Recovering Missing Slices of the Discrete Fourier Transform using Ghosts

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Abstract—The Discrete Fourier Transform (DFT) underpins the solution to many inverse problems commonly possessing missing or un-measured frequency information. This incomplete coverage of Fourier space always produces systematic artefacts called Ghosts. In this paper, a fast and exact method for deconvolving cyclic artefacts caused by missing slices of the DFT using redundant image regions is presented. The slices discussed here originate from the exact partitioning of DFT space, under the projective Discrete Radon Transform, called the Discrete Fourier Slice Theorem. The method has a computational complexity of \( O(n \log_2 n) \) (for an \( n \times N \times N \) image) and is constructed from a new cyclic theory of Ghosts. This theory is also shown to unify several aspects of work done on Ghosts over the past three decades. The paper concludes with an application to fast, exact, non-iterative image reconstruction from a highly sparse set of views.

Index Terms—Discrete Radon Transform, Mojette Transform, Discrete Tomography, Image Reconstruction, Discrete Fourier Slice Theorem, Ghosts, Number Theoretic Transform, Limited Angle, Cyclic Ghost Theory

I. INTRODUCTION

The Discrete Fourier Transform (DFT) is an important tool for inverse problems, where the discrete Fourier representation of an object is used as a mechanism to recover that object. The Discrete Fourier Slice Theorem (FST) property of the DFT, independently developed by Grigoryan and others [1–4], is especially important for discrete tomographic inverse problems, where a discrete object can be recovered exactly from its discrete projected “views” or projections [5].

The Discrete FST provides an exact partitioning of two dimensional (2D) DFT space into a finite number of one dimensional (1D) discrete “slices” of equal length. These slices have discrete slopes \( m \) and \( s \) with \( m, s \in \mathbb{N}_0 \), wrapping around DFT space in both rows and columns, while tiling the space perfectly without interpolation [6]. The partitioning is valid for \( N \times N \) spaces, where \( N = 2^n \), \( p \) is prime, \( n \geq 1 \) and \( p, n \in \mathbb{N}_0 \). This includes prime sizes [7], power of two sizes [8–10] and has also been generalised to composite \( N \) [11, 7]. Rectangular spaces can be handled by zero-padding them to the nearest square size. A total of \( N + N/p \) slices is required to tile all of 2D DFT space exactly [7]. Fig. 1 gives an example of this slice partitioning when \( N \) is prime.

The inverse DFT (iDFT) of the slices corresponds directly to the projections of the Discrete Radon Transform (DRT) [6]. These projections are computed as sums along the lines formed by the vectors \([1, m]\) and \([ps, 1]\), i.e. \( m \) pixels across and one pixel down or one pixel across and \( ps \) pixels down. Fig. 2 gives the equivalent projections for the slices of the DFT shown in Fig. 1. The DRT projections are normally ordered by slope and translate \( t \), and is often referred to as DRT space. Fig. 3(a) shows the DRT space of an image of Lena.

By computing \( N + N/p \) DRT projections with the set of slopes \( m \) and \( s \) as

\[
m = \{ m : m < N, \ m \in \mathbb{N}_0 \}, \tag{1}
\]

\[
s = \{ s : s < N/p, \ s \in \mathbb{N}_0 \}, \tag{2}
\]

and the set of translates or intercepts \( t \) as

\[
t = \{ t : t < N, \ t \in \mathbb{N}_0 \}, \tag{3}
\]
where $x, y \in \mathbb{N}_0$, the 1D DFT of the projections can be placed into 2D DFT space directly (without interpolation) and the inverse 2D DFT utilised to recover the image. The slices are placed along the vectors $[-m, 1]$ and $[1, -ps]$ in 2D DFT space (see Fig. 1). For a prime-sized image, the result is normalised by subtracting the total image sum from each pixel (or equivalently correcting the DC coefficient in DFT space) and dividing by $N$. The reconstruction of the image is exact when there is no noise present in the projections.

It is common that not all projections are available to tile Fourier space in discrete tomographic inverse problems. This effectively means there are missing slices or projections, which introduce artefacts in the reconstruction known as discrete Ghosts [12]. Ghost artefacts have zero-sums in the projected directions of the missing data, so that their corresponding Fourier coefficients are also zero [12]. They may be superimposed on an image while still being invisible in the directions of the missing projections. Fig. 4 gives an example of what a Ghost can look like as an image.

Missing projections in the DRT produce cyclic artefacts on the reconstructed image [13, 14] and occur naturally when the DRT is applied to discrete tomography [13], image/erasure coding [15] and image processing [16]. Fig. 3(b) gives an example of missing projections and Ghosts in the DRT.

This paper presents a fast method for exactly removing Ghost artefacts formed from missing slices of the prime-sized DFT. The method utilises redundant or known image regions within the reconstruction to deconvolve these Ghosts artefacts. The method is constructed from a theory of cyclic Ghosts within the prime-sized Discrete FST that is also developed in this work. This theory for the Discrete FST is constructed using cyclic convolutions and results in algorithms of low computational complexity ($O(n \log_2 n)$ for an $n = N \times N$ image).

A schematic summary of the process in given in Fig. 3(c). Finally, the Ghost deconvolution method is applied to exactly reconstruct images from a highly asymmetric set of rational angle projections that give rise to sets of sparse slices within the DFT.

The paper is structured as follows. Previous work on Ghosts and reconstructing from sparse signals is reviewed in the next section. This is followed by a theory of cyclic Ghosts for the Discrete FST in Sec. III. The convolution and deconvolution techniques for missing slices of the DFT is presented in Sec. IV. Finally, the results for applying these techniques in reconstructing an image exactly from a highly asymmetric set of rational angle projections is presented in Sec. V.

II. PREVIOUS WORK

Bracewell and Roberts [17] introduced the concept of the “invisible distribution”, later referred to as Ghosts by Katz [12] and Cornwell [18], in the context of the Fourier Transform (FT). These distributions are a consequence of non-unique solutions arising from an incomplete Fourier space or “$u, v$ coverage” due to a finite number of measurements. Explicitly, a classical Ghost or invisible distribution exists where a continuous theory, i.e. one that utilises an integral transform, is the mechanism for an inverse problem. For example, the theory for Computed Tomography (CT) is commonly based on the FT, and is therefore ill-posed as a result and always has classical Ghosts [19]. Katz [12] rediscovered the concept of a Ghost and showed that Ghosts must have zero-sums in the directions of the missing projections.

Logan [20] presented the uncertainty principle for projections in the FT and determined that Ghosts are mostly present in the high frequencies. He proved why low-pass filtering of the projections, using filters such as the Ram-Lak [21] or Shepp-Logan [22] filters, is required when reconstructing from projections in the FT. These and other related filters are still being used today in modern CT [23]. Louis [24] reformulated this result in a simpler form using the Hankel Transform.

Katz [12] presented the uncertainty principle for discrete rational angle projections for square image sizes, where a projection is computed as sums along the lines formed by using the vector $[b, a]$ with $a, b \in \mathbb{Z}$. These acyclic projections constitute what is now known as the Mojette Transform (MT) [25]. Fig. 5 shows a simple example of a MT for a $4 \times 4$ image using three projections.

Katz [12] determined that an $N \times N$ image can be reconstructed exactly from a set of $\mu$ rational angle projections $[b_j, a_j]$ if and only if

$$N \leq \max \left( \sum_{j=0}^{\mu - 1} |a_j|, \sum_{j=0}^{\mu - 1} |b_j| \right).$$

(4)
Figure 3. An illustration of the Discrete Radon Transform (DRT) and its cyclic Ghosts for an image of Lena. (a) shows the DRT space of the image of Lena and its one-to-one nature. (b) shows missing projections (black rows in DRT space) and their effect on the reconstructed image. (c) shows the result of deconvolving the Ghosts or “De-Ghosting” an image with Ghost artefacts in order to restore the image when a redundant image area is present.

Figure 4. An example of a Ghost represented as an image. This Ghost is invisible at 24 rational angle projections and may be superimposed on an image without changing the projections of the image for these 24 angles.

Figure 5. An example of a Mojette Transform for a discrete image of size $4 \times 4$ using the three projections $[1, 1]$, $[1, -1]$ and $[1, -2]$. The bold lines within the right-hand grid shows a possible reconstruction path using a corner-based reconstruction method [26].

This is now known as the Katz criterion. It is a statement that the information contained in the projection set needs to be one-to-one with the image data. When the criterion is not met, discrete Ghosts superimpose on the image because of the ambiguity in the projections [12]. These Ghosts look similar to the image in Fig. 4.

Normand et al. [27] extended the Katz criteria to arbitrary convex regions using mathematical morphology. They proved that a set of rational angle projections can only be reconstructed unambiguously if and only if the largest possible Ghost is larger than the convex region being reconstructed, so that there is no ghost present in the region due to the projection set. This less stringent criteria for the MT allows one to reconstruct images from a highly asymmetric set of projections [28]. These projections have a limited coverage in terms of the half-plane, i.e. the interval $[0, \pi)$, but are sufficient for an exact reconstruction in terms of the Katz criterion (4). A number of schemes have been proposed for these types of MT projections, including a Conjugate Gradient method [29] and a Geometric Graph approach [26], but the former is not suitably convergent and the latter is very sensitive to noise. Sec. V will present a fast alternative method based on the slices of the DFT and their Ghosts.

Highly asymmetric projection sets also occur in conventional limited angle tomography. Boyd and Little [30] presented a solution to limited angle tomography by reducing Ghosts through the fusion of multi-modal data to improve Fourier coverage. Techniques that minimise the $L_1$-norm are a common approach to reduce Ghost artefacts in reconstructions [31–34]. Vetterli et al. [35] showed that $N$ spikes can be recovered exactly from $2N + 1$ consecutive Fourier samples by solving a system of equations. Work by Kuba [36] inspired many others (for a recent example see [37]) to develop the theory of “switching components”. They considered the problem of uniquely reconstructing all entries in a matrix from its directed sums, based initially along the matrix rows and columns. These switching components are structures that prohibit unique inversion. They are equivalent to the Ghosts discussed here.

In recent work, Candès et al. [38] showed that an $N \times N$ image can be recovered exactly using convex optimisation of a very small number of projections. However, their method
suffers from high computationally complexity and is still an active area of research [39]. Herman and Davidi [40] discussed the Candès et al. [38] result using Ghosts and showed that Ghosts artefacts may still remain invisible to small number of projections when using their method. Nevertheless, their work paves the way in using Ghosts for analysing the sensitivity of reconstruction algorithms. Inspired by their work, the next section constructs a cyclic theory of Ghosts for the Discrete FST.

III. CYCLIC GHOSTS

In this section, it is shown that Ghosts of the Discrete FST have exact cyclic forms known as \( m \)-Circulant matrices (or circulants).

**Definition 1** \((m\text{-Circulant})^1 [41]\). An \( m \)-Circulant is an \( N \times N \) matrix containing a unique row \( f(j) \) with \( j = 0, \ldots, N - 1 \) replicated on each row, but where each row is cyclically shifted \( (\text{mod} \ N) \) by an additional \( m \) elements to the right.

\( m \)-Circulants represent the slices of the Discrete FST via its Fourier “diagonalisation” property. A 2-Circuit is diagonalised by the DFT as illustrated in (5).

\[
\begin{bmatrix}
 a_0 & a_1 & a_2 & a_3 & a_4 \\
 a_1 & a_2 & a_3 & a_4 & a_0 \\
 a_2 & a_3 & a_4 & a_0 & a_1 \\
 a_3 & a_4 & a_0 & a_1 & a_2 \\
 a_4 & a_0 & a_1 & a_2 & a_3 \\
\end{bmatrix}
\overset{\text{DFT}}{\leftrightarrow}
\begin{bmatrix}
 \lambda_0 & \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 \\
\end{bmatrix}
\]

The diagonalisation is represented by monomial matrices \( M = PA \), where \( A \) is an \( N \times N \) diagonal matrix and \( P \) is an \( N \times N \) permutation matrix with \( N \) non-zero elements representing a discrete line at slope \( m \) (mod \( N \)) [41]. Hence, the lines of slope 1 given in part (b) of Fig. 2 is a 1-Circuit. The lines of slope 2 (as in Fig. 2(c)) will be a 2-Circuit and so on.

Every projection can then be represented by an \( m \)-Circuit and its 2D DFT represents a slice in discrete Fourier space. Equivalently, since the 1D DFT of the unique circuit row are the diagonal values [41], the 1D DFT of the (1D) projections are the slice values, so that only a computation complexity of \( O(N \log_2 N) \) is required to compute each slice. Superimposing a circuit in image space places a slice of corresponding slope in DFT space. The tiling of the diagonals is the same for the lines formed by the vectors \([−m, 1]\) and \([1, −ps]\), so a total of \( N + N/p \) diagonals is required for full tiling of DFT space. The result is a sum of \( N + N/p \) circulants in image space to recover the image. This is known as Circuit Back-Projection (CBP) [42] and is illustrated in Fig. 6.

Fill [4] showed that the Discrete FST can be constructed using circulant matrices. Chandra and Svalbe [42] showed that the construction can be extended to other DFT-like transforms, such as the Number Theoretic Transform (NTT). The NTT will be important in removing the cyclic Ghosts exactly (see discussion at the end of this section).

The circulant matrices also allow one to naturally define and understand Ghosts. In what follows, we restrict ourselves to prime-sized image spaces, so that projections are acquired via the vectors \([1, m]\) and \([0, 1]\) with a total of \( N + 1 \) projections required for an exact reconstruction.

\(^1\)Also referred to as a generalised circulant or \( g \)-circulant [41].
constructed in this paper can be found in the Finite Transform Library written by the authors [43].

**Proposition 2 (Ghost Convolution).** Ghosts formed from the missing projections of the DRT are cyclic convolutions of these projections in the geometry of the DFT.

**Proof:** According to the Discrete FST, each slice is a cyclic smearing or back-projection of its corresponding projection in image space [4, 42]. Within the geometry of the DFT, convolutions can be represented as a sum of \(m\)-Circulants [41]. Thus, the Ghosts that arise due to empty (i.e. zero) projections in the DRT result in an incomplete back-projected image, which are a sum of the missing \(m\)-Circulants, and are convolutions in image space.

**Proposition 3 (Ghost Kernels).** The Ghost convolution kernels required for De-Ghosting are the vectors \([1, m]\), which sum to zero along its vector for each of the missing projections \(j = 0, \ldots, \aleph - 1\).

**Proof:** The projections of Ghosts are zero-valued at their corresponding \(m\)-values, so the projection vectors \([1, m]\) must also be Ghosts at their \(m\)-values in order that Ghosts annihilate with shifted versions of themselves. The kernels apply to each bin in the missing projection or Ghost row (using Prop. 2). An example of a Ghost kernel is shown in Fig. 8(a)-(c).

Using Prop. 3, the De-Ghost filter is constructed in the following way:

1. Create a 2D convolution kernel for each unknown slope \(m_j\) by placing a \(+1\) at the image origin and \(-1\) at \((1, m_j)\).
2. Convolve these kernels in 2D to obtain the De-Ghost filter.

The convolution of the kernels may be done either in 2D (in Fourier or image space), or as a set of 1D convolutions on the projections via the Projection Convolution Theorem (PCT) [6] (see Fig. 8). The advantage of the latter is efficiency, especially when the number of known projections \(\mu \ll N\).

### A. Projection Convolution Theorem

The PCT states that a 2D convolution is equivalent to the 1D convolution of each projection or slice in Fourier space. Thus, the 2D Ghost convolution can be computed as a series of 1D convolutions on the known projections. For Ghost convolution, when the number of Ghosts \(\aleph\) is close to \(N\), DRT space is sparse and the 2D convolution is computed over a small number of 1D signals.

The PCT can be interpreted as a consequence of the Discrete FST. Since the slices tile DFT space exactly and are the DFT of the projections, cyclically convolving the slices of two objects is equivalent to cyclically convolving the objects themselves. Thus, in order to utilise the PCT, one needs to know the projections of the two objects. In this case, the projections of the Ghost kernels are particularly convenient, which makes a PCT approach very efficient.

**Proposition 4 (Kernel Projections).** The projections of the 2D convolution kernel \([1, m]\), with a positive term at the origin and a negative term at the coordinate \((1, m_j)\), will have the positive term at the zeroth translate and the negative term at translate \(t_j\) as

\[
t_j = (m_k - m_j) \pmod{N}
\]

for each projection \(m_j\) with \(j = 0, \ldots, N - 1\) and \(\ell = 0, \ldots, \aleph - 1\). For the \(j = N\) projection, negative term is always at \(t_j = 1\). Note that negative values \(-x\) modulo \(N\) are equivalent to the value \(N - x\) (mod \(N\)).
Pro...
circulants defined by the rows $A, B, C$ and $D$ of the filter in Fig. 11. These circulants represent the 1D convolutions of the rows of the filter. The resultant image row is back-substituted and the filter translated up or down to recover the next row. In this case, row $I(3,0)$ is back-substituted into $I(3,0)$ in order to make it a redundant row and the filter is translated upwards to the next row and Eq. (8) is repeated for (II). The process is repeated for all image rows until all the Ghosts are removed.

In the 1D convolution method, the $m$-values of the Ghosts need to be $-m_N$ (i.e. $N - m_i$ rather than $m_i$) when convolving the Ghosts to undo the right shift of the $m$-Circulants. A total of $\mathcal{N} + 1$ rows need to be convolved, resulting in a computational complexity of $O(Q\mathcal{N})$. When $\mu = \min(P,Q) + 1$ and $\mu \ll N$, the algorithm has a computational complexity of $O(n \log_2 n)$ (where $n = N^2$). A visual interpretation to constructing Ghosts, that unifies the Ghost recovery algorithm of Chandra et al. [13], Latin squares approach of Chandra and Svalbe [14] and the 2D convolution approach of this paper, can be made using $n$-gons (see the thesis of Chandra [47]).

More work has to be done with both methods when the projections or slices contain noise or inconsistencies as these also become convolved during the De-Ghost process. The convolution approach requires a very good estimation of noise prior to De-Ghosting, so that the estimates may be used to deconvolve their effects on the results. Future work includes generalising the approach to arbitrary missing discrete Fourier coefficients. Recent work by Svalbe et al. [48–50] discussed the minimal extent of cyclic Ghosts, which may prove useful in this endeavour. In the next section, the De-Ghost method is applied to the discrete inverse problem of determining a reconstruction from a set of highly asymmetric rational angle projections.

V. Application: Discrete Reconstruction

Chandra et al. [13] utilised their Ghost removal technique to exactly reconstruct an image from rational angle (noise-free) discrete 1D projections of the MT. The projections sets in their work required covering the half-plane, i.e. the interval $[0, \pi)$. In this section, the De-Ghost method will be applied to reconstructing from a set of rational angle projections with arbitrary coverage of the half-plane, such as those within a quadrant or the interval $[0, \pi/2)$, provided there are $\min(P,Q) + 1$ projections for a $Q \times P$ image. Both sets require that the projections satisfy the Katz criterion (4) for exact reconstruction.

In the De-Ghost reconstruction algorithm, a $Q \times P$ image is zero padded into an $N \times N$ space, where $N \geq \max(P,Q)$. A total of $\min(P,Q) + 1$ projections are required to ensure that the number of redundant image rows is equal to the number of missing slices $\mathcal{N}$, since there will be $N + 1$ slices within DFT space. An efficient new method for generating $\min(P,Q) + 1$ highly asymmetric rational angles is presented in the Appendix. Assuming that $\min(P,Q) + 1$ of this type of projections have been acquired, the remaining parts of this section will demonstrate the performance of the De-Ghost algorithm in reconstructing these rational angle projections exactly, where a large number of Ghosts are present.
faster than the method of Chandra et al. [13], whose computations took in the order of hours to complete on the same machine.

Further work needs to be done to handle inconsistencies within projections while utilising De-Ghosting. Also, a study of the small variations within the rational angle multiplicity may be critical in understanding whether some projections are more “important” than others. The information in an image is known to be distributed non-uniformly amongst its discrete projections. The variance for a DRT projection \( m \), with equivalent MT projection \([b, a]\), scales inversely as \((a^2 + b^2)\).

**CONCLUSION**

A theory for missing slices of the Discrete FST was constructed that allowed description of Ghost artefacts in the 2D DFT. The theory was used to construct new Ghost convolution and deconvolution methods, having a computational complexity of \(O(n \log_2 n)\) with \(n = N^2\), which can be utilised in recovering missing slices in the DFT (see Props 1 to 4). The methods required the use of the NTT in order to avoid numerical overflow and loss of precision problems. This De-Ghost method was then used to solve the discrete inverse problem of reconstructing exactly from highly asymmetric rational angle projections (see Figs 12 and 14).

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**APPENDIX**

**GHOST ANGLE SETS**

Chandra et al. [51] showed that any MT projection, acquired along a rational vector \([b, a]\), can be directly and efficiently mapped to a prime-sized DFT space exactly as

\[
m \equiv ab^{-1} (\text{mod} N),
\]

where \(a, b \in \mathbb{Z}\) with \(\gcd(a, b) = 1\) and \(b^{-1}\) is the multiplicative inverse of \(b\). The mapping (9) results from solving \(mb \equiv a \pmod{N}\). Each Mojette bin or translate \(t_M\) is then placed into a DRT translate \(t_R\) as

\[
t_R = \begin{cases} 
 b^{-1} t_M \pmod{N}, & \text{if } \gcd(a, N) > 1 \\
 a^{-1} t_M \pmod{N}, & \text{if } \gcd(b, N) > 1
\end{cases}
\]

(10)

An efficient way to generate the values \(a\) and \(b\) to minimise the \(L_1\)-norm of the rational vectors \([b, a]\). This can be computed as

\[
b_3 = \left[ \frac{b_1 + a_1 + N}{a_2} \right] b_2 - b_1, \quad a_3 = \left[ \frac{b_1 + a_1 + N}{a_2} \right] a_2 - a_1,
\]

(11)

where \(\lfloor \cdot \rfloor\) is the floor (round-down) operator, beginning the computation with \([b_1, a_1] = [0, 1]\) and \([b_2, a_2] = [1, N]\) until \([b_3, a_3] = [1, 1]\) [51]. Positive \([b, a]\) values represent the first octant of the half-plane with the other octants produced by \([-b, a], [a, b]\) and \([a, -b]\) vectors [16]. The resulting \(m\) values from these rational vectors can be compared with values.
The multiplicity is a consequence of the one-to-many nature of vectors related to the distribution of prime numbers and vectors from uniform coverage. The unevenness of the rational set are given in Fig. 14.

\[ N_{\text{total}} \text{ number of projections required to reconstruct these projections exactly.} \]

Once chosen, the theory of cyclic Ghosts can be applied to the Riemann hypothesis [53]. Once

\[ B = |a|(Q - 1) + |b|(P - 1) + 1. \]

Minimising \( L_1 \)-norm of the rational angles allows one to acquire \( N + 1 \) projections of certain coverage with reduced redundancy. The De-Ghost methods discussed in this work does not depend on how the rational angles are generated or selected.

This method can be extended to cover a fraction of the half-plane by limiting the number of octants used within the projection set. One then can obtain a set that sparsely covers the half-plane, but one that can still completely cover DFT space. This is because there exists a multiplicity of possible rational vectors for each finite angle \([1, m]\) across the half-plane. The multiplicity is a consequence of the one-to-many nature of the rational mapping in Eq. (9). A total of \( \min(P, Q) + 1 \) projections can then be selected from this set that satisfies Katz criterion (4), while still being highly asymmetric. This multiplicity is shown as graphs in Fig. 13 for a single quadrant and the half plane.

The multiplicity of the rational mapping appears to be relatively “flat” in a discrete sense, but with small variations. Graph 13 is reminiscent of curves obtained by Svalbe and Kingston [52] when observing the “unevenness” of the rational vectors from uniform coverage. The unevenness of the rational vectors is related to the distribution of prime numbers and the Riemann hypothesis [53]. Once \( \min(P, Q) + 1 \) projections are chosen, the theory of cyclic Ghosts can be applied to reconstruct these projections exactly. It effectively reduces the number of projections required to \( \min(P, Q) + 1 \), rather than a total \( N + 1 \) in the traditional case. Examples of each projection set are given in Fig. 14.

**REFERENCES**


Jeanpierre Guédon was born in Grosbreuil, France in 1962. He received the M.S degree and Ph.D degree from Ecole Centrale de Nantes and University of Nantes in 1986 and 1990 respectively, working on sampling in tomography with Yves Bizais. In 1991-92, he was a post-doc at CDRH FDA Rockville MD, USA with Kyle Myers and Bob Wagner. In 1994, he was assistant professor at Polytech Nantes then professor since 2002.

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