SCALE EFFECTS AFFECTING TWO-PHASE FLOW PROPERTIES IN HYDRAULIC JUMP WITH SMALL INFLOW FROUDE NUMBER

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Abstract: The transition from supercritical to subcritical flow is characterised by a strong dissipative mechanism, a hydraulic jump. Herein a comparative analysis of physical data is presented with a focus on a broad range of two-phase flow parameters. The results show that, for hydraulic jumps with Fr₁ = 5.1, the void fraction data obtained with Re < 4×10⁴ could not be scaled up to Re = 1×10⁵. Most air-water flow properties measured with Reynolds numbers up to 1.25×10⁵ could not be extrapolated to large-size prototype structures without significant scale effects in terms of bubble count rate, turbulence, bubble chord time distributions and bubble cluster characteristics. The findings have some implications of civil and sanitary engineering designs, because most hydraulic structures, storm water systems and water treatment facilities operate with Reynolds numbers larger than 10⁶ to over 10⁸.

Key words: Hydraulic jumps, physical modelling, dimensional analysis, scale effects, air entrainment, two-phase flows, bubble clustering.

INTRODUCTION

A hydraulic jump is the sudden and rapid transition from a high velocity flow into a slower motion in an open channel (HENDERSON 1966) (Fig. 1 & 2). It is characterised by some substantial energy losses and air entrainment in the jump roller (RAJARATNAM 1967, MONTES 1998). Figure 1 presents a hydraulic jump downstream of a dam spillway during a recent flood event and Figure 2A shows a laboratory model. Traditionally, physical hydraulic models are used to optimize a design and to predict the behaviour of prototype flow situations. The model is investigated in a laboratory under controlled conditions, and the model flow conditions are said to be similar to those in the prototype if the model displays similarity of form, motion and forces (HENDERSON 1966, LIGGETT 1994). In most hydraulic structures including hydraulic jump stilling basins, it is impossible to satisfy simultaneously all the dynamic similarities with geometrically-scaled physical models using air and water (RAO and KOBUS 1971, WOOD 1991, CHANSON 1997). It was recently argued that the notion of scale effects is critically linked with the definition of key parameters.

In this contribution, the validity of the Froude similitude is tested for Fr₁ = 5.1 with respect to the two-
phase flow properties, including the distributions of void fraction, bubble count rate, interfacial velocity, turbulence intensity and integral turbulent time scales, as well as the probability distribution functions of bubble chord times and clustering properties. The comparison is based upon a comparative analysis of geometrically scaled data obtained with the same type of phase detection probes.

**DIMENSIONAL CONSIDERATIONS AND PHYSICAL MODELLING OF TWO-PHASE FLOW PROPERTIES IN HYDRAULIC JUMP**

Any fundamental analysis of hydraulic jumps is based upon a large number of relevant equations to describe the two-phase turbulent flow motion. Physical modelling may provide some insights into the flow motion if a suitable dynamic similarity is selected (NOVAK and CABELKA 1981, LIGGETT 1994). For a hydraulic jump, the relevant dimensional parameters include the air and water physical properties and constants, the flume characteristics, the inflow conditions, and the local two-phase flow properties. Considering a jump in a smooth horizontal rectangular channel with an inflow depth \( d_1 \) and velocity \( V_1 \), a simplified dimensional analysis yields:

\[
\frac{C}{V_1}, \frac{V'}{V_1}, \frac{\rho d_1}{V_1^2}, \frac{D_{ab}}{d_1}, \frac{N_c d_1}{V_1}, ... = F \left( \frac{x-x_1}{d_1}, \frac{y}{d_1}, \frac{z}{\sqrt{g d_1}}, \rho \frac{V_1}{\mu}, \frac{\rho V_1^2 d_1}{\sigma}, \frac{x_1}{d_1}, \frac{V_1}{d_1}, \frac{v_1'}{d_1}, \frac{\delta}{d_1}, ... \right)
\]

where \( C \) is the void fraction, \( V \) the air-water velocity, \( V' \) a characteristic turbulent velocity, \( F \) the bubble count rate defined as the number of bubbles detected per second in a small control volume, \( D_{ab} \) a characteristic bubble size, \( N_c \) the number of bubble clusters per second, \( x \) the longitudinal coordinate, \( y \) the vertical elevation above the invert, \( z \) the transverse coordinate measured from the channel centreline, \( \rho \) and \( \mu \) the water density and dynamic viscosity respectively, \( \sigma \) the surface tension between air and water, \( x_1 \) the longitudinal coordinate of the jump toe, \( W \) the channel width, \( v_1' \) a characteristic turbulent velocity at the inflow, \( \delta \) the boundary layer thickness of the inflow (Fig. 2B). Equation (1) expresses the air-water turbulent flow properties at a position \((x,y,z)\) within the hydraulic jump as functions of the inflow properties, fluid properties and channel geometry using the upstream flow depth \( d_1 \) as the characteristic length scale. In the right hand side of Equation (1), the 4th, 5th and 6th terms are respectively the upstream Froude number \( Fr_1 \), the Reynolds number \( Re \) and the Weber number \( We \).

In a hydraulic jump, the momentum considerations demonstrate the significance of the inflow Froude number (BÉLANGER 1841, LIGHTHILL 1978) and the selection of the Froude similitude derives implicitly from basic theoretical considerations (LIGGETT 1994, CHANSON 2012).

The Froude dynamic similarity is commonly applied in the hydraulic literature (HENDERSON 1966, NOVAK and CABELKA 1981, CHANSON 2004) including in the present analysis. The Reynolds
number is another relevant dimensionless number because the hydraulic jump is a turbulent shear flow (ROUSE et al. 1959, RAJARATNAM 1965, HOYT and SELLIN 1989). Further the Π-Buckingham theorem implies that any dimensionless number can be replaced by a combination of itself and other dimensionless numbers. That is, the Froude, Reynolds or Weber number may be replaced by the Morton number Mo since:

$$Mo = \frac{g \mu}{\rho \sigma^3} = \frac{We^3}{Fr^2 Re^4}$$  \hspace{1cm} (2)

Equation (1) may be rewritten as:

$$C, \frac{V}{V_i}, \frac{v'}{V_i}, \frac{F dl}{d_1}, \frac{D_{ab}}{d_1}, \frac{N_c d_1}{V_i}, ... = F \left( \frac{x-x_1}{d_1}, \frac{y}{d_1}, \frac{z}{d_1}, \frac{V_i}{\sqrt{g d_1}}, \frac{g \mu}{\rho \sigma^3}, \frac{x_1}{d_1}, \frac{W}{d_i}, \frac{v_i'}{d_1}, \frac{\delta}{d_1}, ... \right)$$  \hspace{1cm} (3)

When the same fluids (air and water) are used in models and prototype as in the present study, the Morton number Mo becomes an invariant and this adds an additional constraint upon the dimensional analysis. Equation (3) yields:

$$C, \frac{V}{V_i}, \frac{v'}{V_i}, \frac{F dl}{d_1}, \frac{D_{ab}}{d_1}, \frac{N_c d_1}{V_i}, ... = F \left( \frac{x-x_1}{d_1}, \frac{y}{d_1}, \frac{z}{d_1}, \frac{Fr_1}{d_1}, \frac{Re}{d_1}, \frac{x_1}{d_1}, \frac{W}{d_i}, \frac{v_i'}{d_1}, \frac{\delta}{d_1}, ... \right)$$  \hspace{1cm} (4)

Physically it is impossible to fulfil simultaneously the Froude and Reynolds similarity requirements, unless working at full scale. For example, the air entrainment process is adversely affected by significant scale effects in small size models (RAO and KOBUS 1971, CHANSON 1997). The Reynolds number was selected herein instead of the Weber number because the present study focuses on the scaling of prototype hydraulic jumps with Reynolds numbers from $10^6$ to in excess of $10^9$ (e.g. Fig. 1). For such large flows, surface tension is considered of lesser significance compared to the viscous effects in the turbulent shear regions (CAIN and WOOD 1981, WOOD 1991, CHANSON 1997, ERVINE 1998). Lastly the Froude and Morton similarities imply that $We \propto Re^{4/3}$ (Eq. (2)).

**Methodology**

A number of data sets were re-analysed in the present study (Table 1). All data sets were geometrically similar based upon a Froude similitude with undistorted scaling ratio, and the same types of intrusive phase-detection probes were used (Table 1, column 7). The geometric scaling ratio was $L_{scale} = 3.3$ between the largest and smallest series of experiments ($d_1 = 0.04$ and 0.012 m respectively) where $L_{scale}$ is the geometric scaling ratio defined as the ratio of prototype to model dimensions. Note that all the experiments were conducted in hydraulic jumps with partially-developed inflow conditions. The experimental flow conditions were selected as $Fr_1 = 5.1$ with identical upstream distance $x_1/d_1$ between
gate and jump toe, and the two-phase flow measurements were performed in the developing air-water flow region at cross-sections such that \((x-x_1)/d_1 \leq 15\). For \((x-x_1)/d_1 > 15\), the flow aeration (e.g. void fraction, bubble count rate) was drastically reduced at the lowest Reynolds numbers and the two-phase flow properties could not be recorded with the phase-detection probe technique.

Importantly all experimental data sets were collected in similar facilities, with the same type of instrumentation as well as signal processing techniques. A broad range of air-water flow parameters were systematically tested, including velocity, turbulence, integral time scale and bubble clustering data analyses. The latter data sets were never analysed in terms of dynamic similarity to date. Lastly the range of re-analysed data sets spanned over a wider range of Reynolds numbers (i.e. an order of magnitude herein) than any previous studies;

**BASIC RESULTS**

In the jump roller, the vertical distributions of void fraction presented a local maximum in the turbulent shear layer while the distributions of bubble count rate exhibited a sharp maximum in the shear region (Fig. 2B). Both features are sketched in Figure 2B and seen in Figure 3. Typical comparative results are presented in Figures 3 and 4 in terms of void fraction and bubble count rate. The physical data showed drastic scale effects in the smaller hydraulic jumps in terms of void fraction and bubble count rate distributions. The results highlighted consistently a more rapid de-aeration of the jump roller with decreasing Reynolds number for a given inflow Froude number for \(Re < 68,000\), an absence of self-similarity of the void fraction profiles in the turbulent shear layer for \(Re < 40,000\) (Fig. 3A), and an increasing dimensionless bubble count rate with increasing Reynolds number for a given inflow Froude number (Fig. 3B). In Figure 3A, the void fraction profiles in the air-water shear layer followed an analytical solution of the air bubble advection equation (Eq. (2)) for \(Re = 125,000\) and 68,000 but were basically flat for \(Re = 25,000\) and 38,000. In a hydraulic jump, the impingement point acts as a localised source of air entrainment and a solution of the air bubble entrainment equation is (CHANSON 2010):

\[
C = \frac{q_{air}}{q} \sqrt{\frac{4 \pi D^g X'}{X'}} \left( \exp \left( -\frac{(y'-1)^2}{4 D^g} \right) + \exp \left( -\frac{(y'+1)^2}{4 D^g} \right) \right)
\]

where \(X' = (x + u_r / V_l y)/d_1\), \(y' = y/d_1\), \(u_r\) is the bubble rise velocity, and \(D^g\) is a dimensionless diffusion coefficient. Equation (5) is restricted to the air-water shear layer and it is compared with experimental data in Figure 3.

The vertical distributions of bubble count rate showed a maximum in the shear region (Fig. 2B & 3). The dimensionless maximum bubble count rate data are summarised in Figure 4 as functions of the Reynolds number \(Re\). The results highlighted a monotonic increase in maximum bubble count rate
F_{\text{max}}d_1/V_1. No asymptotic trend was observed within the range of investigated flow conditions (Fig. 4). This is illustrated in Figure 4 showing the dimensionless maximum bubble count rate in the air-water shear layer as a function of the Reynolds number for several dimensionless longitudinal locations. The data highlighted a monotonic trend as well as the absence of an asymptotic limit, demonstrating some scale effects in terms of bubble count rates.

The velocity distribution followed closely a self-similar profile which was close to a wall jet velocity distribution, as discussed by RAJARATNAM (1965) and CHANSON and BRATTBERG (2000). The results for Re = 38,000 and 130,000 highlighted the self-similarity (Fig. 5A). A relevant parameter is the shear layer thickness, although its quantitative estimate is not trivial. Based upon the velocity profile data, the mixing zone may be conceptualised as a boundary layer region, or inner layer, where 0 < V < V_{\text{max}} with V_{\text{max}} the maximum velocity in a cross-section, and a shear zone, or outer layer, above (RAJARATNAM 1976, GEORGE et al. 2000). Such a model is sketched in Figure 1B using the terminology introduced by GEORGE et al. (2000) based upon similarity considerations. Although there is no theoretical justification, the maximum velocity V_{\text{max}} in a wall jet may be normalised in term of the outer layer length scale (GEORGE et al. 2000, TACHIE et al. 2004). Herein it is proposed to link the maximum velocity with the inner layer thickness which characterise the vertical extent of the boundary shear stress. The present data are shown in Figure 5B and the (limited) data set followed a power law trend:

\[
\frac{V_{\text{max}}}{V_1} = 0.663 \left( \frac{\delta}{d_1} \right)^{-0.128}
\]  

Introducing the thickness \( \delta \) of the inner layer and the length scale \( b \) (Fig. 1B) defined as the elevation where \( V = V_{\text{max}}/2 \), the two-phase flow data are reported in Figure 5C, where the results are compared with the characteristic height \( Y_{90} \) of the roller defined as the location where the void fraction C equals 0.90 as well as monophase wall jet data. The data showed an expanding shear zone (inner and outer layers) with increasing distance from the jump toe. The inner and outer layer length scales, \( \delta \) and \( b \) respectively, were significantly larger than monophase flow observations.

The turbulent properties (Tu, T_{xx}) showed however some scale effects (Fig. 6) where Tu is the turbulent intensity: \( Tu = \nu'/V \), \( \nu' \) is the root mean square of the longitudinal velocity component, and T_{xx} is the auto-correlation integral time scale. Herein Tu was calculated based upon the correlation analysis of the dual-tip probe signals (CROWE et al. 1998, CHANSON and TOOMBES 2002). In the air-water shear layer, the turbulence intensity was larger and the integral time scales were smaller for the largest Reynolds numbers, at the same given dimensionless location and for an identical Froude number. Figure 6 includes also the distribution of auto-correlation time scales, indicating a monotonic increase in turbulent time scales with increasing distance from the invert. Further the bubble chord time distributions were not scaled according to a Froude similitude (Fig. 7). Comparatively larger bubble
chord times were observed at low Reynolds numbers. This is seen in Figure 7 presenting the normalised probability distribution functions of dimensionless bubble chord times. The present results supported the earlier findings (CHANSON and GUALTIERI 2008, MURZYN and CHANSON 2008), and they extended the findings to a broader range of air-water flow properties and Reynolds numbers.

**Bubble cluster properties**

When two bubbles are closer than a particular time/length scale, they can be considered a group of bubbles: i.e., a cluster. The characteristic water time/length scale may be related to the water chord statistics or to the near-wake of the preceding particle. Herein the latter approach was applied following CHANSON et al. (2006) and CHANSON (2010). Two bubbles were considered parts of a cluster when the water chord time between two consecutive bubbles was less than the lead bubble chord time. That is, when a bubble trailed the previous bubble by a short time/length, it was considered to be in the near-wake of and could be influenced by the leading particle.

The effects of the Reynolds number were also tested on the bubble clustering properties for Fr1 = 5.1. Some results are shown in Figure 8 in terms of the dimensionless number of clusters per second ($N_c d_1 / V_1$), the percentage of bubble in clusters and the number of bubbles per cluster. The data indicated a relatively significant proportion of clustered bubbles, ranging from 26% to 42% (Fig. 8B). Importantly the dimensionless properties of bubble clusters in the air-water shear layers were not scaled according to a Froude similitude for Fr1 = 5.1. The comparative analysis showed that the dimensionless number of clusters per second, the percentage of bubbles in cluster and the number of bubbles per clusters increased monotonically with the Reynolds number at a given dimensionless location $(x-x_1)/d_1$ and for a given Froude number Fr1 = 5.1. This is seen in Figures 8A to 8C.

In hydraulic jumps, the level of clustering may give a measure of the magnitude of bubble-turbulence interactions and associated energy dissipation. The present findings highlighted that the clustering affected a comparatively greater proportion of bubbles at high Reynolds numbers, indicating that the interactions between entrained bubbles and vortical structures were not scaled accurately with the Froude similarity.

**DISCUSSION**

The II-Buckingham theorem implied that only two dimensionless numbers are relevant to investigate air entrainment in hydraulic jumps using the same fluids in physical models and prototype (Eq. (4)). The selection of the Froude similitude was based upon some basic theoretical considerations (LIGHTHILL 1978, LIGGETT 1994, CHANSON 2012). A key feature of the hydraulic jump is the turbulent shear region. The jump toe is both a source of vorticity for the shear layer, as well as a source of entrapped air. The entrained 'bubbles' are convected in a region of high shear stress where bubble-
turbulence interactions take place, including bubble breakup, vortex trapping of bubbles, bubble coalescence, modifications of vortical structures and of their properties. Since a Froude similarity was selected, the Reynolds number differed between the various physical models for an identical Froude number: i.e., the scaling ratio of the Reynolds number was equal to \( L_{\text{scale}}^{3/2} \). In turn the scaling of turbulent shear stress tensor and vorticity vector differed from the velocity scaling ratio \( L_{\text{scale}}^{1/2} \) for a Froude similitude.

The comparative analysis implied that, for hydraulic jumps with \( \text{Fr}_1 = 5.1 \), (a) the void fraction data obtained with \( \text{Re} < 40,000 \) could not be scaled up to \( \text{Re} = 1 \times 10^5 \), and (b) the bubble count rate data, turbulence properties, bubble chords and clustering properties with Reynolds numbers up to 125,000 could not be extrapolated to large-size prototype structures without significant scale effects in terms of bubble count rate, turbulence and bubble chord time distributions. The findings are summarised in Table 2 and they have some major implications of civil, environmental and sanitary engineering designs, because most hydraulic structures, storm water systems and water treatment facilities operate with Reynolds numbers within ranging from \( 10^6 \) to over \( 10^8 \).

For completeness, CHANSON (2007) tested the effect of the relative width \( W/d_1 \), with all other relevant parameters (\( \text{Fr}_1, \text{Re}, \text{Mo} \)) being constant. That is:

\[
C = \frac{\dot{V}}{V}, \frac{\nu'}{V}, \frac{F}{d_1}, \frac{D_{ab}}{d_1}, \frac{N_c}{d_1}, ..., = f\left(\frac{W}{d_1}\right) \tag{7}
\]

where the inflow Froude and Reynolds numbers were constant : i.e., \( \text{Fr}_1 = 5.1 \) and 8.5, \( \text{Re} = 70,000 \) to 95,000. The results showed that the relative channel width had no effect on the air-water flow properties at the centreline of the channel for \( W/d_1 > 10 \), including terms of the void fraction, bubble count rate and bubble chord time distributions. Since all the data tested in Table 1 satisfied \( W/d_1 > 10 \), the present results are believed also to be independent of the relative flume width.

**CONCLUSION**

The hydraulic jump is a complex phenomenon that remains incompletely understood. In the present study, a re-analysis of physical data was conducted with a focus on the air-water flow properties in hydraulic jumps with \( \text{Fr}_1 = 5.1 \). The Froude similarity was tested for a range of Reynolds numbers \( 2.5 \times 10^4 < \text{Re} < 1.3 \times 10^5 \). The analysis was performed over a broad range of two-phase flow parameters including the distributions of void fraction, bubble count rate, velocity, turbulence intensity, integral time scale and bubble clustering properties. The comparative results demonstrated that, for hydraulic jumps with \( \text{Fr}_1 = 5.1 \), the void fraction data obtained with \( \text{Re} < 4 \times 10^4 \) could not be scaled up to \( \text{Re} = 1 \times 10^5 \). The bubble count rate data, turbulence properties, bubble chords and clustering properties with Reynolds numbers up to \( 1.25 \times 10^5 \) would not be up-scaled to large-size prototype structures without...
significant scale effects in terms of bubble count rate, turbulence and bubble chord time distributions. The basic results are summarised in Table 2.

The findings have some implications of civil engineering, because most hydraulic structures operate at Reynolds numbers ranging from $10^6$ to over $10^8$. In a physical model, the flow conditions are said to be similar to those in the prototype flow conditions if the model displays similarity of form, similarity of motion and similarity of forces. The present results demonstrated quantitatively that the dynamic similarity of two-phase flows in hydraulic jumps at relatively small Froude numbers cannot be achieved with a Froude similarity unless working at full-scale.

ACKNOWLEDGMENTS

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REFERENCES


Table 1 - Physical modelling of air-water flow properties in hydraulic jumps at relatively small Froude numbers based upon an undistorted Froude similitude with air and water

<table>
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<th>$d_1$ (m)</th>
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<th>Re</th>
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Notes: Hydraulic jumps with partially-developed inflow conditions; all experiments performed with tap water; $C_{max}$: maximum void fraction in the shear layer; $F_{max}$: maximum bubble count rate in the shear layer; $W$: channel width; (--): data unavailable.

Table 2 - Physical scaling of hydraulic jump with small inflow Froude number: basic recommendations

<table>
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<th>Air-water flow property</th>
<th>Criterion to minimise scale effects</th>
<th>Remarks</th>
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<td>Self-similarity for $Re &gt; 4 \times 10^4$</td>
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<td>Time-averaged velocity $V$</td>
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<td>Bubble chord time $t_{ch}$</td>
<td>$L_{scale} = 1$</td>
<td>Scale effects unless at full-sale</td>
</tr>
<tr>
<td>Turbulence intensity $Tu$</td>
<td>$L_{scale} = 1$</td>
<td>Scale effects unless at full-sale</td>
</tr>
<tr>
<td>Auto-correlation integral time scale $T_{xx}$</td>
<td>$L_{scale} = 1$</td>
<td>Scale effects unless at full-sale</td>
</tr>
<tr>
<td>Bubble cluster properties</td>
<td>$L_{scale} = 1$</td>
<td>Scale effects unless at full-sale</td>
</tr>
</tbody>
</table>

Note: experiments performed for $Fr_1 = 5.1, 2.5 \times 10^4 \leq Re \leq 1.3 \times 10^5$ with air and Brisbane tap water.
Fig. 1 - Hydraulic jump stilling basin downstream basin downstream of the Paradise dam spillway on 30 Dec. 2010 on the Burnett River (QLD, Australia) - $Q \approx 6,300 \text{ m}^3/\text{s}$, $W = 335 \text{ m}$, $Re \approx 2 \times 10^7$ - Flow direction from left to right

(A) General view

(B) Details of the jump roller
Fig. 2 - Air entrainment in a breaking hydraulic jump

(A) $d_i = 0.0395$ m, $Fr_1 = 5.1$, $Re = 1.3 \times 10^5$, shutter speed: 1/100 s - Flow from left to right

(B) Definition sketch, including vertical distributions of void fraction $C$, bubble count rate $F$ and interfacial velocity $V$ in the jump roller

Figure 3 - Dimensionless distributions of void fraction and bubble count rate in the hydraulic jump for $Fr_1 = 5.1$, $x_1/d_1 = 42$, $W/d_1 \geq 12$ and $Re = 25,000$, 38,000, 68,000 & 125,000

(A, Left) Void fraction data
(B, Right) Bubble count rate data

(A1) $(x-x_1)/d_1 = 4$
(A2) $(x-x_1)/d_1 = 8$

(B1) $(x-x_1)/d_1 = 4$
(B2) $(x-x_1)/d_1 = 8$

(A3) \( \frac{x-x_1}{d_1} = 12 \)  

(B3) \( \frac{x-x_1}{d_1} = 12 \)
Figure 4 - Effects of the Reynolds number on the maximum void bubble count rate $F_{\text{max}d_1/V_1}$ in the turbulent shear layer.
Figure 5 - Interfacial velocity profile in the hydraulic jump roller

(A) Vertical distributions of interfacial velocity for $3.5 < (x-x_1)/d_1 < 8$ - Comparison with the wall jet equations for $(x-x_1)/d_1 = 3.5$ & $Re = 130,000$

(B) Dimensionless relationship between the maximum velocity $V_{max}$ and inner layer thickness $\delta$ - Comparison with Equation (6)
(C) Longitudinal distribution of the inner layer thickness $\delta$ and length scale $b$ (sketch above) defined as the elevation where $V = \frac{V_{\text{max}}}{2}$ - Comparison with the roller upper surface elevation $Y_{90}$ and monophase wall jet data (GEORGE et al. 2000, TACHIE et al. 2004)
Figure 6 - Dimensionless distributions of void fraction, turbulence intensity and integral time scale in the hydraulic jump for $Fr_1 = 5.1$, $x_1/d_1 = 42$, $W/d_1 \geq 12$ and $Re = 38,000$ & $125,000$

(A, Left) Turbulence intensity data  
(B, Right) Integral time scale data

(A1) $(x-x_1)/d_1 = 4$

(B1) $(x-x_1)/d_1 = 4$

(A2) $(x-x_1)/d_1 = 8$

(B2) $(x-x_1)/d_1 = 8$
(A3) \( \frac{y}{d_1} \) vs. \( \frac{C, Tu}{4} \) for \( Re = 38,000 \) and \( Re = 125,000 \)

(B3) \( \frac{y}{d_1} \) vs. \( \frac{C, T_{xx, V_1/d_1}}{d_1} \) for \( Re = 38,000 \) and \( Re = 125,000 \)

Figure 7 - Dimensionless probability distribution functions of bubble chord time $t_{ch} \sqrt{g/d_1}$ in the developing shear layer of hydraulic jumps at $F = F_{\text{max}}$ for $Fr_1 = 5.1$, $x_1/d_1 = 40$, $B/d_1 \geq 10$ and $Re = 25,000, 38,000, 68,000 \& 125,000$

(A) $(x-x_1)/d_1 = 4$

<table>
<thead>
<tr>
<th>$Re$</th>
<th>$y/d_1$</th>
<th>$C$</th>
<th>$F_{\text{max}}d_1/V_1$</th>
<th>$N_{ab}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25,000</td>
<td>1.44</td>
<td>0.115</td>
<td>0.25</td>
<td>1598</td>
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<tr>
<td>38,000</td>
<td>1.50</td>
<td>0.170</td>
<td>0.47</td>
<td>2491</td>
</tr>
<tr>
<td>68,000</td>
<td>1.76</td>
<td>0.334</td>
<td>1.10</td>
<td>2252</td>
</tr>
<tr>
<td>125,000</td>
<td>1.24</td>
<td>0.203</td>
<td>1.33</td>
<td>4804</td>
</tr>
</tbody>
</table>

(B) $(x-x_1)/d_1 = 12$

<table>
<thead>
<tr>
<th>$Re$</th>
<th>$y/d_1$</th>
<th>$C$</th>
<th>$F_{\text{max}}d_1/V_1$</th>
<th>$N_{ab}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>38,000</td>
<td>1.67</td>
<td>0.058</td>
<td>0.22</td>
<td>1144</td>
</tr>
<tr>
<td>68,000</td>
<td>2.42</td>
<td>0.159</td>
<td>0.62</td>
<td>900</td>
</tr>
<tr>
<td>125,000</td>
<td>1.24</td>
<td>0.079</td>
<td>0.71</td>
<td>2561</td>
</tr>
</tbody>
</table>
Figure 8 - Effects of the Reynolds number on the bubble cluster properties in the air-water turbulent shear layer of hydraulic jumps at the locations where $F = F_{max} (y = y_{Fmax}) - Fr_1 = 5.1$, $(x-x_1)/d_1 = 4$ & 12

(A, Left) Dimensionless number of cluster per second $N_c \times d_1/V_1$

(B, Right) Percentage of bubbles in clusters

(C) Number of bubbles per cluster