ABSTRACT
The paper investigates the interaction between propagation buckling and lateral buckling in deep subsea pipelines. Lateral buckling is a possible global buckling mode in long pipelines while the propagation buckling is a local mode that can quickly propagate and damage a long segment of a pipeline in deep water. A numerical study is conducted to simulate buckle interaction in deep subsea pipelines. The interaction produces a significant reduction in the buckle design capacity of the pipeline. This is further exasperated due to the inherent imperfection sensitivity of the problem.

INTRODUCTION
The relentless demand for energy resources has shifted hydrocarbon exploration to deep frontier subsea regions. Examples of recent deep subsea fields are; the Perdido fold belt oil fields at depth of 2300m in the Gulf of Mexico and Lula-Mexilhão gas fields at a depth of 2145m at Santos basin off the coast of Brazil. It is expected that 25% of offshore petroleum production will be in deep water by 2015. Hydrocarbon production in deep water requires long pipelines (several hundred kilometres) and the design of such pipelines poses many engineering challenges.

Although lateral buckling is not essentially a failure mode, it can precipitate failure through excessive bending that may lead to fracture, fatigue or propagation buckling. In deep water, the catastrophic propagation buckling can quickly transform the pipe cross-section into a dumb-bell shape that travels along the pipeline, as long as the external pressure is high enough to sustain propagation. A number of experimental, analytical and numerical studies have been conducted by many researchers on; upheaval buckling [1-3], lateral buckling [4-6] and propagation buckling [7-9] of pipelines.

So far, the buckle interaction between lateral buckling and propagation buckling has received very limited attention [10]. Buckle interaction is a possible scenario in deep water. The current trends towards deep water operations justify an
assessment of the effects of this interaction on the integrity of the pipeline, which is the subject of this paper.

A numerical study is conducted for lateral buckling. Propagation buckling and its imperfection sensitivity is studied in the next section. The interaction between lateral buckling and propagation buckling is presented in terms of design curves. The effect of the interaction is summarised in an interaction curve that shows the % reduction in buckling capacity of the pipeline. Two model aluminium pipes with diameter-to-thickness ratio (D/t) of 28.57 and 42.86 are used for comparison. The nominal properties of these model pipes are given in Table 1. A third pipe given in Table 1 (D/t=34.9) is used for verification.

LATERAL BUCKLING OF SUBSEA PIPELINES

A pipeline is a slender structure that travels long distances. The hydrocarbon contents in the pipeline usually are at high temperature (80°C or higher) and high internal pressure (10 MPa or higher). Both the rise in temperature and internal pressure result in longitudinal expansion of the pipeline. The seabed friction acts to restrain this expansion which results in the build-up of axial compression in the pipe that may eventuate in buckling. A pipeline resting on the seabed will buckle laterally (in the horizontal plane). The axial compression force, N, in the pipeline due to restrained longitudinal expansion is given by [1]

$$N = EA\alpha\Delta T_e$$  \hspace{1cm} (1)

where the effective temperature change, $\Delta T_e$ accounts for the combined effects of temperature $\Delta T$ and internal pressure $\rho$ (also see [1])

$$\Delta T_e = \Delta T + \frac{\rho D(1-2\nu)}{4tE\alpha}$$  \hspace{1cm} (2)

Above eqs based on linear material behaviour while, $E$ is elasticity modulus, $A$ is cross-section area, $D$ is the pipe’s outer diameter, $t$ is the wall thickness, $\nu$ is Poisson’s ratio and $\alpha$ is the coefficient of thermal expansion.

Nonlinear FE shell modelling of the two pipes showed in Table 1 is conducted using ANSYS [11]. Thin shell (Shell-181) elements with 5 through-thickness integration points and von-Mises elastoplastic material definition (Table 1) with isotropic hardening are used. The model accounts for possible ovalization of the pipe’s cross section under lateral buckling. Nonlinear spring elements (COMBIN39) are used to account for the lateral drag and vertical stiffness of the seabed. Fig (1) shows the bilinear constitutive model adopted for the seabed lateral drag where the peak force $F_y$ is assumed to be mobilized at a lateral displacement equal to the pipe’s diameter $D$. Assuming rigid seabed, the vertical springs were assigned a substantially higher stiffness than lateral springs.

An initial geometric imperfection with amplitude $\Delta_o=4D$ and a sinusoidal half wave length $\lambda=200D$ is assumed. In order to have adequate thermal feed-in length for the evolution of lateral buckling, the length of the FE model of the pipe is taken as $4\lambda$. Due to symmetry, a half-model ($2\lambda$) is used in the analysis. The axial compression force is applied through incrementing the longitudinal displacements at the far end of the pipe. The resulting axial force and bending moment are obtained by integrating the induced reactions at the near end (the crown). Fig 2 shows the normalised axial compression force $P/P_y$ in the buckle and the normalised crown’s bending moment $M/M_p$ against normalised crown’s lateral displacement $(\delta-\Delta_o)/D$ for the two pipes, where

$$P_y = \sigma_y \pi D_o t$$  \hspace{1cm} (3)
As shown in Fig 2 the axial response reaches a peak and slowly falls to a plateau as the lateral displacement increases. However the moment response is monotonically increasing with the growth of the lateral displacement.

\[ M_p = \sigma_y D_0^2 t \]  \hspace{1cm} (4)

\[ D_0 = D - t \]  \hspace{1cm} (5)

**BUCKLE PROPAGATION IN DEEP SUBSEA PIPELINES**

Buckle propagation is a snap-through phenomenon that can be triggered by a local buckle, ovalization, dent or corrosion in the pipe wall. The resulting buckle quickly transforms the pipe cross-section into a dumb-bell shape that travels along the pipeline as long as the external pressure is high enough to sustain propagation. Figure 3 shows a typical buckle propagation response obtained from testing a 3m long aluminium pipe with D/t=25 in a hyperbaric chamber [7]. The response shown in Fig 3 is depicted in terms of the applied hydrostatic pressure against the pipe’s volume change \( \Delta V/V \) and is characterised by; the pressure at which the snap-through takes place (the initiation pressure \( P_I \)) and the pressure that maintains propagation (the propagation pressure \( P_p \)) which is a small fraction of \( P_I \).

**Figure 3.** Buckle propagation in hyperbaric chamber test of a 3m long aluminium pipe (D/t=25)

The elastic collapse pressure, \( P_c \), represents an upper-bound on \( P_I \) while Palmer and Martin [9] pressure, \( P_{PM} \), gives a lower-bound on \( P_p \). These two pressures, \( P_c \) and \( P_{PM} \), are given by

\[ P_c = \frac{2E}{(1-\nu^2)} \left( \frac{t}{D} \right)^3 \]  \hspace{1cm} (6)

\[ P_{PM} = \pi \sigma_y \left( \frac{t}{D} \right)^2 \]  \hspace{1cm} (7)
Nonlinear finite element analysis of propagation buckling was conducted and shown to agree reasonably well with experimental results [7]. Buckle propagation of the two pipes used in Sec 2 is conducted using nonlinear finite element analysis with ANSYS thin shell-181 element. Frictionless contact and target elements (ANSYS element 174 and 170) are used to define the contact between the inner surfaces of the pipe wall. A von-Mises elastoplastic material definition with isotropic hardening was adopted. In order to control the nonlinear analysis, a small dent $\Omega=0.1\%$ over a small circular surface area (20mm diameter) is introduced at the pipe’s mid-length. $\Omega$ refers to the dent in the pipe (local imperfection) which is defined as

$$\Omega = \frac{\Delta D}{D} , \Delta D = D_{\text{max}} - D_{\text{min}}$$

where $D_{\text{max}}$ and $D_{\text{min}}$ correspond to maximum and minimum pipe diameter respectively.

Fig (4) shows the predicted finite elements propagation response of the two pipes with $D/t=28.57$ and 42.86 (Table 1). The response is shown in terms of normalised applied external pressure ($P/P_c$, Fig 4a) and the applied pressure ($P$, Fig 4b) against normalised distortion of the pipe ($\Delta D/D$). In order to clearly distinguish the response of each pipe at buckle initiation, Fig 4b shows the response up to $\Delta D/D=0.2$. It is clear from this figure that propagation pressure $P_p$ is a small fraction of initiation pressure $P_I$ (around 20% for both pipes according to Fig 4a). This necessitates substantial increase in material and installation cost of deep subsea pipelines since the design is governed by $P_p$.

The initiation pressure $P_I$ represents a snap-through instability; it is expected to be very sensitive to imperfection (such as dent for example). By increasing the initial imperfection from intact ($\Omega=0.1\%$) to dented pipe with $\Omega=1\%$ and $\Omega=1.5\%$, Fig 4b shows a drastic reduction in $P_I$ of 17 and 33% for $D/t=42.86$ and 28.57 respectively. On the other hand, Fig 4 shows that $P_p$ is insensitive to imperfection. The catastrophic nature of buckle propagation and its acute imperfection sensitivity highlights the importance of investigating possible buckle interactions between global (upheaval or lateral) and local (propagation) buckles in deep subsea pipelines. It is for this reason that Albermani et al [7, 12] proposed a textured pipeline that exhibits superior buckle propagation capacity and insensitivity to imperfection.

**BUCKLE INTERACTION**

As shown in Section 2, lateral buckling can induce excessive bending in pipelines. The resulting bending may precipitate catastrophic buckle propagation that damages a substantial length of deep subsea pipeline. This is further
exasperated by the severe imperfection sensitivity of propagation response. In this section, a FE study is conducted to investigate buckle interaction for the two pipes with \( \frac{D}{t} = 28.57 \) and 42.86 shown in Table 1.

First, the FE modelling is verified against available experimental study on the interaction between bending and external pressure [13]. A steel pipe 1m long (L) with \( \frac{D}{t} = 34.9 \) (\( L/D = 31.5 \), Table 1) was used in the reported experimental study. Using symmetry, a shell FE model of the pipe using half-length (500mm) and half cross-section is generated in ANSYS [11] using a total of 2250 SHELL-181 elements (18 elements in circumferential direction and 125 elements along the length). A bilinear material model (Table 1, \( \frac{D}{t} = 34.9 \)) using von-Mises plasticity with isotropic hardening is adopted. To control the numerical solution, a localised wrinkled initial imperfection \( \bar{\omega} \) [14] is imposed on the compression side of the pipe at mid-length as shown in Fig (5a)

\[
\bar{\omega} = -\frac{D}{2} \left[ a_o + a_i \cos \left( \frac{\pi x}{N \lambda} \right) \right] \cos \left( \frac{\pi x}{\lambda} \right)
\]

\[0 \leq x \leq N \lambda, \quad (9)\]

Where the imperfection half-wave length \( \lambda = 0.165D \), number of half-waves \( N = 11 \), with a base amplitude \( a_o = 0.0025 \) and 20% amplitude bias, \( a_i/a_o \), towards mid-span. The load is applied at two stages according to the experiment [13]. At the first stage, a couple is incrementally applied at the far end of the pipe until the desired bending/curvature is achieved. This is followed by incremental application of external hydrostatic pressure while maintaining the desired curvature. Fig (5b) shows the normalised moment-curvature response from the experiment and the current FE simulation. The curvature \( k \) is normalized by critical curvature \( k_c \); \( \frac{\kappa}{\kappa_c} = \frac{t}{D_o^2} \) \( \frac{M}{M_p} \) \( \frac{\kappa}{\kappa_c} \)

\( \frac{p_o}{\sigma_y} \left( \frac{t}{D_o} \right) \) \( (11) \)

As seen in Fig (7b), the bending moment is incrementated beyond the plastic moment capacity and held constant at a normalised curvature around 0.55 followed by the application of the external hydrostatic pressure. The onset of failure during the experiment was reported at an applied hydrostatic pressure of \( 0.3p_o \) accompanied by 20% drop in moment (Point B in Fig 7b), where \( p_o \) is given by

The FE results are in good agreement with the experimental results with a predicted collapse at \( 0.28p_o \) (Point A) accompanied by 14% drop in moment.

\[ M/M_p = \frac{k}{\kappa_c} \]

![Figure 5(a) Exaggerated FE model view of the assumed wrinkled initial imperfection in the vicinity of mid-length. (b) Comparison of FE results and experiments on interaction of bending and external hydrostatic pressure for pipe \( \frac{D}{t} = 34.9 \) in Table 1](image)
INTERACTION OF LATERAL AND PROPAGATION BUCKLING

As shown in Fig. 2 the highest bending moment under lateral buckling is in the vicinity of the uplifted crown point. For this reason, the FE model used to investigate the interaction between upheaval and propagation buckling is based on a crown segment of a dented pipe with a length L=1000mm (500mm on either side of the crown point, Fig 6). The two pipes with D/t=28.57 and 42.86 (Table 1) are used in the interaction study. This gives L/D of 26-39 which is comparable to that of the experimental study presented in Fig (5b).

![Figure 6](image)

**Figure 6.** A crown segment of the pipeline (used in FE model) under the combined actions of lateral buckling (axial force P and bending moment M) and external pressure $\rho$

A half-length and half-section model is used as shown in Fig 7a. A shell FE model using SHELL-181 elements and a bilinear material model (Table 1, $D/t=28.57$ and 42.86) using von-Mises plasticity with isotropic hardening is adopted. Similar localised wrinkled initial imperfection as described in Sec 4 (eq 14 and Fig 7a) is assumed. Making use of symmetry (Fig 7a), the lateral (X) displacement and the rotation about the longitudinal axis (Z) are restrained along L1 and L2. Similarly, the longitudinal displacement (Z) and rotation about X-axis are restrained along L4 at mid-length and the vertical Y- displacement is restrained along L3 at the far end. The loading shown in Fig 7a is applied in three steps. In the first step, axial compression load is incremented to the maximum lateral buckling load obtained in Fig 2a. In the second loading step, the axial load is maintained (which is conservative) while a couple is incremented at the far end of the pipe. The couple is incremented to the desired $M/M_p$ ratio (Fig 2b). In the third step of loading, while maintaining the axial and bending load achieved in the previous two steps, external hydrostatic pressure is incremented until a propagating buckle is initiated at $p_I$ (Fig 7b). The resulting initiation pressure $p_I$ is compared to the initiation pressure of a dented pipe (same amount of dent $\Omega$) subjected to external hydrostatic pressure alone, $P_I$, as discussed in Sec 3. It is worth noting that the effect of the axial compression load (step 1) on the final results is negligible. This is expected since the lateral response is dominated by bending.

![Figure 7](image)

**Figure 7** (a) FE model of the interaction between lateral and propagation buckling (loading and constraints) (b) FE deformed shape of buckle propagation due to the interaction between lateral buckling and external pressure.

DISCUSSION ON RESULTS

Due to the interaction between the lateral and propagation buckling, the resulting initiation pressure $p_I$ is lower than the initiation pressure, $P_I$, of a similarly dented pipe subjected to external hydrostatic pressure alone (See Fig.4). Interaction between lateral and propagation buckling for the two pipes ($D/t=28.57$ and 42.86) is represented as interaction curves shown in Fig 8 and in conjunction with Fig 2. The initiation pressure $P_I$ of the dented pipes (no interaction, $M/M_p=0$, Fig 4b) is 1.74 and 4.53 MPa for $D/t$ 42.86 and 28.57,
respectively. As the pipe undergoes lateral buckling (due to restrained thermal expansion), rapid growth in bending moment as the pipe deforms laterally results in a drastic reduction in initiation pressure \( p_i \). According to Fig 8, when 90% of the pipe’s bending capacity is exhausted, the resulting reduction in initiation pressure is 17-21% for \( D/t=42.86 \) and 28.57 respectively. As seen from Fig 8, a steeper reduction in initiation pressure is obtained as \( M/M_p \) approaches 1. Due to interaction, higher reduction in buckle initiation capacity is expected at lower \( D/t \) (deep subsea applications). As discussed in Sec 3, buckle initiation is a snap-through instability that is very sensitive to geometric imperfection, this imperfection sensitivity together with the possibility of buckle interaction, impose sever design limitations on deep subsea pipelines.

**CONCLUSIONS**

The paper has presented an FE analysis of lateral buckling and buckle propagation. It is shown that the snap-through propagation buckling is very sensitive to initial imperfection. The lateral buckling response for two model pipes with different \( D/t \) ratios are presented and compared. A pipe segment in the vicinity of the crown point of lateral is used to study buckle interaction between lateral buckling and propagation buckling. The calculated axial compression load and bending moment from lateral buckling are fed to this segment model followed by the incremental application of external hydrostatic pressure on the pipe to obtain propagation response. Due to the interaction between lateral and propagation buckling, a substantial reduction in initiation pressure is expected, particularly at lower \( D/t \) ratios. An interaction curve is presented for each of the pipe models considered. Higher reduction in initiation pressure is expected under upheaval/propagation interaction in comparison to lateral/propagation interaction. The acute imperfection sensitivity coupled with buckle interaction need to be considered in the design of deep subsea pipelines.

![Figure 8 FE results showing reduction in buckle initiation pressure due to interaction of lateral buckling and propagation buckling](http://proceedings.asmedigitalcollection.asme.org/)

**Table 1** Nominal properties of the studied model pipes

<table>
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<tr>
<th>( D/t )</th>
<th>D (mm)</th>
<th>t (mm)</th>
<th>E (GPa)</th>
<th>( E_t ) (MPa)</th>
<th>( \sigma_y ) (MPa)</th>
<th>( \frac{\sigma_y}{E} )</th>
<th>( P_y ) (kN)</th>
<th>( M_p ) (kN-mm)</th>
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<td>9.35</td>
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<td>11.57</td>
<td>222.75</td>
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REFERENCES


11. ANSYS 14.0 Release, A.I.: 275 Technology Drive, Canonsburg, PA 15317.

