Development of a Multi-Component Model Structure for Autogenous and Semi-Autogenous Mills

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Sustainable Minerals Institute, Julius Kruttschnitt Mineral Research Centre
Abstract

Autogenous (AG) and semi-autogenous (SAG) grinding mills have become popular in the mining industry for their ability to provide a high reduction ratio and deal with high tonnage projects in a compact plant layout. AG mills exclusively use ore as grinding media, while in SAG mills steel balls are added as a supplement. For more than 40 years researchers have been developing mathematical models to assist in the design and optimization of AG/SAG mill operations.

The existing mathematical models to date are incapable of dealing with multi-component feeds. In other words, they consider a homogenous feed, which is an unrealistic scenario for some mining operations. Occasionally, the run of mine feed is constituted of different ore types which have different physical characteristics. The performance of AG/SAG mills is intrinsically related to the feed material as it determines the mill load (grinding media) constitution. Hard ores usually build up in the mill contents and limit throughput. However, these well-known effects have not been investigated in detail or considered in any of the current models.

Therefore, a research programme was conducted to understand, describe and model the interactions between the various components inside an AG/SAG mill in terms of particle breakage, transportation and classification, and to provide a true multi-component model structure that considers size and component distributions (A two-dimensional structure - 2D).

During this research a comprehensive testwork program was conducted to investigate the effects of multi-component feeds on autogenous and semi autogenous grinding performance. Tests were conducted at laboratory, pilot and industrial scale, using artificial mixtures and real ore blends. During the laboratory investigations, a new SAG Locked Cycle Test procedure was developed. Two pilot plant campaigns were carried out using real multi-component ores. Additionally, the entire mill contents of an autogenous industrial mill was measured in terms of size and mineral distribution, during a survey conducted at LKAB operations in Sweden.

The breakage and transport in AG/SAG mills was found to depend on the physical properties and proportion of the different components in the mill feed. Consequently, the overall performance of an AG/SAG mill (throughput, product size and power draw) is dependent on the feed composition and it could be predicted using a multi-component model.
The new multi-component model structure (2D) was developed by modifying Leung’s AG/SAG model (1987) to accommodate simultaneous iterations with different ore types, allowing for the specification of the data on a component by size basis. The new model is based on the independent breakage and transport of components with substantially different physical properties and on the assumption of fully liberated (i.e. independent) components. The model was refined and validated using multi-component data obtained through the extensive experimental campaign. The model is capable of accurately describing the assay-by-size data, as well as the changes in throughput capacity, mill load and product caused by variations in the mill feed composition.
Declaration by author

This thesis is composed of my original work, and contains no material previously published or written by another person except where due reference has been made in the text. I have clearly stated the contribution by others to jointly-authored works that I have included in my thesis.

I have clearly stated the contribution of others to my thesis as a whole, including statistical assistance, survey design, data analysis, significant technical procedures, professional editorial advice, and any other original research work used or reported in my thesis. The content of my thesis is the result of work I have carried out since the commencement of my research higher degree candidature and does not include a substantial part of work that has been submitted to qualify for the award of any other degree or diploma in any university or other tertiary institution. I have clearly stated which parts of my thesis, if any, have been submitted to qualify for another award.

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Bueno, M et al. (2010) – Partially incorporated in Chapter 3

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**Contributions by others to the thesis**

Mr. Michal Andrusiewicz assisted in programming the code of the new multi-component model and implementing it into the MDK simulation platform.

**Statement of parts of the thesis submitted to qualify for the award of another degree**

None.
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Autogenous, grinding, mill, multi-component, ore, blending, mixture, pilot, industrial, AG, SAG, modelling, simulation

Australian and New Zealand Standard Research Classifications (ANZSRC)

091404 Mineral Processing/Beneficiation 100%

Fields of Research (FoR) Classification

0914 Resources Engineering and Extractive Metallurgy 100%
Contributions to Knowledge

I hereby declare that the content of this thesis is original, to the best of my knowledge and belief. The material presented in this work has not been submitted, either in whole or in part, for another degree at this university or anywhere else. The subjects that comprise the original contributions to this field of knowledge are:

1) A locked-cycle test for characterization of autogenous and semi-autogenous grinding of mixtures of ores.
2) Methodology to conduct pilot plant tests with multi-component ore feeds at different blends, obtaining composition by size data in mill feed, load and discharge.
3) Survey and mass balancing of an AG-Pebble mill circuit in terms of particle size and magnetite/silicate composition in every stream and the mill load.
4) Novel pilot and lab scale experimental data which demonstrate the effect of multi-component feed on performance of AG/SAG mills in three separate applications.
5) Improved understanding of breakage interactions and mass transport of independent components inside AG/SAG mills and guidelines for efficient mill operation with multi-component feeds.
6) Effective size by component (2D) AG/SAG mill modelling structure that incorporates multiple ore types, relating their appearance function and distribution in the mill feed to the overall breakage rates, transport and mill power draw.
# Table of Contents

Abstract .......................................................................................................................... i  
Declaration by author ................................................................................................. iii  
Publications during candidature ................................................................................ iv  
Acknowledgements ....................................................................................................... vii  
Contributions to Knowledge ...................................................................................... ix  
Table of Contents .......................................................................................................... x  
List of Figures ................................................................................................................ xv  
List of Tables ................................................................................................................ xxii  
List of Abbreviations ..................................................................................................... xxv  
Dedication ...................................................................................................................... xxvi  

## Chapter 1  Introduction ............................................................................................... 1  
1.1  Objective and Hypotheses ...................................................................................... 2  
1.2  Thesis Structure ..................................................................................................... 2  

## Chapter 2  Literature Review ..................................................................................... 4  
2.1  Introduction ........................................................................................................... 4  
2.2  Modelling of Comminution Processes .................................................................. 5  
2.2.1  Matrix Model .................................................................................................... 5  
2.2.2  Kinetic Model .................................................................................................. 6  
2.2.3  Perfect Mixing Model ...................................................................................... 7  
2.3  AG/SAG Mill Steady-State Models ....................................................................... 9  
2.3.1  Breakage Rates ............................................................................................... 12  
2.3.1.1  Breakage Mechanisms ............................................................................ 13  
2.3.1.2  Modelling of Breakage Rates .................................................................. 15  
2.4  Appearance Function ............................................................................................ 27
A.4 LKAB plant survey data ........................................................................................................ 188
A.5 LKAB pilot plant data........................................................................................................ 188
A.6 LKAB SAG-LCT data........................................................................................................ 188
Appendix B Averaging “Axb” parameters and mill throughput.............................................. 189
List of Figures

Figure 2.1 – First order plot for breakage rate (redrawn after Loveday, 1967) ................................................. 6

Figure 2.2 – Mechanisms of Perfect Mixing Model equation (after Napier-Munn et al, 1996) ............... 8

Figure 2.3 – Schematic diagram of AG/SAG mill process mechanisms (after Napier-Munn et al., 1996) ................................................................................................................................. 12

Figure 2.4 – Back-calculation of breakage rates using different appearance functions (after Leung, 1987) ................................................................................................................................. 13

Figure 2.5 – Main breakage mechanisms in AG/SAG mills and the resultant daughter fragment size distributions (redrawn after Delboni, 1999) ................................................................................................................................. 14

Figure 2.6 – Weight of tagged particle as a function of time (after Manlapig et al., 1980) ............ 14

Figure 2.7 – Typical shape for the specific rate of breakage in SAG mill. (after Austin et al., 1986) .................................................................................................................................................. 16

Figure 2.8 – Log breakage rate vs. log size (after Stanley, 1974) .............................................................. 17

Figure 2.9 – Graphic representation of the average SAG breakage rate distribution ...................... 18

Figure 2.10 – Breakage rates at each spline knot for a soft ore (after Morrell and Morrison, 1989) 19

Figure 2.11 – The influence of operating variables on breakage rates (after Morrell et al., 1994) ... 19

Figure 2.12 – Effect of load volume on the breakage rate distribution (after Morrell et al., 2001) .. 22

Figure 2.13 – Simulated load-throughput response (after Morrell et al., 2001) ......................... 23

Figure 2.14 – Comparison between pilot and full-scale AG/SAG mill breakage rate distributions (after Morrell, 2004) .................................................................................................................... 23

Figure 2.15 – Breakage rate response to ball load, ball size, total load and mill speed (after Morrell, 2004) ........................................................................................................................................... 24

Figure 2.16 – The relationship between $t_n$ vs. breakage parameter $t_{10}$ (after Narayanan 1985) ....... 29

Figure 2.17 – Description of a shell and particle trajectories (redrawn after Delboni, 1999) ........... 36

Figure 2.18 – Particle packing during attrition breakage and the surface between shells with different speeds (re-drawn after Delboni, 1999) ......................................................................................... 37
Figure 2.19 – The old and new breakage model fitted to 42 measured points from drop weight tests on Mt Coot-tha quarry material (after Shi and Kojovic, 2007) ..............................................................39

Figure 2.20 – Illustration of grate classification treatment as an exit classifier (after Austin et al., 1986) ..................................................................................................................41

Figure 2.21 – Classification function proposed by Leung (redrawn after Leung, 1987) ...............43

Figure 2.22 – Predicted slurry hold-up vs. observed in 19 full-scale AG and SAG mills (after Morrell, 2004) .................................................................................................................46

Figure 2.23 – First-order feed-size disappearance plots for the minerals under different grinding conditions (after Venkataraman and Fuerstenau, 1984) ..................................................................52

Figure 2.24 – Cumulative feed-size breakage distribution function for the minerals under different grinding conditions (after Venkataraman and Fuerstenau, 1984) ..............52

Figure 2.25 – Breakage rate function normalized with specific energy for calcite and quartz (after Venkataraman and Fuerstenau, 1984) ........................................................................53

Figure 2.26 – First order feed disappearance plot for dolomite ground alone and in hematite-dolomite mixtures of different compositions (after Fuerstenau et al, 1986) ......................54

Figure 2.27 – First order plots for the disappearance of hematite feed particles ground alone and in hematite-dolomite mixtures of different compositions (after Fuerstenau et al, 1986) .............54

Figure 2.28 – The effect of mixture composition on the amount of fines generated (dolomite and hematite) as a function of grinding time (after Fuerstenau et al, 1986) ...................................................55

Figure 2.29 – First order feed disappearance plots for dolomite and quartz in terms of specific energy for various hematite-dolomite mixtures (after Fuerstenau et al, 1986) .......................55

Figure 2.30 – Composition of the recycle material and circuit product as a function of cycle number in the locked-cycle experiments (after Fuerstenau and Venkataraman, 1988) ..........60

Figure 2.31 – Computed values of the breakage rate functions for the top-size fraction for calcite and quartz in the locked-cycle test (after Fuerstenau and Venkataraman, 1988) .................60

Figure 2.32 – Evolution of the circulating load as a function of the cycle number in the lockedcycles experiments (after Fuerstenau and Venkataraman, 1988) .............................................61
Figure 2.33 – Total circulating load as a function of the number of grinding cycles. Comparison between experimental data and simulated values by Algorithms I and II (after Kapur and Fuerstenau, 1989) ................................................................. 62

Figure 2.34 – Experimental and simulated composition of recycle as a function of the number of grinding cycles using Algorithm II (after Kapur and Fuerstenau, 1989) ................................................................. 62

Figure 2.35 – Experimental and simulated composition of screened product as a function of the number of grinding cycles using Algorithm II (after Kapur and Fuerstenau, 1989) ................. 63

Figure 2.36 – Evolution of total circulating load with grinding cycles for feeds of different composition (after Kapur and Fuerstenau, 1989) ............................................................................. 63

Figure 2.37 – Experimental and simulated recycles for the locked-cycle grinding of pure limestone and quartz feeds (after Kapur et al, 1992) ........................................................................ 66

Figure 2.38 – Experimental and simulated recycles for 1:1 and 3:1 limestone-quartzite mixtures (after Kapur et al, 1992) ........................................................................................................... 66

Figure 2.39 – Experimental and simulated recycles for the locked-cycle grinding of a 1:1 limestone-quartzite mixture with a step change to feed ration of 1:3 (after Kapur et al, 1992) .............. 67

Figure 2.40 – Relationship between slurry hold-up (fs) and flowrate (after Stange, 1996) .............. 70

Figure 2.41 – Classification model regression results (after Stange, 1996) ........................................ 71

Figure 2.42 – Mill load and product size distributions (Meas. Vs Fitted) (after Stange, 1996) ......... 71

Figure 3.1 – Locked-cycled test flowsheet ............................................................................................ 75

Figure 3.2 – Mill dynamics as a function of cycle number for a locked-cycle test using 1:1 QTZ/BIF mixture ......................................................................................................................... 79

Figure 3.3 – Normalized mill throughput as a function of the amount of soft material present in the fresh feed .................................................................................................................... 80

Figure 3.4 – Composite product size distribution according to the percentage of soft material in the mill ........................................................................................................................... 81

Figure 3.5 – BIF Product Size Distribution according to the percentage of soft material in the mill 82

Figure 3.6 – Quartz product size distribution according to the percentage of soft material in the mill ........................................................................................................................................... 82
Figure 3.7 – Comparison of the normalized throughput curve obtained using the laboratory and pilot mill .................................................................................................................................................................. 84

Figure 3.8 – The mill product size distribution for the different feed composition ................................................. 84

Figure 3.9 – Comparison of the SAG-LCT product size distributions for the pilot and laboratory mills .................................................................................................................................................................. 85

Figure 3.10 – Comparison of SAG-LCT component product size distributions in the pilot and laboratory mills (1:1 blend) .................................................................................................................. 86

Figure 3.11 – Simulated vs. experimental SAG-LCT equilibrium conditions ................................................................. 87

Figure 4.1 – Pilot plant circuit flowsheet and the pilot AG mill .......................................................................................... 90

Figure 4.2 – UG2 and Waste particles density distributions .................................................................................. 91

Figure 4.3 – Blending system and expected throughput response for each blend .................................................. 92

Figure 4.4 – Mill operation from empty to steady-state .................................................................................. 93

Figure 4.5 – AG mill load being discharged ................................................................................................................. 95

Figure 4.6 – AG mill feed size distributions and F80s .......................................................................................... 97

Figure 4.7 – The 80% passing size in the AG mill load for Tests 1, 3 and 5 .......................................................... 97

Figure 4.8 – Chromite and silicate distributions by size in the mill feed and load for Tests 1 and 5 98

Figure 4.9 – The 80% passing size in the AG mill trommel oversize (T80) and undersize (P80) for Tests 1, 3 & 5 ........................................................................................................................................ 99

Figure 5.1 – Sorting plant and KA2 milling circuit flowsheets showing the survey sampling points .................................................................................................................................................................. 106

Figure 5.2 – Mill load dumping process .................................................................................................................. 109

Figure 5.3 – Mill charge screening setup and flowsheet .................................................................................. 110

Figure 5.4 – Sampling the screened mill load size fractions .................................................................................. 110

Figure 5.5 – Multi-component distributions by size .......................................................................................... 111

Figure 5.6 – Experimental and mass balanced data .......................................................................................... 112

Figure 5.7 – LKAB pilot plant and AG mill used in the P9O project trials .......................................................... 113

Figure 5.8 – Magnetic separator used to upgrade the LKAB average +30 mm mill feed ore ....... 114
Figure 5.9 – LKAB pilot mill feeding system ................................................................. 115
Figure 5.10 – Emptying and collecting the mill contents .................................................. 116
Figure 5.11 – Experimental conditions obtained during the LKAB pilot plant trials, and key
operating results .......................................................................................................................... 116
Figure 5.12 – SAG-LCT results compared to pilot and industrial survey test outcomes .......... 118
Figure 6.1 – Bulk breakage rates calculated using the bulk feed approach ................................ 122
Figure 6.2 – Simulations of mill throughput using the bulk rates approach .......................... 122
Figure 6.3 – Ore specific breakage rates calculated by the two identical mills approach in relation to
the proportion of soft component ................................................................................................. 123
Figure 6.4 – Simulations of mill throughput using the two identical mills approach ............... 124
Figure 6.5 – Mill load composition in relation to mill fresh feed composition ......................... 124
Figure 6.6 – Ore specific breakage rates calculated using the two variable length mills approach in
relation to the proportion of soft component ................................................................................ 125
Figure 6.7 – Simulations of mill throughput using the two variable length mills approach .......... 125
Figure 6.8 – Multi-component modelling concept on JKSimMet and Exp vs. Fit results for LKAB
pilot Test 1 ................................................................................................................................. 127
Figure 6.9 – Pilot Test 1 (ave. LKAB ore) product size distribution (Exp vs Fit) ...................... 128
Figure 6.10 – Calculated Ore Specific Breakage Rates for Tests 1, 4 (50% +30 Waste) and 5
(cleaner ore) ............................................................................................................................... 129
Figure 6.11 – Flowsheets for ‘soft’ chromite and ‘hard’ silicate component streams showing Exp vs.
Fit data .......................................................................................................................................... 130
Figure 6.12 – Ball load representations for ‘soft’ and ‘hard’ component streams in Test 5 .......... 131
Figure 6.13 – Comparison of breakage rates for ‘soft’ and ‘hard’ component streams in Test 5 ... 132
Figure 6.14 – Quality of model fitting ‘soft’ and ‘hard’ component streams in Test 5 ............. 133
Figure 6.15 – Simulation of ‘soft’ and ‘hard’ component streams in Test 1 using the Test 5 model
parameters and the Test 1 fitted parameters ............................................................................... 134
Figure 6.16 – Breakage rates fitted for chromite and silicate components in Test 1 and Test 5 .... 135
Figure 7.1 – AG/SAG Model Structure (after Leung, 1987) ................................................................. 137
Figure 7.2 – Multi-Component AG/SAG mill model structure ............................................................. 138
Figure 7.3 – Schematic diagram of AG/SAG mill process mechanisms (after Napier-Munn et al., 1996) ............................................................................................................................................... 139
Figure 7.4 – The effect of multi-component ores in the calculation of breakage energy .............. 142
Figure 7.5 – Mill grate classification function (Napier-Munn et al. 2005) ............................................. 143
Figure 7.6 – LKAB pilot mill discharge function (experimental and fitted) ........................................ 144
Figure 7.7 – Multi-component model iteration method with multiple components in feed .......... 145
Figure 7.8 – Typical AG/SAG breakage rate function ........................................................................ 146
Figure 7.9 – Magnetite and silicate breakage rates, fitted to LKAB pilot plant data .................. 147
Figure 7.10 – Simplified AG/SAG mill charge shape (after Morrell, 1992) ....................................... 148
Figure 8.1 – Effect of feed blend on magnetite and silicate breakage rates ........................................ 153
Figure 8.2 – Fitted and measured mill discharge assay-by-size data for Tests 1, 4 and 5 .......... 155
Figure 8.3 – Fitted and measured load composition size-by-size ...................................................... 156
Figure 8.4 – Mill load and product size distributions (exp vs. fit) for Test 1 ............................... 157
Figure 8.5 – Fitted and measured mill discharge assay-by-size data for Case 1 ..................... 158
Figure 8.6 – Fitted and measured mill discharge assay-by-size data for Case 2 ..................... 158
Figure 8.7 – Breakage rates of magnetite and silicate modelled as a function of feed composition. 159
Figure 8.8 – Discharge function parameter $X_m$ modelled as a function of feed composition ........ 159
Figure 8.9 – Simulation outcomes and model response to feed blend (%hard in +30mm feed) ..... 160
Figure 8.10 – Comparison of product and Load size distributions for 2D vs. 1D models .......... 162
Figure 8.11 – KA2 mill load composition (exp vs. sim) ................................................................. 162
Figure 8.12 – Magnetite and Silicate breakage rates calculated using industrial and pilot mill data ............................................................................................................................................... 163
Figure 8.13 – Chromite and silicate breakage rates, fitted to pilot plant data from Tests 1 and 5 .. 164
Figure 8.14 – The effect of feed composition on chromite and silicate breakage rates .......... 164
Figure 8.15 – Measured and fitted assay-by-size data for mill discharge and load for Test 5 ....... 165
Figure 8.16 – Measured and fitted mill discharge and load particle size distributions for Test 5 ... 166
Figure 8.17 – Simulated mill throughput, energy consumption, product size and dilution according to blend for Anglo pilot AG milling .............................................................................................................. 167
Figure 8.18 – Mill load response to feed composition................................................................. 168
Figure B.1 – Inverse weighting vs. simple arithmetic weighting of Axβ parameters.............. 189
List of Tables

Table 2.1 – Summary of Major Advances in Modelling the AG/SAG Mill Operations ......................... 10
Table 2.2 – Average breakage rates for AG and SAG mills (after Leung, 1987) ....................... 18
Table 2.3 – Breakage rate regression coefficients used in JKSimMet Variable Rates AG/SAG Model ........................................................................................................................................ 21
Table 2.4 – Major Contributions to Understanding of the Multi-component Grinding Process ......47
Table 2.5 – Variation of Bond Work Index with blend composition (after Yan and Eaton, 1993) ..58
Table 2.6 – Experimental and computer simulated Bond Work Indices (after Yan and Eaton, 1993) ........................................................................................................................................ 58
Table 2.7 – Bond grindability results using mixtures of ceramic raw material (after Ipek et al, 2005) ........................................................................................................................................ 59
Table 2.8 – Pendulum parameters for binary system (after Stange, 1996) ..................................... 68
Table 2.9 – Summary of Pilot Plant performance. (after Stange, 1996) ......................................... 69
Table 2.10 – Measured and predicted dolerite vs. size in mill load (after Stange, 1996) ............... 70
Table 2.11 – Predicted and measured load compositions (after Stange, 1996) ............................. 72
Table 2.12 – Characteristics of three ore types used in Macpherson blend tests by Mcken and Chiasson (2006) ........................................................................................................................................ 73
Table 2.13 – Grindability test summary on Macpherson test blends (after Macken and Chiasson, 2006) ........................................................................................................................................ 73
Table 3.1 – Comminution characterization data for quartz and BIF ........................................... 78
Table 3.2 – Hard component build up behaviour in circulating load during the mixture tests....... 80
Table 3.3 – Size distribution parameters according to the presence of soft material in the fresh feed ........................................................................................................................................ 81
Table 3.4 – Pilot SAG-LCT mill dimensions ................................................................................. 83
Table 3.5 – Build-up of hard BIF material in the mill load for laboratory and pilot mill tests ....... 84
Table 4.1 – Pilot AG mill specifications ......................................................................................... 90
Table 4.2 – Breakage testing data.......................................................................................................................... 91
Table 4.3 – Multi-component data for Tests 1 and 5................................................................................................. 100
Table 4.4 – Experimental conditions and mill performance.......................................................................................... 101
Table 5.1 – Ore and waste characterization tests results............................................................................................. 106
Table 5.2 – AG mill and trommel parameters ............................................................................................................. 107
Table 5.3 – AG Mill operating conditions during the surveys......................................................................................... 107
Table 5.4 – LKAB pilot mill specifications................................................................................................................ 113
Table 5.5 – Mill feed composition during the pilot trials ............................................................................................. 114
Table 5.6 – Summary of the SAG Locked-Cycle Test standard procedures and operating conditions ......................... 117
Table 6.1 – Comparison of $X_g$, $X_m$ and trommel parameters for ‘soft’ and ‘hard’ component streams
in Test 5................................................................................................................................................................. 132
Table 6.2 – Experimental data of Test 1 vs. the simulated results using Test 5 breakage rates and the
re-fitted breakage rates respectively ..................................................................................................................... 134
Table 7.1 – Comparison between the new multi-component AG/SAG model (2D) vs. Leung (1D)
model..................................................................................................................................................................... 149
Table 8.1 – LKAB magnetite and silicate characterization results................................................................................... 152
Table 8.2 – Mill feed composition during LKAB pilot trials ......................................................................................... 152
Table 8.3 – Experimental and fitted data for Test 1.......................................................................................................... 154
Table 8.4 – Build-up of hard component (Exp vs. Fit)..................................................................................................... 155
Table 8.5 – Model variance comparison: F-test inputs and outcomes............................................................................ 157
Table 8.6 – Linear equations relating the magnetite and silicate breakage rates to the amount of hard
silicate in the +30 mm fresh feed........................................................................................................................... 159
Table 8.7 – Comparison of experimental data and model fitting results for KA2 AG mill using 1D
and 2D models respectively ................................................................................................................................... 161
Table 8.8 – %Hard silicate in KA2 AG mill feed and load (exp vs. sim) ....................................................................... 162
Table 8.9 – Mill power calculations (1D, 2D and measured)........................................................................................ 163
Table 8.10 – Build-up of hard silicate (exp vs. fit)........................................................................................................165
### List of Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A*b</td>
<td>JK drop weight test ore harness index</td>
</tr>
<tr>
<td>AG</td>
<td>Autogenous grinding</td>
</tr>
<tr>
<td>ALS</td>
<td>Australian Laboratory Services</td>
</tr>
<tr>
<td>BIF</td>
<td>Banded Iron formation</td>
</tr>
<tr>
<td>BWI</td>
<td>Bond ball mill work index</td>
</tr>
<tr>
<td>Chr</td>
<td>Chromite</td>
</tr>
<tr>
<td>Cs</td>
<td>Mill critical speed</td>
</tr>
<tr>
<td>deg.</td>
<td>Degree</td>
</tr>
<tr>
<td>Ecs</td>
<td>Specific comminution energy</td>
</tr>
<tr>
<td>Exp</td>
<td>Experimental</td>
</tr>
<tr>
<td>Fit</td>
<td>Fitted</td>
</tr>
<tr>
<td>HPGR</td>
<td>High pressure grinding roll</td>
</tr>
<tr>
<td>JK</td>
<td>Julius Kruttschnitt</td>
</tr>
<tr>
<td>JKDWT</td>
<td>JK Drop Weight Test</td>
</tr>
<tr>
<td>JKMRC</td>
<td>Julius Kruttschnitt Mineral Research Centre</td>
</tr>
<tr>
<td>JKRBT</td>
<td>JK Rotary Breakage Test</td>
</tr>
<tr>
<td>L80</td>
<td>Mill load 80% passing size</td>
</tr>
<tr>
<td>LCT</td>
<td>Locked Cycle Test</td>
</tr>
<tr>
<td>LKAB</td>
<td>Luossavaara-Kiirunavaara Aktiebolag</td>
</tr>
<tr>
<td>MDK</td>
<td>Model Developers Kit</td>
</tr>
<tr>
<td>OS</td>
<td>Oversize</td>
</tr>
<tr>
<td>P80</td>
<td>Product 80 % passing size</td>
</tr>
<tr>
<td>Prod</td>
<td>Product</td>
</tr>
<tr>
<td>QTZ</td>
<td>Quartz</td>
</tr>
<tr>
<td>RBT</td>
<td>Rotary breakage tester</td>
</tr>
<tr>
<td>RoM</td>
<td>Run-of-mine</td>
</tr>
<tr>
<td>RPM</td>
<td>Rotations per minute</td>
</tr>
<tr>
<td>SAG</td>
<td>Semi-autogenous grinding Locked Cycle Test</td>
</tr>
<tr>
<td>SAG-LCT</td>
<td>Semi-autogenous grinding</td>
</tr>
<tr>
<td>SD</td>
<td>Standard deviation</td>
</tr>
<tr>
<td>SG</td>
<td>Specific gravity</td>
</tr>
<tr>
<td>Sil</td>
<td>Silicate</td>
</tr>
<tr>
<td>Sim</td>
<td>Simulated</td>
</tr>
<tr>
<td>Std Dev</td>
<td>Standard deviation</td>
</tr>
<tr>
<td>T80</td>
<td>Trommel 80 % passing size</td>
</tr>
<tr>
<td>ta</td>
<td>Abrasion parameter</td>
</tr>
<tr>
<td>tph</td>
<td>Tonnes per hour</td>
</tr>
<tr>
<td>Trom.</td>
<td>Trommel</td>
</tr>
<tr>
<td>UCT</td>
<td>University of Cape Town</td>
</tr>
<tr>
<td>US</td>
<td>Undersize</td>
</tr>
<tr>
<td>WI</td>
<td>Work index</td>
</tr>
<tr>
<td>XRF</td>
<td>X-ray fluorescence</td>
</tr>
</tbody>
</table>
Dedication

I dedicate this work to the memory of my beloved sister Karina, an angel, who showed me the true meaning of love and life.

May we meet in heaven when my time comes.
Chapter 1  Introduction

In many mining operations the Run-of-Mine (RoM) feed has multiple ore types that may have different crushing and grinding characteristics such as: competence, toughness, abrasion resistance, and grindability; as measured by standard ore characterisation parameters such as JKMRC A*b values and Bond Work Index (Wi). Some multi-component ores can be reduced to a combination of hard and soft components and their varying ratio has a substantial impact on AG/SAG mill operation (Stange 1996). However, the existing AG/SAG mill models use a single ore component to describe the feed and product stream, handling only one set of ore hardness parameters A and b. Therefore, these models assume a uniform feed.

Since the grinding media of autogenous mills are controlled by the feed ore size and composition (Delboni, H. & Morrell 1996), the varying ratio of different components in the fresh mill feed affects the mill charge and consequently the whole breakage process. Delboni (1999), after developing a new AG/SAG mill model that describes the influence of mill charge composition on breakage performance, recommended investigating multi-component ore interactions in order to develop a model structure that could account for preferential accumulation of hard ore types in the mill load. However, this remains a well-known effect that has rarely been quantified or modelled.

It is also important to point out that a component, in comminution modelling approach, is one or a group of ores that hold distinct physical characteristics like hardness/toughness, specific gravity and breakage behaviour.

Usually the ‘hard’ component acts as grinding media to break the ‘soft’ component and remains in the mill the longest. It has been found that using the weighted average of A and b parameters from various components in the existing AG/SAG mill model may generate a significant bias in the simulated mill performance (JKTech 1998; Appendix B).

The mill power prediction is also affected when the feed has components with different specific gravities. For example, the build-up or absence of a high SG component when the AG/SAG mill is treating a multi-component feed can cause the mill power prediction, and hence energy available to the charge, to be significantly in error. Hence there is a clear need to develop a model structure that allows the user to handle a multi-component input and output.

Research is therefore required to understand, describe and model the interactions among the various components inside an AG/SAG mill in terms of particle breakage, transportation and classification. This model should be designed to easily link with a multi-component classification model, and ultimately, link with a multi-component flotation model.
1.1 Objective and Hypotheses

The aim of this thesis is to:

- Investigate the influence of feed composition on the overall AG/SAG mill performance.
- Study the breakage interactions and mass transport of individual ore components inside the mill.
- Develop an effective size by component (2D) AG/SAG mill modelling approach that incorporates multiple ore types, relating their appearance function and distribution in the mill feed to the overall breakage rates, transport and mill power draw.
- Implement the model in the MDK (Model Developers Kit) JKSimMet platform to facilitate model validation.

Hypothesis:

- The varying ratio of significantly different ore components with different physical properties in the fresh mill feed affects the overall AG/SAG mill response in terms of throughput, product size and power draw. Secondly, these effects cannot be accurately modelled using average bulk parameters, but can be with a discrete component by size (2D) model.

1.2 Thesis Structure

This thesis comprises 9 chapters, including the introduction as Chapter 1.

Chapter 2 reviews the published mathematical models of AG/SAG mill operation. The author also exposes the evident problem in simulating the behaviour of mixtures or multi-component ores and describes the major contributions towards understanding this issue, which were mainly focused on ball mill operation.

Chapter 3 introduces the new SAG Locked Cycle Test that was developed to investigate the behaviour of mixtures in a SAG process. The test procedure is described in detail as well as the results of experiments conducted using mixtures of quartz and BIF (banded iron formation) ore. The key findings and trends are discussed.

Chapter 4 describes the pilot plant campaign conducted at the Anglo American facilities in South Africa using the multi-component UG2 ore. Different blends of hard and soft (silicate:chromite) and coarse:fine particles in the AG mill fresh feed were tested. The effects of multi-component feed on the pilot scale AG mill are presented in this chapter, and can be used as a guideline for multi-component feed blending for AG milling.
Chapter 5 presents the comprehensive experimental campaign conducted at LKAB iron ore operations in Sweden, where the control of AG mills fed by ores with harder and softer components can be challenging. This work included a sampling survey at the Kiruna KA2 mill concentrator, where the entire AG mill contents were removed and assayed by size to quantify the build-up of the hard waste material in the mill. This was supplemented by pilot and locked-cycle laboratory tests at a wide range of feed blends in order to facilitate the development of an appropriate multi-component AG mill model. This chapter reports the experimental campaign requirements and methodologies, as well as the major outcomes from the industrial, pilot and laboratory multi-component data analysis.

Chapter 6 describes the modelling and simulation exercises using the JKSimMet software and the multi-component experimental data shown in Chapters 3, 4 and 5. The preliminary modelling findings described in this chapter provided the insights necessary for realistic simulations and development of the multi-component model structure which is presented in Chapter 7.

Chapter 7 introduces the new multi-component model structure and provides detailed explanation on the approaches adopted to model the different mechanisms present in AG/SAG mills. The main model features and assumptions made to describe the behaviour of mixtures of materials with different physical properties in AG/SAG mills are discussed.

Chapter 8 presents results from testing and validation of the new multi-component model, and a comparison against the Leung (1987) model. Simulation exercises were conducted to describe the changes in throughput capacity, mill load and product caused by changes in the mill feed composition. The simulation results were then compared against the experimental data to assess the model accuracy.

Chapter 9 summarizes and discusses the major outcomes of this thesis, providing recommendations for future research work.
Chapter 2 Literature Review

For more than 40 years, many authors have carried out research in modelling of autogenous (AG) and semi-autogenous (SAG) mills. Most developments found in the literature can be classified into two main groups of phenomenological models; 1) the JKMRC approach is based on the Perfect Mixing Model (Whiten 1974), and 2) Austin and Klimpel’s approach using the Kinetic Model (Austin & Klimpel 1964).

Although several authors have investigated the phenomena of grinding ore mixtures in laboratory and pilot scale, there was only one attempt at creating a multi-component model for AG/SAG mills (Blois, Stange & Steynberg 1994; Stange 1996), which is the focus of this thesis. The model used the population balance frame work proposed by Austin et al (1987) and incorporated some features of the JKMRC model, but it was not successful.

Therefore, the present literature reviews the published mathematical models of AG/SAG mill operation, as well as the major contributions towards understanding the evident problem of multi-component grinding, which mainly focused on ball mill operation.

2.1 Introduction

The first researchers trying to model AG/SAG operations used empirical approaches to predict the specific energy requirements. They were motivated by the opportunity of reducing the large amounts of energy consumed in comminution. Therefore, the energy-size relationships or “laws of comminution” (Bond 1952; Hukki 1961; Kick 1885; Rittinger 1867) were largely used to estimate the specific energy based on characteristics of the comminuted material.

Due to the difficulty in operating AG mills at steady state, a group of researchers was motivated to develop models to predict and control their behaviour. These models used statistical correlations (Kelly, FJ 1970; Nagahama, Sirois & Pickett 1969) and moved towards more comprehensive approaches for process simulation.

The first successful mechanistic AG/SAG model was published by Stanley (1974a), which was developed through a number of surveys on both pilot and full scale mills. The main assumption was that AG/SAG mills are mixers where breakage and transport occur. Likewise, when rocks collide inside the mill, they produce broken particles that are distributed into finer size fractions.
Since then, two major groups of phenomenological models for AG/SAG mills have been developed: the perfect mixing models which include the JKMRC AG/SAG model (Leung, 1987) which is very robust and has been successfully used in many design and optimization studies (Morrison et al., 1989; Morrell, 1992b), as well as the kinetic models which were introduced by Austin et al (1977) and rely on data of batch grinding tests done in laboratory and empirical relationships to scale the results to industrial mills.

As this thesis aims to develop a new multi-component model for AG/SAG mills using the perfect mixing model equations, this approach is described in detail with some additional discussions on contributions based on the kinetic model methodology. Moreover, the major findings on grinding of mixtures and attempts to develop multi-component models are also included in this chapter.

2.2 Modelling of Comminution Processes

Epstein (1948) introduced the concept that breakage in a comminution process is determined by the probability of a particle being selected for breakage (selection function or breakage rate) and the size distribution of the broken particles (breakage or appearance function). This basic idea is present in the matrix model, the kinetic model and the perfect mixing model, representing the most widely accepted and used models for describing the comminution processes in tumbling mills.

2.2.1 Matrix Model

In the matrix model (Broadbent & Callcott 1956), the mass balance at any size during batch grinding is described using matrix algebra, as shown in Equation 2.1. In order to model industrial continuous operation, this concept was further developed by accommodating breakage and classification in the same equation as shown in Equation 2.2 (Lynch 1977a).

\[
p = (BS + I - S)f
\]

\[
p = (I - C)(BS + I - S)[I - C(BS + I - S)]^{-1}f
\]

where,

- \( f \) : feed and product flowrate vectors, respectively
- \( p \) : classification function matrix
- \( B \) : breakage function matrix
- \( I \) : identity matrix
- \( S \) : selection function matrix
Although this approach has been employed in mathematical models for industrial ball and rod mill circuits (Lynch, 1977b), it has not been used in the modelling of AG/SAG mill operations mainly because it assumes all components have the same residence time in the mill (Stanley, 1974).

### 2.2.2 Kinetic Model

Loveday (1967) conducted batch grinding experiments using mono-sized samples and observed that breakage rates obey the first order law (i.e. the longer the grinding period the greater the size reduction). Therefore, the disappearance of mass \( m \) from top size interval \( (1) \) with time \( t \) could be expressed in Equation 2.3, where \( S_1 \) is the specific rate of breakage.

\[
\frac{dm_1(t)}{dt} = -S_1 m_1(t)
\]  
\[\text{(2.3)}\]

Since the mass in a batch mill is constant and \( S_1 \) is independent of time, the integral of the Equation 2.3 yields:

\[
m_1(t) = m_1(0) \exp(-S_1 t)
\]  
\[\text{(2.4)}\]

or taking logs:

\[
\log m_1(t) = \log m_1(0) - \frac{S_1 t}{2.303}
\]  
\[\text{(2.5)}\]

Therefore, a plot of \( m_1 \) against grind time \( t \) should be a straight line of slope \(-S_1/2.303\), as show in Figure 2.1.

![Figure 2.1 – First order plot for breakage rate (redrawn after Loveday, 1967)](image_url)
Austin et al (1976) stated that Equation 2.5 could be used to determine the breakage rates ($S_i$) of a narrow size fraction, by measuring the mass decay of mono-sized ball mill charges at increasing grinding periods. The breakage function was also measured with tests conducted with mono-sized samples at shorter grinding times. This procedure was standardized and became the basis of the kinetic model. Using the assumption that the mill charge was fully mixed, the population balance equation utilized by the kinetic model to describe the mass flow to and from a size fraction gives:

$$\frac{dm_i(t)}{dt} = -S_im_i(t) + \sum_{j=1, i>1}^{i} b_{ij}S_jm_j(t) \quad j \geq i \geq 1 \quad (2.6)$$

where

$m_i(t)$: mass fraction in the $i^{th}$ size fraction
$t$: grinding time
$S_j$: breakage rate for the $j^{th}$ size fraction
$B_{ij}$: breakage function

### 2.2.3 Perfect Mixing Model

The perfect mixing model was introduced by Whiten (1974) and relies on the assumption of perfectly mixed mill contents. The model is also based on breakage rates, the distribution of a single size after breakage (appearance function) and the particles transport inside the equipment. It is described by the basic equations that follow:

$$\frac{ds_i}{dt} = f_i - p_i + \sum_{j=1}^{i} a_{ij}r_jS_j - r_is_i \quad (2.7)$$

$$p_i = d_is_i \quad (2.8)$$

where

$f_i$: mass flow of size fraction $i$ in the feed
$p_i$: mass flow of size fraction $i$ in the product
$s_i$: mass of size fraction $i$ in the load
$r_i$: breakage rate of size fraction $i$
$a_{ij}$: appearance function, i.e. the fraction of size j material broken into size i
$d_i$: discharge rate of size fraction $i$

In steady state conditions $ds / dt = 0$, thus:
\[ p_i = d_i s_i = f_i - p_i + \sum_{j=1}^{i} a_{ij} r_j s_j - r_i s_i \]  

If the case where \( j = i \) is taken out of the summation, Equation 2.9 can be re-written as follows:

\[ p_i = f_i + \sum_{j=1}^{i-1} a_{ij} r_j s_j - (r_i s_i - a_{ii} r_i s_i) \]

or

\[ p_i = f_i + \sum_{j=1}^{i-1} a_{ij} r_j s_j - r_i s_i (1 - a_{ii}) \]

The diagram presented in

Figure 2.2 illustrates the steady state condition of the perfect mixing model in terms of a size fraction \( i \).

![Diagram](image)

**Figure 2.2 – Mechanisms of Perfect Mixing Model equation (after Napier-Munn et al, 1996)**

In operating mills, the size distributions and flow rates can be directly measured by means of sampling and mass flow measurement instruments, respectively. On the other hand, the appearance function, breakage rates and discharge rates are obtained through laboratory test or back-calculation.
The discharge rate can be directly calculated from Equation 2.8, using the product and load size distributions, and this is usually done in the case of pilot plant mills, where determining the mill load size distribution is relatively practical. However, for industrial size mills this procedure becomes extremely difficult, expensive and is rarely done. Therefore, in these cases, an independent discharge rate is used to back calculate the load using the mill product sized distribution data. Alternatively, the breakage rates and discharge function can be described as a single term \((r/d)\), combining Equations 2.7 and 2.8, and can be calculated using an appearance function and the measured product and size distributions.

Combining the breakage rate and discharge functions into a single term is the particular advantage of the Whiten perfect mixing model. However, the discharge function is assumed to remain constant as milling conditions change. This is a reasonable assumption over average ball milling regimes, but not appropriate for SAG mills.

### 2.3 AG/SAG Mill Steady-State Models

The mathematical modelling of comminution equipment was initially focused on crushers and rod/ball mills, the so called traditional comminution circuits. When AG/SAG mills started to become more and more popular in the mid 70’s, the research focus moved to this new comminution circuit configuration.

Although the current AG/SAG models have improved during 40 years of research and incorporate a great amount of knowledge gathered during this period, there is no model that can successfully handle multi-component feeds. Stange (1996) made an attempt that will be discussed later in this chapter, but his model did not succeed in reproducing experimental results.

In order to summarize and clarify the work done so far, the most significant AG/SAG mill model developments are listed chronologically in Table 2.1.
Table 2.1 – Summary of Major Advances in Modelling the AG/SAG Mill Operations

<table>
<thead>
<tr>
<th>Date</th>
<th>Author</th>
<th>Development</th>
</tr>
</thead>
<tbody>
<tr>
<td>1972</td>
<td>Wickham</td>
<td>First application of the perfect mixing equation (Whiten 1974), using a simplified matrix form, in which both load and discharge rate are ignored. Two non-ore specific appearance functions were considered, one for impact and the other for abrasion.</td>
</tr>
<tr>
<td>1974</td>
<td>Stanley</td>
<td>Used a full version of perfect mixing equation considering the mill load. Combined Wickham’s appearance functions for intermediate sizes, allowing a gradual transition from the impact to abrasion. Breakage rates were back-calculated and correlated with the mill operating conditions. Discharge and classification functions were modelled using pilot plant data.</td>
</tr>
<tr>
<td>1975</td>
<td>Gault</td>
<td>Developed a simple dynamic model using Wickham's framework.</td>
</tr>
<tr>
<td>1977</td>
<td>Austin et al</td>
<td>Proposed an AG/SAG model form based on the kinetic equation. The batch grinding data were correlated to the performance of pilot plant and industrial mills using the mean residence time of the particles within the load.</td>
</tr>
<tr>
<td>1979</td>
<td>Weymont</td>
<td>Modified the basic form of Austin et al model by using ore-specific appearance functions and breakage rates, determined from laboratory tests. Modelled the classification function as an ideal classifier and the mass transfer mechanisms using an empirical relationship. The model was used to simulate the operation of industrial mills.</td>
</tr>
<tr>
<td>1981</td>
<td>Duckworth</td>
<td>Developed a dynamic model for control of AG/SAG mills using Stanley's model.</td>
</tr>
<tr>
<td>1984</td>
<td>Barahona</td>
<td>Further developed Weymont's model by implementing an empirical function for grate classification, developed from pilot milling data. The mass transfer relationship was also improved by taking into account the water contained in the mill feed.</td>
</tr>
<tr>
<td>1986</td>
<td>Narayanan</td>
<td>Implemented an ore specific appearance function, obtained using Whiten and Narayanan’s method (1985; 1983), in the same perfect mixing model structure used by Wickham. Normalized the breakage rates according to the mill volume as proposed by Whiten (1974).</td>
</tr>
<tr>
<td>Date</td>
<td>Author</td>
<td>Development</td>
</tr>
<tr>
<td>-------</td>
<td>-------------------</td>
<td>--------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>1987</td>
<td>Leung</td>
<td>Used the perfect mixing model in the same form as Barahona. Introduced a combined ore-dependant impact and abrasion appearance function. The impact was determined using the twin-pendulum method and the abrasion using a standard tumbling test. The breakage rates were fitted using mill performance and ore parameters.</td>
</tr>
<tr>
<td>1989</td>
<td>Morrell</td>
<td>Developed a phenomenological mass transfer relationship based on work carried out on industrial mills and implemented the equation into Leung's model.</td>
</tr>
<tr>
<td>1992</td>
<td>Mutambo</td>
<td>Used more than 63 pilot plant data and 2 industrial mills, where the load size distribution had been measured, in order to correlate the fitted breakage rates to the mill operating conditions.</td>
</tr>
<tr>
<td>1996</td>
<td>Morrell and Morrison</td>
<td>Investigated the dependence among breakage rates together with operating variables using Mutambo database and implemented a mass transfer relationship developed by Morrell and Stephenson (1995). This is the JKMRC’s so-called variable rates model.</td>
</tr>
<tr>
<td>1996</td>
<td>Stange</td>
<td>Developed a multi-component model that used Austin et al (1977) kinetic model structure, combined with Leung’s ore specific breakage function.</td>
</tr>
<tr>
<td>1997</td>
<td>Valery</td>
<td>Developed a dynamic model that relates the breakage rates to the mill charge composition and uses a new method to calculate the comminution energy.</td>
</tr>
<tr>
<td>1999</td>
<td>Delboni</td>
<td>Developed a mechanistic model that relates the frequency and energy associated with two different breakage mechanisms (impact and attrition) to the mill charge composition and motion.</td>
</tr>
<tr>
<td>2004</td>
<td>Morrell</td>
<td>Developed a new set of empirical equations that relate breakage rates to operating conditions, using a similar approach as Mutambo (1992) and Morrell and Morrison (1996), and implemented a mass transfer relationship developed by Latchireddi (2002).</td>
</tr>
</tbody>
</table>
The AG/SAG mill process mechanisms, illustrated in Figure 2.3, are described by all these existing models using a selection function or breakage rate curve, an appearance or breakage function and a discharge function. Therefore, the next sub-section will describe the different approaches towards modelling each of these components.

![Figure 2.3 – Schematic diagram of AG/SAG mill process mechanisms (after Napier-Munn et al., 1996)](image)

2.3.1 Breakage Rates

There are quite a few different definitions of what the breakage rate represents and these controversies originated from the definition of a breakage event. The perfect mixing model defines it as the fraction of material broken per unit time, i.e. the frequency of breakage events. However, the breakage rate can also be described as the number of breakage events that occur to each particle per unit of time.

Particles inside the mill experience interactions according to a distribution of energy levels (Mishra & Rajamai 1994; Mishra & Rajamani 1994; Rolf & Simonis 1990), which may cause breakage. Therefore, a breakage event can be defined as any interaction within the mill that causes a measurable size reduction.

Although DEM simulation can provide a good estimate of the frequency and intensity of interactions inside industrial mills (Djordjevic et al. 2006), it is not guaranteed to be accurate (Powell & Morrison 2007). In addition, there is no suitable method to directly measure the breakage rates in industrial mills and therefore these had to be calculated with great dependency on the assumed appearance function. Leung (1987) demonstrated this interdependence by calculating the breakage rates using different appearance functions, as shown in Figure 2.4. This is an indication that a multi-component model should allow for ore-specific breakage rates, as different ore types present in the mill feed may have unique appearance functions.
Figure 2.4 – Back-calculation of breakage rates using different appearance functions (after Leung, 1987)

2.3.1.1 Breakage Mechanisms

Previous investigations on AG/SAG milling (Austin et al. 1977; Digre 1969; Goldman & Barbery 1988; Kelly, EG & Spottiswood 1990; Loveday & Naidoo 1997; Manlapig, Seitz & Spottiswood 1980; Stanley 1974b; Wickham 1972) have considered impact, attrition, chipping and abrasion to be the main breakage mechanisms present in AG/SAG mills.

Impact breakage is normally considered to occur when ore particles are crushed by direct impacts from steel balls or larger ore particles. Attrition is defined as the breakage of particles being nipped between balls and larger ore particles rolling and sliding against each other. Chipping is the wearing of edges and sharp corners, also considered to be the first stage of abrasion, which is the rubbing action of rounded rock particles against each other, balls and mill liner.

Impact and attrition are high energy breakage mechanisms and produce a normal product size distribution, while chipping and abrasion are low energy interactions that result in a bi-modal product size distribution. Figure 2.5 illustrates these breakage mechanisms and their resultant daughter fragment size distributions.
Manlapig et al. (1980) conducted a series of tests in a 1.8m diameter pilot plant batch mill, measuring the mass loss of tagged ore pieces, that allowed these different types of breakage to be identified. This investigation showed that four different stages of size reduction occurred in chronological order, as shown in Figure 2.10.
It can be seen that initially, the rate of mass disappearance is relatively high due to the chipping mechanism that rapidly wears off the rough surface of the fresh feed fragments. Then abrasion takes place at a constant and lower rate, and fine grains are slowly torn away from the surface of rounded particles. Once the particles become smaller, they experience higher specific energy levels and fracture by impact. The last period is characterized by a constant size reduction by attrition, which depends on the probability of a particle being nipped.

Although the mill charge is theoretically well mixed and every particle should receive the same amount of energy, particles experience distinct specific energy levels according to their mass, generating different product size distributions. Therefore, certain breakage mechanisms will predominate in different size fractions. This size dependency has been considered in a number of approaches given to the modelling of the breakage rate distribution, which will be discussed in the next section.

2.3.1.2 Modelling of Breakage Rates

There are two main approaches used in modelling the breakage rates. Austin and his co-workers adopted the kinetic model and determined breakage rates empirically, using data obtained from standard batch grinding tests using mono-sized samples (Austin et al. 1977; Barahona 1984; Weymont 1979). On the other hand, JKMRC researchers (Whiten 1974) used the perfect mixing model equations to back calculate the breakage rates using experimental data (Duckworth 1981; Gault 1975; Leung 1987; Morrell & Morrison 1996; Mutambo 1992; Stanley 1974a). However, both groups agree on the same approximate form of the breakage rate vs size relationship.

The typical form of the breakage rate function derived by Austin et al (1986) is shown in Figure 2.7. Region 1 was determined by the nipping action of the grinding media on smaller particles, i.e. attrition. While abrasion was said to dominate region 2, where particles were too large to be nipped but too strong to be easily broken by the grinding media. In region 3, the particles were considered large enough to break by impact due to their own fall.
Figure 2.7 – Typical shape for the specific rate of breakage in SAG mill. (after Austin et al., 1986)

The breakage rate distribution was described using Equation 2.12, originally derived by Austin et al (1984) for modelling ball mills. The slowing down effect in coarser fractions was expressed by empirical multiplicative factors $Q_i$, ranging from 1 to 0 as particle size increases.

$$S_i = a \left( \frac{x_i}{x_0} \right)^\alpha Q_i$$

$$Q_i = \left[ \frac{1}{1 + (x_i/\mu)^\Lambda} \right]$$

(2.12)

where

- $S_i$ : breakage rate of particles in size fraction i
- $x_0$ : 1 mm default particle size
- $x_i$ : bottom limit of size fraction i
- $\alpha$ : material dependant parameter
- $a$ : parameter dependant on material and mill operating conditions.
- $\mu$ : parameter dependent on ball diameter and mill diameter
- $\Lambda$ : parameter dependant on material properties under specific operating conditions

Stanley (1974a) back calculated the breakage rates using pilot plant test data, in which the entire mill load was sized. The resulting twin-peak distribution is shown in Figure 2.8. The peak associated with smaller sizes was considered to be the abrasion limit, and the second peak at coarser size was called crushing limit. Particles below the abrasion limit would break predominantly by impact, i.e. crushing/attrition. Above the crushing limit, particles would break predominantly by abrasion. In between the peak, a mixture of both breakage mechanisms would occur.
Stanley also formulated equations for describing the position and magnitude of these peaks. Although the breakage mechanisms attributed to the breakage rate distribution by Stanley and Austin are quite similar, their parameterizations were significantly different.

Gault (1975); Wickham (1972) and Narayanan (1985) adopted an alternative method that combines the breakage rate and discharge function into a single term \((r_i/d_i)\), that can be calculated given an appearance function and the measured product size distribution. The approach assumes a constant mill load, avoiding the difficult task of measuring the mill contents, and although it was successful for modelling ball mills, it failed to describe AG/SAG operations (Morrell, 1989).
Leung (1987) developed a more versatile model, introducing equations for discharge function and an ore specific appearance function. The new model structure was used to back-calculate the average breakage rates at five spline knots (128, 44.8, 16, 4.0 and 0.25 mm) for AG and SAG mills respectively. The obtained values, listed in Table 2.2, were considered to be constant, i.e. independent of operating conditions, and cubic spline interpolation was used to obtain a smooth breakage rate distribution at any given size as shown in Figure 2.9.

<table>
<thead>
<tr>
<th>Spline Knots</th>
<th>Ln (Breakage Rate)</th>
<th>Ln (Breakage Rate)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Index</td>
<td>mm</td>
<td>AG</td>
</tr>
<tr>
<td>R1</td>
<td>128</td>
<td>3.37</td>
</tr>
<tr>
<td>R2</td>
<td>44.8</td>
<td>1.98</td>
</tr>
<tr>
<td>R3</td>
<td>16.0</td>
<td>3.32</td>
</tr>
<tr>
<td>R4</td>
<td>4.0</td>
<td>4.04</td>
</tr>
<tr>
<td>R5</td>
<td>0.25</td>
<td>2.63</td>
</tr>
</tbody>
</table>

Figure 2.9 – Graphic representation of the average SAG breakage rate distribution

However, Morrell and Morrison (1989) showed a systematic effect of ball charge on breakage rates, which varied for different ore types and ball size distribution. Figure 2.10 shows this relationship for a soft ore. Therefore, Leung’s assumption of constant breakage rates was not valid when the operating conditions varied significantly.
Morrell et al. (1994) further investigated the variation of breakage rates with operating conditions of AG/SAG mills, demonstrating the influence of variables such as ball charge, ball size, feed coarseness (F80) and mill critical speed (Cs), as shown in Figure 2.11.

Figure 2.10 – Breakage rates at each spline knot for a soft ore (after Morrell and Morrison, 1989)

Figure 2.11 – The influence of operating variables on breakage rates (after Morrell et al., 1994)
Mutambo (1992) compiled an extensive data set that included 52 pilot tests and two industrial surveys, where the entire mill charge was sized (Morrell 1989). The data were used to fit Leung’s model and derive empirical relationships between operating conditions and breakage rates. Multiple linear regression techniques (Kojovic 1988) were then used to calculate the coefficients of simple linear equations with the following general form.

\[ R_n = a + b \text{BL} + c \text{Wi} + d (BL \text{Wi}) + e \text{F}_{80} + f \text{RR} + g \text{Rf} \]  

(2.13)

where

- \( R_n \) : breakage rates at standard knot sizes 1 to 5 (i.e. 0.25, 4.0, 16.0, 44.0 and 128 mm)
- \( \text{BL} \) : ball load (\%)
- \( \text{Wi} \) : work index (kWh/t), related to Leung’s laboratory abrasion parameter, \( t_a \)
- \( \text{F}_{80} \) : size at which 80\% of the total feed mass passes
- \( \text{RR} \) : recycle ratio (\%)
- \( \text{Rf} \) : \( \text{RR} \text{R}_{80}/\text{F}_{80} \)
- \( \text{R}_{80} \) : size at which 80\% of the total recycle mass passes

a, b, c, d, e, f and g are the coefficients determined using least-squares regression.

After carrying out validation using a dataset of 91 industrial surveys to predict the product size and mill power draw, Mutambo concluded that the performance of AG/SAG mill can be simulated with some degree of confidence and it is mainly determined by the ball load, feed coarseness, and ore hardness.

Morrell and Morrison (1996) adopted a more comprehensive approach and investigated the intrinsic interdependence between breakage rates at different sizes, in combination with the effect of selected operating variables. The work resulted in the following set of equations that was later implemented in the so-called JKSimMet “Variable Rates Model”.
\[
\ln(R1) = (k_{11} + k_{12} \ln(R2) - k_{13} \ln(R3) + J_B(k_{14} - k_{15}F_{80}) - D_r)/S_b \tag{2.14}
\]
\[
\ln(R2) = k_{21} + k_{22} \ln(R3) - k_{23} \ln(R4) - k_{24}F_{80} \tag{2.15}
\]
\[
\ln(R3) = S_b + (k_{31} + k_{32} \ln(R4) - k_{33}R_T)/S_b \tag{2.16}
\]
\[
\ln(R4) = S_b(k_{41} + k_{42} \ln(R5) + J_B(k_{43} - k_{44}F_{80})) \tag{2.17}
\]
\[
\ln(R5) = S_a + S_b(k_{51} + k_{52}F_{80} + J_B(k_{53} - k_{54}F_{80}) - 3D_B) \tag{2.18}
\]

where

\( R_n \) : breakage rates (hr\(^{-1}\)) at 5 standard knot sizes
\( k_{ij} \) : coefficients determined using least-squares regression
\( J_B \) : mill volume occupied by grinding balls and associated voids (%)
\( S_a \) : \( \ln\left(\frac{\text{RPM}}{23.6}\right) \), RPM scaling factor
\( S_b \) : \( \ln\left(\frac{\text{Nex}}{0.75}\right) \), fraction of critical speed scaling factor
\( D_B \) : \( \ln\left(\frac{\text{D}}{90}\right) \), ball diameter scaling factor
\( R_r \) : \( \frac{\text{tph recycled material–20+4mm}}{\text{tph fresh feed+ tph recycled material–20+4mm}} \), recycle ratio

The regression coefficients \( (k_{ij}) \), shown in Table 2.3, were calculated using the JKMRC’s current database at the time, consisting of 63 pilot plant data sets.

**Table 2.3 – Breakage rate regression coefficients used in JKSImMet Variable Rates AG/SAG Model**

<table>
<thead>
<tr>
<th>j</th>
<th>( k_{ij} )</th>
<th>( k_{2j} )</th>
<th>( k_{3j} )</th>
<th>( k_{4j} )</th>
<th>( k_{5j} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.504</td>
<td>4.682</td>
<td>3.141</td>
<td>1.057</td>
<td>1.894</td>
</tr>
<tr>
<td>2</td>
<td>0.397</td>
<td>0.468</td>
<td>0.402</td>
<td>0.333</td>
<td>0.014</td>
</tr>
<tr>
<td>3</td>
<td>0.597</td>
<td>0.327</td>
<td>4.632</td>
<td>0.171</td>
<td>0.473</td>
</tr>
<tr>
<td>4</td>
<td>0.192</td>
<td>0.0085</td>
<td>--</td>
<td>0.0014</td>
<td>0.002</td>
</tr>
<tr>
<td>5</td>
<td>0.002</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

The rates were divided into two groups, R4 and R5 which represent the grinding media, and R1, R2 and R3 forming the product size fractions. It is also evident that the finer size rates are functions of the coarser size rates, but they are interrelated in a complex manner. The effects of operating conditions on both breakage rates groups are described in detail elsewhere (Delboni, H. 1999; Morrell & Morrison 1996).
Although the Variable Rates Model equations relate the breakage rate to a number of operating conditions, they are not sensitive to the rock load mass and/or volume. For a given ore and a set of operating conditions, the appearance function \( (a) \) and breakage rate \( (r) \) are constant and therefore, the perfect mixing model structure predicts that throughput \( (\text{tph}) \) is linearly related to the load mass \( (s) \) because \( \text{tph} \propto r.s.(1-a) \).

However, data from both pilot and industrial scale has shown that this is not true in practice. Generally throughput reaches a maximum level in the range of 20-40% mill filling, depending on ore hardness and operating conditions (Morrell et al. 2001).

In order to better describe the load-throughput response, Morrell et al. (2001) derived an adjustment factor for the breakage rate distribution. The corrections were based on hypotheses around energy input and utilization by the load for breakage, and related to the charge motion and shape. The effect of this adjustment factor is illustrated in Figure 2.12, and indicates that high loads reduce the breakage rates at coarser sizes and increase the rates at finer sizes. Simulations using this correction provided more realistic load-throughput responses, as shown in Figure 2.13.

![Figure 2.12 – Effect of load volume on the breakage rate distribution (after Morrell et al., 2001)](image-url)
Figure 2.13 – Simulated load-throughput response (after Morrell et al., 2001)

Although the Variable Rates Model had addressed the effect of scale via the inclusion of an RPM term in the rates equation, which varies with $D^{0.5}$ for a fixed fraction of critical speed, Morrell (2004) confirmed and explained this effect in a more cogent and accurate fashion. The author analysed and compared the results of 11 pilot tests to survey data from the industrial mills that were commissioned to treat the same ores. Figure 2.14 illustrates the effect of scale on the breakage rate distribution.

Figure 2.14 – Comparison between pilot and full-scale AG/SAG mill breakage rate distributions (after Morrell, 2004)
Morrell used this set of 11 pilot tests and matching full scale survey data to derive new breakage rate empirical relationships that were directly related to mill rotational speed and charge composition. The resulting equations, described below, were able to reproduce the effect shown in Figure 2.14 and provide responses to ball load, ball size, total load and mill speed as illustrated in Figure 2.15.

\[
\ln(R_i) = k_{i1} + k_{i2}J_b D_b + k_{i3} \omega + k_{i4} J_t
\]  

(2.19)

where

- \( r_i \): breakage rate values for standard knot sizes 1 – 5
- \( J_b \): ball load (%)
- \( J_t \): total load (balls plus rocks) volume (%)
- \( D_b \): make up ball size
- \( \omega \): mill rotational rate
- \( k_{i1-i4} \): regression coefficients for rates i = 1 – 5

Morrell did not publish the fitted values of the regression coefficients, so to be used the relationship has to be independently fitted to a data set of a similar form.

Figure 2.15 – Breakage rate response to ball load, ball size, total load and mill speed (after Morrell, 2004)
Besides these empirically derived relationships presented so far, there are other mechanistic approaches (Delboni, H. 1999; Valery 1997) that directly relate the breakage rates to the mill charge composition and motion. These will be briefly described because they have potential application in future developments in multi-component modelling.

Valery (1997) modelled the breakage rates as a function of mill speed, load size distribution and composition (rock and balls), using a relationship based on projected surface areas of the grinding media. The mill charge was arbitrarily divided into three groups according to particle size: Coarse grinding media particles (> 50 mm), slurry or fine particles (< 16 mm) and intermediate or transition fraction (particles between 16 and 50 mm). Equations 2.20 and 2.21 described the breakage rate curve for coarse and fine particles respectively, and they were joined using splines to describe the intermediate size interval (-50+16 mm).

\[ R_i = K_{i_i} \cdot N_h \left( \frac{0.5n_i x_i^2 + \sum_{j=1}^{i-1} n_j x_j^2 + \sum_{j=i+1}^{z} nb_j x_j^2}{0.5n_i x_i^2 + \sum_{j=i+1}^{q} n_j x_j^2} \right) \]  \hspace{1cm} (2.20)

and for sizes smaller than 16 mm:

\[ R_i = K_{a_i} \cdot \frac{1}{N_h} \left( \sum_{j=1}^{p} n_j x_j^2 + \sum_{j=1}^{z} nb_j x_j^2 \right) \]  \hspace{1cm} (2.21)

where

- \( R_i \) : breakage rate of size class i (1/time)
- \( K_{i_i} \) : impact breakage rate constant of size class i
- \( K_{a_i} \) : attrition breakage rate constant of size class i
- \( N_h \) : rotational rate of the mill (rev/time)
- \( n_i \) : number of particles in size interval i (i = 1, 2, 3,…, z)
- \( x_i \) : geometric mean of size interval i
- \( nb_i \) : number of balls in size interval i
- \( q \) : size class corresponding 16 mm
- \( p \) : size class corresponding 50 mm

Although these equations directly calculate the breakage rate distribution, the constants \( K_{i_i} \) and \( K_{a_i} \) still have to be fitted using at least one set of steady-state survey data.

Delboni, H. (1999) adopted an approach based on the mechanistic description of interactions between contactors (grinding media) and “contactees” (impacted or nipped particles). The equations developed to describe both impact and attrition breakage frequency considered that the total number of breakage events associated with the contactors was distributed amongst the mill charge particles according to a partition term. The impact/attrition breakage rates equations have the same form and were therefore estimated as follows:
\[ r_{i}^{\text{imp/att}} = \Gamma_{i}^{\text{imp/att}} P_{i}^{\text{imp/att}} K_{i}^{\text{imp/att}} \] (2.22)

where

\[ r_{i}^{\text{imp/att}} \]: impact/attrition breakage frequency of size fraction i 
\[ \Gamma_{i}^{\text{imp/att}} \]: specific impact/attrition generation rate 
\[ P_{i}^{\text{imp/att}} \]: impact/attrition partition term 
\[ K_{i}^{\text{imp/att}} \]: impact/attrition calibration constants

The average number of impact/attrition contacts per particle (\( \Gamma_{i}^{\text{imp}} \)) comprises the ratio between the total number of impact/attrition contacts generated (\( N_{s_{i}}^{\text{imp/att}} \)) and the total number of particles which forms the grinding charge (\( \varepsilon \)). Numerically this relationship was expressed as follows:

\[ \Gamma_{i}^{\text{imp/att}} = \frac{\sum_{i}^{n} N_{s_{i}}^{\text{imp/att}}}{\varepsilon} \] (2.23)

where

\[ N_{s_{i}}^{\text{imp/att}} \]: number of impact/attrition contacts per unit time given 
\[ n \]: number of shells 
\[ \varepsilon \]: total number of grinding charge particles with an equivalent size equal to the characteristic media size (C), given by:

\[ \varepsilon = \left[ \frac{\sum_{i}^{m} \left( s_{i} + s_{i}^{b} \right)}{\frac{\text{CMS}^{3} \pi}{6}} \right] \] (2.24)

where

\[ s_{i} \]: volume of all ore particles contained in size fraction i 
\[ s_{i}^{b} \]: volume of all steel balls contained in size fraction i 
\[ m \]: size fraction which contains media size lower limit value 
\[ \text{CMS} \]: characteristic media size

The values for \( N_{s_{i}}^{\text{imp/att}} \), \( n \), \( m \) and CMS parameters were derived using the “discrete shell” model developed by Morrell (1993). The impact (\( K_{i}^{\text{imp}} \)) and attrition (\( K_{i}^{\text{att}} \)) calibration constants were expected to be independent of operating conditions, ore type and equipment design characteristics. However, the results obtained from backcalculation using pilot and plant survey data showed that this is not true and therefore average values were calculated.
Although empirical and mechanistic models describe the variation of breakage rates with a number of operating conditions, none of them is capable of describing the differential behaviour between breakage rates of hard and soft ores. The effect of ore hardn
ess and blending on breakage rates has not been investigated in detail nor modelled in the published work. In view of this, this thesis aims to contribute towards this problem.

2.4 Appearance Function

The appearance function or breakage distribution function describes the size distribution from a breakage event. Different forms have been used, some are ore-independent and others relate the energy applied to ore fragmentation. These functions provide different descriptions for high energy (impact/attrition) and low energy (abrasion/chipping) breakage.

2.4.1 High Energy

The initial developments (Austin et al. 1977; Duckworth 1981; Gault 1975; Stanley 1974a; Wickham 1972) used ore-independent appearance functions, i.e. fixed size distributions for any ore at any operation condition, which was a great limitation when modelling the behaviour of different ore types. These authors based their impact/attrition appearance function on the form proposed by Broadbent and Callcott (1956):

$$B(x, y) = \frac{1 - e^{-(\frac{x}{y})}}{1 - e^{-1}} \quad (2.25)$$

where

$B(x, y)$ : describes the cumulative percentage passing size ‘x’, resulting from the breakage of a particle of size (y)

Leung (1987) proposed an ore specific appearance function that relates the specific energy applied to a particle and the resulting fragmentation. The relationship was derived using results of twin pendulum breakage tests (Narayanan, S. S. & Whiten 1983, 1988) conducted on a wide range of ore types. The derived equation is described as follows:

$$t_{10} = A(1 - e^{-bEcs}) \quad (2.26)$$

where

$t_{10}$ : cumulative percentage passing one tenth of the original particle size

Ecs : specific comminution energy

$A, b$ : ore dependent parameter, determined from single particle breakage tests
Leung’s AG/SAG model associated a specific energy (Ecs) input for each size fraction using an average energy level that was related to the mill diameter and a characteristic particle size which was considered to be the geometric mean of the charge top 20% by mass, calculated as follows:

\[ S_{20} = \sqrt[3]{P_{100} \cdot P_{98} \cdot P_{96} \cdot \ldots \cdot P_{80}} \]  \hspace{1cm} (2.27)

where

- \( S_{20} \): characteristic particle size
- \( P_n \): size at which n% passes

The potential energy associated to the characteristic particle size defined the mean energy level:

\[ E_1 = \frac{1}{6} \pi (S_{20})^3 \rho g D \]  \hspace{1cm} (2.28)

where

- \( E_1 \): mean energy level
- \( m \): particle mass
- \( g \): gravitational constant
- \( D \): mill diameter
- \( \rho \): solids density

Therefore, the specific energy input for the top size fraction was given by:

\[ Ecs_1 = \frac{6E_1}{\pi x_1^3 \rho} \]  \hspace{1cm} (2.29)

where \( x_1 \) is the mean size of the top size fraction in the mill charge.

By combining Equations 2.28 and 2.29, the specific energy for the top size fraction can be expressed by the following equation:

\[ Ecs_1 = \frac{S_{20}^3 g D}{x_1^3} \]  \hspace{1cm} (2.30)

The specific energy for smaller sizes is then calculated using a relationship originally proposed by Austin et al (1977), as follows:

\[ Ecs_i = \frac{Ecs_1}{(x_i / x_1)^{1.5}} \]  \hspace{1cm} (2.31)
Equation 2.31 is then used to correlate the specific energy levels to the corresponding fragmentation, represented by the $t_{10}$ parameter. The complete size distribution was described using the relationship between $t_{10}$ and other t parameter values ($t_{75}$, $t_{50}$, $t_{25}$, $t_4$, and $t_2$), which is shown in Figure 2.16. Narayanan and Whiten had previously found that this relationship remained constant over a wide range of ore types, and used cubic spline functions to describe the t curves by interpolation. However, Leung described these curves using the following equation:

$$P = 1 - (1 - x) e^{(0.676x - \ln(1 - t) A)}$$

(2.32)

where

$P$ : cumulative fraction passing size $x$

$x$ : normalized size

$t$ : cumulative fraction passing at $x = 0.1$

and

$$G = 0.880 \ln(10x) e^{0.676x} - \ln(-\ln(1 - x))$$

(2.33)

$$A = 1 + 0.0898 \ln(10x) - 0.429(x - 0.1)$$

(2.34)

Figure 2.16 – The relationship between $t_n$ vs. breakage parameter $t_{10}$ (after Narayanan 1985)
The ore dependency was described by the constants A and b that were originally obtained using the twin pendulum device to conduct single particle breakage tests on selected samples. The test consisted of breaking particles of different sizes at a range of specific energies and measuring the resultant fragmentation. Therefore, the test provided a relationship between energy, breakage and particle size.

The twin pendulum device was later replaced by a drop weight tester, JKDWT, (Napier-Munn et al. 2005) that has become an industry standard and allowed a wider range of particle sizes to be tested at higher specific energy levels. More recently, a rotary breakage tester device (JKRBT) was introduced for conducting faster impact breakage characterization tests that generate statistically similar breakage parameters (A and b) to the traditional JKDWT (Kojovic et al. 2008; Shi, Fengnian et al. 2009). Both JKDWT and JKRBT devices have been used for ore characterization during this research and a detailed description of these devices and test procedures can be found elsewhere (Napier-Munn et al. 2005) (Shi, Fengnian et al. 2009).

2.4.2 Low Energy

Wickham (1972) was the first to introduce an abrasion (low energy) appearance function, considering that in a $\sqrt{2}$ screen series a particle would wear down to 35.4%, i.e. $(1/\sqrt{2})^{3}$ of its original mass and appear in the next smaller $\sqrt{2}$ size fraction. The other 64.6%, i.e. abraded mass, would be distributed over the other size fractions. Additionally, the standard size distribution of the abraded material was estimated from a series of tumbling tests using silica pebbles. Stanley (1974), Gault (1975) and Duckworth (1981) also adopted the same concepts to generate their abrasion appearance functions. Stanley, however, arbitrarily assumed a five size interval gap between the abraded cores and the abraded material, which he described using a Rosin-Rammler particle size distribution.

Leung (1987) developed an experimental procedure to directly generate an ore dependent abrasion appearance function. The standard test consisted of tumbling a 3kg of particles in the -55+38mm range for 10 min in a 300 mm by 300 mm laboratory batch mill. Since the product had a bimodal size distribution, an arbitrary single multiplication factor was introduced to correlate the experimental $t_{10}$ values to the other $t_{n}$ values and generate an adjusted entire size distribution. The proposed ore specific abrasion parameter ($t_{a}$) was calculated as 10% of the measured $t_{10}$ value to better describe this bimodal abrasion appearance function. Unlike his impact/attrition (high energy) appearance function, the abrasion was not energy related.
2.4.3 Combined Appearance Function

The appearance functions of different breakage modes have to be combined in a manner that reflects their relative contribution to the AG/SAG milling process. As previously discussed, particles of different sizes will experience different breakage mechanisms. Various authors have therefore established methods to combine the various appearance functions as a function of particle size.

Wickham (1972) established a simplistic sharp delineation between abrasion and impact breakage according to particle size, i.e. the abrasion appearance function was used above a certain size and the impact appearance function below it. Stanley considered abrasion to be the dominant mechanism for coarse particles down to a limit, and impact for finer particles up to a limit. He also suggested that there is a transition zone between the abrasion and impact limits, and therefore combined the different appearance function in a gradual form as follows:

\[ B = \alpha B_1 + (1 - \alpha)B_2 \]  \hspace{1cm} (2.35)

where

- \( B \) : combined appearance function
- \( B_1 \) : abrasion appearance function
- \( B_2 \) : impact appearance function
- \( \alpha \) : factor < 1, calculated from the relative position within the transition zone

Austin and his co-workers (Austin et al. 1977; Barahona 1984; Weymont 1979) considered three different breakage mechanisms to occur (self breakage, ball breakage and pebble breakage) and combined them using the following weighted appearance function:

\[ b_{ij} = \frac{b(s)_{ij}r(s)_j + b(p)_{ij}r(p)_j + b(B)_{ij}r(B)_j}{r(s)_j + r(p)_j + r(B)_j} \]  \hspace{1cm} (2.36)

where

- \( b(s) \) : appearance function for self breakage
- \( b(p) \) : appearance function for pebble breakage
- \( b(B) \) : appearance function for ball breakage
- \( r(s) \) : breakage rate for self breakage
- \( r(p) \) : breakage rate for pebble breakage
- \( r(B) \) : breakage rate for ball breakage
- \( i \) and \( j \) : size interval \( i \) and \( j \) (\( i \) or \( j = 1, 2, 3...n \) and \( j > i \))

The self breakage appearance function, \( b(s) \), was directly measured and the other two components, \( b(p) \) and \( b(B) \), were empirically modelled using results from laboratory tumbling tests and the empirical relationship with the following form.
\[ B_{ij} = \phi_j \left( \frac{x_{i-1}}{x_j} \right)^\gamma + \left(1 - \phi_j\right) + \left( \frac{x_{i-1}}{x_j} \right)^\beta, \quad 0 \leq \phi_j \leq 1 \text{ and } i > j \]  \hspace{1cm} (2.37)

where

- \( B_{ij} \): size discredited cumulative breakage distribution function
- \( x_i \): size interval i
- \( \phi_j, \beta \): experimentally determined parameters

Leung combined his low and high energy appearance functions using a weighted average equation that he applied to the entire particle size range, calculated as follows:

\[ a = \frac{t_{LE} a_{LE} + t_{HE} a_{HE}}{t_{LE} + t_{HE}} \]  \hspace{1cm} (2.38)

where

- \( a \): combined appearance function
- \( a_{LE} \): low energy appearance function
- \( a_{HE} \): high energy appearance function
- \( t_{LE} \): \( t_{10} \) value for low energy breakage
- \( t_{HE} \): \( t_{10} \) value for high energy breakage

The combined appearance form proposed by Leung implies that the low energy (abrasion) breakage dominate at coarse size fractions whilst the high energy (impact) at finer sizes. This function can also shift towards a preponderance of abrasion or impact, depending on the ore characterization results.

### 2.4.4 Mechanistic Breakage Energy Calculation

Although Leung was the first to adopt a mechanistic relationship to calculate a mean energy level that was related to mill diameter and a characteristic particle size, his method was simplistic and arbitrary. The assumption used to scale the specific energy to smaller size fractions had no physical meaning either. Therefore, other authors conducted further research to better describe the available comminution energy and its utilization in the different breakage mechanisms (Delboni, H. 1999; Valery 1997).
Valery (1997) proposed a new method to calculate the available impact comminution energy, its variation with the mill charge composition, and the energy absorbed by rock particles in the mill. The specific comminution energy was calculated using the concept of an effective grinding media with mean size and density, falling from a height - related to the charge shape and motion - and impacting a bed of particles. Additionally, an energy absorption factor that caters for rock and ball collisions was used to ensure that the impact energy is absorbed by the rock particle due to the high strength and elastic nature of steel. This relationship was derived as follows:

\[ E_{cs_i} = \frac{g m_{si} \cdot \rho_{mi} \cdot g \cdot h}{\rho_o \cdot x_i \cdot 3600 \cdot \psi_e} \quad (2.39) \]

where

- \( E_{cs_i} \) : specific comminution energy
- \( g m_{si} \) : effective grinding media size for size fraction \( i \)
- \( \rho_{mi} \) : density of grinding media for size fraction \( i \)
- \( g \) : gravitational acceleration
- \( h \) : mean drop height, determined from charge geometry as described by Morrell (1994)
- \( \rho_o \) : ore specific gravity
- \( \rho \) : ore specific gravity
- \( i \) : \( \sqrt{2} \) size fraction \( i \) (\( i = 1, 2, 3, ... z \)), where \( i = 1 \) is the top size fraction
- \( x_i \) : geometric mean of \( \sqrt{2} \) size fraction \( i \)
- \( \psi_e \) : energy absorption factor related to the presence of steel grinding media, given by:

\[ \psi_e = \frac{\left(\sum_{i=1}^{\sqrt{2}} v_{bi}\right) \rho_b + \left(\sum_{i=1}^{q} v_{oi}\right) \rho_o}{\left(\sum_{i=1}^{q} v_{oi}\right) \rho_o} \quad (2.40) \]

where

- \( v_{oi} \) : volume of ore in size class \( i \)
- \( v_{bi} \) : volume of grinding balls in size class \( i \)
- \( \rho_b \) : specific gravity of steel balls
- \( \rho_o \) : ore specific gravity
- \( q \) : finest size fraction of rock media (assumed to be 16 mm)

This model assumes that ore particles in size fraction ‘i’ can only be broken by ore particles greater than \( x_i \) and steel grinding balls, which constitute the effective grinding media size (\( g m_{si} \)). Additionally, half of the rocks within a coarse size fraction (\( x_i \geq 50 \) mm) were considered to be larger than the remaining half and therefore could also cause breakage within that size fraction. Therefore, the effective grinding media size for size fraction \( i \) was calculated as follows:
\[
\text{gms}_i = \left( \frac{\sum n_i x_i^2 + \sum_{j=1}^{i-1} n_j x_j^2 + \sum_{j=1}^{z} n_{b_j} x_{b_j}^2}{2n_i + \sum_{j=1}^{i-1} n_j + \sum_{j=1}^{z} n_{b_j}} \right)^{0.5} \quad \text{for } x_i \geq 50 \text{ mm} \\
\text{gms}_i = \text{gms}_{i-1} \quad \text{for } x_i < 50 \text{ mm}
\]

where,

\( n_{i or j} \)  \quad \text{number of particles in √2 size fraction i or j (i or j = 1, 2, 3, ... z)}

\( n_{b_j} \)  \quad \text{number of balls in √2 size fraction j (j = 1, 2, 3, ... z)}

Since the grinding media can be constituted by a mixture of rocks and ball, its density was calculated as a weighted average, given by:

\[
\rho_{m_i} = \left( \frac{\sum_{i=1}^{i-1} \rho_{o_i} v_{o_i} + \sum_{j=1}^{z} \rho_{b_j} v_{b_j}}{\sum_{i=1}^{i-1} v_{o_i} + \sum_{j=1}^{z} v_{b_j}} \right)
\]

where

\( v_{o_i} \) : volume of ore in size class i

\( v_{b_i} \) : volume of grinding balls in size class i

\( \rho_{b} \) : specific gravity of steel balls

Although Valery’s model has some arbitrary assumptions around what constitutes grinding media, it was capable of providing more realistic predictions when compared against Leung’s model. (Valery 1997).

Delboni (1999) introduced a mechanics model that decoupled impact from attrition and described the motion of charge in terms of the breakage frequency and energy associated to each size reduction mechanism. The discrete shell power model developed by Morrell (1994) was used to calculate the bulk power associated with each of these mechanisms as well as the basis to establish relationships that associate the mill charge motion with specific comminution energy for all size fractions of the mill charge. The main derived equations are described below and more detailed information on this methodology can be found elsewhere (Delboni, H. 1999; Delboni, H. & Morrell 1996; Delboni, H & Morrell 2002).

The average specific energy per impact was derived as follows:

\[
E_{cs_i}^{\text{imp}} = K_i \left( \frac{\sum n_i \rho_{s_i}^{\text{imp}}}{\sum n_i N_{s_i}^{\text{imp}} * F} \right) * \left( \frac{x_i}{\text{CMS}} \right)^2 * 1000
\]

where

\( E_{cs_i}^{\text{imp}} \) : specific energy per impact associated with size fraction i
\( x_i \): mean size of particle in size fraction \( i \) (impactee)

\( m_i \): mass of impactee particle in size fraction \( i \)

\( CMS \): characteristic media size (impactor)

\( n \): number of shells

\( P_{s_{\text{imp}}} \): impact power associated with shell ‘\( s \)’ (kW), calculated using D-Model (Morrell, 1994)

\( N_{s_{\text{imp}}} \): number of impact contacts per unit time given by shell ‘\( s \)’ (1/h), calculated as:

\[
N_{s_{\text{imp}}} = \frac{\omega_s r_s \cdot L}{CMS \cdot CMS} \cdot 3600
\]  

(2.44)

where

\( \omega_s \): angular velocity of shell ‘\( s \)’

\( r_s \): average radius of shell ‘\( s \)’

\( L \): mill internal length

Since it was assumed that a particle in size fraction ‘\( i \)’ can only be broken by steel balls or another particle larger than or equal to \( x_i \), the input energy which causes breakage should vary with size. Therefore, this was described by the weighting function – \( K_i \), as follows

\[
K_i = \frac{A_{o_{i+}} (CMS_{o_i})^3 \rho_o + A_b (CMS_b)^3 \rho_b}{A_t (CMS_c)^3 \rho_c}
\]  

(2.45)

where

\( CMS_{o_i} \): characteristic media size of particles equal to or coarser than size \( i \)

\( CMS_b \): characteristic media size of the ball charge

\( CMS_c \): characteristic media size of the grinding charge

\( A_{o_{i+}} \): surface area of particles equal to or larger than \( i \)

\( A_b \): total surface area of ball charge

\( A_t \): total area of the grinding media

\( \rho_o \): ore density

\( \rho_b \): steel balls density

\( \rho_c \): grinding media charge density

The model also catered for the effect of slurry pooling that decreases the impact speed of falling grinding media due to the buoyancy forces. The reduction of total impact energy due to a slurry pool was therefore incorporated in the factor \( F \), calculated as follows:

\[
F = \frac{\rho_s \sum_{i=1}^{n} h_{ti} - \rho_p \sum_{i=1}^{n} h_{pi}}{\rho_s \sum_{i=1}^{n} h_{ti}}
\]  

(2.46)

where

\( \rho_s \): density of the solid charge

\( \rho_p \): slurry density

\( h_{pi} \): slurry pool height corresponding to shell ‘\( i \)’

\( h_{ti} \): total height corresponding to shell ‘\( i \)’ (= free fall plus slurry pool height)
Figure 2.17 illustrates some of the mill charge motion concepts adopted by Delboni to calculate his impact comminution energy

Although Delboni’s method caters for a range of operating conditions and has detailed physical descriptions, it relies on a number of assumptions and had to be calibrated. Using a Hopkinson Bar to measure the specific energy at which the first fracture begin to propagate (Bourgeois, King & Herbst 1992; Briggs & Bearman 1996), Delboni carried out a simple experiment using samples from a mill charge critical size, which was believed to survive impact breakage energy levels in that mill. He compared the simulated specific energy against his measurement of minimum level for breakage to occur and introduced a calibration factor $k = 0.33$, what means that only 33% of the calculated specific energy should result in actual breakage.

Delboni also derived a separate method to directly obtain the energy associated with the attrition breakage mechanism. The average specific energy per attrition event was therefore calculated as follows:

$$E_{Cs_i^{att}} = \frac{\sum_{s=1}^{n} P_{s^{imp}}}{\sum_{s=1}^{n} N_{s^{imp}} m_i CMS} \times \left(\frac{x_i}{CMS}\right)^2$$  \hspace{1cm} (2.47)

where

- $E_{Cs_i^{att}}$: specific energy per attrition event associated with size fraction $i$
- $x_i$: mean size of particle in size fraction $i$ (attritees)
- $m_i$: mass of attritees particle in size fraction $i$
- $CMS$: characteristic media size (attritor)
- $n$: number of shells
- $P_{s^{imp}}$: power absorbed by shell ‘s’ due to shear motion, calculated using D-Model (Morrell 1994)
- $N_{s^{imp}}$: number of attrition contacts per unit time at shell ‘s’ $(1/h)$, calculated as:
\[ N_{s}^{imp} = \frac{(\omega_s - \omega_{s-1})r_s L}{\text{CMS} \text{CMS} \text{CMS}} \Delta_s \times 3600 \]  

where

- \( \omega_s \): angular velocity of shell ‘s’
- \( r_s \): average radius of shell ‘s’
- \( \Delta_s \): length of shell ‘s’
- \( L \): mill internal length

Figure 2.18 illustrates the geometry relationship between attritors (CMS) and attritees assumed to occur during attrition breakage, as well as the surface formed between shells that have different rotational speeds and create the conditions for attrition breakage.

His model was calibrated using data from a single ore operation (Alcoa – Pinjarra Hills), but once calibrated it was able to provide more accurate and realistic simulations when compared against Leung’s model across a few other datasets. Although his model provides a more mechanistic description than others, it uses a single ore breakage function parameters and therefore was not able to describe multi-component ores.

### 2.4.5 Modelling the Effect of Particle Size

A typical AG/SAG mill feed comprises a primary crusher discharge and may contain particles from 300 mm down to minus 1 mm. This difference has a significant influence on modelling, as coarser particles tend to be easier to break than smaller ones (King & Bourgeois 1993).
However, the effect of particle size is ignored in the breakage model described in Equation 2.26, which was introduced by Leung (1987) and adopted by various authors that developed AG/SAG mill models (Delboni, H. 1999; Morrell 2004; Morrell & Morrison 1996; Mutambo 1992; Stange 1996; Valery 1997). Equation 2.26 is used to fit data from impact breakage tests with one set of ‘average’ A and b parameters, which is used in the AG/SAG modelling assuming that particles of different sizes would break equally when subjected to the same impact energy.

Banini (2002) conducted a series of drop weight tests on various ore types, using a wide range of particle sizes and specific energy levels. The results confirmed the effect of particle size that was incorporated in a new breakage model using the following expression:

$$t_{10} = 100 \left(1 - \left(1 \div \left(1 + \left(\frac{\ln(E_{sv} + 1)}{\alpha_{av} d^{-n}}\right)^{\beta_m}\right)\right)\right)$$

(2.49)

where

- $E_{sv}$: volumetric specific input energy (kWh/m$^3$)
- $d$: particle size (mm)
- $\alpha_{av}$, $\beta_m$ and n: model parameters fitted to the drop weight test data.

More recently, Shi, F. and Kojovic (2007) modified the breakage probability model developed by Vogel and Peukert (2002) to describe the breakage index $t_{10}$ in relation to material property, particle size and net cumulative impact energy, and derived the following equation:

$$t_{10} = M\{1 - \exp[-f_{mat} \cdot x \cdot k(E_{cs} - E_{min})]\}$$

(2.50)

where

- $M$: maximum $t_{10}$ value
- $f_{mat}$: material breakage property
- $k$: number of impacts
- $E_{cs}$: Mass-specific impact energy
- $E_{min}$: Mass-specific threshold energy

The model structure is very similar to Equation 2.26 since $M$, $(f_{mat} \cdot x)$ and $k \cdot (E_{cs} - E_{min})$ replaces A, b and $E_{cs}$, respectively. This parameter convertibility, which is not present in Banini’s model, is convenient because the product $A \times b$ (= 3600$\cdot$M$\cdot$f_{mat}$\cdot$x, when $E_{min}$ is set to zero) has become an industry standard indicator of ore hardness.
According to Shi and Kojovic (2007) Equation 2.26 predicted the $t_{10}$ values slightly better than Equation 2.50 when each size fraction was treated separately. Hence, each size fraction has its own set of A and b values. However, the new model was capable of handling the effect of particle size on $t_{10}$ and provided a better fit when all sizes were treated together, as illustrated in Figure 2.19. One of the reasons for why the fit in the Shi and Kojovic graph is so much better is because their model has an extra fitted parameter $f_{\text{mat}}$.

![Figure 2.19 – The old and new breakage model fitted to 42 measured points from drop weight tests on Mt Coot-tha quarry material (after Shi and Kojovic, 2007)](image)

2.5 Discharge Function

The discharge function is the specific rate at which the particle load flows out of the mill per unit of time. In the perfect mixing model, the mill discharge flow rate is calculated as the product between the discharge function and the load contents for each size fraction, as follows:

$$p_i = d_i s_i$$  \hspace{1cm} (2.51)

where

- $p_i$ : mass flow rate of product in size fraction $i$
- $s_i$ : load mass of size fraction $i$
- $d_i$ : discharge rate of size fraction $i$

The grate discharge system present in AG/SAG mills acts like any other classifier in that the rates at which material pass through vary with particle size. The discharge rate, $d_i$, is zero for above grate size material and increases to a maximum as particle size decreases. Water and smaller particles flow through and out of the mill at the maximum discharge rate. Therefore, the discharge function is described by two mechanisms:
- Mass transport through the mill
- Classification by the grate screen

Stanley described the discharge function \( (d_i) \) establishing a maximum value of mass transport through the mill \( (d_{\text{max}}) \) and used a grate classification function \( (c_i) \) to modulate it, using the following relationship:

\[
d_i = d_{\text{max}} \times c_i
\]  

(2.52)

The grate classification function could be modelled using a simple partition curve that represents the proportion of the mill load which reports to the product. For this purpose, Stanley (1974) suggested the use of the reduced efficiency curve, which was described by Lynch (1977b) as follows:

\[
E = \frac{e^{\alpha} - 1}{e^{\alpha x_i} + e^\alpha - 2}
\]  

(2.53)

where

- \( x_i \): particle size
- \( d_{50c} \): particle size that has equal probability to report to any of the products
- \( E \): reduced efficiency
- \( \alpha \): parameter that describes the degree of curvature

Despite suggesting the use of the reduced efficiency curve, Stanley used the following relationships with reference to grate size:

\[
c_i = 1 \quad \text{for } x_i \leq x_{\text{dmax}}
\]  

(2.54)

\[
c_i = \frac{(\ln x_i - \ln x_g)^2 (2 \ln x_i - 3 \ln x_{\text{dmax}} + \ln x_g)}{(\ln x_g - \ln x_{\text{dmax}})^3} \quad \text{for } x_i > x_{\text{dmax}}
\]  

(2.55)

where

- \( c_i \): classification function at size fraction \( i \)
- \( x_i \): particle size
- \( x_g \): grate aperture size
- \( x_{\text{dmax}} \): maximum particle size below which no classification occurs

The mass transport mechanism was described using an equation to relate the \( d_{\text{max}} \) value with mill operating conditions and the charge size distribution, as follows:

\[
d_{\text{max}} + k_1 (d_{\text{ps}} - 70) = k_3 \frac{wt_1}{wt_2} - k_2
\]  

(2.56)
where

\[ wt_1 \] : weight of the +37.9 mm fraction in the mill load
\[ wt_2 \] : weight of the +4.74 mm fraction in the mill load
\[ dps \] : discharge percent solids
\[ kn \] : fitted parameters

Austin et al (1986) modelled the grate classification as completely separate classifier, as shown in Figure 2.20. Particles larger than the great aperture would not discharge while any material smaller than the grate would undergo classification.

![Figure 2.20 – Illustration of grate classification treatment as an exit classifier (after Austin et al., 1986)](image)

The mass balance around the classifier was given as:

\[ F_p i = F(1 + C') w_i(1 - c_i) \]  \hspace{1cm} (2.57)

where

\[ F \] : mill feed rate
\[ C' \] : total recycle ratio
\[ w_i \] : mass of size i in the load
\[ p_i \] : weight fraction of size i in the product
\[ c_i \] : fraction of size i material returned to the mill load

Austin and his co-workers used data from pilot-scale tests which had measurements of the mill load and product size distribution to calculate the discharge rates. The results were analysed and the following partition curve equation was derived to model the mill grate classification function (Austin et al. 1986):
\[ c_i = \frac{1}{1 + \left(\frac{X_{50}}{x_i}\right)^{\lambda_g}} \]  

(2.58)

where

\( x_i \) : upper size of size interval \( i \)

\( X_{50} \) : particle size that has equal probability to report to any of the products

\( \lambda_g \) : parameter that describes the degree of curvature

In order to describe the hold-up of size fractions smaller than the grate opening, Austin et al (1977), Weymont (1979) and Barahona (1984) used limited data to derive an empirical mass transfer relationship, given by:

\[ L = m_1 F^{m_2} \]  

(2.59)

where

\( L \) : Fraction of mill internal volume occupied by below grate size solids and water

\( F \) : mill volumetric feed rate divided by the internal volume

\( m_1, m_2 \) : constants, originally estimated as 0.33 and 0.63 respectively (Austin et al, 1977)

Austin et al (1986) later developed a more sophisticated relationship that was described using the following empirical power function.

\[ \frac{f_s}{f_{so}} = \left(\frac{F_v}{F_{vo}}\right)^{N_m} \]  

(2.60)

where

\[ f_s = \frac{W(\sum_{i=g}^{ig} w_k)}{\rho_s VC_c} \]  

(2.61)

and

\[ F_{vo} = k_m \phi cA_g D^{3.5} \frac{L}{D} \]  

(2.62)

where

\( f_s \) : fraction of mill volume occupied by slurry

\( F_v \) : volumetric flow rate

\( f_{so} \) : fraction filling giving flow rate \( F_{vo} \)

\( W \) : solids hold up mass

\( V \) : mill volume

\( \rho_s \) : solids density

\( w_k \) : fraction of \( W \) in size class \( k \)

\( i_g \) : size interval for grate size
Leung (1987) adopted the empirical mass transfer relationship described in Equation 2.59 to model the mass transport to the mill, but using a slightly larger database he estimated $m_1 = m_2 = 0.37$. Additionally, the grate classification was modelled using a very simple relationship which is an approximation of a typical classification S curve, given by:

$$C_i = \begin{cases} 
1 & \text{for } x_i \leq x_m \\
\frac{\ln(x_i) - \ln(x_g)}{\ln(x_m) - \ln(x_g)} & \text{for } x_i \text{ between } x_m \text{ and } x_g \\
0 & \text{for } x_i \geq x_g 
\end{cases}$$

(2.63)

where

- $c_i$ : classification function at size fraction i
- $x_i$ : particle size
- $x_g$ : grate aperture size
- $x_m$ : maximum particle size below which no classification occurs

This equation assumes that particles above the grate size ($x_g$) would be retained within the mill load, particles smaller than a certain size ($x_m$) would flow freely through the grate and intermediate sizes would have a varying degree of classification. The general form of this classification function is shown in Figure 2.21.

![Classification function proposed by Leung (redrawn after Leung, 1987)](image)

Leung’s model combined the mass transfer relationship and the classification functions to determine the quantity of pulp discharge though the grates using the following equation:
\[ D = d_{\text{max}} \times C \]  \hspace{1cm} (2.64)

Where

- \( D \): quantity of pulp discharge through the grates
- \( d_{\text{max}} \): maximum discharge rate
- \( C \): classification function

The value of \( d_{\text{max}} \) was calculated iteratively until the predicted pulp load satisfied the empirical mass transfer relation given in Equation 2.59.

The first mechanistic model of mass transport through a mill with grate discharge was proposed by Moys (1986). The model considered the grinding media as a packed bed and the grate to be a series of orifices and was derived using classical fluid flow equations, as follows:

\[
V = \frac{E \pi R^{0.5} Q \left( \frac{1 + 2.5 \beta L K^{1.25}}{Q^{0.25}} \right)^{1.6}}{8 \beta K^2} - 1
\]  \hspace{1cm} (2.65)

where

- \( K = C_d \pi^2 (1.5)^{1.2} N_g \delta^2 R^{2.5} \)
- \( R \): mill internal radius
- \( L \): mill internal length
- \( \delta \): grate orifice diameter
- \( N_g \): orifices/m² evenly distributed
- \( \beta = 1.462 / R^{0.5} \rho K_Q \)
- \( K_Q = \alpha \left( \frac{D_b E}{\mu (1 - E)^2} \right) \)
- \( D_b \): mean grinding media diameter
- \( \rho, \mu \): slurry specific gravity and viscosity
- \( V \): volume of the mill charge
- \( C_d \): orifice discharge coefficient
- \( Q \): volumetric mill discharge rate

Although Moys assumed that the charge forms a flat surface sloping from the feed end down towards the discharge and this is unlikely to be the case in AG/SAO mills, the model established the most important mill design and operational parameters to determine the slurry flow through a mill (Morrell, 1989).

Morrell and Morrison (1989) carried out investigations using the data available from Stanley (1974a), Austin et al (1976), Weymont (1979), Barahona (1984) and Leung (1987) in addition to his data from Pinjarra SAG mills and pointed out that the empirical mass transfer relationship adopted by Leung is mill specific. He therefore related the discharge function to mill operating conditions and charge characteristics, deriving an improved empirical mass transfer relationship of the form:
\[ Q = K \phi_c^a E^b \lambda^c L_p^d A^e D^f \]  

(2.66)

where

- \( Q \): volumetric discharge rate
- \( \phi_c \): the fractional mill critical speed
- \( E \): the porosity factor (\%) (% of charge > 4 times the grate aperture)
- \( \lambda \): \( 1 - \) recycle fraction
- \( L_p \): the fraction mill pulp filling
- \( A \): the open area of the grate (m\(^2\))
- \( D \): the mill diameter (m)
- \( a, b, c, d, e, f \) and \( K \) are constants

Morrell concluded that the above equation better described the discharge rates than Leung’s equation over a wide range of mill diameters and pulp filling levels. Accordingly, the new relationship significantly improved the quality of the estimations, both in terms of the observed changes in mill charge mass and product fineness.

Morrell and Stephenson (1996) further investigated the slurry discharge capacity of AG/SAG mills associated with the effect of grate design, mill speed and charge volume. In their approach two equations were developed, one to describe the flow through the grinding media and the other to flow via the slurry pool, in cases where this was present. The equations are shown below.

\[ J_{pm} = k_m Q_m^{0.5} \gamma^{1.25} A^{-0.5} \phi^{0.67} D^{-0.25} \]  

(2.67)

\[ J_{pt} = k_t Q_t \gamma A^{-1} D^{-0.5} \]  

(2.68)

where

- \( J_{pm} \): net fraction slurry hold-up in the grinding media interstices
- \( Q_m \): flow rate through the grinding media zone
- \( \gamma \): mean relative radial position of the grate apertures
- \( A \): total area of the apertures (open area)
- \( \phi \): fraction of critical speed
- \( D \): mill diameter
- \( J_{pt} \): net fraction slurry hold-up in the pool zone
- \( Q_t \): flow rate through the pool zone
- \( k_m, k_t \): constants

The resultant mass transfer equations were tested over an extensive data base comprising both pilot and industrial mills, and proved to be able to predict the data consistently either for open or closed circuits. Moreover, Morrell and Stephenson equations were adopted in a number of AG/SAG models, including the JKSimMet Variable Rates Model (Morrell & Morrison 1996) and the mechanistic models developed by Valery (1997) and Delboni (1999).
However, the range of grate designs they used for their experimental program was quite limited and did not take into account the effect of pulp lifters. Latchireddi (2002) addressed this deficiency by conducting a detailed laboratory and pilot study of the influence of grate design and pulp lifters. He therefore improved the Morrell and Stephenson equations and incorporated the influence of pulp lifter depth (k). The result was the following general equation:

\[
J_s = \eta^n_1 A^n_2 J^n_3 \phi^n_4 Q^n_5 D^n_6
\]  

(2.69)

where

- \( J_s \): net fractional hold-up inside the mill
- \( A \): fractional open area
- \( J_t \): fractional grinding media volume
- \( \phi \): fraction of critical speed
- \( Q \): slurry discharge
- \( \gamma \): mean relative radial position of the grate holes
- \( \eta \): coefficient of resistance which varied depending on weather flow was via the grinding media or the slurry pool
- \( n_1 - n_6 \): model parameters found to be functions of pulp lifter size and modeled as

\[
n_i = n_g - k_i e^{(-k_j A)}
\]  

(2.70)

where

- \( n_g \): parameter values for grate-only condition
- \( k_i, k_j \): constants
- \( k \): depth of the pulp lifter expressed as a fraction of mill diameter

Morrell (2004) developed a new AG/SAG model that incorporated the Latchireddi mass transfer equations, which he fitted to 19 full-scale mill data sets, obtaining accurate predictions of the slurry hold-up as shown in Figure 2.22.

![Figure 2.22 – Predicted slurry hold-up vs. observed in 19 full-scale AG and SAG mills (after Morrell, 2004)](image-url)
The various empirical and mechanistic models discussed here describe the mass transport and discharge in AG/SAG mills according to a number of operating conditions. However, none of them is capable of describing the differential behaviour of ore components with different physical properties (e.g. specific gravity and shape). The effect of multi-component ores on mill discharge rates has not been investigated in detail nor modelled in the published work and therefore, this thesis also aims to address this issue.

2.6 Multi-Component Grinding

The phenomenon of preferential breakage is a reality in the comminution field, since heterogeneous ores are present in many mines in the world. Although the behaviour of multi-component feeds in grinding systems has been the focus of research for more than 40 years, this issue is far from being successfully resolved.

The main focus of this thesis is to understand the behaviour of mixtures when ground in AG/SAG mills, leading to the development of a mathematical model of this process. Very little work related to multiple components in AG/SAG mills was found in the literature; however there is some good and useful information relevant to ball mills. As a quick reference of the research conducted, the most relevant works are listed chronologically in Table 2.4. The outcomes of some of these investigations will be further discussed in this section.

<table>
<thead>
<tr>
<th>Year</th>
<th>Author</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>1957</td>
<td>Holmes, JA &amp; Patching, SWF</td>
<td>Preliminary investigation of differential grinding – grinding of quartz-limestone mixtures</td>
</tr>
<tr>
<td>1962</td>
<td>Fuerstenau, DW &amp; Sullivan, DA</td>
<td>Comminution of mixtures in ball mills</td>
</tr>
<tr>
<td>1962</td>
<td>Fuerstenau, DW &amp; Sullivan, DA</td>
<td>Analysis of comminution of mixtures</td>
</tr>
<tr>
<td>1963</td>
<td>Somasundaran, P &amp; Fuerstenau, DW</td>
<td>Preferential energy consumption in tumbling mills</td>
</tr>
<tr>
<td>1964</td>
<td>Kinasevich, RS &amp; Fuerstenau, DW</td>
<td>Research on mechanism of comminution in tumbling mills</td>
</tr>
<tr>
<td>1976</td>
<td>Tanaka, T &amp; Selby, DW</td>
<td>Kinetic approach to interference effects in the grinding of binary mixtures</td>
</tr>
<tr>
<td>1978</td>
<td>Cross, M &amp; Owst, AP</td>
<td>Computer simulation of the comminution of mixtures of materials</td>
</tr>
<tr>
<td>1984</td>
<td>Fuerstenau, DW et al.</td>
<td>Simulation of closed-circuit mill dynamics by locked-cycle grinding of mixtures</td>
</tr>
<tr>
<td>Year</td>
<td>Author</td>
<td>Title</td>
</tr>
<tr>
<td>------</td>
<td>--------</td>
<td>-------</td>
</tr>
<tr>
<td>1984</td>
<td>Venkataraman, KS &amp; Fuerstenau, DW</td>
<td>Application of the population balance model to the grinding of mixtures of minerals</td>
</tr>
<tr>
<td>1986</td>
<td>Fuerstenau, DW et al.</td>
<td>Grinding of mixtures of minerals: kinetics and energy distributions</td>
</tr>
<tr>
<td>1987</td>
<td>Choi, WZ et al.</td>
<td>Size reduction/liberation model of grinding including multiple classes of composite particles</td>
</tr>
<tr>
<td>1988</td>
<td>Fuerstenau, DW &amp; Venkataraman, KS</td>
<td>The comminution of multi-component feeds under batch and locked-cycle conditions - kinetics, simulation and energy-distribution</td>
</tr>
<tr>
<td>1988</td>
<td>Kapur, PC &amp; Fuerstenau, DW</td>
<td>Energy split in multi-component grinding</td>
</tr>
<tr>
<td>1988</td>
<td>Weller, KR et al.</td>
<td>Multi-component models of grinding and classification for scale-up from continuous small or pilot scale circuits</td>
</tr>
<tr>
<td>1989</td>
<td>Kapur, PC &amp; Fuerstenau, DW</td>
<td>Simulation of locked-cycle grinding tests using multi-component feeds</td>
</tr>
<tr>
<td>1992</td>
<td>Fuerstenau, DW et al.</td>
<td>Energy split and kinetics of ball mill grinding of mixture feeds in heterogeneous environment</td>
</tr>
<tr>
<td>1992</td>
<td>Kapur, PC et al.</td>
<td>Simulation of locked-cycle grinding of multi-component feeds and its implications for stability and control of industrial comminution circuits</td>
</tr>
<tr>
<td>1993</td>
<td>Yan, D &amp; Eaton, R.</td>
<td>Breakage properties of ore blends</td>
</tr>
<tr>
<td>1994</td>
<td>Blois, MDS et al.</td>
<td>The evaluation, by steady-state simulation, of alternative grinding circuits</td>
</tr>
<tr>
<td>1995</td>
<td>Cho, HC &amp; Luckie, PT</td>
<td>Investigation of the breakage properties of components in mixtures ground in a batch ball-and-race mill</td>
</tr>
<tr>
<td>1996</td>
<td>Stange, W</td>
<td>The modelling of binary ore behaviour in FAG/SAG milling</td>
</tr>
<tr>
<td>2005</td>
<td>Ipek, H et al.</td>
<td>Ternary-mixture grinding of ceramic raw materials</td>
</tr>
<tr>
<td>2005</td>
<td>Ipek, H et al.</td>
<td>The Bond work index of mixtures of ceramic raw materials</td>
</tr>
<tr>
<td>2005</td>
<td>Ipek, H et al.</td>
<td>Dry grinding kinetics of binary mixtures of ceramic raw materials by Bond milling</td>
</tr>
<tr>
<td>2008</td>
<td>Coello, AL et al.</td>
<td>Grindability of lateritic nickel ores in Cuba</td>
</tr>
</tbody>
</table>
2.6.1 Early Multi-Component Grinding Investigations

The first reported investigation on the grinding of mixtures was conducted by Holmes and Patching (1957). Grindability tests were performed using crushed minus 3.35 mm pure components as well as mixtures of quartz and limestone in a range of blends. The Maxon, Cadena and Bond method was used to analyse the grindability of the mixtures. The proportion of each component in each size fraction was determined by chemical analysis for carbon dioxide.

The size reduction was assumed to be first order with respect to the volume of each component in the mill. The specific rate of reduction, \( k \), was calculated for each component. The grinding performance was compared in terms of the ratios \( k_L/k_Q \), \( k_L'/k_Q' \), \( k_L/k_L' \) and \( k_Q/k_Q' \), where:

\[ k_L, k_Q, k_L', k_Q' \]: Specific rate of reduction for pure limestone, pure quartz, the limestone component and the quartz component of the mixture respectively.

The specific rate of reduction of both components was found to be lower when ground in a mixture. No significant change in the product size distributions of the components was observed when their proportions in the feed changed. They also concluded that the rate of breakage was not affected by the blend ratios. The effects of changes in the feed mixture composition were found to be buffered by changes in the circulating load composition.

Fuerstenau and Sullivan (1962a; 1962b) also investigated the comminution of quartz and limestone mixtures. Grinding tests were carried out on mono-sized samples (4.75 x 2.36 mm) of pure materials and various blends, using ball and rod mills. In order to determine the fraction of energy consumed by each component in the mixture, the authors used the single-comminution-event hypothesis proposed by Schuhmann (1960), which is a derived from of the Charles energy-size reduction equation (Charles 1957). This relationship uses two parameters to describe the breakage, the size modulus and the distribution modulus, the first relates to the applied energy and is derived from product size distribution; the latter is a kind of appearance function and indicates the hardness of the material.

The results showed that the distribution modulus is a property of the material and is the same whether it is ground separately or as part of a mixture; and, the size modulus of a material in a mixture is determined by the fraction of the total energy which is utilized in grinding each component. In addition, the energy consumption by a given mineral in a ball mill has been shown to be determined by its volume fraction in the mill; and for rod mills, energy is consumed preferentially by the material with lower grindability.
Somasundaran and Fuerstenau (1963) also investigated the extent to which grinding energy is consumed by one component more than another. The tests were carried out using samples of 1:1 quartz/limestone mixtures of 4.75 x 2.36 mm, and mixtures of 4.75 x 2.36 mm quartz with 300 x 150 μm limestone, as well as the opposite. The grinding time was used as a measurement of the energy usefully employed for comminution, and the Charles-Schuhmann approach was applied.

Batch tests were performed in ball and rod mills and the product was analysed to determine the proportion of each component in the various size fractions. This information was used in the calculation of the energy consumed by each component. For the equal size mixtures the energy consumed by each component was almost the same. However, when grinding the different size fractions, the coarser component consumed a greater proportion of the energy. It was more noticeable in the rod mill, where the harder component (quartz) consumed 2.5 times as much energy as limestone during the longer batch tests. This effect was considered to result from the wedge action of the rods inside the mill.

Tanaka and Selby (1976), contested the previous approach and introduced the kinetic approach (Austin & Klimpel 1964), in the analysis of grinding of binary mixtures. The study was carried out using the data obtained from Fuerstenau and Sullivan (1962b) for batch rod milling of quartz/limestone mixtures. The authors used a derived form of the general equation for first-order batch grinding, i.e. a Rosin-Rammler distribution equation (Austin, Klimpel & Luckie 1972), and used a factor called degree of interaction in the grinding of mixtures (ξ and η), defined as ξ = k_q(x,t)/k_l and η = k_l(x,t)/k_q. Therefore:

\[ 1 \geq \xi \geq \frac{k_q}{k_l} \]  
\[ 2.71 \]
and

\[ 1 \geq \eta \geq \frac{k_l}{k_q} \]  
\[ 2.72 \]

where k is the rate constant; l and q refer to limestone and quartz respectively. When ξ = η = 1, means that the mixture is ground as if it were a single material, and when ξ = k_q/k_l = 1/η, means that component behaves as a separate material without interaction.

The interactions between components were considered by comparing the experimental size distributions of mill product with those distributions predicted by the Rosin-Rammler equations for each component weighted according to its volume fraction in the mixture. The results showed that the interactions were complex and sensitive to particle size; however, the proposed method enabled the size distribution of the components to be calculated from the overall size distribution and the characteristics of the components for single mineral grinding.
Venkataraman and Fuerstenau (1984), also reported the application of the population balance grinding kinetics model on grinding of mixtures. The investigation was carried out for dry ball mill grinding of calcite, hematite and quartz, as single components and as binary mixtures. To determine the amount of net energy consumed by each component, a modified form of Charles-Schuhmann energy-size reduction equation was used, as follows:

$$E = m_1 c_1 X_{m_1}^{-\alpha_1} + m_2 c_2 X_{m_2}^{-\alpha_2}$$  \hspace{1cm} (2.73)

where subscripts 1 and 2 refer to the components in the binary mixture, $E$ is the energy consumed per unit weight of mixture (kWh/t), $m$ is the mass fraction of the component mineral, $\alpha$ is the distribution modulus of component, $X_m$ is the size modulus of the mineral when ground in a binary mixture. This information was used in order to normalize the breakage rates in terms of specific energy (energy consumed per mass of mineral).

Experiments were performed grinding mono-sized samples (1.7 x 1.18 mm) in a 25.4 x 29.2 cm ball mill (D x L), running at 60% of its critical speed, using 30 kg of 25.4 mm dia. steel balls, and loaded such that 100% of its voids was filled with the mono-sized sample. The specific gravity of calcite, hematite and quartz is 2.73, 5.2 and 2.66, respectively; and the Mohr's hardness is 3, 6 and 7, respectively. Calcite-Quartz mixtures give a combination of minerals with comparable densities, but with different hardness, and the Hematite-Quartz give the opposite comparison. The first mixture had the mill content and size distribution composition analysis by using HCl to leach the calcite; and the last by using a specific gravity bottle.

The reported results have confirmed the validity that the first-order disappearance kinetics for the minerals when ground alone or as components of binary mixture. It has been shown that the absolute value of the breakage rate function of a mineral in dry ball mill grinding is time-independent but environment-dependent, i.e. the $k_1$ depends on whether the mineral is ground alone or in a mixture (Figure 2.23). However, the breakage distribution function of the components remains the same when ground in a mixture or alone (Figure 2.24). In addition, the first-order breakage rates can be normalized in terms of specific energy for the minerals, which is calculated using the modified Charles energy-size reduction relation; the energy and size independent breakage rate ($k^E_i$), shown in Figure 2.25, simplifies the computer simulation. Finally, the method enables the prediction of the energy consumed by each component using the product size distribution and breakage parameters.
Figure 2.23 – First-order feed-size disappearance plots for the minerals under different grinding conditions
(after Venkataraman and Fuerstenau, 1984)

Figure 2.24 – Cumulative feed-size breakage distribution function for the minerals under different grinding conditions
(after Venkataraman and Fuerstenau, 1984)
The results indicate that the breakage rate of dolomite increases as its proportion in the feed decreases, and for any blend, it increases with time, becoming nonlinear with time. On the other hand, the breakage rate of hematite remained first order, i.e. linear with time, and independent of mixture composition. This is presented on Figure 2.26 and Figure 2.27. The same trend was found in the production of fines (-37 μm), and shown in Figure 2.28. The calculated values of specific energy, using the modified Charles equation, were slightly higher than those measured. In addition, the breakage rates normalized in terms of specific energies remained the same for a component when ground alone or in a mixture, and the nonlinearity disappeared, as shown in Figure 2.29.
Figure 2.26 – First order feed disappearance plot for dolomite ground alone and in hematite-dolomite mixtures of different compositions (after Fuerstenau et al, 1986)

Figure 2.27 – First order plots for the disappearance of hematite feed particles ground alone and in hematite-dolomite mixtures of different compositions (after Fuerstenau et al, 1986)
Figure 2.28 – The effect of mixture composition on the amount of fines generated (dolomite and hematite) as a function of grinding time (after Fuerstenau et al, 1986)

Figure 2.29 – First order feed disappearance plots for dolomite and quartz in terms of specific energy for various hematite-dolomite mixtures (after Fuerstenau et al, 1986)
Kapur and Fuerstenau (1992; 1988), where motivated to integrate the energetic and kinetic aspects of multi-component feed grinding into a unified framework and developed a new method to quantify the energy split. They used the concept of grinding paths, which is tracked by dividing the ground particle size distribution data into suitably chosen coarse, medium and fine fractions. The constancy of these grinding paths allowed them to exploit the breakage rate normalized in terms of power.

The definition of splitting factors, \( S_1 = \frac{E_{1m}}{E_{1a}} \) and \( S_2 = \frac{E_{2m}}{E_{2a}} \), where \( E_{1m} \) and \( E_{2m} \) are the energies consumed in grinding components 1 and 2 in a mixture, as well as \( E_{1a} \) and \( E_{2a} \) are the energies consumed when the components are ground alone; allowed Equation 2.73 to be rearranged into the following form:

\[
E_m = m_1 S_1 E_{1a} + m_2 S_2 E_{2a}
\]  

(2.74)

\( E_m \), \( E_{1a} \) and \( E_{2a} \) can be directly measured and, therefore, the problem reduces to find the energy split factor \( S \). Taking the rate of breakage of the top-size and considering the breakage rate function time-independent, the energy consumption can be defined as \( E = \frac{k_{1,t}}{k_{1}^{E}} \), which is valid when the material is ground alone, \( E = \frac{k_{1,t}}{k_{1}^{E}} \) or in a mixture, \( E = \frac{k_{1,t}}{k_{1}^{E}} \). Once the reduced breakage rate function \( (K_1^E) \) remains the same, the split factors can be re-written as \( S_1^k = \frac{K_{1(1m)}}{K_{1(1a)}} \) and \( S_2^k = \frac{K_{1(2m)}}{K_{1(2a)}} \), which allows the estimation of \( S \) in terms of breakage rates. Finally, the energy required to grind the mixture can be written as:

\[
E_m = m_1 \frac{k_{1(1m)}}{k_{1(1a)}} E_{1a} + m_2 \frac{k_{1(2m)}}{k_{1(2a)}} E_{2a}
\]  

(2.75)

For the cases where the breakage rate is not time-independent, the energy split factor had to be changed to be a function of time. As \( E(t) = W(t) / K_1^{E} \), where \( W(t) = \ln \frac{M_{1(t)}}{M_{1(0)}} \) and \( M_1 \) is the weighted mass fraction on the top-size. The splitting factor is obtained by dividing \( E_{1m} \) by \( E_{1a} \) (t), then \( S_1^{W}(t) = \frac{W_{1m}(t)}{W_{1a}(t)} \) for component 1.

Moreover, a simpler way of calculating the splitting factor was proposed based on the zero-order production of fines when a single-size feed is ground (Herbst & Fuerstenau 1968). In the initial grinding times, the rate of production of fines passing in the \( i \)th size screen, \( Y_i \), is given, for component 1 when ground alone and in a mixture respectively, by:
\[ Y_{1(1a)} = k_{1(1a)}B_{11(1m)} \]  

and 

\[ Y_{1(1a)} = k_{1(1a)}B_{11(1m)} \]  

where \( B_{11} \) is the cumulative feed size breakage function. Therefore, the ratio of the two rates becomes the energy split factor, \( S_{1}^{Y}(t) = \frac{Y_{1(1m)}}{Y_{1(1a)}} \).

The main implication of the energy split factor is that it permits simulation of the grinding kinetics of individual components in a mixture feed. This methodology was applied using data from the author’s previous tests of grinding mixtures. In most of cases, it was possible to predict the product size distribution, with good agreement to the experimental data. This methodology can also estimate the energy consumed in grinding the individual components, which cannot be measured directly, and the degree of disproportionate split of energy.

More recently the kinetic approach has been successfully applied in investigations on grinding mixtures of coal (Cho & Luckie 1995), as well as on mixture of ceramics raw materials (Ipek, Ucbas & Hosten 2005b). Theses authors report results that agree with previous studies described above.

### 2.6.2 Bond Grindability Tests Using Multi-component Feeds

Yan and Eaton (1993) performed Bond tests using a mixture of ores and also carried out an investigation of the breakage properties using the kinetic approach, as described before. The tests were performed using two gold ores from the Meekatharra district in Western Australia, which are Mickey Doolan (hard) and Vivians (soft).

Five full Bond tests were carried out, one on each of the pure ores and three blends, the results are presented in Table 2.5. They tried to predict the Bond Work Index from the results of simulations of batch grinding, using the breakage rate and breakage function. The results are shown in Table 2.6.
### Table 2.5 – Variation of Bond Work Index with blend composition
*(after Yan and Eaton, 1993)*

<table>
<thead>
<tr>
<th>Vivians vol./vol.</th>
<th>Vivians wt./wt.</th>
<th>Test sieve size (µm)</th>
<th>F80 (µm)</th>
<th>P80 (µm)</th>
<th>Grams per revolution</th>
<th>Bond Work Index (kWh/t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>1.00</td>
<td>106</td>
<td>2100</td>
<td>73</td>
<td>3.31</td>
<td>6.60</td>
</tr>
<tr>
<td>0.75</td>
<td>0.69</td>
<td>106</td>
<td>2440</td>
<td>82</td>
<td>1.90</td>
<td>11.00</td>
</tr>
<tr>
<td>0.50</td>
<td>0.42</td>
<td>106</td>
<td>2621</td>
<td>80</td>
<td>1.60</td>
<td>12.30</td>
</tr>
<tr>
<td>0.25</td>
<td>0.20</td>
<td>106</td>
<td>2669</td>
<td>82</td>
<td>1.46</td>
<td>13.50</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>106</td>
<td>2762</td>
<td>85</td>
<td>1.42</td>
<td>14.00</td>
</tr>
</tbody>
</table>

### Table 2.6 – Experimental and computer simulated Bond Work Indices
*(after Yan and Eaton, 1993)*

<table>
<thead>
<tr>
<th>Material</th>
<th>Bond Work Index (kWh/t)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experimental</td>
</tr>
<tr>
<td>Mickey Doolan</td>
<td>14.0</td>
</tr>
<tr>
<td>50:50 Blend</td>
<td>12.0</td>
</tr>
<tr>
<td>Vivians</td>
<td>6.6</td>
</tr>
</tbody>
</table>

The authors concluded that there is an interaction between the components of the blend which affects their individual breakage rates, also that the harder material seems to have greater influence on the overall breakage properties and Bond Work Index (BWi). In addition, they believe that it should be possible to predict the BWi of a blend of any composition, given the breakage parameters of the pure components and how these indices change with the ore blend.

Ipek and his co-workers (2005a) also investigated the behaviour of mixtures on the Bond Test Work Index test. Bond grindability tests were performed using pure and mixtures of three different ceramic raw material, quartz, Kaolin and potassium-feldspar (K-feldspar). The results, presented in Table 2.7, revealed that when the softer material (kaolin) is in the mixture, the work indices become higher than for the hard component, which contradicts the previous findings (Yan & Eaton 1993). The authors believe that it happens because the presence of relatively softer kaolin particles protects harder quartz and feldspar from being ground, but no physical interpretation of this phenomena was given.
Table 2.7 – Bond grindability results using mixtures of ceramic raw material (after Ipek et al, 2005)

<table>
<thead>
<tr>
<th>Sample</th>
<th>F (µm)</th>
<th>P (µm)</th>
<th>Gbp g/rev</th>
<th>Exp Wi (kWh/t)</th>
<th>Exp average Wi (kWh/t)</th>
<th>Calc (kWh/t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quartz (Q)</td>
<td>2659.5</td>
<td>124.2, 122.8</td>
<td>2.273, 2.173</td>
<td>11.21, 11.55</td>
<td>11.38</td>
<td>–</td>
</tr>
<tr>
<td>Kaolin (K)</td>
<td>2599.3</td>
<td>114.8, 113.3</td>
<td>2.247, 2.374</td>
<td>10.80, 10.24</td>
<td>10.52</td>
<td>–</td>
</tr>
<tr>
<td>Feldspar (F)</td>
<td>2655.3</td>
<td>120.3, 120.8</td>
<td>2.231, 2.246</td>
<td>11.15, 11.12</td>
<td>11.14</td>
<td>–</td>
</tr>
<tr>
<td>1:01 Q-K</td>
<td>2630.4</td>
<td>117.4, 116.2</td>
<td>1.942, 1.902</td>
<td>12.32, 12.45</td>
<td>12.39</td>
<td>10.95</td>
</tr>
<tr>
<td>1:01 Q-F</td>
<td>2657.5</td>
<td>122.5, 121.8</td>
<td>2.262, 2.239</td>
<td>11.16, 11.21</td>
<td>11.19</td>
<td>11.26</td>
</tr>
<tr>
<td>1:01 K-F</td>
<td>2626.3</td>
<td>109.4, 117.8</td>
<td>1.807, 1.884</td>
<td>12.50, 12.66</td>
<td>12.58</td>
<td>10.83</td>
</tr>
<tr>
<td>1:01:01 Q-K-F</td>
<td>2635.9</td>
<td>118.8, 120.4</td>
<td>1.991, 1.976</td>
<td>12.16, 12.34</td>
<td>12.25</td>
<td>11.01</td>
</tr>
<tr>
<td>1:02:01 Q-K-F</td>
<td>2628.4</td>
<td>117.9, 120.3</td>
<td>1.965, 1.959</td>
<td>12.24, 12.43</td>
<td>12.34</td>
<td>10.89</td>
</tr>
</tbody>
</table>

2.6.3 Locked-Cycle Tests

Fuerstenau and his co-workers (1988; 1984), motivated to study the comminution behaviour of multi-component feeds in closed circuit continuous grinding circuits, carried out locked-cycled ball mill grinding experiments using quartz (hard) and calcite (soft) mixtures.

The tests were performed using a two-minute cycle time and a 254 mm dia. x 292 mm long mill, loaded with 30 kg of 25.4 mm dia. steel balls and running at 54 rpm. The mill was fed with a quartz/calcite mixture at constant proportion of 1:1 by volume. Both materials were in the minus 1.18 mm range, with natural size distribution produced by a roll crusher. After each cycle, the product was sieved on a 210 µm screen for 10 min and the oversize material returned to the mill together with fresh feed to make up the initial charge (1150 cm³). The proportion of calcite and quartz in the recycle material and product was determined by leaching the samples with hydrochloric acid, dissolving the calcite. The cycles were repeated until the steady state, i.e. when the amounts of quartz/calcite in the product and recycle stream were constant.

The results have shown that the amount of hard component in the recycle and product slowly builds up, reaching steady state after 25 cycles, as show in Figure 2.30. In addition, it confirmed that the $k_i$ (first order breakage rates) values are strongly composition dependant, which agrees with their previous findings. Figure 2.31 shows the estimated values of $k_i$, i.e. for the top size. The circulating load was found to be a function of cycle number and unexpectedly kept oscillating during the test, which is presented in Figure 2.32.
Figure 2.30 – Composition of the recycle material and circuit product as a function of cycle number in the locked-cycle experiments (after Fuerstenau and Venkataraman, 1988)

Figure 2.31 – Computed values of the breakage rate functions for the top-size fraction for calcite and quartz in the locked-cycle test (after Fuerstenau and Venkataraman, 1988)
Kapur and Fuerstenau (1989) used these results to develop models for simulation of locked-cycle grinding tests using multi-component feeds. The balance equation of grinding kinetics (Herbst & Fuerstenau 1980) was applied in the development of two algorithms. Algorithm I can be written as:

\[ R_c(n) = r_c F_c(n) \phi_c + R_c(n-1) \phi'_c \]  

(2.78)

and

\[ R_q(n) = r_q F_q(n) \phi_q + R_q(n-1) \phi'_q \]  

(2.79)

where the subscripts c and q stand for calcite and quartz, R is the amount of material in the mill product retained on the closing size, r is the weighted fraction of the feed already retained on the closing screen, n is the cycle number and \( \phi / \phi' \) are the mill transfer functions for make-up and recycle feeds, respectively. The transfer functions are calculated using experimental data from the first three cycles.

From data on batch grinding of binary mixture feeds (Venkataraman & Fuerstenau 1984), correction factors for the effect of the interaction of hard and soft particles were estimated by a trial-and-error approach using the first five cycles data. The interaction factors \( I_c \) and \( I_q \) were incorporated into the transfer functions as to modify them in step with the changing composition of the mill contents in each cycle, resulting in the algorithm II, described as:

Figure 2.32 – Evolution of the circulating load as a function of the cycle number in the locked-cycles experiments (after Fuerstenau and Venkataraman, 1988)
\[ R_c(n) = r_c F_c(n) \phi_c[l_c(n)] + R_c(n-1) \phi'_c[l_c(n)] \]  
(2.80)

and

\[ R_q(n) = r_q F_q(n) \phi_q[l_q(n)] + R_q(n-1) \phi'_q[l_q(n)] \]  
(2.81)

Both algorithms were applied to predict the circulating load, as well as mill load and product composition against the experimental results, which can be seen in Figure 2.33, Figure 2.34 and Figure 2.35, respectively. In addition, simulation runs were performed assuming binary calcite-quartz feeds in weight ratios of 1:0, 3:1, 2:1, 1:1, 1:2, 1:3; the calculated total circulating load as a function of grinding cycle and blend composition is presented in Figure 2.36.

![Graph](image1)

**Figure 2.33** – Total circulating load as a function of the number of grinding cycles. Comparison between experimental data and simulated values by Algorithms I and II (after Kapur and Fuerstenau, 1989)

![Graph](image2)

**Figure 2.34** – Experimental and simulated composition of recycle as a function of the number of grinding cycles using Algorithm II (after Kapur and Fuerstenau, 1989)
Figure 2.35 – Experimental and simulated composition of screened product as a function of the number of grinding cycles using Algorithm II (after Kapur and Fuerstenau, 1989).

Figure 2.36 – Evolution of total circulating load with grinding cycles for feeds of different composition (after Kapur and Fuerstenau, 1989).
The developed algorithm interfaces directly with the locked-cycle tests, which simplifies its execution, and provides quite realistic results when compared with the experimental data. The relationships do not allow the simulation of size distribution, a much more sophisticated approach and data would be necessary; however it is not crucial when studying mill dynamics. The problem of non-availability of a viable theory or accurate empirical correlation for describing the interactions between hard and soft components have been overcome by applying an empirical interaction factor whose parameters were fitted by a trial-and-error approach. Both algorithms provided similar trends, which allowed the understanding of general features of the grinding circuit dynamics.

Kapur and his co-workers (1992) developed a more general and versatile simulation model for multi-component feeds that incorporated an energy split factor for mixture grinding, which had been published (Kapur & Fuerstenau 1988) and can be employed for variable grinding times. Locked-cycle grinding data for quartz-limestone (calcite) mixtures feed in 1:3, 1:1 and 3:1 proportions (mass) was used to validate the algorithm.

Like the previous work (Kapur & Fuerstenau 1989), the mill transfer function used in the simulation scheme was based on the breakage rate / breakage distribution function solution to the population balance model, i.e. the G-H solution (Kapur 1988). Through the introduction of energy split factors, S(k), which are charge composition/time-dependent, the algorithm for cycle-wise simulation of locked-cycle tests becomes:

\[ R_c(n) = r_c \sum_{j=1}^{n} F_c(j)\phi_c \left[ \sum_{k=j}^{n} S_c(k) \right] \] (2.82)

and

\[ R_q(n) = r_q \sum_{j=1}^{n} F_q(j)\phi_q \left[ \sum_{k=j}^{n} S_q(k) \right] \] (2.83)

where the subscripts c and q stand for calcite and quartz, R is the amount of material in the mill product retained on the closing size, r is the weighted fraction of the feed already retained on the closing screen, n is the cycle number, \( \phi \) is the mill transfer function. The algorithm can also be written in form of G-H (i.e. breakage rate and breakage distribution function, respectively) as follows:

\[
\phi_c \left[ t \sum_{k=j}^{n} S_c(k) \right] = \exp \left[ G_c t \sum_{k=j}^{n} S_c(k) + H_c \frac{t^2}{2} (n - j + 1)^2 \right] \quad (2.84)
\]

Also an empirical equation is proposed to relate the energy split factor \( S_c \) with the composition of the mill charge:

\[
\frac{S_c^*-S_c(k)}{S_c^*-1} = W_c M_c^a(k) + (1-W_c) M_c^b(k) \quad (2.85)
\]

where \( S_c^* \) is greater than one and is the maximum possible split factor as the amount of calcite tends to zero, \( W_c \) lies between zero and one; \( a \) and \( b \) are exponents. The same equation can be written for quartz, as follows:

\[
\phi_q \left[ t \sum_{k=j}^{n} S_q(k) \right] = \exp \left[ G_q t \sum_{k=j}^{n} S_q(k) + H_q \frac{t^2}{2} (n - j + 1)^2 \right] \quad (2.86)
\]

and

\[
\frac{S_q^*-S_q(k)}{S_q^*-1} = W_q M_q^c(k) + (1-W_q) M_q^d(k) \quad (2.87)
\]

where \( S_q^* \) represents the minimum split factor as the quartz amount in the mixture gets closer to zero, \( W_q \) lies between zero and one; \( c \) and \( d \) are exponents.

The grinding parameters \( G \) and \( D \) are estimated separately from results of locked-cycles tests using the single-component. This is done through a direct search technique, which optimizes these parameters in order to reduce the root mean square error between experimental and computed values of the recycle stream. Figure 2.37 shows that simulated recycles for pure components fits the experimental results pretty well for both materials. Once the values of \( G_s \) and \( H_s \) are defined, another search is performed to find the best values of \( W_s, S^*_s \) and factors in Equations 2.85 and 2.87. Figure 2.38 shows the good agreement between the simulated and experimental recycles for two mixtures (1:1 and 3:1). The last and most demanding validation exercise for this simulation procedure is presented in Figure 2.39, where the feed composition changes significantly in the thirteenth cycle and the model was capable of tracking this.
Figure 2.37 – Experimental and simulated recycles for the locked-cycle grinding of pure limestone and quartz feeds (after Kapur et al, 1992)

Figure 2.38 – Experimental and simulated recycles for 1:1 and 3:1 limestone-quartzite mixtures (after Kapur et al, 1992)
Once the model was validated, a series of simulations were performed in order to visualize the effect of several disturbances on the circulating load. The results indicated that the most direct consequences are on the circulating load. In the case of AG/SAG mill the same phenomena would be expected to occur in the mill internal circulating load due to the effect of grate classification.

The problem becomes much worse when these disturbances act in addition, which is one of the main reasons why it is difficult to maintain stability in circuits that process heterogeneous ores. The authors also believe that their simulator should be capable of handling a simultaneous combination of disturbances, although they have just presented results based on imposition of only one type at a time.

Though the above methodology looks promising, their model relies on the fitting of many parameters (at least six). Taking this into consideration, it becomes difficult to consider realistic simulations of scenarios out of the range of experimental condition in which the models were initially fitted. In order to clarify this issue, tests using the simulated conditions should be carried out to compare their simulations against the experimental data. Moreover, no industrial application was published.
2.6.4 AG/SAG Investigations

Although multi-component feeds have a significant effect on AG/SAG circuits, this issue has not been investigated in such detail as for ball mills. Moreover, in AG/SAG mills the interaction between the components is much more relevant than in a ball mill, since the hard material may act as grinding media. Despite that, only a few researchers have reported work on the effect of blends in AG/SAG mills.

Stange (1996) investigated the behaviour of binary ores and attempted to develop a multi-component mathematical model for AG mills. The author combined some features of the JKMRC model (Leung 1987) with Austin’s model (1987) framework, which he considered more rigorous, and created a hybrid model.

The breakage rate function applied in Stange’s model was that published by Austin (1986), which quantifies the rate at which particles of a given size are broken and is considered ore/mineral type independent. The approach used by Leung (1987), as previously described, was adopted for the breakage/appearance function, that characterizes the size distribution generated when a particular size of rock is broken.

The binary ore system investigated was the classical example of Palabora Mining Company where very hard dykes of dolerite are present in a soft ore deposit. The parameters of the breakage function, shown in Table 2.8, were measured using the twin pendulum test, and then used to calculate the breakage function for different particle sizes in a 1.75 m diameter mill.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Ore</th>
<th>Dolerite</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>61.6</td>
<td>45.8</td>
</tr>
<tr>
<td>b</td>
<td>1.61</td>
<td>0.72</td>
</tr>
<tr>
<td>A*b</td>
<td>99.2</td>
<td>33.0</td>
</tr>
<tr>
<td>ta</td>
<td>0.80</td>
<td>0.22</td>
</tr>
</tbody>
</table>

The mass transfer function reported by Austin (1987) was used in the model, as well as a classification function developed by the author himself, which is available in the MicroSim simulator (Stange et al., 1998) and can be described as:
\[ c_i = \left[ \frac{A_g}{A_g + A_p} \right] \left[ \frac{1}{1 + \left( \frac{d_{\text{grate}}d_{50}}{x_i} \right)^\lambda} \right] + \left[ \frac{A_g}{A_g + A_p} \right] \left[ \frac{1}{1 + \left( \frac{d_{\text{port}}d_{50}}{x_i} \right)^\lambda} \right] \] (2.88)

where:

\( A_g \) : fraction of the mill cross-section area that is grate open area
\( A_p \) : fraction of the mill cross-section area that is pebble port area
\( x_i \) : particle size
\( d_{\text{grate}} \) : grate aperture size
\( d_{\text{port}} \) : pebble port size
\( d_{50} \) : particle size that has equal probability to report to any of the products
\( \lambda \) : parameter related to the sharpness of classification

The described model was tested using data from a fully autogenous pilot mill campaign. The tests were carried out in a 1.75 m dia. x 0.54 m length mill, running at 75% of critical speed. The mill had grate aperture of 15 mm and pebble ports of 50 x 70 mm, for pebble crushing. The operational procedure was to find the feed rate that kept the mill load volume at 35%, and once achieved the plant was operated for an additional four hours, to ensure steady-state, before taking the sample cuts. Five runs were performed, increasing the amount of dolerite in the mill feed in each run, and the summary of these tests is shown in Table 2.9.

<table>
<thead>
<tr>
<th>Run</th>
<th>Added Dolerite (%)</th>
<th>Circuit</th>
<th>Throughput (kg/hr)</th>
<th>AG Specific Energy (kWh/t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>Single stage, closed circuit AG</td>
<td>1100</td>
<td>7.67</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>Primary AG, pebble crushing, secondary closed circuit ball milling</td>
<td>1950</td>
<td>4.82</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>Primary AG, pebble crushing, secondary closed circuit ball milling</td>
<td>1650</td>
<td>5.36</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
<td>Primary AG, pebble crushing, secondary closed circuit ball milling</td>
<td>1400</td>
<td>5.81</td>
</tr>
<tr>
<td>5</td>
<td>18</td>
<td>Primary AG, pebble crushing, secondary closed circuit ball milling</td>
<td>1200</td>
<td>6.58</td>
</tr>
</tbody>
</table>

Samples were analysed to provide a full set of data, i.e. size distributions, solid/liquid ratios and mass flowrates in all streams. Moreover, mineralogical analysis of the mill load was performed to determine its composition; however the same was not done for the mill product.
The models were calibrated by fitting the mass transfer relationship using all data sets, as shown in Figure 2.40; and the classification functions from data set 2, is illustrated in Figure 2.41. Once this was completed, the selection function parameters were adjusted to obtain the best-fit to the measured mill product size distribution for Run 2. The fit is shown in Figure 2.42 and it is clear that the model prediction was not accurate. The predicted and experimental percent of dolerite in each size class of the mill load is presented in Table 2.10 for Run 2, showing a significant discrepancy. The error is also evident in the other runs, including the total load mass prediction, as shown in Table 2.11.

Table 2.10 – Measured and predicted dolerite vs. size in mill load (after Stange, 1996)

<table>
<thead>
<tr>
<th>Size Class</th>
<th>Measured % Dolerite</th>
<th>Model % Dolerite</th>
</tr>
</thead>
<tbody>
<tr>
<td>+25mm</td>
<td>29.2</td>
<td>10.8</td>
</tr>
<tr>
<td>+40mm</td>
<td>23.3</td>
<td>9.9</td>
</tr>
<tr>
<td>+60mm</td>
<td>38.9</td>
<td>9.9</td>
</tr>
<tr>
<td>+80mm</td>
<td>35.0</td>
<td>9.2</td>
</tr>
<tr>
<td>+100mm</td>
<td>22.7</td>
<td>8.7</td>
</tr>
<tr>
<td>+150mm</td>
<td>41.9</td>
<td>8.5</td>
</tr>
</tbody>
</table>

Figure 2.40 – Relationship between slurry hold-up (fs) and flowrate (after Stange, 1996)
Figure 2.41 – Classification model regression results (after Stange, 1996)

Figure 2.42 – Mill load and product size distributions (Meas. Vs Fitted) (after Stange, 1996)
Table 2.11 – Predicted and measured load compositions (after Stange, 1996)

<table>
<thead>
<tr>
<th>Run</th>
<th>% Dolerite</th>
<th>Load kg</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>+25mm</td>
<td>+40mm</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measured</td>
<td>29.2</td>
<td>23.3</td>
</tr>
<tr>
<td>Model</td>
<td>10.8</td>
<td>9.9</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measured</td>
<td>66.5</td>
<td>54.8</td>
</tr>
<tr>
<td>Model</td>
<td>18.4</td>
<td>17.0</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measured</td>
<td>35.9</td>
<td>54.9</td>
</tr>
<tr>
<td>Model</td>
<td>25.7</td>
<td>23.9</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measured</td>
<td>62.1</td>
<td>71.9</td>
</tr>
<tr>
<td>Model</td>
<td>32.7</td>
<td>30.7</td>
</tr>
</tbody>
</table>

Stange also reported that he tried to fit the Palabora data using Leung’s model in JKSimMet, but the model was able to acceptably fit just two out of the three quantities (mill load mass, mill product size distribution and mill load distribution). Although, it was not reported where Leung’s model failed, Stange inferred that the AG/SAG models available at that time could not describe the multi-component grinding.

Stange’s work clearly shows that both his and Leung’s model were unable to accurately predict the build-up of hard material in the mill load, which supports the need for the development of a model framework capable of handling multi-component feeds. Stange added that his model failed to correctly describe the behaviour of AG/SAG mills when abrasion breakage dominates (mode of breakage of hard ores), even when the ore was not a binary system.

McKen and Chiasson (2006), reported some interesting findings on the use of the Macpherson test (Macpherson 1989) to study the behaviour of ore blends. The investigation was carried out using three different ore types, whose main comminution characteristics are presented in Table 2.12. Grinding tests using binary and ternary mixtures were performed. The first test used a blend of 46% Ore 1 and 54% Ore 2. The other investigated case was a blend of the three ores on a proportion of 18:22:60, respectively. The results for this investigation are shown in Table 2.13.
Table 2.12 – Characteristics of three ore types used in Macpherson blend tests by Mcken and Chiasson (2006)

<table>
<thead>
<tr>
<th>Sample</th>
<th>S.G. (g/cm³)</th>
<th>DWT A x b</th>
<th>RWI (kWh/t)</th>
<th>BWI (kWh/t)</th>
<th>Expected Throughput (Kg/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ore 1</td>
<td>4.49</td>
<td>174</td>
<td>5.3</td>
<td>9.0</td>
<td>39</td>
</tr>
<tr>
<td>Ore 2</td>
<td>3.24</td>
<td>33</td>
<td>15.9</td>
<td>12.8</td>
<td>5</td>
</tr>
<tr>
<td>Ore 3</td>
<td>3.04</td>
<td>28</td>
<td>21.6</td>
<td>23.3</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 2.13 – Grindability test summary on Macpherson test blends (after Macken and Chiasson, 2006)

<table>
<thead>
<tr>
<th>Sample</th>
<th>RWI (kWh/t)</th>
<th>BWI (kWh/t)</th>
<th>Macpherson Test (kg/h)</th>
<th>S.G. (g/cm³)</th>
<th>AWI (μm)</th>
<th>Feed (kWh/t)</th>
<th>Charge (kWh/t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blend 1</td>
<td>11.5</td>
<td>10.5</td>
<td>13.4</td>
<td>6.5</td>
<td>166</td>
<td>3.7</td>
<td>3.3</td>
</tr>
<tr>
<td>Blend 2</td>
<td>18.8</td>
<td>18.2</td>
<td>4.1</td>
<td>20.7</td>
<td>144</td>
<td>3.3</td>
<td>3.0</td>
</tr>
</tbody>
</table>

The above results suggest that the ball and rod mill work indices for the blends were pretty close to the weighted average calculated from the blend ratios, but Macpherson throughput rates were not. Although it is not mathematically correct to average throughput (t/h) with respect to blends based on the proportions of mass (Appendix B), the results indicate that interactions between components in an AG mill environment are much more relevant than in a ball mill. Another important finding was that the comminution behaviour of Blend 2 was controlled by the hard component during the AG milling test, since its expected throughput was about the same as for Ore 3 alone (hard).

2.7 Conclusions

The research conducted in the past 40 years provides an extensive understanding of the many complex interactions determinant for the AG/SAG grinding process. A number of population balance models provided simple mass balance relationships, which can describe these interactions in steady state conditions.

However the quality of these predictions relies on the capability of these models in describing the breakage and transport events in the AG/SAG grinding process. Basically there are three different ways of looking into this, i.e. the empirical, fundamental and phenomenological approaches. The former constitutes the most consolidated approach, and has two main streams, the “kinetic models” and the “perfect mixing models”. Although they have different methods for parameterization, both proved to be able to handle the modelling of AG/SAG mills with similar effectiveness.
The existing JKMRC models incorporate the ore specific breakage characteristics according to different mechanisms, using a single ore component to describe the feed and handling only one set of ore hardness parameters $A$, $b$ and $t_a$ (i.e. they assume a uniform feed). However, in many mining operations the Run-of-Mine (RoM) feed has multiple ore types that may have different competencies and physical properties. In view of this, the need for a multi-component AG/SAG mill model is evident.

The literature reveals a single attempt to develop a multi-component AG/SAG mill model (Stange, 1996), which has failed to provide accurate results. The model was tested with data obtained from AG pilot mill tests carried out using the Palabora binary (hard – soft) ore. The model parameters were adjusted to obtain the best-fit to the experimental data, but it failed to reproduce the measured mill product size distribution and the amount of dolerite in each size class of the mill load. Stage’s model introduced the use of multi-component (2D) model structure and ore-specific appearance function parameters. However, the model used single breakage rates and discharge function parameters, which is likely to be the main deficiency in his model.

Although the literature lacks further detailed investigations on the effect of multi-component ores in AG/SAG mill operations, there are several published research works focused on grinding of ore mixtures in ball mills. The outcomes of these works have almost no direct application to the study of autogenous and semi-autogenous mills, but they provide significant contribution towards the understanding of multi-component grinding. The works carried out using ball mills established robust experimental procedures to investigate the behaviour of materials when ground alone or in a mixture. The results of these investigations revealed, among others, the effect of blend on breakage rates and the accumulation of hard materials in the mill load, which are two major foci of the present thesis.

Therefore, there is a clear need for carrying out a detailed investigation on the breakage interactions and mass transport of independent components inside AG/SAG mills, in order to develop an effective multi-component AG/SAG model. The new modelling approach should be able to accommodate simultaneous iterations with different ore types which constitute the mill feed, relating their appearance function and distribution in the mill feed to the overall breakage rates, transport and mill power draw.
Chapter 3  The SAG Locked Cycle Test (SAG-LCT)

In order to investigate the grinding behaviour and the interaction of different ore components in a mill, a new SAG locked-cycle test (SAG-LCT) was developed. The investigation was carried out in a 600 mm diameter laboratory mill, using quartz and BIF (banded iron formation) samples, tested as pure components and in binary mixtures. Tests using a 1.8 m diameter pilot mill were also conducted to understand the effect of scale. The mill load and comminution behaviour was found to be controlled by the hard component, as expected, but in a highly non-linear manner. The other key finding was the marked differences in product size distribution for each component when they were ground alone and in mixtures. The SAG-LCT results were modelled with reasonable accuracy using the same equations proposed by Kapur and Fuerstenau (1989). The SAG-LCT test methodology as well as the important behaviours and trends are discussed in this chapter.

3.1 Introduction

To better understand the operation of different ore components in a semi-autogenous grinding process, an experimental method that simulates a SAG mill operating at constant mill load was required. Therefore a locked-cycle SAG tumbling test was developed by combining elements of the well established locked-cycle tests for ball mills (Fuerstenau, Venkatarman & Williams 1984; Holmes & Patching 1957) and the currently used AG/SAG laboratory tests, i.e. Macpherson (MacPherson 1977) and SAGDesign (Starkey, Hindstrom & Nadasdy 2006).

The SAG-LCT method aims to represent an open circuit SAG mill in steady-state continuous operation, where the harder component is expected to accumulate in the mill load due to the internal grate classification. The SAG-LCT flowsheet is represented in Figure 3.1.

![Figure 3.1 – Locked-cycled test flowsheet](image-url)
3.2 SAG-LCT Methodology

The mill used for the SAG-LCT is 600 mm in diameter and 200 mm in length (inside liners), fitted with 8 equally spaced lifters (40 x 40 x 15 mm, face angle 58°). It is driven by a 3 kW motor coupled to an electronic variable speed drive. The mill was operated in dry grinding mode at 42.2 rpm (75% of critical speed), loaded with 13.2 kg of 45 mm diameter stainless steel balls (corresponding to approximately 5% of the mill volume) and a mass of ore equivalent to 20% of the mill volume. The 25% total volumetric load corresponds to a standard semi-autogenous mill grinding load. These mill operating conditions were rigorously kept constant in all the experiments.

The feed size distribution was scaled down from a typical AG/SAG pilot scale feed size distribution to a top size of 53 mm. The feed size was kept the same for both components during all tests, in order to minimize the effect of size on the grinding rates. Both materials had been pre-sieved into narrow size fractions for characterization testwork, and the required feed size distribution was artificially made up by adding fixed amounts from each size fraction.

After running a few preliminary tests to quantify the grinding effectiveness of the selected operating conditions, the grinding cycle time was set to 20 minutes, which provided a reasonable circulating load for both hard and soft materials when ground alone. At the end of each cycle the ground material was classified using a closing sieve of 13.2 mm aperture, which in this experiment represents the mill grate classification. This sieving procedure was carried out using a Gilson® Hydraulic Screen device, as well as extra scalping sieves. The oversize material could be easily hand-sorted into two components. The proportion of each component in the mill circulating load could thus be directly measured.

The total mass of finished product (i.e. material below 13.2 mm) was replaced with an equal mass of new feed material after each cycle. This make up feed was prepared to match the target feed sized distribution and the blend under investigation, by weighing out fixed ratios of the \( \sqrt{2} \) fractions between 53 and 13.2 mm, as well as riffled sub-samples of minus 13.2 mm material.

Since the grinding time per cycle was kept constant, the mill circulating load was free to reach an equilibrium value according to the mill feed hardness and composition. The convergence criterion for this test was the stabilisation of the net amount of undersize material (g) produced during the cycle time (min), giving an overall specific grindability (g/min). The test was considered at steady state when the specific grindabilities from the last three cycles were within \( \pm 5\% \) of each another, and the ratio of the individual components in the mill recycle and product was constant.
This standardized protocol of the SAG-LCT allows the investigation of different operational conditions using a relatively small amount of material in a full range of sizes, making the sample preparation relatively easy. The reproducibility of the SAG-LCT is also expected to be one of its main advantages, since the experimental conditions are well controlled.

The SAG-LCT requires a relatively small quantity, approximately 100 kg per test, and is designed to generate results that can provide a better understanding of the blend effect on the mill grindability, as well as the mill load and product size distribution and composition.

The only downside in carrying out the test is the amount of work involved. Each test can take up to 18 cycles and approximately 30 man-hours to complete (i.e. including sample preparation, grinding, as well as screening and composition analysis).

3.3 Material Sourcing and Characterization

Materials with appreciable differences in hardness that could be separated using their physical properties were required in order to carry out this study. After trying a few different mixtures of materials and methods for separating and/or quantifying their proportions, quartz and an iron ore were found to be the materials best suited to this investigation.

The quartz was found to be soft (high A*b), which was unexpected, and the three ore types of magnetite had different hardness characteristics, from very soft to hard. Therefore, the hard iron ore type, which in fact is a banded iron formation ore (BIF), was chosen to be used with quartz in a hard/soft mixture.

During the experimental campaign five trials were carried out using pure materials as well as mixtures having Quartz/BIF ratios of 3:1, 1:1 and 1:3. In addition, a repeat test using a 1:1 mixture was performed to verify the reproducibility of the experiment.

Small samples of both materials were acquired for preliminary characterization using the JKRBT (Kojovic et al. 2008; Shi, F. et al. 2008a, 2008b; Shi, Fengnian et al. 2009) to quantify their impact resistance. A magnetic separator was used to verify the separation efficiency, which was higher than 95% across all size fractions. After this preliminary investigation, a much larger amount of both materials was sourced for more detailed characterization. The very pure quartz was collected in an old deactivated platinum mine in Bundaberg, North Queensland, and the BIF material was shipped from an iron ore mine in South Australia. All material (~1.6 tonnes) was crushed down and sized to generate the characteristic size fractions necessary for conducting JKDWT tests (Napier-Munn et al. 2005) on both samples. The results for the comminution characterization tests are shown in Table 3.1.
Table 3.1 – Comminution characterization data for quartz and BIF

<table>
<thead>
<tr>
<th>Material</th>
<th>SG</th>
<th>A*b</th>
<th>t_a</th>
<th>BWI (kWh/t)¹</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quartz</td>
<td>2.65</td>
<td>94</td>
<td>1.01</td>
<td>16.4</td>
</tr>
<tr>
<td>BIF</td>
<td>3.35</td>
<td>39</td>
<td>0.37</td>
<td>15.5</td>
</tr>
</tbody>
</table>

¹ at 150 micron closing size.

The parameter t_a in Table 3.1 is an abrasion ore parameter used in the current JKSimMet AG/SAG model and larger t_a values indicate softer materials. Table 3.1 shows that the BIF material is denser and harder to crush than the quartz material. However, the Bond Ball Mill Work Index tests suggest the Quartz is 5% harder in terms of grindability down to a fine size.

3.3.1 Screening and Composition Analysis

When the mill is running at steady-state, the test is stopped and the mill circulating load and product from the last three cycles is sieved using the Gilson and Ro-Tap screens respectively. This is followed by a size-by-size composition analysis of both the mill load and product.

Since the amount of product generated in the last three cycles is relatively large (~15 kg), a careful procedure had to be established to conduct the sieving and composition analysis of these samples, in order to generate high quality data. Therefore the three products were scalped at 3.35 mm using 300 mm sieves for 10 minutes using the Ro-Tap. All oversize material was sieved down to 3.35 using 200 mm analysis sieves for 10 minutes. The -3.35 mm material of each cycle was riffled into two parts: one half was mixed and the other separately stored, and then the mixed product was sub-sampled using a rotary divider. This sub-sample was weighed, wet screened at 38 μm, oversize dried, re-weighed and then sieved down to 38 μm using 200 mm analysis sieves for 10 minutes.

The size-by-size composition analysis of the mill product (-13.2 mm) was conducted by magnetic separation, using the differences in magnetic susceptibility of quartz and BIF. However, the analysis for the mill load (+13 mm) was done by hand sorting, taking advantage of the large size of those particles and their differences in colour (i.e. white quartz and black BIF).
3.4 Results and Discussion

Due to the different hardness and grindabilities of the components, the composition of the oversize material returned to the mill from the grate classification changed after every cycle. Figure 3.2 shows the mineral composition of the mill circulating load and product and the overall grindability, as a function of cycle number for a test carried out using a 1:1 quartz/BIF mixture. It shows that the amount of hard material (BIF) in the mill circulating load asymptotically builds-up with the cycle number; this change is slow and takes around 18 cycles to reach steady state. Furthermore, the grindability of each component stabilises when they reach the right levels to meet the steady state condition, i.e. the product composition matches the feed.

![Mill Circulating Load and Product Composition](image1)

![Grindability](image2)

Figure 3.2 – Mill dynamics as a function of cycle number for a locked-cycle test using 1:1 QTZ/BIF mixture
The results also clearly show that the more soft material present in the fresh feed, the more significant is the hard material build-up in the circulating load. This can be explained by the fact that the hard material should reach a certain volume inside the mill in order to compensate for its slower breakage rates until reaching steady state. This behaviour can be observed in Table 3.2.

<table>
<thead>
<tr>
<th>Start</th>
<th>Final</th>
<th>BIF build-up (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quartz (%)</td>
<td>BIF (%)</td>
<td>Quartz (%)</td>
</tr>
<tr>
<td>25</td>
<td>75</td>
<td>13</td>
</tr>
<tr>
<td>50</td>
<td>50</td>
<td>31</td>
</tr>
<tr>
<td>75</td>
<td>25</td>
<td>58</td>
</tr>
</tbody>
</table>

One of the major findings of the SAG-LCT investigation was related to the overall SAG mill throughput, which was shown to be strongly dependent on composition. However, it does not have a linear behaviour according to the ratio of components in the fresh feed. In fact, it has been shown that it is mainly controlled by the presence of the hard component in the mill. For the selected ore types, quartz and BIF, there is little difference in throughput between the pure hard material and blends with over 50% hard material, as shown in Figure 3.3. Only once the fraction of soft ore in the feed exceeds 50% does the throughput rate begin to ramp up to that of pure soft material.

Figure 3.3 – Normalized mill throughput as a function of the amount of soft material present in the fresh feed
The throughput rates were normalized with respect to the highest rate achieved when processing pure quartz (i.e. pure quartz has the highest normalised throughput of 1). The reproducibility of the data is demonstrated by the duplicate runs at 50% of each ore, suggesting a relative standard deviation of 2%. This relative standard deviation was applied to the other tests as shown by the error bars in Figure 3.3.

The product size distribution for the individual components did not show significant differences in their P80, though the trends were as expected. Essentially reducing the proportion of hard BIF in the feed reduces the P80. However, the cumulative percentage passing 150μm suggests that the product becomes somewhat coarser in the fine sizes range, as the amount of softer material increases in the fresh feed. Quartz appears to be harder to break below 600μm, which may be related to the sand sized grains that are liberated in impact breakage, consistent with the higher Bond Work index. These results are presented in Table 3.3, as well as in Figure 3.4, Figure 3.5 and Figure 3.6.

Table 3.3 – Size distribution parameters according to the presence of soft material in the fresh feed

<table>
<thead>
<tr>
<th>% QTZ</th>
<th>P80 (mm)</th>
<th>Cum % Passing 150μm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Combined</td>
<td>Quartz BIF</td>
</tr>
<tr>
<td>0</td>
<td>10.44</td>
<td>- 10.44</td>
</tr>
<tr>
<td>25</td>
<td>9.85</td>
<td>8.45 10.26</td>
</tr>
<tr>
<td>50</td>
<td>9.00</td>
<td>7.51 10.08</td>
</tr>
<tr>
<td>75</td>
<td>9.10</td>
<td>8.90 9.67</td>
</tr>
<tr>
<td>100</td>
<td>8.03</td>
<td>8.03 -</td>
</tr>
</tbody>
</table>

Figure 3.4 – Composite product size distribution according to the percentage of soft material in the mill
Figure 3.5 – BIF Product Size Distribution according to the percentage of soft material in the mill

Figure 3.6 – Quartz product size distribution according to the percentage of soft material in the mill
The behaviour observed in the product size distributions can be explained in two ways. Firstly, the more soft material in the mill, the less competent and dense material (BIF) available to act as grinding media causing breakage. Secondly, as the hard and soft components have different breakage properties, the BIF tends to break mainly by abrasion and attrition, while the quartz by impact. Once the quartz is broken to sizes below the grate size, the product passes quickly through the grate (screen). The lower residence time, coupled with the apparently hard sand sized grains, results in a relatively coarse product when compared to feed blends with higher proportions of BIF. As a consequence, the hard material tends to generate much finer material and the soft component generates more material retained in the -600+38μm.

The difference in the competence of the coarse and fine breakage of the materials used in this test result in responses in the product size distributions that may not be matched for other blends of ores, so the response by size may not be generic to the majority of ore types.

### 3.5 Large scale SAG-LCT

Due to concerns regarding the scalability of the SAG-LCT results, an additional set of experiments was conducted using the same materials (i.e. quartz and BIF) and a larger 1.8 m diameter batch pilot mill, which has its dimensions shown in Table 3.4. The test conditions and methodology were kept exactly the same as for the small scale laboratory mill, except for the grinding cycle time that was reduced from 20 to 7 minutes because the pilot mill provides more energy and therefore breakage in a shorter period of time.

<table>
<thead>
<tr>
<th>Mill (inside shell)</th>
<th>Lifters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter (mm)</td>
<td>Length (mm)</td>
</tr>
<tr>
<td>1800</td>
<td>300</td>
</tr>
</tbody>
</table>

Three tests were conducted using the larger pilot mill, one using a 1:1 mixture of quartz and BIF and other two with the pure components. The major objective of this exercise was to verify the impact of scale on the mill throughput curve and the mill charge composition when equilibrium is reached.

Although the normalized mill throughput curve as a function of fresh feed was similar for both pilot and laboratory mills, the dominance of the hard material in the pilot mill was not as significant as for the laboratory mill. Therefore, the pilot mill tests potentially provide a response curve that is more representative of full-scale mills, as shown in Figure 3.7.
Figure 3.7 – Comparison of the normalized throughput curve obtained using the laboratory and pilot mill

The build-up of hard BIF material in the mill load for the 1:1 blend, (i.e. the overall charge composition at equilibrium), was 3% higher in the laboratory mill than in the pilot mill, as shown in Table 3.5. This is expected since the differential breakage response of the hard and soft materials in a higher energy environment is less significant.

Table 3.5 – Build-up of hard BIF material in the mill load for laboratory and pilot mill tests

<table>
<thead>
<tr>
<th>Mill (Internal Diameter)</th>
<th>Fresh Feed (%BIF)</th>
<th>Mill Load (%BIF)</th>
<th>Build-up (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laboratory (0.6 m)</td>
<td>50</td>
<td>69</td>
<td>38</td>
</tr>
<tr>
<td>Pilot (1.8 m)</td>
<td>50</td>
<td>66</td>
<td>32</td>
</tr>
</tbody>
</table>

The same trend in product size, seen in the laboratory mill tests (coarser for quartz and finer for BIF) was also observed in the pilot mill test, as shown in Figure 3.8.
However, there was a noticeable difference between the product size distributions obtained using the pilot and the standard laboratory mill, as shown in Figure 3.9. This difference can be attributed to the difference in breakage mechanisms of each mill. The larger pilot mill provides more high energy (impact) breakage, generating a coarser product, while the laboratory mill is predominantly dominated by abrasion and generates a finer product. This difference was more significant for the soft material (quartz), because the harder material (BIF) has more resistance to impact and breaks predominantly by abrasion even in the larger mill.

![Figure 3.9 – Comparison of the SAG-LCT product size distributions for the pilot and laboratory mills](image)

The product size distributions for each component in the 1:1 tests, shown in Figure 3.10, confirms that quartz (softer material) is significantly more affected by the higher breakage energy in the pilot mill than BIF (harder material) was. Quartz easily breaks with higher energy impacts and generates a coarse product, while BIF is more resistant to impact breakage, and tends to primarily break down by abrasion in both mills.
It is clear that larger mills do provide more energy and consequently more breakage of both hard and soft components, but the harder BIF material is very resistant to breakage despite the extra energy in the 1.8m diameter mill.

Therefore, the 600 mm diameter mill used in the standard SAG-LCT can be used to provide an indication of the mill throughput response and the steady state mill load equilibrium for multi-component feeds or blends. However, the product size distribution generated in the smaller mill will certainly be different to that generated in larger industrial mills.

Although the mill response is sensitive to energy, the SAG-LCT could be scaled to any mill size if the effect of energy is incorporated properly. Kojovic (2011) introduced an encouraging approach for this purpose, based on the SAG-LCT concept, to model the mill throughput response using the ore breakage parameters ($A$, $b$ and $t_a$) and the size specific energies.

### 3.6 Modelling the SAG-LCT

Due to the significant amount of time, effort and material required to conduct the SAG-LCT until equilibrium is reached, an investigation was initiated to determine if the test outcomes could be modelled.
Kapur and Fuerstenau (1984) introduced the locked-cycle test methodology using ball mills and later, applied the balance equation of grinding kinetics (Herbst & Fuerstenau 1980) to develop two algorithms that could predict the results of their tests with reasonable accuracy (Kapur & Fuerstenau 1989). Their algorithm I, which was previously described in Equations 2.78 and 2.79, can describe the amount of material in the mill product retained on the closing size and was therefore adopted to simulate the SAG-LCT data.

The mill transfer functions for make-up and recycle feeds (ϕ and ϕ’) had to be calibrated using experimental data from the first three cycles, i.e. the measured amount and proportions of each material retained on the closing size. However, the model proved to be quite robust and once adjusted it could describe the experimental results with reasonable accuracy. Figure 3.11 compares the simulation results with the experimental data for the 600 mm laboratory test using a 1:1 mixture of quartz and BIF.

![Mill Load, i.e. Screen O/S composition (exp vs sim)](image)

![Circulating Load](image)

Figure 3.11 – Simulated vs. experimental SAG-LCT equilibrium conditions
These modelling results suggest that the Kapur and Fuerstenau modelling approach can be used as an alternative tool to simulate the SAG-LCT final equilibrium conditions after running only a few grinding cycles. This can be especially useful when constrained by the amount of sample or time available.

3.7 Conclusions

A new SAG laboratory locked-cycle test procedure has been developed and applied to investigate the behaviour of mixtures of two materials with different physical properties (i.e. hardness and density) in laboratory and pilot scale mills.

It has been shown that the mill overall grindability (throughput) is affected by the proportion of hard material in the feed. The response was found to be non-linear in both 600 mm and 1800 mm diameter mills. The controlling effect of the hard component is dominant up to a 1:1 ratio of components, particularly in the 600 mm diameter mill, where the throughput only increases significantly for blends with more than 50% soft ore in the feed. The trend is less non-linear for the 1.8 m diameter mill, attributed to the increased available energy. However, as expected, the product size distributions become coarser as the amount of soft material in the mill feed increases, and the throughput is increased at similar energy levels.

The potential for modelling the blend response for a SAG mill, using laboratory locked-cycle test data, has been demonstrated. However, the interactions still have to be modelled to confidently describe the response, given only the ore hardness parameters and milling conditions (Kojovic 2011). Extra milling tests (i.e. the pure components plus a single mixture) using different materials are required in a first instance to provide actual data on the interaction.

Further work is required to assess the behaviour of the size specific breakage rates in the mill in order to develop more robust models to predict the behaviour of blends of hard/soft materials in AG/SAG mills, as discussed later in this thesis.

The raw data recorded during the SAG-LCT experiments can be found in Appendices A.1 and A.2.
Chapter 4  Multi-Component Pilot Plant Tests

A pilot scale AG milling campaign was conducted at the Anglo American pilot plant in South Africa. Different blends of hard and soft ore (silicate and chromite) and coarse and fine particles in the AG mill fresh feed were tested. The effects of multi-component feed on the pilot scale AG mill operation are presented in this chapter, providing useful guidelines in using multi-component feed blends in AG milling.

4.1  Introduction

In November 2009 an AG milling pilot plant campaign was carried out using multi-component feeds, as part of a research program within the AMIRA P9O project at the Anglo American pilot plant facility in South Africa. Three trials were performed to quantify the blend response of different ore types (normal ‘Run-of-Mine’ UG2 ore and ‘Waste’ rocks), as well as the ratio of coarse (+60 mm) and fine (-60 mm) material in the feed. The resulting feed materials had significant differences in the proportions of silicate (hard) and chromite (soft) components.

The major objectives of the pilot plant campaign were to:

- Obtain a multi-component set of data, which will be used to validate the new AG/SAG multi-component model being developed as part of the P9O project.
- Provide operational blending limits for the Anglo American operations wishing to consider AG milling of UG2 ores.

The UG2 platinum ore has a friable mineral-bearing seam predominantly composed of chromite. The hanging- and foot-walls are predominately reasonably competent silicate. The ore is, in theory, well-suited to autogenous grinding and this has been successfully practiced. However, if the fraction of Waste (from the hanging- or foot-wall) is too low, then the mill throughput reduces. This has driven the operations to switch to high ball load – run-of-mine (RoM) ball milling. Anglo American would prefer to return to the more economic AG milling, but clear operating limits are required to assess the longer-term viability in relation to their various ore-bodies and mining techniques. For Anglo American, this work was aimed at seeking to understand the limits to AG operation on such ores and provide design parameters for AG milling.
4.1.1 The Pilot Mill

Anglo American research facilities are equipped with a fully instrumented Magotteaux pilot mill, which has a variable speed drive which provides some flexibility in its operation. Figure 4.1 shows the open circuit flowsheet used during the trials and a photograph of the Magotteaux AG mill, the dimensions of which are presented in Table 4.1.

![Pilot plant circuit flowsheet and the pilot AG mill](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internal mill diameter (m)</td>
<td>1.26</td>
</tr>
<tr>
<td>Mill belly length (m)</td>
<td>2.2</td>
</tr>
<tr>
<td>Feed trunnion diameter (m)</td>
<td>0.35</td>
</tr>
<tr>
<td>Feed and Discharge cone angle (°)</td>
<td>0</td>
</tr>
<tr>
<td>Grate aperture (mm)</td>
<td>17</td>
</tr>
<tr>
<td>Grate opening area fraction (%)</td>
<td>3</td>
</tr>
<tr>
<td>Trommel aperture (mm)</td>
<td>5</td>
</tr>
</tbody>
</table>

4.1.2 Mill Feeding System

The UG2 feed materials used in this campaign came from the same mine shaft. UG2 is the Run-of-Mine ore, which has a significant amount of chromite in a silicate matrix, and the Waste is the mined gangue material, mostly composed of silicate, obtained from the hanging- and foot-walls.

The large rocks in the bulk ore were first removed using a 100 mm static grizzly and then crushed in a jaw crusher with a closed side setting of 130 mm. The -100 mm and crusher product were screened at 60 mm to generate two size classes of +60 mm and -60 mm. The size fractions were kept separate. The top size was estimated to be 180 mm.
The ore was then conveyed to either the coarse ore bin (+60-180 mm) or the fine ore bin (-60 mm). The feed rate from the coarse ore bin was controlled by a vibratory feeder discharging onto a weightometer, providing measured batch additions of coarse rocks. The required coarse addition was then controlled by the cumulative sum of the mini-batch additions at timed intervals. The feed rate from the fine bin was controlled by a belt feeder. The ratio of coarse:fine material could be varied according to the requirements of the test. The control of the coarse fraction delivery was excellent, which is a credit to the careful design of this feeding system.

4.1.3 Feed Characterization

Samples of UG2 chromite and Waste silicate collected during the pilot plant campaign were used in a comprehensive ore composition and breakage characterization program. These samples were sorted by density, generating enough particles for characterizing the ore properties of chromite (high density), silicate (low density) and binary (mid density) components using the JK Drop Weight Test – JKDWT (Napier-Munn et al. 2005). The results are summarized in Figure 4.2 and Table 4.2.

![Figure 4.2 – UG2 and Waste particles density distributions](image)

**Table 4.2 – Breakage testing data**

<table>
<thead>
<tr>
<th>Sample</th>
<th>SG</th>
<th>Std dev SG</th>
<th>A</th>
<th>b</th>
<th>A*b</th>
<th>ta</th>
<th>BWI¹</th>
</tr>
</thead>
<tbody>
<tr>
<td>UG2 Low density</td>
<td>3.0</td>
<td>0.1</td>
<td>73.9</td>
<td>1.66</td>
<td>123</td>
<td>0.51</td>
<td>19.5</td>
</tr>
<tr>
<td>UG2 Mid density</td>
<td>3.3</td>
<td>0.1</td>
<td>73.3</td>
<td>1.66</td>
<td>122</td>
<td>0.61</td>
<td>20.4</td>
</tr>
<tr>
<td>UG2 High density</td>
<td>4.0</td>
<td>0.1</td>
<td>81.4</td>
<td>4.29</td>
<td>349</td>
<td>2.38</td>
<td>16.8</td>
</tr>
<tr>
<td>(pure soft component)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Waste Low density</td>
<td>2.9</td>
<td>0.1</td>
<td>71.5</td>
<td>0.86</td>
<td>62</td>
<td>0.34</td>
<td>19.2</td>
</tr>
<tr>
<td>(pure hard component)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Waste Mid density</td>
<td>3.3</td>
<td>0.1</td>
<td>73.1</td>
<td>1.07</td>
<td>78</td>
<td>0.34</td>
<td>19.8</td>
</tr>
</tbody>
</table>

¹ 150 µm closing size
The density distribution data indicated three distinct classes for UG2 ore, but the JKDWT results suggest that there is almost no difference between UG2 low and mid density particles. The data also show that chromite rocks have a low resistance to impact, as indicated by the high $A^*b$ values, and that low density UG2 rocks are less competent than the low density Waste rocks.

### 4.1.4 Testing Scope

In dealing with the constraints of the friability of the UG2 ore, the soft:hard ratio was varied by changing the ratio of UG2 (all -60 mm) to Waste (all +60 mm). The Waste formed the competent autogenous grinding media. Thus, changing the soft:hard ratio also changed the mill feed size distribution, but this could not be avoided. Considering the underlying objectives and available operational controls, three surveys with different feed blends were planned:

- 80% UG2 -60 mm : 20% Waste +60 mm (Test 1)
- >80% (maximum limit to be sought during test) UG2 -60 mm : Waste +60 mm (Test 3)
- 80% UG2 -60 mm : 20% UG2 +60 mm (Test 5)

The aim of the three surveys was primarily to quantify the effect of a varying ratio of hard and soft components when AG milling UG2 ore. The UG2 +60 mm was expected to represent a ‘finer’ and ‘softer’ media when compared to the Waste +60 mm material. This variation is in line with expectations when these materials are blended in the industrial operations.

The expectation was that the highest throughput would be achieved in Test 3, the medium in Test 5 and the lowest in Test 1, as illustrated in Figure 4.3. The second survey aimed at reducing the amount of coarse material, whilst increasing the fine proportion until the throughput reached a maximum. During this gradual ramp-up the mill would be monitored closely to establish the grind curve. The final survey was conducted with the standard UG2 ore as received RoM feed. Adequate steady-state conditions were not achieved in Test 2 and 4 surveys, so the load was not emptied and sized and the results were not used in this study.

![Figure 4.3 – Blending system and expected throughput response for each blend](image)

92
4.2 Experimental Procedures

The mill was rapidly brought to steady state by preferentially feeding the hard material at a ratio expected in the final mill load, so as to rapidly build up the mill load. When the filling level was close to the target, the ratio of hard to soft was steadily reduced and the total feed rate steadily increased until the mill reached the target filling at the desired soft:hard ratio. The stabilisation time was reduced from the typical 12 to 18 hours expectations to only 6 hours, as shown in Figure 4.4.

![Graph showing mill operation from empty to steady-state](image-url)
The surveys began when steady-state conditions had been reached, i.e. when the mill load was stable at the target mill weight and the mill feed rate was constant.

Although the ideal procedure for obtaining the mill feed sample is to perform a belt cut just after crash stopping the system, this was not feasible at the Anglo American pilot plant. The coarse material was fed straight from the feed hopper into the mill via a vibratory feeder and the fines belt was too short and difficult to access. Therefore, the feed stockpiles had to be sub-sampled in order to obtain the feed sample.

One Bobcat® scoop was set aside on a separate part of the RoM pad every time the silos were fed. At the end of each survey three smaller stock piles of ‘UG2 coarse’, ‘UG2 fine’ and ‘Waste’ were generated. All the material from the coarse feed samples was used for size analysis. The ‘UG2 fines’ pile was then flattened out and quartered for sieving. This sampling procedure reflected the feeding system and generated representative samples of the three materials used during these trials.

Another critical measurement of the pilot campaign was the comprehensive analysis of the mill charge (including %solids, size distribution and composition), which was carried out once the material was removed from the mill.

The total charge filling was obtained from physical measurements post each crash stop. The charge load was calculated using the average height to the top of the mill (inside liner). The actual inside liner diameter was also measured when the mill was empty. The combination of charge height and mill diameter provided the key inputs in the calculation of the charge volumetric filling.

The slurry level in the mill charge was also measured. This was done by carefully rotating the mill to an angle at which the slurry could be seen through the hatch, and then height measurements from the slurry level to the top of the mill were taken using a laser meter.

Once the measurements were completed, the mill load was manually removed through the side hatch as per the standard Magotteaux procedure, shown in Figure 4.5. All solids and most of the slurry were recovered and stored in bins. The additional water required to assist cleaning out the mill was accounted for.
For the AG mill in open circuit, only two streams, the trommel undersize and trommel oversize, had to be sampled. The samples were taken at 10-minute intervals for a period of one hour. Before sampling commenced for each test, the cutters were ‘seasoned’ by taking a dummy sample and emptying out the cutter in the same manner as during the test, so as to ensure a steady state ‘dirtiness’ both before and after sampling. Any spilt or spoiled samples were discarded and immediately retaken. Once the samples were successfully taken, they were emptied into a bucket. Immediately after every sample was taken, a backup sample was taken and stored separately from the primary sample.

The sample collection period was divided into two half hours in order to have the mill stability verified. This was done by simply checking the deviation between the results from the first and second half hour. In addition, should any instability be noticed during the first half hour of the sampling period, this sample could be discarded and a third sample collected.

Multiple full cross-section cuts of the trommel undersize stream were sampled using a ‘pelican’ style cutter. The trommel oversize was a small flow of pebbles. Timed samples were collected using 20 litre buckets during the whole 1-hour sampling period, in order to obtain an estimate of the flowrate of the pebbles. This sample was also submitted for size distribution and composition analysis.

All samples collected during the pilot trials were sized down to 1 mm on site, and the minus 1 mm material was split and sent to the Anglo American Divisional Metallurgical Laboratories (DML) for further sieving and preparation for assaying.
The mill feed was dried in the sun and then manually screened using √2 sieves between 180 mm and 1 mm. A sub-sample of the -1 mm material was wet screened at 25 μm. The -25 μm was dried and stored. The +25 μm was dried, weighed and screened using √2 sieves between 1 mm and 25 μm. Samples of the mill feed were sent for further characterization to both JKMRC and Anglo American Research, where ore hardness testing, including specific gravity, JKDWT and Bond tests, were carried out. All the chemical assays were performed by Anglo American Research and the mineralogical analyses were conducted at the University of Cape Town.

Once the mill load samples were removed from the mill, stored in drums and weighed, the slurry (fines) was separated from the pebbles. All pebbles were washed in order to recover the fines that were stuck to the rock surface. The clean pebbles were sieved and stored in 20 litre buckets to be weighed when dry. All the slurry samples were weighed wet to account for the added water, then filtered and dried. Once the fines samples were dry, they were then screened using a 16 mm screen. All the retained material was sieved on the larger sieves and combined with the pebble samples. All the -16 mm particles both from the slurry sample and the pebble sample were weighed and sieved down to 1 mm. The -1 mm samples were sub-sampled to be sent to DML for further processing. Finally, all sized fractions from 180 mm down to 25 μm were sub-sampled then crushed or pulverized for assaying (Cr₂O₃). Samples of all size fractions larger than 16 mm were collected and sent to Anglo American Research for breakage testing.

Similar sizing procedures were applied to the mill product samples (i.e. trommel over- and under-size). Sub-samples of all sized fractions were taken for assaying (Cr₂O₃).

4.3 The Influence of Multi-Component Feed on Mill Performance

The pilot plant campaign was conducted at different feed compositions which caused the AG mill throughput and product size distributions to vary significantly. Figure 4.6 displays the size distributions and F80s of the AG mill feed used in the pilot plant campaign. There was a significant difference in the feed size distribution due to the adopted blending procedure. Nevertheless, this shift in feed size, with a change in the ratio of hard:soft is realistic in an operating sense, so is of direct relevance to the Anglo American operations.

Figure 4.6 shows that as the soft component increased, the feed F80 decreased. Particularly in Test 3, the feed contained far less coarse particles, with the F80 decreasing from 60 mm in Test 1 to 32 mm in Test 3. However, the comparison between Tests 1 and 5 was not compromised because their feed size distributions were similar.
4.3.1 The influence on mill load and product

In line with the variations in feed size and composition, the AG mill load size distribution and composition were significantly different in the surveys. The entire mill load from AG Tests 1, 3 and 5 was removed and analysed. It can be seen in Figure 4.7 that when the amount of soft material in the fresh feed increased, the mill load became finer, with the 80% passing size in the mill load decreasing from 77 mm in Test 1 to 70 mm in Test 5, and further downward to 49 mm in Test 3.
To analyse the compositions, the load samples were assayed for Cr₂O₃, and the Cr₂O₃ assays were converted to chromite using historical data provided by research staff at the Centre for Minerals Research at the University of Cape Town. Chromite was assumed to represent the ‘soft’ component, and the balance of material the ‘hard’ siliceous component. This simplifying assumption ignores issues related to liberation in the comminution stage, i.e. the components are dealt with as independent. The chromite and silicate distributions by size in the mill feed and load, for both Tests 1 and 5, are presented in Figure 4.8, showing that the chromite is mainly present in finer sizes and silicate in coarser fractions. The comparison between feed and load distributions confirms the hypothesis that the hard material builds up in the mill load at coarser sizes. Test 1 mill load also has more competent (silicate) media than Test 5.

Figure 4.8 – Chromite and silicate distributions by size in the mill feed and load for Tests 1 and 5

A similar size-by-size compositional analysis was performed on the mill product (i.e. trommel oversize and undersize), and the data were subsequently used in the multi-component modelling investigations.
Figure 4.9 shows the 80% passing sizes in the trommel oversize (T80) and undersize (P80) in Tests 1, 3 and 5. The mill product size distributions, obtained during the campaign, show that the product became coarser when the amount of soft material in the fresh feed increased. The exception is Test 3, in which P80 is the finest. This was due to the dominant effect of much finer feed in Test 3, which is further discussed in the next section.

![Figure 4.9](image.png)  
**Figure 4.9 – The 80% passing size in the AG mill trommel oversize (T80) and undersize (P80) for Tests 1, 3 & 5**

The chromite and silicate distributions by size in the trommel undersize and oversize for both Tests 1 and 5 were also calculated. They show a similar trend to the mill load: the chromite was mainly present in the finer sizes in both trommel undersize and oversize. In addition, as the soft component in the mill feed increased, the chromite content in the mill product also increased.

A summary of the multi-component data for Tests 1 and 5, containing the calculated head assays and tonnages for both silicate and chromite, is presented in Table 4.3. The dominant presence of the hard component (silicate) in the mill load and trommel oversize is evident. In Test 1, the feed contained 67% silicate. The silicate component increased to 96.6% in the mill load and to 95.3% in the trommel oversize. A similar trend is exhibited in Test 5, with the silicate content increasing from 62.5% in the feed to 88.9% in the load, and to 91.4% in the trommel oversize. Comparing the two tests, Test 1 feed contained more silicate than Test 5, as did the load and trommel oversize (about 5-6% consistently). This provides clear evidence that hard component accumulation in the mill is due to its slow break down characteristics. The data also supports the assertion that any attempt to use one set of averaged breakage parameters to model a multi-component AG/SAG mill is unlikely to give a realistic description of the behaviour of the various components in the mill, particularly if the components have very different properties as in the Anglo American study.
Table 4.3 – Multi-component data for Tests 1 and 5

<table>
<thead>
<tr>
<th>Test</th>
<th>Stream</th>
<th>kg/h</th>
<th>Chromite %</th>
<th>Silicate %</th>
<th>Chromite (kg/h)</th>
<th>Silicate (kg/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>FEED</td>
<td>1639</td>
<td>33.0</td>
<td>67.0</td>
<td>541</td>
<td>1098</td>
</tr>
<tr>
<td></td>
<td>TR O/S</td>
<td>29</td>
<td>3.4</td>
<td>96.6</td>
<td>1.0</td>
<td>28.0</td>
</tr>
<tr>
<td></td>
<td>TR U/S</td>
<td>1610</td>
<td>33.5</td>
<td>66.5</td>
<td>540</td>
<td>1070</td>
</tr>
<tr>
<td></td>
<td>LOAD</td>
<td>1400</td>
<td>4.7</td>
<td>95.3</td>
<td>65</td>
<td>1335</td>
</tr>
<tr>
<td>5</td>
<td>FEED</td>
<td>2550</td>
<td>37.5</td>
<td>62.5</td>
<td>956</td>
<td>1594</td>
</tr>
<tr>
<td></td>
<td>TR O/S</td>
<td>63</td>
<td>8.6</td>
<td>91.4</td>
<td>5.4</td>
<td>57.6</td>
</tr>
<tr>
<td></td>
<td>TR U/S</td>
<td>2487</td>
<td>38.2</td>
<td>61.8</td>
<td>951</td>
<td>1536</td>
</tr>
<tr>
<td></td>
<td>LOAD</td>
<td>1333</td>
<td>11.1</td>
<td>88.9</td>
<td>148</td>
<td>1185</td>
</tr>
</tbody>
</table>

4.3.2 The influence of feed composition on mill throughput and energy consumption

Test 1 had the highest proportion of competent material (Waste) in the fresh feed and consequently the lowest throughput. Test 5 had a similar feed size distribution to Test 1, but used normal UG2 rather than Waste in the +60 mm range. Therefore, the only significant difference between Tests 1 and 5 is the ratio of hard to soft components in the fresh feed. This feed composition difference can be used to decouple the effects of feed size distribution and feed component hardness, and to investigate the influence of feed component hardness on pilot scale AG mill performance. Table 4.4 shows the comparison of mill throughput for the two tests. As the soft component in the AG mill feed increased, at virtually the same feed size distribution, the AG mill throughput increased from 1639 kg/h in Test 1 to 2559 kg/h in Test 5; an increase of 56%. Since Tests 1 and 5 were conducted with almost identical feed size distributions, the increase in throughput was simply attributed to the change in the quantity of the soft component in the feed. This confirms the findings from the SAG LCT tests that an increase in the proportion of the soft component in the feed leads to an increase in mill throughput. A summary of the experimental conditions and mill performance is presented in Table 4.4.
Table 4.4 – Experimental conditions and mill performance

<table>
<thead>
<tr>
<th>Test</th>
<th>1</th>
<th>3</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mill throughput (kg/h)</td>
<td>1639</td>
<td>1750</td>
<td>2550</td>
</tr>
<tr>
<td>F80 (mm)</td>
<td>59.82</td>
<td>32.13</td>
<td>55.64</td>
</tr>
<tr>
<td>Waste +60 mm (%)</td>
<td>22.0</td>
<td>8.6</td>
<td>0.0</td>
</tr>
<tr>
<td>UG2 +60 mm (%)</td>
<td>0.0</td>
<td>0.0</td>
<td>21.6</td>
</tr>
<tr>
<td>UG2 -60 mm (%)</td>
<td>78.0</td>
<td>91.4</td>
<td>78.4</td>
</tr>
<tr>
<td>Mill load solids (kg)</td>
<td>1400</td>
<td>1648</td>
<td>1333</td>
</tr>
<tr>
<td>Mill filling (%)</td>
<td>26.8</td>
<td>29.0</td>
<td>23.3</td>
</tr>
<tr>
<td>L80 (mm)</td>
<td>76.92</td>
<td>48.73</td>
<td>69.58</td>
</tr>
<tr>
<td>% reported to Trom. OS</td>
<td>1.8</td>
<td>2.9</td>
<td>2.5</td>
</tr>
<tr>
<td>Trom. OS T80 (mm)</td>
<td>9.77</td>
<td>10.28</td>
<td>10.02</td>
</tr>
<tr>
<td>Trom. US P80 (mm)</td>
<td>0.25</td>
<td>0.23</td>
<td>0.29</td>
</tr>
<tr>
<td>Mill % Critical Speed</td>
<td>75.7</td>
<td>75.7</td>
<td>75.7</td>
</tr>
<tr>
<td>Mill gross torque (Nm)</td>
<td>776.5</td>
<td>941.1</td>
<td>785.8</td>
</tr>
<tr>
<td>Mill net power (kW)</td>
<td>12.0</td>
<td>14.8</td>
<td>12.1</td>
</tr>
<tr>
<td>Specific energy (kWh/t)</td>
<td>7.3</td>
<td>8.4</td>
<td>4.8</td>
</tr>
<tr>
<td>Size specific energy (kWh/t -106um)</td>
<td>20.7</td>
<td>22.5</td>
<td>17.1</td>
</tr>
</tbody>
</table>

Test 3 used the same Waste in the +60 mm portion of the feed as Test 1, but the ratio of hard:soft decreased from 22:78 in Test 1 to 9:91 in Test 3. Since both size distribution and the proportion of hard component in the feed were different, comparison between Tests 1 and 3 allows us to investigate the combined effects of mill feed size distribution and feed composition.

It was anticipated that the harder +60 mm Waste would provide more effective grinding media than the softer +60 mm UG2 material, and hence allow the mill to reach an even higher throughput in Test 3 than Test 5. Similarly, in comparing Tests 1 and 3, both using Waste as grinding media, it was anticipated that the reduced Waste component in Test 3 feed would reduce the amount of hard material in the mill, leading to a higher mill throughput in Test 3 than Test 1. It was therefore expected that Test 3 would produce the highest mill throughput (refer to Figure 4.3). However, it turned out that Test 3 produced only a marginally higher mill throughput than Test 1 (7% higher), but 31% less throughput than Test 5. Furthermore, Test 3 throughput would be 31% less than for Test 1 (i.e. 1428 kg/h) when adjusted for a power draw of 12 kW (same as for Tests 1 and 2) and keeping the actual measured specific energy.
Table 4.4 shows that the feed in Test 3 was much finer than in Tests 1 and 5, the F80 decreasing from 59.8 mm in Test 1 and 55.6 mm in Test 5, to 32.1 mm in Test 3 (Fig. 4.6). It seemed that the mill “sanded up” in Test 3, due to the lack of coarse grinding media. This is shown by the low 80 percent passing size in the load (L80) of 48mm for test 3. Hence with multi-component feed, not only will the ratio of hard to soft material affect the AG mill throughput, but also the amount of coarse reasonably competent material. If the AG mill feed does not contain sufficient coarse rocks to act as grinding media, even though the soft component in the feed is very high, the AG mill cannot sustain a high throughput. The pilot plant data clearly shows that both the media competency and quantity control throughput for AG milling of UG2 ore.

Table 4.4 also shows that the AG mill specific energy in Test 3 was the highest (8.4 kWh/t), when compared with Test 1 (7.3 kWh/t) and Test 5 (4.8 kWh/t). This indicates that the lack of coarse competent rocks in the feed resulted in a mill load composed of fine and dense chromite particles, which reduced the AG mill energy efficiency.

4.4 Conclusions

A pilot scale AG milling campaign was conducted at the Anglo American pilot plant in South Africa. Different blends of hard and soft ore (silicate and chromite) in the AG mill fresh feed were tested. The data has clearly highlighted some important and significant trends relevant to AG milling of multi-component feed. The main findings are as follows:

- The hard component accumulated in the mill load and trommel oversize, due to its slow breakdown characteristics, and was retained in the coarser size fractions.
- A certain amount of coarse and reasonably competent particles in the AG mill feed was necessary to maintain good AG performance, as these particles act as grinding media. A lack of coarse competent particles will lead the AG mill ‘sanding up’, resulting in decreased throughput and reduced energy efficiency.
- As the soft component increased in the feed, the mill throughput increased, but the mill product became coarser.
- There appears to be an optimal blend of hard to soft, and coarse to fine materials for UG2 AG milling. The optimal blend is expected to be ore-dependent and mill specific. However, the pilot plant campaign provides useful guidelines on the requirements for AG milling.
- The addition of harder Waste silicate might not be necessary if the mine can provide enough coarse UG2 ore. However, the mill stability will be heavily dependent on the RoM composition. The tests indicate that 20% UG2 coarse ore is adequate. Further pilot tests can be conducted to test the minimum coarse fraction required for stable operation.
The silicate component in UG2 is a low grade ore and open circuit AG milling using this ore as media provides very high throughputs, but at a coarser product.

The harder Waste silicate does enable stable mill operation, but with a throughput penalty as observed in Test 1. Reducing the percentage of Waste from 20% to 8% barely increased the mill throughput, with the Waste remaining very dominant in the mill load.

If there is inadequate coarse UG2 material in the feed (lower limit still to be determined) then some coarse hard Waste should be added to act as the media. The required percentage can be determined using some further pilot tests or through modelling and simulation using the new multi-component model (as described in Chapter 8).

In operation, the harder Waste component should be seen as a grinding media and its use should be kept to a minimum to avoid dilution of the product grade and reduction in the mill throughput. The optimal feed blend conditions can be determined either through additional pilot tests (that can better describe the mill sensitivity to these hard media additions), or via simulations.

The success of AG milling with UG2 ores suggests that the ball addition in RoM ball mills could possibly be substituted with +100 mm Waste rock. However, further testing and modelling are required to determine the ideal amount for addition of Waste, which would have to be regulated at a constant rate, and adjusted to maintain a stable mill load.

The raw data recorded during these pilot tests can be found in Appendix A.3.
Chapter 5  The Dominance of the Competent

This chapter describes a comprehensive experimental campaign conducted at LKAB iron ore operations in Sweden, where the control of AG mills fed by ores with harder and softer components can be challenging. This work included a sampling survey at the Kiruna KA2 mill concentrator, during which the entire AG mill contents were dumped and assayed by size to quantify the build-up of competent waste in the mills. This was supplemented by pilot and locked-cycle laboratory tests at a wide range of feed blends, in order to facilitate the development of an appropriate multi-component AG mill model. The experimental campaign requirements and methodologies, as well as the major outcomes from the industrial, pilot and laboratory multi-component data analysis, are reported in this chapter.

5.1 Introduction

The LKAB Kiirunavaara mine, in Kiruna - Northern Sweden, process a high grade magnetite ore through fully-autogenous grinding lines, comprising an AG mill followed by a pebble mill. Generally the operation of the AG mills is well controlled, but there are periods when the throughput falls off in an uncontrollable manner.

During the P90 AMIRA site work at the newest KA3 concentrating plant in Kiruna (Powell et al., 2011), the likely cause of this was identified. The ratio of competent waste silicates to softer magnetite was identified as the main control variable, which confirms the previous reports by Samskog et al. (1996). The feed size distribution can be controlled in the short-term, by balancing the plus and minus 30 mm fractions from separate feed bins, but this only helps for a few hours before one fraction runs out. The ore is upgraded in the sorting plant through magnetic separation to reduce the proportion of waste silicates in the feed, and this presents an opportunity to manipulate the ratio in advance.

In order to address this issue and generate data for the development of a robust multi-component model of the AG mill, LKAB agreed to survey the mill, then remove the entire mill load, size it, and measure by assay the fraction of the two main components, silicates and magnetite. This would provide a unique set of multi-component data (including the mill feed, load and product), which was invaluable to progressing the development of a multi-component model of AG and SAG mills. The major objective of the survey was to obtain detailed composition by size data in every stream, including the AG mill load, in order to evaluate the dominance of the competent material and generate full scale data for validation of multi-component models.
During a major maintenance work scheduled for the KA2 concentrator, the AG and pebble mill charges would have to be dropped for relining (a practice that is not wide spread) and this lead to the unique opportunity of measuring an industrial AG mill load composition on a size-by-size basis. Since this is not a trivial exercise, there are only a few reported measurements of industrial mill contents (Morrell 1989; Stanley 1974a). However, those studies did not look at the composition of the ores involved. The only reported case where the composition was examined was the work by Mwansa et al. (2006), where the contents of a high-ball load SAG mill was sized in segments along the mill, and the competence and composition of the coarse end of the size distribution was analysed.

Therefore, a comprehensive multi-component experimental investigation was carefully planned and conducted by JKMRC in partnership with LKAB. This work included a sampling survey at the Kiruna KA2 mill concentrator, where the entire AG mill contents were analysed; and a complementary pilot campaign, during which the effect of varying the ratio of hard and soft components was tested. The pilot campaign was performed at the LKAB mineral processing research facilities in Malmberget, Sweden.

5.2 Ore Characterization

The ore deposit in Kiruna is composed of a single continuous high grade magnetite orebody, which is mined using a sublevel caving method. This mining method inevitably allows some dilution with side-wall gangue – typically hard rock textures of silicates associated with phosphates and magnesium oxides. The RoM has a high percentage of magnetite and is further upgraded in a sorting plant, using magnetic separation techniques, before entering the mill concentrator.

During the surveys at the KA2 concentrator, two representative belt-cut samples from the fresh mill feed, and bulk samples of ore and waste from the pre-concentrator, were collected for ore characterization tests. The results from JKRBT impact breakage tests (Shi, Fengnian et al. 2009), Bond (1952) ball mill grinding tests, the Leung (1987) abrasion test, density measurements and XRF assays conducted on these four samples, as well as the associated results, are presented in Table 5.1. The density measurements were conducted using the dry and wet weight method on 31 x 25 mm particles. XRF assays were carried out on the JKRBT products to determine the magnetite grade of the tested samples.
Samples from the mill feed and load, collected during the pilot campaign trials, were used in a more comprehensive ore breakage characterization program. These included JKDWT tests on pure magnetite and silicate, obtained through a careful sorting procedure, to generate the ore-specific breakage function parameters that are later presented in Table 8.1 and used in the multi-component model simulations.

5.3 Industrial Survey

The KA2 comminution circuit flowsheet and the survey sampling points are shown in Figure 5.1. The mill feed comes from a sorting plant, where the RoM is screened at 30 mm, and both +30 mm and -30 mm streams are concentrated using magnetic separators. The coarse and fine upgraded ore is stored in different silos and fed at independent rates into an AG mill, the specifications of which are presented in Table 5.2. The mill discharge is screened by a trommel, producing pebbles for downstream use. The trommel undersize is classified by a large double screw classifier and the coarser stream returns to the mill. The fines are cleaned using magnetic separators, before feeding a pebble mill which is closed with a cluster of cyclones.

- Table 5.1 – Ore and waste characterization tests results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Survey 1</th>
<th>Survey 2</th>
<th>Bulk Ore</th>
<th>Waste</th>
</tr>
</thead>
<tbody>
<tr>
<td>SG (t/m³)</td>
<td>4.4</td>
<td>4.1</td>
<td>4.5</td>
<td>2.7</td>
</tr>
<tr>
<td>RBT, A</td>
<td>57.5</td>
<td>55.9</td>
<td>57.4</td>
<td>79.0</td>
</tr>
<tr>
<td>RBT, b</td>
<td>1.84</td>
<td>1.76</td>
<td>1.73</td>
<td>0.5</td>
</tr>
<tr>
<td>RBT, A*b</td>
<td>106</td>
<td>98</td>
<td>99</td>
<td>37</td>
</tr>
<tr>
<td>Abrasion, ta</td>
<td>0.54</td>
<td>0.38</td>
<td>0.48</td>
<td>0.23</td>
</tr>
<tr>
<td>BWI@75μm</td>
<td>13.2</td>
<td>13.4</td>
<td>12.9</td>
<td>15.8</td>
</tr>
<tr>
<td>XRF %Magnetite</td>
<td>88.6</td>
<td>87.3</td>
<td>86.5</td>
<td>7.3</td>
</tr>
</tbody>
</table>

Figure 5.1 – Sorting plant and KA2 milling circuit flowsheets showing the survey sampling points
Table 5.2 – AG mill and trommel parameters

<table>
<thead>
<tr>
<th></th>
<th>AG Mill</th>
<th>AG Mill (cont)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inside shell diameter, m</td>
<td>6.5</td>
<td>New lining backing plate thickness, mm 110</td>
</tr>
<tr>
<td>Inside liner diameter, m</td>
<td>6.3</td>
<td>Number of lifter rows 24</td>
</tr>
<tr>
<td>Belly length, m</td>
<td>5.3</td>
<td>New lifter height, mm 165</td>
</tr>
<tr>
<td>Inlet trunnion diameter, mm</td>
<td>1800</td>
<td>New lifter angle, ( \theta ) (from face) 27</td>
</tr>
<tr>
<td>cone angle, deg</td>
<td>0</td>
<td>Average lifter height, mm 120</td>
</tr>
<tr>
<td>Installed motor power, kW</td>
<td>4300</td>
<td>Average lifter angle, ( \theta ) (from face) 45</td>
</tr>
<tr>
<td>Speed, RPM</td>
<td>12.7</td>
<td>Speed, % Critical 75.1%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Grate</th>
<th>Trommel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grate aperture size, mm</td>
<td>30 x 90</td>
<td>Diameter, mm 2000</td>
</tr>
<tr>
<td>Grate % open area</td>
<td>3.80%</td>
<td>Length, mm 2860</td>
</tr>
<tr>
<td>Relative radial position</td>
<td>0.76</td>
<td>Aperture size, mm 6 x 15</td>
</tr>
</tbody>
</table>

Two sampling campaigns were conducted according to JKMRC standard protocols (Napier-Munn et al. 2005) at similar operating conditions, as shown in Table 5.3. The surveys provided samples for sizing and composition analysis from every stream plus the mill contents. Although the entire comminution circuit was surveyed, this chapter covers the results obtained for the AG mill circuit only.

The KA2 circuit feeding system can balance the plus and minus 30 mm fractions to control the feed size distribution and assist with process stability. Operating variables were carefully adjusted by experienced mill operators during these surveys, aiming to keep the mill stable at historical maximum throughput conditions.

Table 5.3 – AG Mill operating conditions during the surveys

<table>
<thead>
<tr>
<th>Operating Variables</th>
<th>Survey 1</th>
<th>Survey 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>New feed, tph</td>
<td>374</td>
<td>370</td>
</tr>
<tr>
<td>AG gross power, MW</td>
<td>2.6</td>
<td>2.8</td>
</tr>
<tr>
<td>% Coarse feed (+30 mm)</td>
<td>29.4</td>
<td>31.5</td>
</tr>
<tr>
<td>AG bearing pressure, bar</td>
<td>28.5</td>
<td>29.5</td>
</tr>
<tr>
<td>AG mill filling, %</td>
<td>n/a</td>
<td>30.5</td>
</tr>
<tr>
<td>% Circulating load</td>
<td>23.2</td>
<td>22.4</td>
</tr>
<tr>
<td>AG pebbles, tph</td>
<td>49.2</td>
<td>40.5</td>
</tr>
<tr>
<td>%-45μm AG Circuit Product</td>
<td>37.2</td>
<td>31.6</td>
</tr>
<tr>
<td>AG Circuit F80 (mm)</td>
<td>37.3</td>
<td>48.9</td>
</tr>
<tr>
<td>AG Circuit P80 (mm)</td>
<td>0.134</td>
<td>0.149</td>
</tr>
<tr>
<td>Net* Specific Energy, kWh/t</td>
<td>6.3</td>
<td>6.9</td>
</tr>
</tbody>
</table>

*discounting no-load power
Every stream sample collected during the survey was weighed wet and dry to determine the percentage of solids, and then sieved down to 12.5 mm in the LKAB laboratories. The sub 12.5 mm was split on a rotary splitter to produce sub-samples which were sent to the JKMRC for further sizing down to 38 microns. Every size fraction of each sample was assayed using the X-ray fluorescence (XRF) technique at the LKAB laboratories in Kiruna and ALS (Australian Laboratory Services) in Brisbane.

At the end of the second survey, the mill was crash stopped and the mill dimensions, grate aperture, liner profile and charge level were measured. Then the preparations to collect the mill contents began.

5.3.1 Dropping and Analysis of Mill Contents

The adopted procedure had to be efficient since the survey was conducted during a major maintenance period and there were strict time constraints. Therefore, the mill contents had to be emptied and moved out from the milling area as quickly as possible, to avoid delays in the tight maintenance schedule.

The mill was fitted with an inching (barring) gear, with which it could be slowly rotated. The mill shell hatch was removed to allow the mill contents to be dropped to the floor. Before emptying the mill, some time was allowed for the fine solids to settle, and the water was then carefully poured out and discarded. The mill charge was then dumped on the floor and the rocks formed a natural barrier for holding the dewatered thick slurry.

All dumped material was moved immediately to an asphalted area, previously lined with plastic canvas. The whole pile was stored there and covered with tarps to offer some protection from rain. Any remaining material inside the mill was hosed out and recovered into a large skip bin placed underneath the mill. This bin was siphoned and naturally dewatered outdoors forming a 20 cm thick cake.

Some material losses were inevitable, but approximately 145 t of material was recovered from a total of 160 t mill load mass (calculated from the mill filling). Fines, carried with water and rocks which got stuck in a passage against the mill foundation, were identified as the major cause of material losses. Figure 5.2 shows (clockwise from the top left image): the mill load water being decanted after allowing the solids to settle; the charge dumped on the floor underneath the mill; the empty mill being relined; and the stored mill load.
The material in the skip bin was composed mostly of fines, and a few sub-samples were taken for sizing and composition analysis. These samples were collected by digging spatially distributed holes on 4 x 6 equally spaced matrix. They were combined, dried, and rotary divided into four smaller sub-samples for further analysis.

The mill charge pile was stored for a couple of months over winter, and during the summer it was sieved in order to determine the particle size distribution. Due to the large mass of material, the screening was performed on a full scale portable double-deck screen, as shown in Figure 5.3. Since the charge was to be screened into several fractions, the screening was performed in 3 different runs on separate days. After each screening, the two screen decks were replaced with new ones with smaller combinations of mesh sizes. The following fractions were produced: +90, 70, 50, 30, 10 and -10 mm.

The material was fed onto the screen using a front end wheel loader. An inbuilt scale in the wheel loader recorded the weight of each scoop which fed the total amount of material to the screen. A dumper was placed below each of the three conveyor belts to catch the material and minimize material losses. The process flowsheet is also shown in Figure 5.3.
When the minus 30 mm material was screened, sample cuts were taken from the over, mid and under size streams, to make up a 100 kg-sample of each. The -30+20 mm and the -20+10 mm fractions were split into three sub-samples: the first for XRF; the second for RBT; the third for backup. A sub-sample of the -10 mm fraction was sent to the JKMRC for further screening and assaying on a size-by-size basis.

The +30 mm fractions formed relatively large piles which were flattened out on a concrete paddock, and rocks were manually and randomly taken by four different people in various places around the pad to make up sub-samples as shown in Figure 5.4. Two samples were taken: one sample for XRF (2 x 200 L drums), and another for backup (2 x 200 L drums). The leftovers were discarded.

A total of three XRF repeat tests were performed on each 200 L sample drum. The replicate results allowed the standard deviation of measured values to be determined and to assess the quality of the data. The average relative error for the assay data of these coarse fractions was only 2%, indicating that the sampling was good and measurements were precise.
5.3.2 Multi-Component Data

XRF assays performed on a size-by-size basis on every sample collected during this survey, including mill feed and load, generated a multi-component dataset, where the proportion of soft magnetite and hard silicate components was determined for all streams and sizes. These results are presented in Figure 5.5.

Size fractions below 10 mm in the mill fresh feed present a higher content of waste than in coarser sizes. This can be due to a lower efficiency in the pre-concentration magnetic separation, or to the mining process generating a higher amount of silicates in the fine size range. Therefore, the RoM ore should be analysed to confirm the real cause.
The preferential accumulation of hard silicates in the coarser sizes of the mill load is evident. The upgraded magnetite grade in the feed is 85%, dropping to 75% in the mill contents. Confirming and measuring this effect in an industrial mill was one of the main objectives of this experiment, since the mill load composition plays the major role in grinding performance and mill throughput. The power draw is also affected by the proportion of the two components in the load, given they have significantly different densities. This effect was also observed in the new SAG locked-cycle test (SAG-LCT), developed as part of this thesis. The results from the SAG-LCT and the industrial survey are in good agreement, and are presented at the end of this chapter.

The silicate component appears mostly in the coarser and finer size fractions in the mill discharge, but less so in the intermediate sizes. This is a typical behaviour of hard materials in autogenous mills, i.e. they do not break by impact, and once rounded, they generate a significant amount of fines by abrasion. Since most of the AG mill pebbles are composed of silicates, it might be interesting to remove some of this waste before feeding them to the subsequent pebble milling stage. The increase in the proportion of magnetite pebbles in the pebble mill would increase the charge density and power draw, possibly improving the overall grinding performance of the secondary grinding stage. However, this concept would have to be validated with tests to investigate the effects of charge composition on pebble milling performance, specifically on the wear rate of the softer magnetite media.

5.3.2.1 Mass Balancing

The measured stream data, i.e. flowrates, percent solids, size distributions and assays, were mass balanced using JKMultiBal software in order to assess the data consistency. An excellent agreement between the experimental and calculated data were obtained, as shown by the stream sizing (experimental points and balanced lines) and the comparative balanced and experimental data plots in Figure 5.6; thereby confirming the high quality of the survey and sample treatment.
5.4 Pilot Plant Trials

The pilot plant campaign was carried out at the LKAB mineral processing research facilities in Malmberget, Sweden, shown in Figure 5.7. The specifications of the pilot mill used in this campaign are listed in Table 5.4. The mill was operated in fully autogenous mode and open circuit configuration during all trials.

<table>
<thead>
<tr>
<th>Pilot Mill</th>
<th>Pilot Mill (cont.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shell diameter, m</td>
<td>1.5</td>
</tr>
<tr>
<td>Mill length, m</td>
<td>1.7</td>
</tr>
<tr>
<td>Diameter (inside liners), m</td>
<td>1.414</td>
</tr>
<tr>
<td>Belly length (inside liners), m</td>
<td>1.571</td>
</tr>
<tr>
<td>Trunnion diameter, mm</td>
<td>0.45</td>
</tr>
<tr>
<td>Feed end cone angle, deg.</td>
<td>0</td>
</tr>
<tr>
<td>Speed, RPM</td>
<td>28.5</td>
</tr>
<tr>
<td>Speed, % critical</td>
<td>80</td>
</tr>
</tbody>
</table>

Figure 5.7 – LKAB pilot plant and AG mill used in the P9O project trials

When the KA2 concentrator was surveyed, a sample of approximately 60 tonnes of each component was collected from the pre-concentrator ore (magnetite) and the +30 mm waste (silicates) streams at the sorting plant. These samples, considered to be representative, were sent to the pilot plant site prior to the testwork period. The magnetite minus 30 mm was used as received in all tests, but the coarse feeds were previously screened into two size fractions; +70 mm and -70+30 mm. The ratio of these size fractions in the mill feed was kept the same for every trial, in order to keep a constant feed size distribution which reproduced the industrial mill feed size.
Table 5.5 shows the five different ratios of hard to soft (waste/magnetite) components in the mill feed trialled during this campaign. The ratio of hard to soft was changed through progressive additions of hard silicates in the +30 mm size fractions. An additional test (5) was carried out using an upgraded +30 mm magnetite ore which had been previously sorted using a magnetic separation rig shown in Figure 5.8.

Table 5.5 – Mill feed composition during the pilot trials

<table>
<thead>
<tr>
<th>Test</th>
<th>+30 mm Waste</th>
<th>+30 mm LKAB ore</th>
<th>-30 mm LKAB ore</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>30</td>
<td>70</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>26</td>
<td>70</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>22</td>
<td>70</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
<td>15</td>
<td>70</td>
</tr>
<tr>
<td>5*</td>
<td>0</td>
<td>30</td>
<td>70</td>
</tr>
</tbody>
</table>

*(cleaner +30 mm ore)*

Figure 5.8 – Magnetic separator used to upgrade the LKAB average +30 mm mill feed ore

The ability to maintain a consistent feed size and composition during the pilot trials was crucial to the success of the campaign. The -30 mm ore was fed to the mill via the fines hopper and conveyor belt, and its feed rate was maintained via a set-point using a process control system. The +30 mm components were manually added to the mill at set time intervals, aiming to maintain a stable feed size distribution in the coarse sizes. This feeding system is shown in Figure 5.9.
The operational philosophy adopted for these trials was to find the feed rate which resulted in a steady operation at a mill load level of 28%. This was particularly difficult, because the relationship between the mill load cell readings and the mill volumetric filling changed according to the amount of light silicates in the load. Therefore, periodical crash stops had to be conducted during every trial to take measurements of the mill filling and calibrate the relationship for different conditions.

Once steady-state conditions were reached, usually after 4-6 hours of operation, the mill was operated for one additional hour before taking any samples. Trommel over- and under- size samples were collected in seven increments taken over a one hour period. The mill was crash-stopped after each sampling survey to measure its volumetric filling and to collect a sample of the -30 mm feed material. The +30 mm feed component samples were obtained at periodic increments by operators who were feeding them during the tests. At the end of trials 1, 4 and 5, the entire mill contents were manually collected for analysis, as shown in Figure 5.10. All samples were sized and the proportion of silicate to magnetite in every size fraction was determined using the XRF technique.
The experimental conditions achieved during these trials are presented in Figure 5.11, which shows that the mill feed size and volumetric filling were consistent from test to test.
The trends obtained during these tests clearly show that the relationship between mill throughput and the presence of hard material in the feed is non-linear and conforms to what would be expected from the correct inverse weighting of capacity (Appendix B). On the other hand, the specific energy follows a linear relationship. This can be explained by the fact that the drop in throughput is followed by a slow decrease in charge density and power draw, at the same volumetric filling, as the amount of silicate increases in the feed and consequently in the mill load. The mill product size also changed with increasing silicate additions, becoming finer due to the large amount of fines generated by the abrasion of this component. In addition the lower charge density, would translate to lower breakage energies, and hence reduced impact breakage.

5.5 SAG-LCT

The SAG-LCT is a laboratory scale test developed as part of this thesis to characterize ore interaction when treating multi-component feeds. Using samples from LKAB ore and the waste collected during the surveys, a series of mixture experiments were undertaken using the SAG-LCT, to generate data to compare to both pilot and industrial results.

The test methodology aimed to represent an open circuit SAG mill steady-state continuous operation, with grate classification. The SAG-LCT procedures and the operating conditions are described Chapter 3 and summarized in Table 5.6.

Table 5.6 – Summary of the SAG Locked-Cycle Test standard procedures and operating conditions

<table>
<thead>
<tr>
<th>Procedure/Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>600 mm diameter mill running at 75% of Cs and 25% volumetric load;</td>
</tr>
<tr>
<td>Semi-Autogenous mode (5% balls provide sufficient grinding energy given the small mill size);</td>
</tr>
<tr>
<td>Feed top size of 53 mm (full size distribution), using different proportions of components;</td>
</tr>
<tr>
<td>Feed size distribution is the same for each component;</td>
</tr>
<tr>
<td>Closing size of 13.2 mm (Gilson screen), which represents the mill grate classification;</td>
</tr>
<tr>
<td>Finished product (-13.2 mm) replaced by equal mass of new feed material;</td>
</tr>
<tr>
<td>Make up using feed material previously sized into √2 sieve series between 53 and 13.2 mm;</td>
</tr>
<tr>
<td>Constant grinding cycle time (20 min);</td>
</tr>
<tr>
<td>Grindability (net grams of undersize / time) is the parameter of convergence (± 5%);</td>
</tr>
<tr>
<td>Full sizing and composition analysis (size-by-size) of Mill Load (Gilson) and last three cycle screen undersize products (Ro-Tap).</td>
</tr>
</tbody>
</table>
The standardized SAG-LCT procedure allows the investigation of the mill response and of the interaction between the different ore components, at a range of different blend conditions, using a relatively small amount of material. The reproducibility is also one of its main advantages, as the experimental conditions were well controlled. The test was designed to generate results which can provide a better understanding of the blend effect on mill grindability, mill load, product composition and product size distribution. The only drawback to this test is the amount of work involved. Each test takes approximately 30 man-hours to be accomplished fully (i.e. complete all grinding cycles, as well as the screening and composition analysis).

The SAG-LCT was used to characterize the interactions between the LKAB magnetite and silicate components, and to evaluate the mill performance response at laboratory scale. Five blends of LKAB ore and waste were tested, and one repeat test on a 1:1 mixture was conducted to verify the reproducibility of the results. The major effects of a varying ratio of hard silicates in fresh feed for these SAG-LCT tests, compared to pilot and industrial results, are presented in Figure 5.12. The feedrates are normalised to the maximum obtained for each mill, in order to conduct direct comparisons of the influence of blend ratios on throughput, in different size mills with dramatically different feedrates.

Figure 5.12 – SAG-LCT results compared to pilot and industrial survey test outcomes
The ability to predict the fraction of each component in the mill load, as a function of the fraction in the feed, is a key aspect to understanding and modelling different components in AG/SAG milling and the SAG-LCT can provide an estimate of the non-linearity of the build-up response for any two components. The lower part of the graphs in Figure 5.12 shows the effect of the build-up of hard material in the mill load relative to the feed.

The analysis of the mill load composition for LCT and pilot tests are in good agreement, despite the difference in scale, showing a similar relative build-up of hard material in the mill load. However, this relative build-up effect was lower for the industrial mill load measurement. One reason for this could be the fact that the breakage energies in the industrial mill are higher, mitigating the differential breakage response between the two components. Therefore, the build-up of a hard material in the mill load is believed to be a function of the components’ hardness differential, as well as the available comminution energy.

The results to date suggest the SAG-LCT is a robust tool for not only assessing ore interactions but also in evaluating the effect of mixtures on mill performance. However, the effect of scale (i.e. energy) should be taken into consideration for quantitative scale-up and a model is required for this purpose.

5.6 Conclusions

An intensive series of laboratory, pilot and industrial scale tests, measuring the response of AG mills to a range of multi-component feeds (consisting of a softer magnetite and harder silicates waste), has been conducted. This included surveying the operation of an industrial mill, then dropping and sizing the entire contents, followed by size-by-size assaying of the mill contents and all streams around the circuit. This is a unique set of data, as no comparable case has been published elsewhere.

Different blends of hard and soft material (silicates and magnetite) were tested in the LKAB pilot and SAG-LCT mill. The data shows the effect of treating multi-component feeds in AG milling. The hard component clearly stands out in the comminution process due to its slow breakdown characteristics. As the proportion of the hard component increases in the feed, the mill throughput decreases, but the mill product became finer. The hard component also tends to accumulate in the mill load and trommel oversize.
The recently developed SAG Locked-Cycle Test proved to be a robust method to characterize interactions and the mill response to multi-component feeds, using LKAB ore components. It can be used as a tool to determine the actual proportion of hard and soft components in the mill contents for a given feed and provide an estimate of the mill throughput response according to blend. The trends observed in the SAG-LCT have been confirmed in the pilot scale tests and industrial survey results for LKAB ore components.

The raw data recorded during the LKAB plant survey, pilot tests and SAG-LCT experiments can be found in Appendices A.4, A.5 and A.6.
Chapter 6  Preliminary Modelling

Modelling and simulation exercises using JKSimMet software and the multi-component experimental data described in Chapters 3, 4 and 5 were conducted as proof of concept. The preliminary findings described in this chapter provide the insights necessary to develop a viable multi-component AG/SAG model which is presented in Chapter 7.

6.1  Modelling the SAG-LCT data

An attempt was made to apply the existing JKSimMet AG/SAG model to the SAG-LCT data, although the data had not originated from a continuous operation, but actually from a locked cycle laboratory test. The SAG-LCT is a semi-batch test, but it is designed to represent a continuous operation in steady state. Therefore, the mill load is considered to be the oversize (recirculating) material, with its size distribution truncated at 13.2 mm – the closing screen size.

The data obtained from the LCT grinding tests, on mixtures of quartz and iron ore (BIF), was used to fit the breakage rates, using Leung’s AG/SAG model 430 in JKSimMet. Several modelling approaches were adopted, and simulations to predict the mill throughput were run for each. The other model parameters such as $X_g$ and $X_m$ were kept constant in all fittings and simulations. Additionally, all simulations were run at the same mill load volume.

6.1.1  The bulk feed approach

The first method was called the bulk feed approach, in which two components were combined in one bulk feed, and one mill was employed in JKSimMet. The combined breakage function $A$, $b$, and $t_a$ parameters (weighted averages) were calculated from the individual component $A$, $b$ and $t_a$ values and the component blend ratios, even though this averaging method is not appropriate as discussed in Appendix B. The traditional fitting method, using bulk feed and product sizing data together with the weighted $A$, $b$ and $t_a$ values, was applied to each different feed blend to generate a set of breakage rates.

The calculated bulk breakage rates, and the models fitted to them, can be seen in Figure 6.1. Third order polynomials were used to model the rates for 4, 16 and 44.8 mm particles, and a linear model was used for the 0.25 mm particles. The 128mm rate was kept constant since the SAG-LCT feed top size was 53mm.
These accessory models were used to describe the effect of breakage rates and assist the current JKSimMet model to better predict the behaviour of blends of hard/soft materials in AG/SAG mills. This approach forced the Leung model rates to reflect the breakage of each component in the presence of the other material. The breakage rates calculated as a function of feed composition, together with the weighted appearance function parameter (A, b and tₐ), were used to predict mill throughput for various feed blends. The simulations outcome was a non-linear throughput response as presented in Figure 6.2.
6.1.2 The two identical mills approach

Two separate mills were employed in JKSimMet to treat each component in the feed separately: one representing the hard component; the other the soft component. Two sets of breakage rates were fitted to their individual ore specific data (i.e. to their own breakage function, throughput, mill charge and size distributions of the feed, product and load).

In the two identical mills approach, the breakage rates were determined for each component in a mixture and linear models were fitted to correlate the calculated rates to the amount of soft quartz in the fresh feed, as shown in as shown in Figure 6.3.

The regressed equations were used to estimate the breakage rates for various proportions of the soft component in the feed blends. Simulations were performed using the estimated breakage rates for each component. The combined mill throughput from the two components was recorded when reaching the identical 25% charge by volume. The predicted mill throughput, in relation to the proportion of soft component, is given in Figure 6.4. It shows a non-linear trend, which agrees with the observed data, but is not accurate.
6.1.3 Two variable length mills approach

Similar to the previous approach, two separate mills were used in JKSimMet. Instead of using identical length mills, this approach used variable mill lengths, scaled according to the proportion of each component in the mill load. The mill load was estimated from the proportion of soft component in the fresh feed, by a second order polynomial function fitted to the experimental data as shown in Figure 6.5.

![Figure 6.4 – Simulations of mill throughput using the two identical mills approach](image1)

![Figure 6.5 – Mill load composition in relation to mill fresh feed composition](image2)
Figure 6.6 shows the ore specific breakage rates calculated using the two variable length mills approach. Simple linear models were fitted to all sizes for quartz and a third order polynomial was used to model the rates for BIF.

The regressed relationship between the breakage rates and the proportion of soft component was used to predict the breakage rates at other feed blend conditions, and JKSimMet simulations were conducted to predict the normalised mill throughput. The results are shown in Figure 6.7.

Figure 6.7 – Simulations of mill throughput using the two variable length mills approach
The simulation outcomes presented in Figures 6.2, 6.4 and 6.7 illustrate that the current JKSimMet model can better handle multi-component simulations when breakage rates are changed in relation to feed composition, but these have to be based on measured data so this approach has no predictive capability. Therefore, a model structure that can account for changes in breakage rates when changing the mill feed composition is critical.

6.2 LKAB Pilot Test Data – Two Mills Approach

The pilot plant campaign using the LKAB ore generated multi-component data (i.e. size and composition distributions) for the five conducted trials, of which three had the entire mill load analysed. The data were used to assess the build-up of hard material in the load, which was presented in Chapter 5, and to show how the varying ratio of hard component in fresh feed affects the breakage rates.

Although JKSimMet v5.2 does not have a multi-component model structure, (i.e. it cannot accommodate assay-by-size data), the breakage rates of each component were calculated using a new modelling concept. The new methodology relies on the use of two mills, as described in Section 6.1.3, with lengths scaled according to the volumetric proportion of each component in the mill contents. Additionally, a weighted ore density was applied to reproduce the measured average charge density, providing more realistic breakage energies. The concept flowsheet, showing measured flowrates and P80s against the model-fitted calculated values, are presented in Figure 6.8. The fitted product sizes and flowrates are in fair agreement with the experimental data.
Figure 6.8 – Multi-component modelling concept on JKSimMet and Exp vs. Fit results for LKAB pilot Test 1

The model was able to reproduce accurately the mass of each component in the mill contents and their product size distributions, as shown in Figure 6.9. However, the predicted mill load size distribution for both components is coarser than the measured values, especially for magnetite. This arises from this model relying on the characteristic mill load ore size ($S_{20}$) to calculate the breakage energies (Leung, 1987). The experimental magnetite mill load size distribution is finer than the bulk load, and therefore, when not including the coarser silicates in calculating the mill contents, the applied energies are too low. Consequently the model needs to adjust its $S_{20}$ to reproduce the real energies experienced by magnetite in the mill. When modelled as a single mill and both components in the load are taken into account, this deficiency will not be an issue.
The Leung (1987) AG/SAG mill model calculates the mean energy level for a given load within a mill, and uses ore characterisation parameters to describe the particles response to these breakage energies. The breakage and transport process within the mill are then modelled in a steady state mass flow format, which uses adjustable breakage and discharge (or “mass transfer” events) rates. These rates can also be interpreted as the mill model response to a presented set of ore types and operational conditions. When the mill feed composition changes, the load and energy distributions are affected, and therefore the breakage rates will reflect these changes. This effect is shown in Figure 6.10.

Figure 6.9 – Pilot Test 1 (ave. LKAB ore) product size distribution (Exp vs Fit)
The shifts in breakage rates are quite consistent as the feed blends change. As the blend shifts towards a harder average (more waste silicates), the breakage rate at the coarse end (+40 mm) decreases while the fine end (-10 mm) rates increase. There is one exceptional point at the coarsest knot point for the Magnetite, which is due to the lack of coarsest material in the feed, so these rates are not robust and are sensitive to small changes in feed and load. Therefore, it is likely that this point is not indicative of a real effect. Although the fitting SDs is small (2 to 5%), it is hard to assess the significance of these differences in the calculate breakage rates, as there was not an opportunity to conduct a repeat test.

The current JKMRC variable rates model (Morrell & Morrison 1996) describes how these breakage rates are affected by operational conditions like mill speed, feed size, recycle ratio, charge and ball levels, but not feed composition. Consequently, the new multi-component model must be able to account for the effect of changes in the mill feed composition.

6.3 Anglo Pilot Test Data – Two Mills Approach

The pilot plant campaign carried out using UG2 ore and reported in Chapter 4 also generated multi-component data. Using this information, the aim of JKSimMet modelling was to find if it is possible to correctly simulate the change in performance when the condition is moved to Test 1, given the rates from Test 5, by accounting for:

- The change in component distribution in the feed, and
- The change in hardness of the hard component.

Test 5 was considered the baseline test, representing the highest throughput. The four surveyed streams were separated into silicate and chromite streams, on a size-by-size basis. This resulted in two sets of independent component data, which were then used to determine the fraction of each component in the feed, load, trommel oversize and undersize.
Two streams were developed in JKSimMet, using Leung’s Model 430, to represent the two component streams: silicate and chromite. The corresponding flowrates, load masses, sizing and density data from the surveys were then entered into the respective flowsheets, as shown in Figure 6.11.

![Figure 6.11 – Flowsheets for ‘soft’ chromite and ‘hard’ silicate component streams showing Exp vs. Fit data](image)

The breakage rate determination was expected to be critical in this evaluation. Fitting the rates using only the information pertaining to each component was questioned as a valid approach, as discussed in the previous section. Scaling the mill length, in proportion to the quantity of each component in the mill, as described in Section 6.1.3, is an alternative approach sometimes used with modelling different components in SAG mills. However, this would result in an unrealistically short “apparent” mill length for the chromite, which was estimated to represent only 5 to 10% of the load in Tests 1 and 5 respectively. Since a length of 10 cm seemed unrealistic, a new approach was tested.

This alternative approach assumes the other component in the mill acts as grinding media, and as such occupies the volume of the mill and therefore is simulated as a pseudo SAG mill. In this way, the same mill dimensions were used for both chromite and silicate streams. The equivalent media was estimated from the load size distribution and volumetric filling in the mill, and represented in the Leung model as the ball load. To compensate for the difference in specific gravity between balls (SG = 7.8 g/cm³) and ore (SG = 2.7 or 4.0 g/cm³, depending on component), the ball size was reduced to reflect the equivalent ore size. The silicate rock’s top size of 180 mm was converted into 125 mm ball size and the chromite top size of 125 mm into 106 mm ball size. Similarly, the actual distribution of media in the standard √2 fractions was estimated from the surveyed loads. Hence in Test 5, the streams were represented in Figure 6.12 as follows:
This approach essentially forces the Leung model rates to reflect the breakage of each component in the presence of the other component assumed to be acting as media. Hence in Test 5, the silicate are being ground with only 1.7% of chromite media in the mill, whilst the chromite is being ground with 21.6% of silicate media in the mill. In each case, the ball load (percentage by volume) represents the volumetric percentage of the other component in the mill load.

### 6.3.1 Model Fitting results

A total of ten parameters were fitted for each component. AG Mill: grate size ($X_g$), fine size ($X_m$), ln(breakage rates) at knots 1 to 5 (R1, R2, R3, R4 and R5). Trommel: efficiency ($\alpha$), corrected cut size ($d_{50c}$) and water split (C). The ore parameters used in the fitting for chromite and silicate components are given in Table 6.2.

The fitted breakage rates for Test 5 are shown in Figure 6.13, and the calculated standard deviation for the fitted values were small (2 to 5%) to be plotted as error bars. Without repeat tests, it is difficult to evaluate the significance of the differences on the natural logarithms of the rates for each component. $X_g$ is consistent with the grate size and the sizing data reported in Chapter 4. $X_m$ is quite high, reflecting the efficient pumping of the pilot plant mill. The trommel parameters are consistent with the aperture and sharp separation expected in the pilot plant, as shown in Table 6.1. The fitting was more accurate for the silicate stream than for the chromite, which is not surprising given the assumptions of this approach. Nevertheless, the quality of the fit was good overall, as reflected in Figure 6.14.
Figure 6.13 – Comparison of breakage rates for ‘soft’ and ‘hard’ component streams in Test 5

Table 6.1 – Comparison of $X_g$, $X_m$ and trommel parameters for ‘soft’ and ‘hard’ component streams in Test 5

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Chromite</th>
<th>Silicate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grate Size - $X_g$ (mm)</td>
<td>13.6</td>
<td>13.6</td>
</tr>
<tr>
<td>Fine Size - $X_m$ (mm)</td>
<td>6.3</td>
<td>6.6</td>
</tr>
<tr>
<td>Sharpness of Efficiency Curve - Alpha</td>
<td>31</td>
<td>33</td>
</tr>
<tr>
<td>Water Split to Fine Product (%)</td>
<td>99.99</td>
<td>99.97</td>
</tr>
<tr>
<td>Corrected D50 - d50c (mm)</td>
<td>4.04</td>
<td>3.96</td>
</tr>
</tbody>
</table>
Figure 6.14 – Quality of model fitting ‘soft’ and ‘hard’ component streams in Test 5

6.3.2 Simulation of Test 1 using Test 5 parameters

A simulation of Test 1, using the Test 5 model parameters, was performed to see if it is possible to accurately show the significant drop in throughput when the mill was fed with a harder +60 mm fraction and somewhat lower chromite grade. The simulation was carried out using actual feed size distribution, and the components were separated as outlined, but the harder Waste silicate component ore parameters were used to match the JKDWT results for Test 1 ($A*b$ of 70 vs. 123 in Test 5). Additionally Test 1 rates were re-fitted using the actual trial experimental data, and the results of simulations using these rates are also compared in Table 6.2 and Figure 6.15.
Table 6.2 summarises the comparison between the experimental and simulated flows and the P80’s of the initial simulation results. Figure 6.15 presents the experimental and simulated stream size distributions. The overall match is reasonable, suggesting that the approach reasonably described the effect of the load on the grinding rates.

**Table 6.2 – Experimental data of Test 1 vs. the simulated results using Test 5 breakage rates and the re-fitted breakage rates respectively**

<table>
<thead>
<tr>
<th>Equipment</th>
<th>Port</th>
<th>Solids (kg/h)</th>
<th>80% passes (mm)</th>
<th>% Passing 150μm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Exp</td>
<td>Sim</td>
<td>Fit</td>
</tr>
<tr>
<td>AG Mill - Chromite</td>
<td>Prod</td>
<td>541</td>
<td>541</td>
<td>541</td>
</tr>
<tr>
<td></td>
<td>Load</td>
<td>65</td>
<td>93</td>
<td>78</td>
</tr>
<tr>
<td>AG Mill - Silicate</td>
<td>Prod</td>
<td>1098</td>
<td>1098</td>
<td>1098</td>
</tr>
<tr>
<td></td>
<td>Load</td>
<td>1335</td>
<td>1467</td>
<td>1336</td>
</tr>
</tbody>
</table>

**Figure 6.15 – Simulation of ‘soft’ and ‘hard’ component streams in Test 1 using the Test 5 model parameters and the Test 1 fitted parameters**
The corresponding simulation shows a much higher load estimate, which suggests the Test 5 rates are not applicable in Test 1, if the ore hardness is in fact different. Fitting the rates to Test 1 marginally improved the accuracy, supporting the fact that the rates fitted to Test 5 data, for a softer silicate component at a lower mill filling, do not completely account for the milling conditions in Test 1. Although the fitting standard deviations were small (2 to 5%), without repeat tests, it is difficult to access the significance of these small changes in breakage rate curves, shown in Figure 6.16. However, it is known to be related to the mill charge composition and a model structure that can iterate using multi-components is required to account for this effect.

![Figure 6.16 – Breakage rates fitted for chromite and silicate components in Test 1 and Test 5](image)

6.4 Conclusions

This preliminary investigation has demonstrated that there is a possibility that the existing JKSimMet AG/SAG model structure can be manipulated to describe multi-component grinding data. However, a major limitation is that this structure cannot accept multiple sets of breakage functions and breakage rates for different feed components.

A number of JKSimMet approaches have been investigated as part of the preliminary modelling. The outcomes show that they can predict the non-linear relationship between the mill throughput and the proportion of soft component in the feed, providing the rates are changed in relation to the feed composition.

As part of the preliminary modelling attempts, the LKAB pilot plant data were also modelled using Leung’s model in JKSimMet, showing that magnetite and silicate have different breakage rates which significantly vary with feed composition. Hence the use of component specific breakage rates must be considered in modelling and simulation in order to accurately represent the AG performance for multi-component feeds.
The Anglo UG2 pilot plant data were used in the most comprehensive evaluation using the current JKSimMet AG/SAG model. This modelling approach uses two separate mill circuits for each component, with each component acting as grinding media for the other. Simulations using the model parameters derived from one test were found to give reasonable predictions of the performance at new set of conditions. This preliminary trial highlighted an encouraging approach for modelling multi-component ores using the current JKSimMet model. However, the simulations were found to be sensitive to the ore parameters and the approach requires prior knowledge of the proportion of each component in the mill contents at steady state, hence rendering it of little practical use in simulating new feed blend scenarios.

The next chapter presents the new multi-component AG/SAG model structure developed in this thesis that can accommodate simultaneous iterations with the different components present in the feed, providing a way to account for the breakage and discharge of multiple ore components as well as the preferential accumulation of hard ore types in the mill load.
Chapter 7  Model Statement and Structure

This chapter introduces the new multi-component model structure and provides a detailed explanation of the approaches adopted to model the key mechanisms present in AG/SAG mills, and behaviour of feed components with different physical properties, including hardness and density. The main model features and assumptions are also discussed.

7.1  Introduction

The AG/SAG Model in the JKSimMet simulation package is based on the Leung (1987) framework, shown in Figure 7.1. This model structure was previously described in Chapter 2 and has been used as a platform to develop the multi-component model presented in this chapter because it has proven to be robust enough to serve as a basis for future developments. Although more advanced AG/SAG mill model structures are currently available, they are either not widely validated (Delboni, 1999; Valery, 1997) or lack published parameters (Morrell, 2004) and therefore, were not considered ideal for this development.

The proposed multi-component AG/SAG mill model is similar to, and based on, the current JKSimMet version of Leung’s model. In the new version, however, the behaviour of each component is described using a separate perfect mixing model equation, as illustrated in Figure 7.2. The major features of this model are listed below:
• More accurate calculation of specific comminution energy (Ecs)
• Independent breakage functions (A, b and tₐ) for each component
• Independent breakage and discharge rates for each component
• Accounts for the effect of blending in the mill power draw calculation

![Multi-Component AG/SAG mill model structure](image)

According to the preliminary modelling findings, the breakage rates for each component vary according to their proportion in the mill fresh feed. The reason behind this variation is that different feed blends result in different compositions in the mill charge, which act as grinding media. Therefore, the model must determine the correct steady-state ratio between hard and soft components in the mill load, in order to provide realistic breakage simulations.

### 7.2 Size Reduction and Throughput

The perfect mixing model proposed by Whiten (1974) to describe the size reduction processes inside an AG/SAG mill at steady state, illustrated in Figure 7.3, can be represented by the following equations:

\[
\begin{align*}
& f - R \cdot s + A \cdot R \cdot s - D \cdot s = 0 \\
& p = D \cdot s
\end{align*}
\]

where:

- \( f \) : feed rate
- \( p \) : product rate
- \( R \) : breakage rate
- \( s \) : mill contents
- \( D \) : discharge function
- \( A \) : appearance or breakage distribution function
The breakage and discharge mechanism functions were combined in the new multi-component model, using the same Equations 7.1 and 7.2 to describe the size reduction and throughput of an AG/SAG mill in steady-state. However, the new model treats each component in the mill feed individually, using a separate set of perfect mixing equations.

### 7.3 Appearance Function

The appearance or breakage distribution function describes the progeny size distribution from each breakage event. According to Leung (1987), this function explains the breakage in terms of high (impact) and low energy (chipping/abrasion) breakage.

The high energy breakage is ore-specific and related to the breakage energy as defined by the equation below:

\[
t_{10} = A(1 - e^{-b \cdot E_{cs}})
\]  

(7.3)

where A and b are ore-specific parameters obtained from the JK Drop Weight Tests (DWT), Ecs is the specific comminution energy and \( t_{10} \) is the progeny percent passing one tenth of the initial particle size. The \( t_{10} \) is used to generate a full product size distribution for each breakage event using cubic splines (Narayanan, S. S. & White 1988).

Leung’s model calculates the specific comminution energy (Ecs) for each size fraction in the mill load using the highest energy reference level (E₁), which is related to the average size of the coarsest 20% of the rock charge by mass (S₂₀), and the relationship proposed by Austin et al (1984) in Equation 7.7. This is represented in the model as follows:
\[ S_{20} = (p_{100} \times p_{98} \times p_{96} \ldots p_{80})^{1/11} \]  
(7.4)

\[ E_1 = \frac{4}{3} \pi (S_{20})^3 \rho g D \]  
(7.5)

\[ \text{Ecs}(1) = \frac{E_1}{(4/3 \pi (X_1)^3 \rho)} \]  
(7.6)

\[ \text{Ecs}(i) = \frac{\text{Ecs}(1)}{(x_i/x_1)^{1.5}} \]  
(7.7)

where \( D \) is the mill diameter in metres, \( \rho \) is the ore density and \( \text{Ecs}(1) \) is the specific comminution energy for the top size \( (X_1) \). The calculated \( \text{Ecs} \) values for each size fraction are then used in Equation 7.3 to obtain a \( t_{10} \) for each size fraction.

The low energy breakage function is also ore-specific and defined by a single parameter \( t_a \), obtained from a mill tumbling test that is typically carried out in combination with the JKDWT. The value of \( t_a \) is assumed to be the same for all size fractions and is used to define the abrasion progeny.

The high and low appearance functions are combined proportionally, using the following relationship:

\[ a = \frac{t_{LE} \times a_{LE} + t_{HE} \times a_{HE}}{t_{LE} + t_{HE}} \]  
(7.8)

where, \( a_{LE} \) and \( a_{HE} \) = low and high energy appearance functions

\[ t_{LE} \) and \( t_{HE} \) = low and high energy t values

Since \( t_{LE} \) is considered to be the same for all sizes and \( t_{HE} \) increases as particles size decreases, the combined appearance function implies that abrasion will dominate for coarse particles and impact breakage for fines. More detailed information on the Leung’s appearance function structure and related breakage tests can be found in Chapter 2 and elsewhere (Leung 1987; Napier-Munn et al., 2005).
The new model adopted Leung’s appearance function structure, but introduced the use of independent breakage function parameters (A, b and tₐ) for each ore component. These can be obtained using the standard breakage characterization procedures (i.e. JKDWT or JKRBT and Leung’s abrasion test). However, the mill feed ore samples to be tested, need to be carefully sorted into their different components, at least between 13.2 and 63 mm. Many sorting techniques can be used for this purpose, depending on which characteristic is more appropriate for distinguishing and separating them (e.g. colour, density or magnetic properties). In cases where there is no correlation between rock competency and easily measureable physical properties, the selection is likely to be more difficult and should be conducted in a joint effort between geologists, mine engineers and metallurgists.

The energy calculation in the new model was also refined by using the specific gravity values of each component, as well as their distribution and proportion in the mill load. The bulk SG in the mill load coarsest 20% size (S₂₀) is used in Equation 7.5 to calculate the average energy level (E₁), which is considered to be the same for all components in the mill. Then the SG of each component is used in Equation 7.6 to obtain the Ecs for the top size particles, which are then scaled for smaller sizes using the same relationship shown in Equation 7.7. In this way, the breakage function is calculated using Equations 7.3 and 7.8, with distinct specific energies and ore property parameters (A, b and tₐ) inputs. This is a more realistic way of describing the breakage for each ore type in the mill, and has contributed significantly to the model integrity. Figure 7.4 shows a worked example of the effects of multi-component ores in the calculation of breakage energy.
Figure 7.4 – The effect of multi-component ores in the calculation of breakage energy

This modification in the calculation of breakage energy is particularly relevant in cases when soft and hard components have different specific gravities (e.g. magnetite and silicate or chromite and silicate). In these cases, the build-up of a hard component in the mill load affects the charge density and consequently the nominal breakage energy level in the mill ($E_1$). Additionally, for a given energy level ($E_1$), the heavier component experiences relatively lower specific energy (kWh/t) levels than the lighter component.

7.4 Transport and Discharge

According to Leung (1987), the mill discharge rate for each particle size is a product of the maximum discharge rate through the grate ($hr^{-1}$), multiplied by the grate classification function for each size. This is described using the following equation:
\[ d_i = D_{\text{max}} c_i \] (7.9)

where:

- \( d_i \): discharge rate of size class \( i \) (hr\(^{-1}\))
- \( c_i \): classification function value for size class \( i \) (0 to 1)
- \( D_{\text{max}} \): maximum discharge rate (hr\(^{-1}\))

The grate classification function \((c_i)\) has a simple shape, characterised by two or three distinct regions, as shown in Figure 7.5, depending on whether pebble ports are used or not.

![Figure 7.5 – Mill grate classification function (Napier-Munn et al. 2005)](image)

Particles up to size \( X_m \) presented to the grate will always pass through it. \( X_g \) is the grate aperture and \( X_p \) is the pebble port size and \( f_p \) is the pebble port fraction of the total open area.

However, the multi-component experimental data obtained during this research has suggested that the maximum discharge rates \((D_{\text{max}})\) and \( X_m \) parameters may be different for each component, due to the effects of rock density and shape. For example, the chart in Figure 7.6 shows the discharge function where silicate has a higher discharge rate than magnetite.
Therefore, scale factors for $D_{\text{max}}$ and $X_m$ were used in the new multi-component model to account for a differential behaviour in transport and discharge of each component. At this stage, $D_{\text{max}}$ and $X_m$ can be either fitted or calculated using experimental data when available, and a worked example is show in Chapter 8. However, further research is required to understand and model this effect according to the volumetric flowrate, the proportions of each component, and the differences in specific gravity.

Because of the scatter on the calculated discharge rates at the small sizes, which is due to the mathematical sensitivity of the calculation $d = p/s$ (where $s$ is a very small number), one may argue whether the $D_{\text{max}}$ of each component is actually different. However, the significance of this difference was statistically verified in Chapter 8 by comparing the overall model variance when the $D_{\text{max}}$ is independent or kept the same for each component.

The maximum discharge rate through the grate per unit of time ($D_{\text{max}}$) is found iteratively, in order to satisfy the following empirical relationship used by Leung’s AG/SAG model, and relates the hold-up of slurry to the total mill feed volumetric flowrate (Austin et al. 1977):

$$L = m_1 F^{m_2}$$

(7.10)

where:

- $m_1, m_2$: constants ($m_1 = m_2 = 0.35$ as default)
- $L$: fraction of mill occupied by below grate size material
- $F$: normalized volumetric slurry feed rate, in mill fulls per minute ($Q_f/V_{\text{mill}}$)
- $Q_f$: total volumetric feed slurry rate ($m^3$/min)
- $V_{\text{mill}}$: mill internal volume, $\pi D^2 L/4$ ($m^3$)
However, in the new multi-component model, the $D_{\text{max}}$ is scaled for each component after each iteration, as shown in Figure 7.7.

![Diagram showing the iteration method for a multi-component model](image)

**Figure 7.7 – Multi-component model iteration method with multiple components in feed**

Although the model adopted $m_1 = m_2 = 0.35$ (Leung 1937) as default values, it allows the user to specify these constants when data (mill load measurements) or models are available to calculate them. In Chapter 8, the model fitting and simulations conducted for the LKAB pilot mill used $m_1 = 0.23$ and $m_2 = 0.33$, which are values that were calculated using the actual experimental data. For all other cases, the values fitted by Morrell (1989) to an extended database ($m_1 = m_2 = 0.57$) were used.

The validity of the discharge model represented by Equation 7.10 has been questioned by several users of the current JKSimMet AG/SAG model, but the implications of its use appears to be small given the good model validation results presented in Chapter 8.

### 7.5 Breakage Rates

The breakage rates $R$ can be back-calculated using Equations 7.1 and 7.2, given the feed and product size distributions and flowrates, the rock load distribution and mass, as well as the measured ore breakage function. Cubic splines at five knots (R1-R5) are used to describe the breakage rate distribution function, which is related to particle size and usually takes the form shown in Figure 7.8.
The breakage rate distributions are affected by a number of operating conditions. Morrell and Morrison (1996) modelled the effect of ball charge, mill filling, feed size distribution and mill speed on each breakage rate. The resulting five empirical equations are embedded in the current JKSimMet Variable Rates AG/SAG model. However, there are other elements (e.g. lifter profile and mill feed composition) which might influence these rates and they are not accounted for in the current JKSimMet model.

Breakage rates have been a controversial topic of discussion between researchers, given their differing interpretations of what constitutes a breakage event. Rather than considering it as the rate of breakage events occurring to each particle per unit of time (Morrell 1989), it can be simply interpreted as being the mass transfer rate (1/h) from coarse to smaller size fractions, since the perfect mixing model is a mass balance equation. Thus, different materials are likely to experience different breakage rates within the same mill.

Since the calculated breakage rates are dependent on the appearance function used (Leung 1987), which is a function of the ore competency as measured by the JKDWT (or JKRBT), this is a machine-ore interaction parameter. Therefore, components with significantly different properties would be expected to have distinct breakage rates under the same operating conditions. This hypothesis has been found true based on the application of the new model which allowed the back-calculation (or fitting) of component specific breakage rates. For example, the breakage rate curves for magnetite and silica, shown in Figure 7.9, suggest a systematic difference below 10 mm and a marked shift around 50 mm. This particular example is discussed in more detail in Chapter 8.
These ore specific breakage rates change according to the proportions of each component in the mill fresh feed, as described in the next chapter. However, no generic model has been derived to describe these variations, only for particular case studies using experimental data. Moreover, there were no adequate data to verify whether or not the variable rates model equations (Morrell and Morrison, 1996) apply for ore-specific breakage rates, once all the experimental work has been conducted under fixed mill operating conditions. As a result, these breakage rate equations have not been adopted in the new multi-component AG/SAG model.

7.6 Mill Power

The tumbling mill power draw model used by the current JKSimMet AG/SAG model (Morrell 1992), was also adopted in the multi-component model. Morrell’s power model was based on a conceptualized mill charge shape as shown in Figure 7.10, with the mill gross power draw having two components, net power and no-load power.
By integrating between the limits $\theta_s$ and $\theta_t$ and between $r_i$ and $r_m$, Morrell showed the net power ($P_{net}$) can be expressed as:

$$P_{net} = 2\pi g L \rho \int_{r_i}^{r_m} \int_{\theta_r}^{\theta_s} N_r r^2 \cos \theta \, d\theta \, dr$$  \hspace{1cm} (7.11)$$

The no-load power draw is estimated by the empirical relationship:

$$\text{No-Load Power (kW)} = 3.345 \,(D^3 L N_m)^{0.861}$$  \hspace{1cm} (7.12)$$

where

- $D$ : mill diameter (m)
- $L$ : mill length (m)
- $N_m$ : mill rotation rate (rev/s)
- $N_r$ : rotation rate at radial distance $r$ (rev/s)
- $r_m$ : mill radius (m)
- $r_i$ : charge surface radius (m)
- $\theta_s$ : angular position of the shoulder (radians)
- $\theta_t$ : angular position of the toe (radians)
- $\rho$ : charge density (t/m$^3$)
It is clear that the net power draw estimate using this model is affected by the charge density, which is directly related to the ore specific gravity. The current AG/SAG JKSimMet model does not account for the build-up of hard components in the mill contents, and therefore, considers the ore specific gravity in the mill load to be the same as in the feed. This can lead to significant errors when hard and soft components have significant differences in specific gravity. For example, in the LKAB operation treating magnetite and silicates, the difference between the feed blend and charge density could be as high as 15% depending on the blend. This translates to a 10% difference in mill power draw.

The new multi-component AG/SAG model can describe the build-up of the hard components, providing more realistic estimations of the true charge density, based on the balance between components and their respective specific gravities. Therefore, the effect of blending on mill power draw can now be described more accurately than has previously been possible.

7.7 Conclusions

By upgrading the current JKSimMet Leung’s AG/SAG model structure from the single feed description (1D) to a multi-component (2D) data structure, allowing for the specification of the feed on a component by size basis, a new multi-component model has been developed. The model relies on independent breakage and discharge rates for each component, and when combined, still observes the underlying hold-up vs. discharge rate relationship for the bulk solids and water. The main features of the new model are compared against Leung’s model in Table 7.1.

Table 7.1 – Comparison between the new multi-component AG/SAG model (2D) vs. Leung (1D) model

<table>
<thead>
<tr>
<th>Features</th>
<th>Leung’s Model (1D)</th>
<th>New Model (2D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Framework</td>
<td>1D Perfect Mixing Model</td>
<td>2D Perfect Mixing Model</td>
</tr>
<tr>
<td>Appearance function</td>
<td>Single ore specific parameters (A, b and tₐ)</td>
<td>Multiple ore specific parameters</td>
</tr>
<tr>
<td>Breakage energy calculation</td>
<td>Uses feed bulk density</td>
<td>Uses load bulk density and component specific density</td>
</tr>
<tr>
<td>Transport and discharge</td>
<td>Single discharge rates</td>
<td>Multiple discharge rates, using component specific Xₘ and Dₘₐₓ parameters</td>
</tr>
<tr>
<td>Breakage rates</td>
<td>Single breakage rates</td>
<td>Multiple breakage rates</td>
</tr>
</tbody>
</table>
The model should find applications in the optimization of existing circuit operations with multi-component ores, and in the field of geometallurgical mine-to-mill optimization exercises. When other multi-component models are available (e.g. HPGR, ball mill and classifiers), it can be used to assess flowsheet alternatives to optimise the grinding performance specific to each ore component.

The model was refined and validated using multi-component data obtained through pilot tests and an industrial mill survey at LKAB in Sweden. The methodology for testing the model and considerations regarding the outcomes of such exercise, highlighting its capabilities and limitations are presented in the following chapter.
Chapter 8  Model Validation

The new multi-component AG/SAG model was validated and compared against the Leung AG/SAG model. Simulation exercises were conducted to describe the changes in throughput capacity, mill load, and product, caused by changes in the mill feed composition. The simulation results were then compared against the available experimental data to assess the model accuracy.

8.1 Introduction

The new multi-component AG/SAG model was programmed in FORTRAN and implemented into a simulation platform known as the MDK (Model Developers Kit), a Microsoft Excel™ based software framework designed to allow users to perform circuit simulations using multi-component models during the model development phase.

The MDK is able to run a range of process unit models, ranging from those which consider particle size only (1D) through to the full multi-component models which consider size and mineral assay (2D). The MDK framework was designed to allow users to configure mineral processing circuits by linking multi-component models, to calibrate the models to plant survey data, and to run multi-component simulations of the circuit (Andrusiewicz et al., 2011).

The experimental data obtained during the pilot and industrial surveys, described in Chapters 4 and 5, was used to fit the new AG/SAG multi-component model in the MDK. The model responses to changes in feed blend were then simulated and compared with the measured trends. This chapter presents the results of these validation exercises.

8.2 LKAB Pilot Plant

The multi-component data obtained during surveys at the LKAB Kiruna concentrator in Sweden, and the related pilot plant campaign using mixtures of LKAB magnetite ore and silicate waste (described in Chapter 5) were used to develop and validate the proposed multi-component AG/SAG model.

The ore deposit in Kiruna is composed of a single continuous high grade magnetite orebody, mined using a sublevel caving method, which inevitably leads to some dilution with gangue – typically hard rock textures of silicates associated with phosphates and magnesium oxides. The mined ore has a high percentage of magnetite and is further upgraded in a sorting plant (using magnetic separation techniques) before entering the mill concentrator.
The characterization results from JKMRC DWT (Napier-Munn et al. 2005), Bond (1952) ball mill grinding tests, density measurements, and XRF assays conducted on each component, are presented in Table 8.1.

Table 8.1 – LKAB magnetite and silicate characterization results

<table>
<thead>
<tr>
<th>Measured Parameter</th>
<th>Magnetite</th>
<th>Silicate</th>
</tr>
</thead>
<tbody>
<tr>
<td>SG (t/m³)</td>
<td>4.9</td>
<td>2.6</td>
</tr>
<tr>
<td>DWT, A</td>
<td>68</td>
<td>69</td>
</tr>
<tr>
<td>DWT, b</td>
<td>1.67</td>
<td>0.66</td>
</tr>
<tr>
<td>DWT, A*b</td>
<td>114</td>
<td>45.3</td>
</tr>
<tr>
<td>Abrasion, tₐ</td>
<td>0.59</td>
<td>0.13</td>
</tr>
<tr>
<td>BWI@75μm</td>
<td>13.2</td>
<td>15.8</td>
</tr>
<tr>
<td>XRF % Magnetite</td>
<td>95.7</td>
<td>4.3</td>
</tr>
<tr>
<td>Ore Characteristic</td>
<td>Soft</td>
<td>Hard</td>
</tr>
</tbody>
</table>

The pilot mill was operated in fully autogenous mode and open circuit configuration during all trials. Table 8.2 shows the five different ratios of hard to soft (waste/magnetite) components in the mill feed trialled during this campaign. The ratio of hard to soft was changed through progressive additions of hard silicates in the +30 mm size fractions. An additional test (test T5) was carried out using an upgraded +30 mm magnetite ore and passed through a magnetic separation rig to reduce the silicate content.

Table 8.2 – Mill feed composition during LKAB pilot trials

<table>
<thead>
<tr>
<th>Test</th>
<th>+30 mm Waste (%)</th>
<th>+30 mm LKAB ore (%)</th>
<th>-30 mm LKAB ore (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>0</td>
<td>30</td>
<td>70</td>
</tr>
<tr>
<td>T2</td>
<td>4</td>
<td>26</td>
<td>70</td>
</tr>
<tr>
<td>T3</td>
<td>8</td>
<td>22</td>
<td>70</td>
</tr>
<tr>
<td>T4</td>
<td>15</td>
<td>15</td>
<td>70</td>
</tr>
<tr>
<td>T5*</td>
<td>0</td>
<td>30</td>
<td>70</td>
</tr>
</tbody>
</table>

*upgraded +30 mm ore

The operational philosophy, adopted in the pilot plant trials, was to find the feed rate which resulted in a steady operation at a mill load level of 28%. Once steady-state conditions were reached at this target load level, samples of feed, charge and product were collected and then analysed for size and composition distributions.
Experimental data from three trials (T1, T4 and T5), where the entire mill contents were measured, were used to calibrate the model. A total of six parameters were fitted for each component: namely the fine size ($X_m$), plus 5 breakage rates at the standard size knots 1 to 5 (R1, R2, R3, R4 and R5). The scale factors used to estimate the maximum discharge rate ($D_{\text{max}}$) for each component were based on experimental data, which indicate that the $D_{\text{max}}$ for silicate was consistently 1.2 times higher than that for magnetite.

Since the grinding charge size and composition distributions change according to the feed blend, the calculated breakage rates for both components were also affected by the feed composition. This effect is demonstrated in Figure 8.1, which shows the calculated breakage rates of magnetite and silicate for Test 1, 4 and 5 (respectively 50, 10 and 15 percent hard silicate in the +30 mm feed). The error bars were not plotted because repeats test were not conducted and the calculated standard deviation for the breakage rates were small (2 to 5%).

**Figure 8.1 – Effect of feed blend on magnetite and silicate breakage rates**

The silicate breakage rates at coarse sizes were lower than magnetite in all three trials, explaining the build-up of hard material in the mill. However, the opposite effect was observed at smaller sizes, where at the same energy levels, light silicate particles experience higher levels of specific energy (kWh/t) than dense magnetite particles.
The breakage rates of silicate at small sizes were not greatly affected by changes in feed blend, but magnetite breakage rates were progressively reduced, due to a reduced level of energy provided by the less dense silicate dominated grinding media. The breakage rates were suppressed near the critical size (44 mm) for both materials. The silicate breakage rates in the coarse sizes also decreased as the charge built up with less dense silicate rocks (Test 4). The high breakage rate for silicate rocks in Test 5 is suspected to be an artefact of the model fitting, since there would have been few silicate rocks in the +30mm fraction of the upgraded feed. However, the magnetite breakage rates at coarser sizes increased from Test 1 to Test 5.

The effect of blend on breakage rates requires further investigation, and the new multi-component model is an ideal platform for this study. Meanwhile, the current model can still be used for simulation, once the effect of blend on breakage rates is defined using a few sets of pilot data.

8.2.1 Modelling outcomes

The multi-component model accurately reproduced the experimental results once the breakage and discharge rates parameters had been fitted to measured data. A comparison between measured and fitted data for both mill product and load in Tests 1, 4 and 5 is shown in Table 8.3 (bulk) and Figure 8.2 (assay-by-size).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Test 1</th>
<th>Test 4</th>
<th>Test 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solids (tph)</td>
<td>exp</td>
<td>fit</td>
<td>exp</td>
</tr>
<tr>
<td>% Solids</td>
<td>75</td>
<td>75</td>
<td>95</td>
</tr>
<tr>
<td>Liquid (tph)</td>
<td>0.61</td>
<td>0.61</td>
<td>0.10</td>
</tr>
<tr>
<td>Solids SG (t/m3)</td>
<td>4.48</td>
<td>4.45</td>
<td>4.04</td>
</tr>
<tr>
<td>Pulp SG (t/m3)</td>
<td>2.41</td>
<td>2.41</td>
<td>3.51</td>
</tr>
<tr>
<td>Vol. Flowrate (m³/h)</td>
<td>1.03</td>
<td>1.03</td>
<td>0.55</td>
</tr>
<tr>
<td>% – 0.045 mm</td>
<td>47.0</td>
<td>52.3</td>
<td>4.3</td>
</tr>
<tr>
<td>P80 (mm)</td>
<td>0.122</td>
<td>0.130</td>
<td>69.7</td>
</tr>
</tbody>
</table>
Figure 8.2 – Fitted and measured mill discharge assay-by-size data for Tests 1, 4 and 5

It is clear from Table 8.4 that the multi-component model was also successful in reproducing the build-up of hard silicate within the mill contents, which is an important effect that has implications in every other outcome of the calculations, including the mill power draw. The accuracy was also excellent when describing this phenomenon in terms of size-by-size composition, as shown in Figure 8.3.

Table 8.4 – Build-up of hard component (Exp vs. Fit)

<table>
<thead>
<tr>
<th>TEST</th>
<th>Feed</th>
<th>Load Exp</th>
<th>Load Fit</th>
<th>Feed +30 mm Exp</th>
<th>Load +30 mm Exp</th>
<th>Load +30 mm Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>13.3</td>
<td>24.9</td>
<td>25.3</td>
<td>14.6</td>
<td>31.7</td>
<td>30.9</td>
</tr>
<tr>
<td>T4</td>
<td>27.9</td>
<td>69.4</td>
<td>68.1</td>
<td>53.1</td>
<td>83.8</td>
<td>84.2</td>
</tr>
<tr>
<td>T5</td>
<td>11.4</td>
<td>17.5</td>
<td>16.9</td>
<td>10</td>
<td>20.6</td>
<td>20.1</td>
</tr>
</tbody>
</table>
Ultimately, the model was able to describe well the mill load and product size distributions for magnetite, silicate, and bulk material for all trials. Figure 8.4 illustrates the model fitting results for Test 1 as an example.
Figure 8.4 – Mill load and product size distributions (exp vs. fit) for Test 1

Although the model has proved to be robust enough to reproduce the multi-component experimental data with accuracy, the question could still be raised as to whether this is related to its inherent capabilities or due to the use of extra fitting parameters. That means that the use of component specific breakage rates and discharge function parameters had to be justified. Therefore, an F-test has been conducted to compare the variances of adjusting the new full 2D model to Test 1 data, against two other calibration options while retaining individual appearance functions: Case 1 – multiple breakage rates and single discharge function and Case 2 – single breakage rates as well as single discharge function. In all cases, the breakage function parameters (A, b and tₐ) were component specific (2D) and the model structure kept the same. Table 8.5 shows the inputs and outcomes of the F-test analysis.

Table 8.5 – Model variance comparison: F-test inputs and outcomes

<table>
<thead>
<tr>
<th></th>
<th>Full 2D Model</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>sum of squared errors</td>
<td>209.5</td>
<td>367.5</td>
<td>855.7</td>
</tr>
<tr>
<td>data points</td>
<td>184</td>
<td>184</td>
<td>184</td>
</tr>
<tr>
<td>fitted parameters</td>
<td>14</td>
<td>12</td>
<td>7</td>
</tr>
<tr>
<td>degrees of freedom</td>
<td>170</td>
<td>172</td>
<td>177</td>
</tr>
<tr>
<td>Variance (s²)</td>
<td>1.23</td>
<td>2.14</td>
<td>4.83</td>
</tr>
<tr>
<td>F ratio</td>
<td>1.73</td>
<td>3.92</td>
<td></td>
</tr>
<tr>
<td>Significance of F</td>
<td>99.98%</td>
<td>99.99%</td>
<td></td>
</tr>
</tbody>
</table>
From Table 8.5, the calculated significance of $F$ for both cases 1 and 2 is higher than 99% level, and we may conclude that there is good evidence that the new full multi-component (2D) model provides the most valid description of the process and hence lowest variance. The fitted and measured mill discharge assay-by-size data for Cases 1 and 2 are shown in Figure 8.5 and Figure 8.6 respectively, clearly illustrate the alternative model options are not valid.

![Figure 8.5 – Fitted and measured mill discharge assay-by-size data for Case 1](image)

![Figure 8.6 – Fitted and measured mill discharge assay-by-size data for Case 2](image)

8.2.2 Simulation – model predictions

The variation in breakage rates with feed composition, shown in Figure 8.1, was modelled separately using simple linear equations based on the fitted breakage from Tests 1, 4 and 5, providing a set of equations allowing simulation of different feed blend conditions. Figure 8.7 shows the breakage rates of magnetite and silicate modelled as a function of feed composition, and Table 8.6 shows the regressed equations, where $X$ is the percent by mass of hard silicate in the +30 mm fresh feed.
Figure 8.7 – Breakage rates of magnetite and silicate modelled as a function of feed composition

Table 8.6 – Linear equations relating the magnetite and silicate breakage rates to the amount of hard silicate in the +30 mm fresh feed

<table>
<thead>
<tr>
<th>Breakage Rates</th>
<th>Magnetite</th>
<th>Silicate</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>$-0.0092X + 2.2467$</td>
<td>$-0.0019X + 2.7708$</td>
</tr>
<tr>
<td>R2</td>
<td>$-0.0083X + 5.8538$</td>
<td>$-0.0009X + 6.0455$</td>
</tr>
<tr>
<td>R3</td>
<td>$-0.0076X + 2.3145$</td>
<td>$-0.008X + 2.5082$</td>
</tr>
<tr>
<td>R4</td>
<td>$0.0018X + 2.2249$</td>
<td>$0.0038X + 1.3092$</td>
</tr>
<tr>
<td>R5</td>
<td>$0.0196X + 2.3353$</td>
<td>$0.0128X + 2.3850$</td>
</tr>
</tbody>
</table>

Similarly, the discharge charge function parameter $X_m$ was also modelled using experimental data and the regressed model response is plotted in Figure 8.8, which shows that the $X_m$ for silicates is consistently higher than that for magnetite due to the differences in SG and both are a function of mill total throughput. However, the scale factors for estimating maximum discharge parameter ($D_{max}$) of each component from bulk $D_{max}$ were fixed at 1.2 for silicate and 1.0 for magnetite, as these ratios were observed to be consistent with the calculated $D_{max}$ values in Tests 1, 4 and 5.

Figure 8.8 – Discharge function parameter $X_m$ modelled as a function of feed composition
The aim of this exercise was to verify the multi-component model response to blend, in terms of mill throughput, product size, energy consumption and mill load composition. The simulation procedure followed the operating protocol adopted during the pilot campaign. Therefore, the simulation was run for the average LKAB ore as is, and then for increasing increments of hard waste in the +30 mm feed. Additionally, three upgraded +30 mm feeds were simulated; one at the same upgraded magnetite grade as piloted, and the other two, at higher and lower magnetic separation efficiencies. The simulation outcomes and the various model responses to blends are illustrated in Figure 8.9, where the dashed line represents the average grade LKAB ore results.

Figure 8.9 – Simulation outcomes and model response to feed blend (% hard in +30mm feed)

The simulation results were realistic, describing well the known and expected trends such as:

- Non-linear mill throughput response,
- Linear relationship between specific energy and feed blend,
- Increase in the amount of fines with more competent material, which is ground predominantly by abrasion, and
- Build-up of hard material in the mill load.
The modelling outcomes were also in strong agreement with the measured data. The only exception was the mill product size, which was coarser (P80) but at a marginally higher percentage passing 45µm. This discrepancy is not unexpected since the model does not take into account the inherent difference in silicate and magnetite grindability below 3.35mm, only their measured impact resistance in the 13.2 to 63 mm size range. Moreover, the current SAG mill model platform does not predict the fine end of the distribution adequately due to the smallest knot size at 250 µm forcing linear rates fit below this size. This pilot work has 90 percent of the product finer than 250 µm so the bulk fit is too coarse. An improved base model with finer knot sizes is required to address this issue, which is being addressed in separate model development (Kojovic et al. 2012).

### 8.3 LKAB Industrial AG Mill

The LKAB KA2 concentrator in Kiruna was surveyed to obtain detailed multi-component data for every stream and mill load, as described in Chapter 5. Every sample collected during this campaign was analysed for percent solids, size distribution and assay-by-size (XRF). The data were used in this study to evaluate the new multi-component model’s (2D) ability to describe an industrial mill operation, as well as to compare the model fitting outcomes against the current JKSimMet AG/SAG model (1D).

When fitted to a single survey condition the 2D and 1D models were almost equivalent when applied to the bulk data. The new model (2D) was more accurate when calculating the mill product and load size distributions, but the original model (1D) was marginally more accurate in reproducing the amount of fines in the mill load, as shown in Table 8.7.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Product</th>
<th></th>
<th>Load</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>exp</td>
<td>fit 1D</td>
<td>fit 2D</td>
<td>exp</td>
</tr>
<tr>
<td>Solids (tph)</td>
<td>453</td>
<td>453</td>
<td>453</td>
<td>155</td>
</tr>
<tr>
<td>% Passing 0.045 mm</td>
<td>25.2</td>
<td>30.1</td>
<td>28.0</td>
<td>4.8</td>
</tr>
<tr>
<td>80 % passing size (mm)</td>
<td>0.430</td>
<td>0.615</td>
<td>0.520</td>
<td>72.5</td>
</tr>
</tbody>
</table>

The real benefit of the new model became evident when it demonstrated the ability to reproduce the distributions of magnetite and silicate in both the mill load and product, while the original model was limited to bulk data, as shown in Figure 8.10.
The ability to quantify the product size distribution for different components provides critical information relevant to downstream processes such as classifiers, magnetic separators and secondary milling. However, multi-component models for other processing units are required for a complete the circuit flowsheet simulation. This is an objective for future research within the JKMRC and AMIRA P9P projects.

Another valuable feature of the new model structure is the ability to describe the build-up of hard component material in the mill load. The results for KA2 mill are presented in Table 8.8 and Figure 8.11 on a size-by-size basis.

Table 8.8 – % Hard silicate in KA2 AG mill feed and load (exp vs. sim)

<table>
<thead>
<tr>
<th></th>
<th>Feed Exp</th>
<th>Feed Sim</th>
<th>Load Exp +30 mm</th>
<th>Load Sim +30 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>% Hard Silicate</td>
<td>13.3</td>
<td>22.0</td>
<td>22.9</td>
<td>14.8</td>
</tr>
<tr>
<td></td>
<td>14.8</td>
<td>24.5</td>
<td>25.9</td>
<td></td>
</tr>
</tbody>
</table>

Figure 8.11 – KA2 mill load composition (exp vs. sim)
In the case of the LKAB ore, where hard and soft components have significant differences in SG, the ability to predict the mill load composition, and consequently the charge density, will have a significant impact on mill power calculations and throughput. Table 8.9 shows the mill power calculations for both the 1D and 2D models. The new model predictions are more accurate because they account for a larger amount of light silicate in the mill load (i.e. lower total charge density).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>1D</th>
<th>2D</th>
<th>Meas.</th>
</tr>
</thead>
<tbody>
<tr>
<td>% Volumetric Total Load (%)</td>
<td>30.3</td>
<td>30.8</td>
<td>30.5</td>
</tr>
<tr>
<td>Total Charge Density (t/m³)</td>
<td>3.6</td>
<td>3.3</td>
<td>-</td>
</tr>
<tr>
<td>Net Power (kW)</td>
<td>2176</td>
<td>2020</td>
<td>-</td>
</tr>
<tr>
<td>No Load Power (kW)</td>
<td>319</td>
<td>319</td>
<td>-</td>
</tr>
<tr>
<td>Gross Power (kW)</td>
<td>2961</td>
<td>2772</td>
<td>2767</td>
</tr>
</tbody>
</table>

The breakage rates calculated for the industrial mill were also compared to the pilot mill breakage rates previously adjusted to Test 1 data. Both the industrial survey and the pilot test were carried out using the feed size and composition (i.e. average LKAB ore, 85% magnetite and 15% silicate) and similar mill operating conditions. The comparison made evident a consistent effect of scale on the breakage rates of both hard and soft components, as shown in Figure 8.12. This confirms previous findings on scale-up effect using bulk data are also valid in the multi-component (2D) environment, although the actual trends are slightly different. Therefore, since the new model has limited scaling up capabilities as it is based on the Leung Model, no attempt was made to simulate the industrial performance using the pilot data.

![Figure 8.12 – Magnetite and Silicate breakage rates calculated using industrial and pilot mill data](image-url)
8.4 Anglo Pilot Plant

The pilot plant test results and ore characterisation data, described in Chapter 4, were first used to calibrate the new multi-component model, which was then used to simulate the effect of adding coarse waste to the mill feed.

A total of six parameters were fitted for each component: fine size ($X_m$) and breakage rates at size knots 1 to 5 (R1, R2, R3, R4 and R5). The breakage rates calculated for the two different feeding conditions are plotted in Figure 8.13, and it is clear that chromite has higher breakage rates than silicate in most sizes and under both conditions. Although the fitting standard deviations were very small (2 to 5%), it is difficult to determine the significance of this difference without repeat tests.

![Figure 8.13 – Chromite and silicate breakage rates, fitted to pilot plant data from Tests 1 and 5](image)

The breakage rates of the individual components were affected by the feed composition, as shown in Figure 8.14. With hard waste in the +60 mm feed, the breakage rates at fine sizes increased for both materials. At coarse sizes, the rates for chromite decreased, while there was no change in the silicate breakage rates.

![Figure 8.14 – The effect of feed composition on chromite and silicate breakage rates](image)
Once the multi-component model parameters were adjusted, it was possible to accurately reproduce the experimental data in terms of particle size distribution and composition by size, in both the mill load and product, as shown in Figure 8.15 and Figure 8.16. The build-up of hard silicate in the mill load was also captured by the model, as shown in Table 8.10.

![Graph showing particle size distribution and composition](image-url)

**Figure 8.15 – Measured and fitted assay-by-size data for mill discharge and load for Test 5**

**Table 8.10 – Build-up of hard silicate (exp vs. fit)**

<table>
<thead>
<tr>
<th>Feed % Hard Silicate</th>
<th>Feed Exp</th>
<th>Feed Fit</th>
<th>Feed +60 mm Exp</th>
<th>Feed +60 mm Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>62.4</td>
<td>88.9</td>
<td>91.5</td>
<td>85.0</td>
<td>95.6</td>
</tr>
<tr>
<td>63.5</td>
<td>95.4</td>
<td>95.3</td>
<td>92.7</td>
<td>99.2</td>
</tr>
</tbody>
</table>

165
8.4.1 Simulating the addition of hard waste

For any given mill, there is an optimal feed blend of hard and soft components and coarse and fine materials. The optimal blending conditions are ore-dependent and mill-specific. However, simulations, using the model calibrated with the pilot plant data, can provide a guideline for selecting the optimum feed blend for a given AG mill operation, as illustrated here for the Anglo pilot mill.

The new multi-component model was used to simulate different additions of hard silicate waste in the coarse feed (+60 mm) and to identify the response for each blend. The simulations were run at a constant mill load and using a fixed feed size distribution, starting with pure UG2 ore (i.e. zero addition of waste) and then for increasing increments of hard waste in the +60 mm feed size. Figure 8.17 illustrates the simulated mill throughput, energy consumption, product size and dilution in platinum grade, as well as the recommended minimum and maximum additions of waste as dashed lines.
The suggested optimum range for blending is specific to the tested ore and presented only as a guideline for lowest reduction in throughput and dilution, assuming that a minimum of 5% hard waste is required in the feed. This range depends on ore characteristics and both upstream and downstream limitations (e.g. mine capacities, flowsheet, installed mill power, required grind size, flotation constraints, etc.). Once the objectives and limitations have been established, the new model can assist in determining an optimum blend for a particular application.

In order to assess these effects on a real circuit with recycle loads and downstream processes, a multi-component flowsheet simulator with other processing unit models is required. The MDK platform (Andrusiewicz et al. 2011) can link the new model to any multi-component model of other unit operations, but these still need to be developed.

Though the model suggests the maximum throughput is most likely to occur when there is no addition of hard waste, this condition is not expected to provide sufficient media for autogenous milling. The extremely soft UG2 ore experiences high breakage rates at coarse sizes, so in reality, it cannot provide sufficient energy to sustain a stable mill load. In reality the discharge function changes considerably with the load becoming finer (Powell & Valery 2006), but the model has no response to mill contents. The base model used to establish this multi-component mill model does not have an adequate mill transport function, so it cannot predict the build-up of this soft fine load. The model assumes that all the product can be discharged, which is not the case in a real mill with a sand build-up.

Figure 8.17 – Simulated mill throughput, energy consumption, product size and dilution according to blend for Anglo pilot AG milling
The need to maintain coarse grinding media in the mill means the operation is highly dependent on the feed rate of coarse silicate particles, as the lack of these can lead the AG mill to ‘sand up’, resulting in decreased throughput and reduced energy efficiency. Additionally, the high throughput (achieved prior to reaching the sanding limits) results in a coarser product, which can be problematic downstream, particularly as chromite and silicate have high Bond work indices. This is one of the reasons why RoM ball mills have been adopted for grinding UG2 ore.

This mill instability was observed during the pilot trials when searching for the minimum amount of hard waste required to keep the mill stable in Test 3, as described in Chapter 4. Figure 8.18 shows the mill load response during the tests using UG2 ore and waste material in the +60 mm feed. In both tests, the mill was rapidly brought to steady state by preferentially feeding more coarse material to quickly build up the mill load. It is clear that the use of hard silicate waste reduced the stabilization time, from 8 to 5 hours, although it limited the maximum mill throughput (~2.6 vs. 1.7 tph).

![Figure 8.18 – Mill load response to feed composition](image-url)
Therefore, some waste is desirable for improved stability and a finer grind. The mill's response to this instability cannot be simulated by the new model since it assumes steady-state conditions. A dynamic version of the model would be required to simulate this instability. However, the model indicated that the breakage rates of coarse chromite are very high, which is the cause of this problem.

In operation, the harder waste component should be viewed as grinding media and its use should be kept to a minimum to avoid grade dilution and significant reduction in the mill throughput. The simulations have shown that additions between 15 to 25 percent of waste in the mill coarse feed will produce the following side effects:

- A slightly reduced mill throughput (97 – 94% of maximum simulated throughput)
- A small increase in specific power (5 – 10%)
- A reduction in product size, smaller P80 and more fines below 150 µm
- Very little dilution in platinum grades (1 – 3%)

As a potential operating strategy, the ball addition in the RoM ball mills could be substituted with coarse waste rock, which would then be trickled in at a constant rate and adjusted to maintain a stable mill load. However, the energy of waste rocks and balls is very different, so for the same dimensions and speed the mill would draw less power and have a lower throughput.

8.5 Conclusions

Based on an extensive programme of laboratory, pilot and site testwork, data has been collected to develop a new multi-component AG/SAG mill model. This model is based on the independent breakage of components of substantially different competence. The model is able to predict the mill overall performance with two components and has been used to investigate a particular challenge facing the industry; the current inability to predict the fraction of competent ore required in the feed to Autogenous mills.

The new multi-component AG/SAG model was fine-tuned and validated using the comprehensive multi-component data obtained through LKAB and Anglo pilot tests and an industrial mill survey.

It has been shown that the new model is capable of correctly describing the changes in throughput capacity, mill load and product caused by changes in the mill feed composition.
It is known that a small fraction of competent ore is required to maintain a load of grinding media in an AG mill. However knowing and maintaining the required quantity of this competent ore is a challenge, yet it is critical to the successful operation of the mill. The new model was fitted to pilot tests conducted on UG2 Platinum ore, and then used to test the influence of varying the feed blend of competent waste and soft ore. The model showed that using 15% waste should be adequate, generating only a 3% drop from the theoretical maximum in throughput, a finer grind, and a relatively small 1.5% reduction in platinum grade. These figures, however, assume that the waste is barren, but in reality waste contains a low sub-economic grade which can add value to the grinding media. Additionally since waste is an unavoidable by-product of narrow seam mining, it is readily available. An upper limit of 25% waste is indicated in order to limit the resultant reduction of throughput and product grade.

It should be relatively easy to incorporate the features of the updated AG/SAG model structure currently under development at the JKMRC (Kojovic et al. 2012) into the new multi-component model. This would improve the model scale-up capabilities for it to be used to simulate the required competent ore fraction in production mills.

Although the validation presented in this chapter was related to AG milling, the model should be applicable to SAG mills, because it uses the same method as the current JKSimMet model to account for the extra energy provided by steel balls.
Chapter 9  Conclusions and Recommendations

This chapter provides an overall summary of the thesis in light of research and findings presented in the thesis. Conclusions and comments regarding the strengths and limitations of the new multi-component AG/SAG model and SAG Locked Cycle Test are presented. Potential applications of the research findings are suggested and recommendations for future research directions are discussed.

9.1 Summary of Testwork

During this research a comprehensive test work program was conducted to investigate the effect of multi-component feeds in autogenous and semi autogenous grinding. Tests were conducted in laboratory, pilot and industrial scales, using artificial mixtures and real ore blends.

The SAG Locked Cycle Test (SAG-LCT) has proved to be a robust method for characterizing the interactions between soft and hard ore components in binary feed blends. The test is conducted under controlled conditions, using a relatively small amount of material (approximately 100kg for a test program) and is highly reproducible. The results obtained using this laboratory test showed that the hard ore component builds up in the mill contents and limits the mill throughput. These effects have been confirmed at pilot and industrial scale, showing that this test can provide indicative results that correlate to real operations.

The ability to predict the fraction of each component in the mill load, as a function of the fraction in the feed, is a key aspect to understanding and modelling different components in AG/SAG milling and the SAG-LCT can provide an estimate of the non-linearity of the build-up response for any two components. The absolute build-up of hard material was lower in the full-size mill for the one directly comparable set of test work. This indicates that the full predictive application of the SAG-LCT will require improved scale-up modelling – an area deserving further research.

An AG mill pilot scale campaign was conducted at Anglo American facilities in South Africa, using different blends of hard and soft (silicate:chromite) ore in the fresh feed, and the data clearly shows the effect of feed composition on AG mill performance are in line with the SAG-LCT findings. The results have confirmed the importance of coarse and competent ore in AG mill operation. The detailed multi-component data generated in the pilot plant campaigns have been used to quantify the size-by-size response of hard and soft components in the mill product and load. This comprehensive information has been used in the model development and validation.
Additionally, an intensive series of integrated laboratory, pilot and industrial scale tests, measuring the response of AG mills to a range of multi-component feeds, have been conducted using LKAB ore (consisting of a softer magnetite and harder silicates waste). These included surveying the operation of a production mill, then dropping and sizing the entire mill charge, followed by size-by-size assaying of the mill contents and all streams around the circuit. The data represents a world first in terms of the detail generated and was crucial in the development and validation of the new multi-component model.

Overall, the experimental data generated in this research have demonstrated the interactions and transport of individual ore components inside AG/SAG mills, as well as the influence of feed composition on the mill power draw.

9.2 Model Development

A new multi-component AG/SAG mill model was developed using the data collected through an extensive programme of laboratory, pilot and site testwork. Initially, different approaches using the current JKSimMet model were trialled to investigate their ability to describe multi-component grinding data. The major limitation found was the lack of a model structure capable of dealing with multiple sets of breakage functions and breakage rates when the feed has more than one component.

The preliminary modelling exercises demonstrated that simulations were sensitive to ore parameters, and the use of ore specific breakage rates would enable the model to describe the non-linear relationship between the mill throughput and the feed composition. These exercises provided the insights necessary to perform realistic simulations leading to the development of the new multi-component model.

A multi-component model structure (2D) was developed by modifying Leung’s AG/SAG model (1987) to accommodate parallel computation for each ore type, allowing for the specification of the data on a component by size basis. The two key assumptions in the new model are: 1) fully liberated components, and 2) independent breakage and transport functions for each component. When the component responses are combined, the new model still observes the underlying Leung breakage and discharge relationships for the bulk solids and water.

The new AG/SAG mill model approach is applicable to multiple ore types, relating their appearance function and distribution in mill feed to their breakage rates, transport and mill power draw. The model structure adopted a simple 2D data format used in the MDK platform, allowing the specification of the feed, load and product on a component by size basis.
The model should find applications in the optimization of existing circuit operations with multi-component ores, and in the field of geometallurgical mine-to-mill optimization exercises. When other multi-component models are available (e.g. classifiers, HPGR and ball mill), it can be used to assess flowsheet alternatives to optimise the grinding performance specific to each ore component.

9.3 Model Validation

The model was refined and validated using multi-component data obtained through pilot tests and an industrial mill survey at LKAB in Sweden. It has been shown that the model is capable of correctly describing the changes in throughput capacity, mill load and product caused by changes in the mill feed composition, as well as the assay-by-size data.

Both the new multi-component (2D) model and the Leung model in JKSimMet were adjusted to reproduce the LKAB industrial AG mill survey data, and the outcomes demonstrated that the new model provides better fitting results. Due to the fact that it can account for the build-up of hard material in the mill load, the new 2D accurately describes mill charge density and provides more realistic results, especially for mill power draw.

The model enables for the first time the exploitation of the best operating blend for Anglo ore types in AG milling. In respect to this, the new model was fitted to pilot tests conducted on UG2 platinum ore, and then used to test the influence of varying the feed blend of competent waste silicates and soft chromitite ore. The simulation results were realistic and succeeded in determining the amount of competent ore required to maintain a load of grinding media in the AG mill. The model showed that using 15% waste should be adequate, generating only a 3% drop from the theoretical maximum in throughput, a fine grind, and about a 1.5% drop in platinum grade. An upper limit of 25% waste was indicated in order to limit the resultant reduction of throughput and grade.

The model has proved to be robust enough to accurately describe multi-component data and provide realistic simulation results once it is calibrated. Although the validation case studies presented in this thesis were related to AG mill operations, the model should be applicable to SAG mills, as it uses the same method as per the current JKSimMet model to account for the extra energy provided by steel grinding media.
9.4 Summary of Research Contributions

Considering what has already been discussed in the previous sections, the research has made significant new contributions to the field of comminution. The influence of feed composition of the overall AG/SAG mill performance has been confirmed and measured through an extensive experimental programme, addressing one of the research hypotheses.

The unique experimental data generated in this research described the interactions, breakage and transport of individual ore components inside the mill, which lead to the development of a new multi-component (2D) AG/SAG model.

The new multi-component model after calibration has proved to be robust enough to accurately describe the influence of distinct components on AG/SAG mill performance, confirming the second research hypothesis. However, the new model has limited predictive and scalability capabilities due to the simplistic model structure adopted. Therefore, further model developments are recommended to address such limitations, as discussed in the following section.

9.5 Recommendations

The breakage process in autogenous and semi-autogenous mill operations is inherently related to the grinding charge composition, which is directly controlled by the mill feed composition (and size distribution of each component) at a given mill operation condition. The AG/SAG mill feed may consist of multiple ore components with different hardness and density, and a varying ratio of these components can significantly affect the mill performance. Although this is a common scenario in the mining industry, previous AG/SAG mathematical models were limited to describing a homogeneous feed. The new 2D modelling approach described in this thesis has enabled the development of an effective multi-component AG/SAG mill model, which is the first of its kind.

However, the new AG/SAG multi-component model adopted the modelling structure proposed by Leung (1987), which is robust but requires the calibration and modelling of the breakage rates for different operating or feed conditions. Despite significant efforts being spent on collecting multi-component AG/SAG milling data, there were insufficient data sets available to develop generic sub-models between the fitted component-based breakage rates and mass transport parameters in relation to mill geometry and operating conditions. This has limited the capacity of the current model structure to predict other operation conditions and therefore, more testwork is required to allow future developments.
The intensive experimental work conducted in this thesis has generated the first true multi-component datasets in laboratory, pilot and industrial settings, but these were limited to only two different real ore samples and a mixture test using artificial materials. Due to the difficulty (both logistically and financially) of collecting meaningful industrial-scale data, most of the experimental data were generated at laboratory and pilot scales, with only one industrial survey. The tests were also conducted at fixed operating conditions and the only variable studied was the mill feed composition. Therefore, it is recommended that more comprehensive experimental work be carried out using different ores, to investigate the interaction of changes in feed blend with a range of operating variables such as mill load level, fraction of critical speed, and ball filling. Additional industrial surveys at full scale AG/SAG mill operations with multi-component ores should also be conducted, preferably adopting the sample treatment procedures described in this thesis.

The new model structure is an ideal platform for any future developments in the field of multi-component modelling of AG/SAG mills. When a larger number of multi-component data sets is available, the structure could be used to calculate the ore-specific breakage rates and mass transport parameters for a range of conditions. This would allow the development of empirical relationships to relate the calculated model parameters to mill geometry and operational conditions. Additionally, the new structure could easily incorporate some features of more advanced mechanistic AG/SAG models (Delboni, H. 1999; Kojovic et al. 2012; Morrell 2004; Morrell & Morrison 1996; Valery 1997), once additional data is available to validate their model relationships for multi-component ores. This model upgrades could then be used to simulate the effects of feed composition in combination with changes in operating conditions and enable improved scale-up and simulation of capability.

The SAG-LCT results demonstrated that this new laboratory test can provide an indicative description of the blending response in AG mills, which has some correlation to larger and continuous operation mills. However, only one industrial ore was tested using the SAG-LCT, and the apparatus used in the experimental work conducted in this thesis did not have an accurate means for measuring the mill power draw. This makes it difficult to draw direct relationships between the laboratory and industrial results. Therefore, more tests should be conducted using a variety of industrial ores and an improved laboratory apparatus, in order to generate sufficient data to establish correlations between the SAG-LCT results and real operations. The SAG-LCT test has the potential to map the interaction of the relative breakage rates of ores mixtures with different competencies. This could enable scale-up prediction without the need for pilot work, so it is recommended that this methodology be further investigated.
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Appendices

Appendix A  Experimental Data CD

The following files are included in the appendices CD:

A.1  SAG-LCT Data
A.2  Large scale SAG-LCT data
A.3  Anglo Platinum pilot plant data
A.4  LKAB plant survey data
A.5  LKAB pilot plant data
A.6  LKAB SAG-LCT data
Appendix B  Averaging “Axb” parameters and mill throughput

The mathematical validity of simple weighted averages of “Axb” parameters and mill throughput is an issue that has not been addressed in the published literature. Therefore, this appendix aims to describe a point of view shared among some industry experts, including Stephen Morrell.

Simple weighted averages of “Axb” parameters are not appropriate and may generate a significant bias when simulating the mill performance for different feed blends. They are not appropriate because the inferred units of Axb are tonnes/kWh and therefore, an average of Axb based on the mass ratio in the feed is mathematically incorrect. If the average of a parameter is based on a mass weighting, the units of the parameter being averaged has to have mass in its denominator. Axb has kWh in its denominator and tonnes in its numerator. Therefore, the correct average must be based on 1/(Axb) to ensure the units are correct. Once the average of 1/(Axb) has been done the result is then inverted to get back to Axb. The results from blending in this inverse way can be quite different to the (incorrect) simple arithmetic averaging of Axb, as shown in Figure B1.

![Figure B.1 – Inverse weighting vs. simple arithmetic weighting of Axb parameters](image)

The same concept applies for weighted averages of mill throughput according to blend (i.e. mass ratio in the feed). Throughput has hours in its denominator and tonnes in its numerator. By way of explanation, consider two blocks of different ore type, each containing 1000 tonnes: one allowing a treatment rate of 100 t/h and the other 50 t/h. The simple arithmetic mean of the capacity is (100+50)/2 = 75 t/h. However, this is not the true average capacity of the mill. Once the blocks are treated through the mill, the first block should take 10 hours to treat and the second block 20 hours, i.e. total time of 30 hours. Therefore, the true average capacity is 2000/30 = 66.67 t/h. When averaging with respect to the blocks (units of mass), the average should be done with respect to the inverse of capacity (hours/tonne) in order to obtain the true average capacity, i.e. \(((1/100+1/50)/2)^{-1}\) = 66.67 t/h.