Assumptions and Properties of Limiting Pathway Models for Analysis of Epistasis in Complex Traits

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Abstract

For most complex traits, results from genome-wide association studies show that the proportion of the phenotypic variance attributable to the additive effects of individual SNPs, i.e., the heritability explained by the SNPs, is substantially less than the estimate of heritability obtained by standard methods using correlations between relatives. This difference has been called the “missing heritability”. One explanation is that heritability estimates from family (including twin) studies are biased upwards. Zuk et al. revisited overestimation of narrow sense heritability from twin studies as a result of confounding with non-additive genetic variance. They propose a limiting pathway (LP) model that generates significant epistatic variation and its simple parametrization provides a convenient way to explore implications of epistasis. They conclude that over-estimation of narrow sense heritability from family data (‘phantom heritability’) may explain an important proportion of missing heritability. We show that for highly heritable quantitative traits large phantom heritability estimates from twin studies are possible only if a large contribution of common environment is assumed. The LP model is underpinned by strong assumptions that are unlikely to hold, including that all contributing pathways have the same mean and variance and are uncorrelated. Here, we relax the assumptions that underlie the LP model to be more biologically plausible. Together with theoretical, empirical, and pragmatic arguments we conclude that in outbred populations the contribution of additive genetic variance is likely to be much more important than the contribution of non-additive variance.

Introduction

A finding from genome-wide association studies for most complex traits is that the proportion of the phenotypic variance attributable to the additive effects of individual SNPs, i.e., the heritability explained by the SNPs, is substantially less than the estimate of heritability obtained from correlations of relatives using family data. Many explanations for this so-called ‘missing heritability’ have been proposed [1,2,3,4,5]. One explanation is that heritabilities from family (including twin) studies are overestimated. The problem of bias in heritability estimates has been much discussed in the quantitative genetic literature (e.g., [6,7,8,9]). For example, in the classical twin design of monozygotic (MZ) and dizygotic (DZ) twin pairs, there are only three essential statistics that can be estimated, although many more genetic and phenotypic variation in the sample. Therefore, only three variance components can be estimated, although many more genetic and non-genetic causal components of variance can be postulated to influence MZ and DZ resemblance. It is well recognized that estimates of heritability may be biased and that it is difficult to separate additive genetic from non-additive genetic components and to separate genetic from common (or shared) family environment components (e.g., [6,7,8,9]). Estimates of heritability using phenotypic data from very distantly related individuals may have trivial bias from epistatic or common environment components compared to additive genetic components but are subject to very large sampling error. Human studies of distantly related individuals of sufficient size are simply not achievable. For disease traits, ascertainment bias in sampling of families for estimation of recurrence risks has long been recognized as a possible cause of inflated estimates of heritability [10,11]. Lastly, estimates of heritability for disease traits from twin cohorts (collected in restricted clinical settings) may be higher than those estimated from national cohort data, these differences most likely reflecting environmental factors including clinical practice [12].

Recently, Zuk et al. [13] revisited the overestimation of narrow sense (additive) heritability from family studies that could result from confounding with non-additive genetic variance. They referred to the difference between the expected value of the heritability estimated from family data and the ‘true’ heritability as ‘phantom heritabilty’. To illustrate their arguments, they proposed a limiting pathway (LP) model in which there are k pathway phenotypes, which are unobserved intermediate phenotypes. The phenotypically expressed trait value of an individual is the maximum of the individual pathway values. This model, they suggest, may be representative of biological processes that depend on the rate-limiting value among multiple inputs, such as the
levels of components of a molecular complex required in stoichiometric ratios, reactants required in a biochemical pathway, or proteins required for transcription of a gene’.

Under their LP model, each phenotype includes only additive genetic effects but, for \( k > 1 \), non-additive genetic variance is generated for the expressed phenotype, and so the heritability of this phenotype is less than the expected estimate from a classical twin design analysis. As the magnitude of epistasis depends on \( k \), the LP model provides a convenient way to explore the possible contribution of non-additive variation to missing heritability. They use this model to illustrate that over-estimation of heritability from pedigree data is plausible. For example, in human cells protein concentrations can be correlated and have different variances [14]. As complex traits are affected by many genes, individual genes will typically affect the expressed phenotype, and so the heritability of this phenotype is trait dependent.

The basic LP Model

In the basic LP model for continuous traits [13], the final phenotype \( y \) is the sum of all epistatic genetic variance components, with dominance represented by the subscript D. For DZ twins the phenotypic correlation is \( \rho_{DZ} = (\frac{1}{2} \sigma_A^2 + \frac{1}{2} \sigma_D^2 + \frac{1}{2} \sigma_{AD}^2 + ... + \sigma_A^2) / \sigma_p^2 \). As \( \sigma_C^2 \) is assumed to be equal in MZ and DZ twins and \( \sigma_D^2 = 2(\rho_{MZ} - \rho_{DZ}) = (\frac{1}{2} \sigma_A^2 + \frac{1}{2} \sigma_D^2 + \frac{1}{2} \sigma_{AD}^2 + ... + \frac{1}{2} \sigma_A^2) / \sigma_p^2 \) and \( 2\rho_{DZ} - \rho_{MZ} = (\frac{1}{2} \sigma_A^2 - \frac{1}{2} \sigma_D^2 - \frac{1}{2} \sigma_{AD}^2 + ... - \frac{1}{2} \sigma_A^2) / \sigma_p^2 \). Usually heritability under the ACE model is estimated as \( h^2_{ACE} = 2 \rho_{DZ} - \rho_{MZ} \).

As well recognized [7,8,7,9], \( h^2_{ACE} \) is an upwardly biased estimate of the narrow sense heritability \( h^2 = \sigma_A^2 / \sigma_p^2 \). Likewise, when non-additive genetic variance is present \( c_{pop(ACE)}^2 \) is a downwardly biased estimate of \( c^2 = \sigma_C^2 / \sigma_p^2 \), but if \( 2 \rho_{DZ} > \rho_{MZ} \) we can conclude that common environment plays a role (\( c^2 > 0 \)). Also recognized in the quantitative genetics literature [7,6,9] and concluded by Zuk et al. [13], it is impossible to disentangle the contribution of epistasis and common variance based only on twin data. However, there are bounds on some parameters (and hence their estimates). From the equations provided above and by recognizing that variance components are non-negative, that variance components sum to \( \sigma_p^2 \), and that MZ and DZ correlations are bounded between 0 and 1 independent of the value of \( h^2 \), some bounds are \( \sigma_A^2 / 2 \sigma_p^2 \leq (\rho_{MZ} - 2 \rho_{DZ} + \sigma_C^2 / \sigma_p^2) / 2 \), \( 2 \rho_{DZ} - \rho_{MZ} \leq \sigma_C^2 / \sigma_p^2 \), and \( 2 \rho_{DZ} - \rho_{MZ} \leq \sigma_C^2 / \sigma_p^2 \). In the absence of dominance and epistasis the lower bound of \( \rho_{DZ} \) is \( \rho_{DZ} = \frac{1}{2} \rho_{MZ} + \frac{1}{2} \sigma_C^2 / \sigma_p^2 \). We use these bounds to show in circumstances in which a large contribution from variance from epistasis is possible only if there is a large contribution to the variance from common environment. Whether this is plausible is trait dependent.

The classical Twin Model

Like Zuk et al., we explore the LP genetic architecture through the variance components of the classical twin model. Under this model, only three independent parameters can be estimated from sets of MZ and DZ twins. One set of parameters is the phenotypic variance \( \sigma_p^2 \), the MZ correlation \( \rho_{MZ} \) and DZ correlation \( \rho_{DZ} \). For MZ twins \( \rho_{MZ} = (\sigma_A^2 + \sigma_D^2) / \sigma_p^2 \) with \( \sigma_A^2 \) representing the total genetic variance and \( \sigma_D^2 \) the variance attributable to the common environment. The genetic variance can be broken down into additive \( \sigma_A^2 \) and non-additive components \( \sigma_{AD}^2 = \sigma_{AD}^2 + \sigma_{A^2} \), the sum of all epistatic genetic variance components, with dominance represented by the subscript D. For DZ twins the phenotypic correlation is \( \rho_{DZ} = (\frac{1}{2} \sigma_A^2 + \frac{1}{2} \sigma_D^2 + \frac{1}{2} \sigma_{AD}^2 + ... + \sigma_A^2) / \sigma_p^2 \). As \( \sigma_C^2 \) is assumed to be equal in MZ and DZ twins, and \( \sigma_D^2 = 2(\rho_{MZ} - \rho_{DZ}) = (\frac{1}{2} \sigma_A^2 + \frac{1}{2} \sigma_D^2 + \frac{1}{2} \sigma_{AD}^2 + ... + \frac{1}{2} \sigma_A^2) / \sigma_p^2 \) and \( 2\rho_{DZ} - \rho_{MZ} = (\frac{1}{2} \sigma_A^2 - \frac{1}{2} \sigma_D^2 - \frac{1}{2} \sigma_{AD}^2 + ... - \frac{1}{2} \sigma_A^2) / \sigma_p^2 \). Usually heritability under the ACE model is estimated as \( h^2_{ACE} = 2 \rho_{DZ} - \rho_{MZ} \).

As well recognized [7,8,7,9], \( h^2_{ACE} \) is an upwardly biased estimate of the narrow sense heritability \( h^2 = \sigma_A^2 / \sigma_p^2 \). Likewise, when non-additive genetic variance is present \( c_{pop(ACE)}^2 \) is a downwardly biased estimate of \( c^2 = \sigma_C^2 / \sigma_p^2 \), but if \( 2 \rho_{DZ} > \rho_{MZ} \) we can conclude that common environment plays a role (\( c^2 > 0 \)). Also recognized in the quantitative genetics literature [7,6,9] and concluded by Zuk et al. [13], it is impossible to disentangle the contribution of epistasis and common variance based only on twin data. However, there are bounds on some parameters (and hence their estimates). From the equations provided above and by recognizing that variance components are non-negative, that variance components sum to \( \sigma_p^2 \), and that MZ and DZ correlations are bounded between 0 and 1 independent of the value of \( h^2 \), some bounds are \( \sigma_A^2 / 2 \sigma_p^2 \leq (\rho_{MZ} - 2 \rho_{DZ} + \sigma_C^2 / \sigma_p^2) / 2 \), \( 2 \rho_{DZ} - \rho_{MZ} \leq \sigma_C^2 / \sigma_p^2 \), and \( 2 \rho_{DZ} - \rho_{MZ} \leq \sigma_C^2 / \sigma_p^2 \). In the absence of dominance and epistasis the lower bound of \( \rho_{DZ} \) is \( \rho_{DZ} = \frac{1}{2} \rho_{MZ} + \frac{1}{2} \sigma_C^2 / \sigma_p^2 \). We use these bounds to show in circumstances in which a large contribution from variance from epistasis is possible only if there is a large contribution to the variance from common environment. Whether this is plausible is trait dependent.

The basic LP Model

In the basic LP model for continuous traits [13], the final observed phenotype \( P \) is defined as the maximum (or equivalently the minimum) of \( k \) independent intermediate pathway phenotypes, \( P_{max} = \max(P_1,...,P_k) \). The intermediate phenotypes \( P_i \) are completely additive, but the final phenotype \( P_{max} \) is not if \( k > 1 \). Zuk et al. [13] assumed an infinitesimal model for each pathway, so the basic LP model has three parameters: the number of (additive) genetic pathways \( k \), the heritability of each pathway \( h^2_{path} \) assumed to be constant across pathways, and the proportion of environmental variance \( \sigma_{env}^2 \) which is common among full siblings (including MZ and DZ twins) \( c_{path} = \sigma_{path}^2 / \sigma_{path}^2 (c_{sib} \text{ in [13])}. For computational convenience the parameter \( c_{path} \) in the LP model is a proportion of the environmental variance to ensure a range between 0 and 1 independent of the value of \( h^2_{path} \). This should not be confused with the previously defined common variance \( c^2 \) which is proportional to the phenotypic variance.

When \( k > 1 \), the heritability \( h^2 \) of the expressed phenotype differs from the pathway heritability \( h^2_{path} \) likewise the proportion of environmental variance which is due to common environment in the pathway \( (c_{path}) \) is not necessarily equivalent to the analogous quantity at the final phenotype level \( (c) \). The basic LP model generates no dominance variance, but generates additive \( \times \) additive variance between loci from different pathways.
The Extended LP Model

In the basic LP model there are four important assumptions. All pathway phenotypes (i) have the same mean, (ii) the same heritability, and (iii) are independent at the pathway level. As these assumptions are unlikely to be upheld in biological systems, we extend the basic LP model by relaxing some of them. In this extended LP model the means, variances and heritabilities may differ, and are defined by, respectively, \( A_{\text{path}} = (A_1, \ldots, A_k) \), \( \sigma_{\text{path}}^2 = (\sigma_1^2, \ldots, \sigma_k^2) \) and \( h_{\text{path}}^2 = (h_1^2, \ldots, h_k^2) \). A general correlation matrix could be defined for the genetic relationship between both mothers and fathers, 

\[
\begin{pmatrix}
A_{\text{mother}} \\
A_{\text{father}}
\end{pmatrix} = \begin{pmatrix}
(A_1, \ldots, A_k) \\
(A_1, \ldots, A_k)
\end{pmatrix}
\]

so that

\[
\begin{pmatrix}
A_{\text{mother}} \\
A_{\text{father}}
\end{pmatrix} \sim N \left( \begin{pmatrix}
0 \\
0
\end{pmatrix}, \begin{pmatrix}
\sigma_{\text{mother}}^2 & \rho \sigma_{\text{mother}} \sigma_{\text{father}} \\
\rho \sigma_{\text{father}} & \sigma_{\text{father}}^2
\end{pmatrix} \right)
\]

The proportion of environmental variance common for siblings at the pathway level is \( \rho_{\text{path}} \) (assumed to be the same for each pathway, and therefore the proportion of variance explained by common environmental effects for both mothers and fathers is distributed as bivariate Mendelian sampling variance about the mean, \( \sigma_{\text{path}}^2 = (\sigma_1^2, \ldots, \sigma_k^2) \) hence \( \sigma_{\text{path}}^2 = (1 - \rho_{\text{path}}) \sigma_{\text{path}}^2 \). Therefore, the extended LP model is a six parameter model: \( LP_{\text{ext}}(k, A_{\text{path}}, \sigma_{\text{path}}^2, h_{\text{path}}^2, c_{\text{path}}, \rho_{\text{path}}) \). For example, the limiting pathway model LP\(_{(k)}\) \( (2, h_{\text{path}}^2 = 0.8, c_{\text{path}} = 0.5) \) could be modeled with the extended LP model as \( LP_{\text{ext}}(k = 2, A_{\text{path}} = (0,0), \sigma_{\text{path}}^2 = (1,1), h_{\text{path}}^2 = (0.8,0.8), c_{\text{path}} = 0.5, \rho_{\text{path}} = 0) \). Zuk et al. [13] showed that the narrow-sense heritability of the observed phenotype in the population is \( h^2 = \frac{k \sigma_{\text{path}}^2}{\sigma_{\text{path}}^2} \), where \( \rho_{\text{path}} \) is the correlation between the first pathway phenotype and the final phenotype. This definition assumes exchangeability and independence of the intermediate pathway phenotypes \( P_i \). Under the extended LP model, the pathway phenotypes are non-exchangeable and correlated. As the additive model assumes \( P = \mu + \sum_{i = 1}^{k} \beta_i A_i + \epsilon \), we can estimate the pathway coefficients \( \hat{\beta}_i \) by regressing final phenotype \( P \) on the pathway genetic values \( A_i \). The heritability estimate is a function of the regression coefficients, the additive values and the phenotype variance: \( h_{\text{all}}^2 = \text{Var} \left( \sum_{i = 1}^{k} \hat{\beta}_i A_i \right) / \text{Var} \left( P \right) \). Unbiased estimates of \( h_{\text{all}}^2 \) and \( h_{\text{pop}(ACE)}^2 \) are reported as the mean of \( h_{\text{all}}^2 \) and \( h_{\text{pop}(ACE)}^2 \) across simulation replicates. As defined in Zuk et al. [13], the phantom heritability is \( h_{\text{phantom}}^2 = 1 - h_{\text{pop}(ACE)}^2 / h_{\text{all}}^2 \).

Simulation

For all simulations, we generated 50 independent samples of 100,000 families. Each family comprised two parents, an offspring, its MZ twin and its DZ twin. The phenotype of a parent for pathway \( i \) \( (P_i) \) was simulated as \( P_i = A_i + E_i \). Random mating of parents, the phenotype of an offspring \( j \) for pathway \( i \) was simulated as

\[
P_j(f) = A_j^{off(f_{spring})} + C_i + U_j(f)
\]

with \( A_i \), \( E_i \), \( A_j^{off(f_{spring})} \), \( C_i \), and \( U_j(f) \) drawn from their respective multivariate distributions. For monozygotic twins \( A_j^{off(f_{spring})} = A_j^{off(f_{spring})} \). In all simulations, unless stated otherwise, the following parameters were used: \( LP_{\text{ext}}(k = 2, A_{\text{path}} = (0,0), \sigma_{\text{path}}^2 = (1,1), h_{\text{path}}^2 = (0.8,0.8), c_{\text{path}} = 0.5, \rho_{\text{path}} = 0) \). From the final phenotypes of the offspring, the twin correlations \( r_{MZ} \) and \( r_{DZ} \) were calculated, resulting in a heritability based on the ACE model of \( h_{\text{pop}(ACE)}^2 = 2r_{MZ} - r_{DZ} \) and phantom heritability \( h_{\text{phantom}}^2 = 1 - h_{\text{pop}(ACE)}^2 \). The reported \( h^2 \), \( c^2 \), \( \rho_{MZ} \), \( \rho_{DZ} \), and \( h_{\text{phantom}}^2 \) are means across 50 simulation replicates and hence unbiased.

We first performed simulations to study the implications of the basic LP model with respect to common environmental effects. To explore bounds on variance components we simulated a range of basic LP models \((h_{\text{path}}^2 \in [0,1,0,9], c_{\text{path}} = [0,0,9,0], k \in [1,4])\) and calculated \( h^2 \) and \( c^2 \) for each model. From each simulation we estimated \( \rho_{MZ} \) and \( \rho_{DZ} \) and plotted \( h^2 \) and \( c^2 \) as a function of \( \rho_{MZ} \) and \( \rho_{DZ} \).

Subsequently we performed five simulations to study the effect of differences in (i) pathway mean, (ii) variance, and (iii) heritability on phantom heritability, and (iv & v) the effect of correlations between pathways on phantom heritability in the extended LP model. Simulations (i) to (iii) comprise a series of two-pathway models in which one parameter was changed: (i) the difference in pathway mean \((\mu_2 - \mu_1)\). These values are in standard
deviation units since \( \sigma^2_y = 1 \). (ii) The pathway variances differed, \( \sigma^2_{\text{path}} = 0.25 \) and \( \sigma^2_{\text{path}} = 1 \) was varied. (iii) The pathway heritabilities differed, \( h^2_{\text{path}} = 0.8 \) and \( h^2_{\text{path}} = 0.84 \) was varied. (iv) In this case a multiple pathway model was simulated with no common environment effects (i.e., \( c_{\text{path}} = 0 \), in which both the pathway correlations \( r_{\text{path}} \) and number of pathways \( k \) were varied. (v) As (iv), but with 10% of the variance of each pathway attributed to common environment (i.e., \( c_{\text{path}} = 0.5 \) when \( h^2_{\text{path}} = 0.8 \).

Finally, to illustrate the extended LP model, we chose three continuous traits with different ACE-based heritability estimates from studies found in a recent twin research review paper [17]: (i) height in Danish male twins \( (r_{\text{MZ}} = 0.89, r_{\text{DZ}} = 0.47, h^2_{\text{pop,ACE}} = 0.84) \) [18], (ii) triglyceride levels in blood in Swedish female twins from 20–29 years old \( (r_{\text{MZ}} = 0.89, r_{\text{DZ}} = 0.47, h^2_{\text{pop,ACE}} = 0.84) \) [19], and (iii) high-fat dairy intake in UK male and female twins adjusted for age \( (r_{\text{MZ}} = 0.23, r_{\text{DZ}} = 0.13, h^2_{\text{pop,ACE}} = 0.20) \) [20]. Based on these observed values and for different combinations of \( k \in \{2, 4, 10\} \) and \( r_{\text{path}} \in \{0, 0.2\} \), we report the estimates \( h^2_{\text{phantom}} \) and \( c^2 \). As \( r_{\text{MZ}} \) and \( r_{\text{DZ}} \) are not model parameters but model outputs, the input parameters \( h^2_{\text{path}} \) and \( c_{\text{path}} \) were chosen such that estimated \( r_{\text{MZ}} \) and \( r_{\text{DZ}} \), based on the median of 50 simulations, reflected the observed values.

**Results**

**Exploring Bounds of Variance Components**

Although it is impossible to disentangle the contribution of non-additive genetic variance and common variance from twin data, there are some bounds on these parameters as illustrated in Figure 1, generated under the basic LP model, which is shown to yield more extreme non-additive genetic variance than many extended LP models. Each point in Figure 1 represents \( h^2_{\text{n}M} \) or \( c^2 \) as a function of \( p_{\text{MZ}}, p_{\text{DZ}} \) and \( k \).

In comparisons of the left \( (k = 1) \) with the right panels \( (k = 4) \), each combination of \( p_{\text{MZ}} \) and \( p_{\text{DZ}} \) values is consistent with multiple basic LP models. In other words, the number of pathways cannot be derived from a pair of \( p_{\text{MZ}} \) and \( p_{\text{DZ}} \) values alone. As expected, for any \( p_{\text{MZ}} \) and \( p_{\text{DZ}} \) combination, the non-additive variance increases with the number of pathways, resulting in a lower narrow-sense heritability \( h^2_{\text{n}M} \) estimate for an epistatic model \( (k = 4) \) compared to the additive model \( (k = 1) \) (panels A vs B). However, the contribution of environmental variance which is common for siblings \( (c) \) increases as well (panels C vs D). Therefore, as \( 2p_{\text{DZ}} - p_{\text{MZ}} \) increases for complex traits, important contributions from non-additive variance can be achieved only if accompanied by high \( c^2 \). For example, if \( p_{\text{MZ}} = 0.8 \) and \( p_{\text{DZ}} = 0.4 \), an additive model \( (k = 1; h^2 = 0.8) \) implies no contribution of common variance \( (c^2 = 0) \), whereas an highly epistatic model \( (c, k = 4; h^2 = 0.2-0.4) \) is consistent only with \( c^2 > 0.2 \). More generally, for highly heritable traits \( (h^2 > 0.6; k = 1) \) a large amount of epistasis \( (h^2 < 0.4; k = 4) \) is consistent only with \( c^2 > 0.2 \). However, if \( 2c^2 > p_{\text{DZ}} \) then substantial phantom heritability need not be accompanied by high \( c^2 \).

**Properties of the Extended LP Model**

Simulation results of the extended LP model are reported in Figure 2 for the effect of different parameters on the phantom heritability: (i) As the offset in mean between the two pathways increases, the phantom heritability decreases (panel A). One standard deviation difference in mean between pathway phenotypes (if \( k = 2 \)) approximately halves the phantom heritability. Clearly, as differences in offset become large, some pathways contribute little to the final phenotype, effectively decreasing the number of contributing pathways and hence the amount of epistasis. (ii) Differences in phenotypic variance between two pathways had no effect on phantom heritability (result not shown), because large phenotypic variance not only increases the probability of producing a maximum value, but also increases the probability of producing a minimum value. Across individuals both pathways contribute equally to the observed phenotype, but the mean and variance of the observed phenotype increases. Although the correlation between the final phenotype and the pathway phenotypes is higher for the pathway with the higher variance, the contribution of the observed phenotype increases proportionally with the ratio of the two pathway variances, resulting in a constant heritability. (iii) As panel B shows, differences in heritability between pathway phenotypes have only a marginal effect on phantom heritability. (iv and v) Correlations between pathways affect the phantom heritability significantly (panels C and D). Positive correlations between pathways effectively limit the amount of epistasis, resulting in less phantom heritability. The larger the number of pathways, the larger the phantom heritability reduction (for any given pathway correlation). As the correlation between pathways approaches 1, the model approaches an additive single pathway model. This holds irrespective of the amount of common variance assumed, although common variance increases the phantom heritability slightly (panels D vs C). These results show that relaxing the assumptions of equal mean and uncorrelated pathways can substantially reduce the amount of phantom heritability. In contrast, negative correlations increase the amount of epistasis even if only one out of the \( k \) pathways is negatively correlated to the remaining positively correlated pathways, although the relative impact decreases as \( k \) increases (panels C and D).

**Illustration for Three Traits**

Table 1 shows the implications of the (extended) LP model for three continuous traits with increasing estimated heritability: high-fat dairy intake \( (h^2_{\text{pop,ACE}} = 0.2) \), triglyceride levels in blood \( (h^2_{\text{pop,ACE}} = 0.54) \), and height \( (h^2_{\text{pop,ACE}} = 0.84) \). The table illustrates two important points. First, assuming a larger amount of epistasis (i.e., larger \( k \)), not only implies increased phantom heritability and decreased narrow-sense heritability, but also implies a larger contribution of common variance. Second, assuming positive dependence between pathways \( (r_{\text{path}} > 0) \) reduces the amount of epistasis. In other words, increasing the number of pathways has less effect on phantom heritability, narrow-sense heritability, and the contribution of common variance, compared to a model with \( r_{\text{path}} = 0 \). Especially for traits with a large estimated ACE heritability, a high phantom heritability \( (h^2_{\text{phantom}} > 0.61) \) is only compatible with a scenario in which the percentage of common variance is high \( (c^2 > 0.39) \).

Nonetheless, in some scenarios important phantom heritability is expected with negligible \( c^2 \), for example when \( k = 2, r_{\text{path}} = 0 \) we estimate \( h^2_{\text{phantom}} \) to be 0.25 and \( c^2 = 0.07 \). We note that we selected examples with \( p_{\text{MZ}} - 2p_{\text{DZ}} < 0 \) to illustrate potential implications of the LP model. Hill et al. [21] reported an empirical distribution of \( p_{\text{MZ}} - 2p_{\text{DZ}} \) distributed around zero, with interpretation that the distribution reflected sampling variance given the often small sample size. However, direct interpretation of the point estimates suggests that, since \( p_{\text{MZ}} - 2p_{\text{DZ}} < 0 \) in ~50% of cases, in these cases substantial phantom heritability could be
present in the context of zero or weak common environmental variance.

Discussion

Bounds of Variance Components in Twin Studies

Using the basic LP model we explored constraints on combinations of parameters. For additive models (Figures 1A and 1C) all combinations of $r_{MZ}$ and $r_{DZ}$ lie in a region bounded by $r_{DZ} = \frac{1}{2}r_{MZ}$ (if $c = 0$) and $r_{MZ} = r_{DZ}$ (if $c = 1$). For epistatic models (Figures 1B and 1D) the bounds are evident from the wedge shape of permissible combinations of $r_{MZ}$ and $r_{DZ}$ in Figure 1. They show that when $r_{DZ} > \frac{1}{2}r_{MZ}$, substantial non-additive genetic variance can be accompanied only by unreasonably high $\sigma_A^2 / \sigma_E^2$. This implies that, at least when $r_{DZ} > \frac{1}{2}r_{MZ}$ an underlying additive model is more plausible than a highly epistatic architecture. Specifically, as Table 1 illustrates, in highly heritable traits with a small contribution of common variance, phantom heritability is likely to be small.

As noted by Zuk et al. [13] in their supplementary information, the amount of phantom heritability estimated depends on the method of estimation of $h^2_{\text{pop}}$. The expected heritability estimate from regression of offspring phenotype on mid-parental phenotype ($h^2_{\text{pop(PO)}}$) is less than $h^2_{\text{pop(ACE)}}$ under the $k > 1$ basic LP model. Other factors could also contribute to differences between $h^2_{\text{pop(ACE)}}$ and $h^2_{\text{pop(PO)}}$ such as dominance and greater common environment of sibling compared to filial relations. Despite this, empirical observation ([7] pp. 172–173) does not, in general, suggest large differences between $h^2_{\text{pop(ACE)}}$ and $h^2_{\text{pop(PO)}}$, which is not consistent with an important role for phantom heritability (although sampling variation about estimates make it difficult to draw strong conclusions). Deconfounding of genetic and common environmental variance is possible, for the most part, by use of adopted away relatives. Very different estimates of correlations

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**Figure 1.** Mean narrow-sense heritability ($h^2 = \sigma_A^2 / \sigma_P^2$) (panels A and B) and proportion of phenotypic variance which is common for siblings ($c^2 = \sigma_A^2 / \sigma_P^2$) (panels C and D) across 50 simulation replicates color coded as function of MZ/DZ correlation and number of pathways ($k$) under the basic LP model. Given a combination of MZ/DZ correlations, a decrease in narrow-sense heritability (i.e., as $k$ grows), implies an increase in contribution of common environment.

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between adopted away siblings and those raised together is expected if phantom heritability is important, but adoption studies tend to support genetic estimates from twin studies [22].

The Extended LP Model

Zuk et al. proposed a simple and elegant model that allows exploration of the impact of epistasis on estimates of heritability without needing to define epistasis between individual loci. In fact, the basic LP model is a special case of the optimum pathway model proposed by Sewall Wright in 1935 [23], in which the expressed phenotype is the pathway value closest to a defined optimum, which could be, for example, the mean or median, rather than the maximum. These models include the additive model as a special case, but produce different amounts of epistasis as the number of pathways increases. Indeed any non-linear transformation of an additive genetic model, even the infinitesimal model, leads to non-additive variation; but Zuk et al. show that the basic LP model generates a phenotypic distribution close to normal, particularly when $k$ is small. Under the basic LP model all pathway phenotypes have the same distribution and pathwayheritabilities and pathways are uncorrelated. Biologically, these are very strong assumptions, not least since they invoke the infinitesimal model that implies independent contributions from many genomic sites in each pathway, and so we extended the basic LP model to allow correlated pathway phenotypes with different distributions and pathway heritabilities. Phantom heritability was little affected by differences in variance and heritability between pathways. However, differences in mean phenotype and the presence of positive correlations between pathways can decrease the phantom heritability considerably, and negative correlations increase it. Our results show how the predicted importance of phantom heritability depends on implicit model assumptions, such that the problem of phantom heritability could be overstated.- Drawing inferences about epistasis from the LP model.

There is much debate about the relative importance of non-additive versus additive genetic variance [24,25,26,21,27], summarized by Crow [28]. Central to the debate is that mutational studies demonstrate the ubiquity of epistasis in the classical sense, because genes interact in hierarchical systems to generate biological function [26]. However, in quantitative genetics it is the residual variation segregating in populations that determines differences amongst individuals not overall biological function [26]. Fisher suggested that epistasis was not important because usually there would be some scale transformation of phenotypic values to generate additive effects [26]. Indeed, this is the basis of models of complex disease where non-additivity on the observed scale can be transformed to an underlying additive scale. Furthermore, under mutation drift (neutral) models a high proportion of genetic variants are at frequencies near 0 or 1, so the presence of substantial epistatic interactions at the level of gene effects does not in itself generate appreciable epistatic variance, and contributions from epistatic interactions are detected as

Figure 2. Phantom heritability under the extended LP model as a function of (A) differences in mean (sd unit) of two pathway phenotypes, $(\mu_2 - \mu_1)$ (B) changes in $h^2_{path}$, while $h^2_{path} = 0.8$, (C) pathway correlations for different number of pathways, and (D) pathway correlations for different numbers of pathways assuming a total contribution of common environment of 10% for each pathway phenotype. Unless stated otherwise, number of pathways $k = 2$, contribution of common family environment is zero (i.e., $\epsilon_{path} = 0$), pathway heritability $h^2_{path} = 0.8$, pathway correlations $\rho_{path} = 0$. doi:10.1371/journal.pone.0068913.g002
additive variance [28,21]. These arguments are further strengthened under models that consider selection against mutations deleterious for fitness with pleiotropic effects on quantitative traits [29] as the proportion of variants with frequencies near 0 or 1 is even higher. Zuk et al. [13] incorrectly state in their Supplementary Information that the derivation in Hill et al. [21] applies only to pairs of loci, whereas in fact these were used for illustration, and the argument holds for multilocus epistasis. The elegance of the LP model of Zuk et al. [13] is that it is parametrized in terms of variances and so does not depend on the allele frequency distribution. Zuk et al. [13] (supplement page 45) counter Hill et al’s analysis by arguing that most genetic variants contributing to complex traits cannot be at extreme frequencies because these would generate little variance. They illustrate with a two-locus example (their Supplementary Figure 9), but it shows a steep increase in total genetic variance from minor allele frequency of 0 to 0.1, nearing its maximum for minor allele frequency 0.1, where increase in total genetic variance from minor allele frequency of 0

Table 1. Phantom heritability ($\hat{h}^2_{\text{phantom}}$), narrow-sense heritability ($\hat{h}^2$) and percentage of common variance ($c^2$) for three traits assuming varying number of pathways ($k$) and pathway correlations ($\rho_{\text{path}}$).

<table>
<thead>
<tr>
<th>Trait</th>
<th>$r_{\text{MZ}}$</th>
<th>$r_{\text{DZ}}$</th>
<th>$h^2_{\text{ACE}}$</th>
<th>$k$</th>
<th>$\rho_{\text{path}}$</th>
<th>$\hat{h}^2_{\text{phantom}}$</th>
<th>$\hat{h}^2$</th>
<th>$c^2$</th>
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<td>0.13</td>
<td>0.20</td>
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<td>2</td>
<td>0.2</td>
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<td>Triglyceride levels in blood</td>
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<td>0.28</td>
<td>0.54</td>
<td>2</td>
<td>0</td>
<td>0.25</td>
<td>0.41</td>
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Limiting Pathways in Context

The LP model was justified (Zuk et al., p1193 [13]) without reference as: “Here we show that simple and plausible models can give rise to substantial phantom heritability. Biological processes often depend on the rate-limiting value among multiple inputs, such as the levels of components of a molecular complex required in stoichiometric ratios, reactants required in a biochemical pathway, or proteins required for transcription of a gene.” For biochemical pathways, at least, metabolic control theory has shown that ‘rate limiting steps’ are not a relevant concept, for rate of flux is a continuous function of activities at multiple stages of the pathway [33]. In a recent review Suarez and Moyes stated “The days have long passed when it was simply assumed that enzymes possessing allosteric regulatory properties were ‘rate-limiting’ [34]. It is now recognized that control of pathway flux is often distributed among many enzymes.” And Fell’s well-cited review [35] concludes “whatever criticisms might be made about any one of the experimental studies, it is significant that none have
provided support for the existence of unique ‘rate-limiting' enzymes in pathways.' In quantitative genetic analysis of models of such pathways, it has been shown that a substantial proportion of the variance is additive [36]. The LP model was proposed to explain “missing heritability” in complex traits. Methods are now available to estimate variance attributable to all common genotyped SNPs rather than those identified as significant [37,38]. Simulations conducted under the LP model demonstrate that estimates of additive variance attributable to SNPs calculated using GCTA [39] are unbiased and not inflated by epistasis [40]. Applications of these methods to real data show that at least 40% of heritability estimated from family studies remains unexplained [41]. The number of associated common variants detected has increased with sample size [41] e.g., from 9 to 140 for Crohn’s Disease as case sample size increased from 2000 [42] to >2000[43]. The implication is that, to date, studies have been underpowered to detect common variants of realistic effect sizes, but that many exist, given that rare variants are much more prevalent, and that a very large number of rare variants also contribute exist which individually explain little variance but their cumulative contribution may be important. Collection of empirical data to test an additive only model is unlikely to be achievable in humans. In yeast, an elegant study designed to explore contributions of variance from different sources found substantial epistatic variance (median of 30%) for some of the 46 traits studied [44]. However, its relevance to human populations is limited, since all gene frequencies were one half (two-way cross design), conditions under which epistasis is likely to be maximized [21,28]. More relevant insight may be gained from outbred species. For example, in dairy cattle heritability and SNP associated effects are estimated from large numbers of half-sib daughters born, raised and milked at different farms. Therefore, their estimates are unlikely to be confounded with non-additive genetic or shared environmental effects [45]. For milk yield 79% (s.e. 5%) of the additive genetic variance is captured by SNPs [46]. That there is so little missing heritability can be explained by the smaller effective population size leading to longer linkage disequilibrium (LD) blocks than in humans and hence even rare alleles can be predicted by multiple SNPs. Traits that could reasonably be assumed to be under strong natural selection (so that very rare variants play an important role), such as fertility, have lower heritability (40%) and greater missing heritability (55% explained by common SNPs fitted together) [46]. The simplest explanation of why not all variance is explained by the SNPs is that even in livestock some causal variants are rare and in low LD with the SNPs. These results provide evidence that (when \( h^2_{\text{pop}} \) is estimated accurately) additive effects can explain the majority of observed variance in a complex trait in an outbred population.

**Disease Traits**

Zuk et al. [13] expressed phantom heritability as \( h^2_{\text{pop}} - h^2_{\text{all}} \). For quantitative traits \( h^2_{\text{all}} = h^2_{\text{max}} \), where \( h^2_{\text{max}} \) is the proportion of variance in \( P_{\text{max}} \) attributable to additive genetic factors. For disease traits they considered a liability threshold model, but did not assume disease to occur when the liability phenotype \( P_{\text{max}} \) exceeds the threshold truncated by the proportion \( 2\mu \), but instead defined disease to occur when a pathway phenotype exceeds the threshold truncated by the proportion \( 2\mu/k \), generating a total proportion, \( 2\mu \), of affected individuals when summed over all \( k \) pathways. This definition implies additional non-additive genetic variance, i.e., \( h^2_{\text{all}} < h^2_{\text{max}} \).

For example, using the 3-pathway model for Crohn’s Disease [13] with \( h^2_{\text{path}} = 0.476 \) and \( \epsilon_{\text{path}} = 0.16 \) generates \( h^2_{\text{max}} = 0.283 \), but \( h^2_{\text{all}} = 0.186 \) for \( 2\mu = 0.001 \). Under the extended LP model we showed for quantitative traits that results for a multiple pathway model converged to a single pathway model for positively correlated pathways; qualitatively this result also holds for disease traits.

The LP model was justified by Zuk et al as limiting pathways in a biological and biochemical sense. However, the same methodological approach could represent a heterogeneity model, generating a different interpretation of results. Under the LP model the final phenotype is considered the “true” phenotype and the non-additive genetic variance \( h^2_{\text{pop}} - h^2_{\text{all}} \) is real. In contrast, under a heterogeneity model, the pathways are the true phenotypes but inadequacies in phenoyping cause an inability to distinguish between biologically different classes of the observed disease. Hence, under a heterogeneity model, the measurable additive genetic variance \( h^2_{\text{all}} \) may be much less than true additive variance of each subtype, but mostly \( h^2_{\text{pop}} - h^2_{\text{all}} \) could be viewed as “phantom non-additive genetic variance”, since the non-additive genetic variance results only from incorrectly treating multiple phenotypes as a single trait. In common complex genetic disease there have been notable advances in separation of diseases that originally were considered a single diagnostic class, e.g., diabetes, rheumatoid arthritis, breast cancer. Dilution of allelic effect size is a consequence of phenotypic heterogeneity in genetic association studies. For example, differentiation of breast cancer into ER-positive and ER-negative cancers has identified associated loci not possible from combining the case cohorts [47]. In psychiatric nosology it has long been recognized that diagnostic classes are likely to overarch heterogeneous etiology, recently explored in light of results from genomic studies [48]. Indeed, one motivation of genomic studies is to allow genetically informed nosology.

**Conclusion**

The results of Zuk et al. [13] provide a timely reminder of the well-recognized limitations of analyses based on twin and family data, which are often underpowered to separate additive genetic from common environmental effects [49] and non-additive effects. The (extended) LP model provides a useful framework to explore the possible contribution of non-additive genetic variance to complex traits. An important role for non-additive genetic action is attractive because gene interactions are ubiquitous at the functional level, yet this does not necessarily translate to important epistatic variance over and above variance detected as additive effects. For disease traits, empirical data can only be explained by non-additivity on the disease scale, but such non-additivity can be explained by scale transformations without needing to invoke more complex models. Using the framework of the extended LP model, and together with theoretical, empirical, and pragmatic arguments we conclude that although contributions from non-additive variance may be commonplace in complex traits, the contribution of additive genetic variance is likely to be much more important than that of non-additive variance. Ultimately, only empirical results can provide a satisfactory conclusion to the debate of missing heritability, but these may be elusive. Larger sample sizes should afford the power to identify common variants of smaller effect size and two-locus interactions. However, the heavy penalty of multiple testing will not allow exploration of higher order epistatic interactions implied by the LP model. Likewise, large sample sizes are unlikely to identify rare causal variants of small effect, since rare variants are likely to be population specific and large sample sizes from homogenous
ethnic groups simply may not exist. Zuk et al. [13] suggest a methodology for estimation of $h^2_{adl}$, but the required cohorts (large and from isolated populations) are also difficult to achieve. For disease traits the most tractable approach may be collection of large, informatively phenotyped cohorts to provide the building blocks that may allow clustering of cases based of combinations of genetic risk variants to be mapped onto phenotypic heterogeneity.

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Author Contributions

Wrote the paper: SS EMD RSK WGH NRW.

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