Bond (1991) estimator may not be entirely appropriate because the income of lagged two periods is still correlated with the time-varying component of the measurement error. Thus, a more suitable approach is to take the first difference of Equation ___ and use generalized method of moments using income of lagged three or more periods as instrument (Holtz-Eakin, Newey and Rosen 1988). On the other hand, when income mobility is measured in terms of the Hart’s index, Glewwe (2011) also used instruments to consistently estimate the parameters of the following models,
\[ Z_{i1}^* = b_{1LS}^* Z_{i2}^* + c_{1LS}^* + \epsilon_{i1} \quad (15) \]
\[ Z_{i2}^* = b_{2LS}^* Z_{i2}^* + c_{2LS}^* + \epsilon_{i2} \quad (16) \]

Since it can be shown that the temporal correlation of income can be approximated by the parameters of these models, i.e., \( \text{plim} \sqrt{b_{1LS}^* b_{2LS}^*} = \rho(Z_1^*, Z_2^*) \) where \( b_{1LS}^* \) is the OLS slope coefficient derived from regressing \( Y_1^* \) on \( Y_2^* \) while \( b_{2LS}^* \) is the OLS slope coefficient derived from regressing \( Z_2^* \) on \( Z_1^* \), then the problem at hand is to find suitable instruments to consistently estimate \( b_{1LS}^* \) and \( b_{2LS}^* \) using the observed incomes \( (Z_1, Z_2) \) which have measurement errors. In a more general setting and/or in the absence of reliable instruments, one can follow the approach adopted the previous section. In particular, one can conjecture several different forms of measurement error and create synthetic data of measurement errors by drawing from its assumed distribution. In turn, this can be incorporated to the observed income and estimate different income mobility indicators. This approach will allow us to construct bounds for income mobility estimates.

5. Decomposing Income into Permanent and Transitory Components

An individual’s current income can be decomposed into its permanent and transitory components. Permanent income refers to an individual’s (average) income over a long horizon. On the other hand, transitory income refers to income received from unanticipated sources. Transitory income can be either positive or negative but it is expected to average out in the long-run. Understanding whether observed mobility is a result of movements in either permanent or transitory income is important for policy planning. Let us take the case of poverty dynamics as an example. Without distinguishing poverty persistence from transient poverty, policy planners may not be able to properly target intended program recipients. For

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14 Several studies have tackled the issue of measurement error on income mobility estimates using instrumental variables (Abowd and Card 1989, Cowell and Schluter 1998, Meghir and Pistaferri 2004, Khor and Pencavel 2008, Glewwe 2011 and Krebs, Krishna and Maloney 2012). Antman and McKenzie (2007) also tackled the issue of measurement error when estimating income mobility. However, the author’s approach entails transforming the data into cohort-averages.

15 The same approach was adopted by Khor and Pencavel (2008) when estimating income mobility in China using Chinese Household Income Project (CHIP) data. In particular, the authors find that when the mean of the simulated error is approximately 10% of the mean of measured income, the average quintile move in urban China increases by approximately 4% while immobility ratio increases by 5%.

16 The notations used in this section follow that of Jenkins (2011).
instance, it is possible for the transently poor to receive disproportionately more benefits than the persistently poor. In this context, transient poverty poses a significant constraint on the effectiveness of chronic poverty reduction programs (Jalan and Ravallion 1998). Nevertheless, while it makes sense to spend more effort to alleviate the living conditions of the persistently poor, it is also useful to establish environments with risk-coping mechanisms and enhance the security of the transiently poor (Deaton 1991). Moreover, it is also important to examine the dynamics in the transitory component of income as the cumulative effects of temporary income fluctuations among the poor could eventually lead to poverty persistence, especially when there are irreversible asset losses (Baulch and Hoddinott 2000, Hoddinott 2006).

In general, the relevance of decomposing income mobility due to dynamics in permanent and transitory income is manifold. First, it allows us to understand the incentive and security aspects of income mobility. For instance, the prospect of upward or downward mobility in the long-run, provides incentives for individuals to be engaged in productive economic activities. This is referred to as the incentive aspect of income mobility and is primarily concerned with the dynamics of permanent incomes. On the other hand, the security aspect of income mobility is contextualized within the assumption in economic theory that individuals are risk averse (Kaufmann 1980; Sinn 1981). In other words, for an average individual, the ability to predict future income is important when planning consumption behavior. Thus, people become more concerned on the arrangement of expenditures when income streams are fluctuating due to mobility of transitory income (Fachinger and Himmelreicher 2012). Second, decomposing income mobility into its permanent and transitory components allows us to distinguish income mobility as a desirable outcome from income mobility as an indicator of instability (Friedman and Kuznets 1954). In particular, income mobility is desirable when the growth in an individual’s permanent income is negatively correlated with his initial level of income as such type of growth pattern allows the poor to catch-up with the rich (Benabou and Ok 2001). In this context, income mobility makes the distribution of opportunities more equitable in the long-run (Atkinson, Bourguignon and Morisson 1992). On the other hand, income mobility may be perceived as an indicator of socio-economic insecurity when it is

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17 Analogously, cross-sectional estimates of inequality may also be decomposed into inequality due to disparities in permanent income and inequality due to varying income fluctuations. Up to some extent, income inequalities arising from disparities in permanent income may be associated with inequality of opportunities.
mostly driven by fluctuations in short-term income (Rohde, Tang and Rao 2011; Allanson 2008; Creedy and Wilhelm 2002; and Jarvis and Jenkins 1998)\textsuperscript{18}.

How do we decompose income into its permanent and transitory components? There is a wide range of literature discussing permanent-transitory decompositions of income. One of the most commonly used approach is to estimate variance-components models with varying complexity and conditioned on various covariates (Gustavsson 2007; Moffitt and Gottschalk 2002; and Dickens 2000; Zandvakili 2002; Ramos 2003; Geweke and Keane 2000). In particular, suppose that an individual’s income can be expressed as a function of a permanent individual-specific component and an idiosyncratic random component such that

\[ Y_{it} = u_t v_{it} \] 

(17)

or alternatively,

\[ Z_{it} = u_i' + v_{it}' \] 

(18)

For simplicity, I assume that \( u_i' \) is distributed with mean zero and a fixed variance that is constant across all individuals \( \sigma_u^2 \) while \( v_{it}' \) is distributed with mean zero and variance \( \sigma_{vt}^2 \). Assuming that the idiosyncratic component is uncorrelated with the observed income, then the total variance can be decomposed into two components: the variability of individuals’ permanent incomes and the variability of the transitory shocks.

\[ \sigma_t^2 = \sigma_u^2 + \sigma_{vt}^2 \] 

(19)

Thus,

\[ \sigma_{t+r}^2 - \sigma_t^2 = (\sigma_u^2 + \sigma_{vt+r}^2) - (\sigma_u^2 + \sigma_{vt}^2) = \sigma_{vt+r}^2 - \sigma_{vt}^2 \] 

(20)

\textsuperscript{18} Jenkins (2011) argues that fluctuation in transitory income is not a perfect indicator of economic instability. This is because income fluctuations may arise from voluntary choices made by an individual. For example, if an individual voluntarily decides to work shorter hours, the resulting income fluctuations will not necessarily imply insecurity.
Equation 20 implies that, assuming that the variability of individuals’ permanent incomes are constant over time, temporal changes in income inequality can be expressed as a function of temporal changes in inequality of transitory shocks encountered by all individuals. In other words, changes in inequality over time reflect the changes in income risks. However, researchers argue that Equation 20 are not grounded on realistic assumptions and variations of this model have been proposed over the years. Here, we consider three alternative specifications for the model of the transitory and permanent income components.

Alternative Model 1.  \[ Z_{it} = \varphi_t u_i + v'_{it} ; \sigma_t^2 = (\varphi_t)^2 \sigma_u^2 + \sigma_v^2 \]  

Alternative Model 2.  \[ Z_{it} = u_i' + v'_{it} ; v'_{it} = \omega v_{it-p} + \pi \varepsilon_{it-q} + \varepsilon_{it} \]  

Alternative Model 3.  \[ Z_{it} = u_i' + v'_{it} ; u_i' = u_{it-1} + \tau_{it} \]  

The addition of a time-specific term \( \varphi_t \) in the first model allows the relative importance for overall inequality of the permanent and transitory income to change over time. For example, an increasing returns to skilled labour may be analogous to the growing importance of the permanent component. The second model views the transitory component as an ARMA(p, q) process\(^\text{19}\). In general, it portrays instances when the effect of the transitory component is persistent beyond the observation period. For example, the effect of a pilot intervention program may have long lasting effects for some of the selected program beneficiaries. The third model allows the “permanent” component of an individual’s income to vary over time. For example, an accidental injury which causes major health changes can have long-lasting effects on an individual’s long-term income.

While the econometric literature offer several estimation methods to fit the variance component models described above using longitudinal data, there are several issues regarding the use of parametric models. For instance, Shin and Solon (2011) argued that parametric models are in some sense, “arbitrary mechanical constructs” such that the estimates can be sensitive to variations in how the underlying model is specified. In addition, choosing which parametric model is more appropriate in different contexts is not an easy task. Another potential issue about the use of parametric variance components models is that it requires panel data of adequate length to be able to estimate the model parameters consistently.

\(^\text{19}\) Larger values for p or q correspond to instances when the previous income shocks have longer effect on current income. In the econometric literature, many studies set p = 1 and/or q = 1.
However, while panel data is regularly collected in many industrialized countries, it is not collected frequently in developing countries. At best, developing countries with nationwide longitudinal data on income have 3 to 4 waves, on the average. In this context, a simpler alternative approach is warranted. A candidate measure of permanent income is some central tendency of each individual’s incomes over a specific time period. For example, if there are only three waves of panel data say at time $t$, $t+p$ and $t+p+q$, income can be averaged between $t$ and $t+p$, and subsequently throughout $t$, $t+p$, $t+p+q$ to be able to approximate the mobility of the permanent component of income.

6. Summary and Conclusion

This study presents the building blocks for the analysis of income mobility. First, it considers the different definitions of income mobility and how each definition is measured empirically. In general, the existing literature offers three broad perspectives of what income mobility means. In particular, mobility may refer to either movements within the income distribution, (temporal) independence of current income from initial income or as equalizer of long-term incomes. The indicators used to measure these concepts do not necessarily produce qualitatively similar results. Thus, when doing comparative studies, it is important to estimate income mobility using the same perspective to avoid comparing apples with oranges. On the other hand, if the objective is to provide a comprehensive snapshot of the income mobility regime transpiring in a society, it is important to measure mobility using different perspectives. To avoid unwanted complexity in the analysis, one can focus on a small set of indicators that measure different aspects of income mobility.

Second, as different levels of income mobility call for different mix of policies, the study discusses the robustness of income mobility estimates in the presence of measurement error. Previous studies have shown that income data, especially those based on household surveys, are prone to measurement error. The results of a simulation experiment conducted in the study shows that mobility estimates can be severely biased when income is measured with error. In other words, the observed mobility may be artificially driven by mobility in the error terms rather than mobility of the true income. Nevertheless, the characteristics of the measurement error can also have offsetting effects on the severity of bias that it induces on income mobility estimates. For instance, a positive auto-correlation between the measurement errors can offset the bias-increasing effect of measurement errors on mobility estimates. The study also briefly identifies several approaches of correcting for measurement errors. In particular, one way to
make such corrections is to use external data to measure the degree of measurement bias and derive appropriate adjustment factors; another, to rely on finding suitable instruments to estimate income mobility parameters consistently. In the absence of auxiliary information, a general approach that can be adopted is to simulate measurement errors using relevant assumptions. Such simulation studies can help constructing bounds for the proportion of the observed income mobility that can be attributed to measurement errors.

Third, the study also emphasizes that the normative assumption of more income mobility being always a desirable outcome should be examined with caution. In particular, an income mobility regime that is mainly driven by fluctuations in the transitory component of income may represent socio-economic insecurity. For policy planning, it is important to determine whether the observed mobility is a result of changes in permanent or transitory income. In this context, the study reviews several econometric methods to decompose current income into its permanent and transitory components. Much of the proposed procedures rely on characterizing the relationship between the permanent and transitory component of income using parametric models. If the available panel data is not of sufficient length to allow (consistent) estimation of the parameters of these models, one can adopt simpler techniques such as longitudinal-averaging of individual incomes to approximate the permanent component.
7. References


