Quantum features in the scattering of atoms from an optical standing wave

S. Dyrtng and G. J. Milburn
Department of Physics, University of Queensland, Queensland 4072, Australia
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We consider the scattering of an atom as it passes through a classical standing wave, without neglecting the center-of-mass kinetic energy. In the large detuning limit we show that this may be modeled as a nonlinear pendulum. In the quantum description fractional revivals of the initial state occur. These revivals are manifest as nonclassical peaks in the distribution of momentum orthogonal to the direction of beam propagation. This quantum scattering could be achieved with cooled well-collimated beams.

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The past ten years have seen a resurgence of interest in quantum dynamics. Of particular interest has been that transition from the linear behavior of quantum dynamics to the nonlinear, possibly chaotic, behavior of classical systems. The quantum system will typically mimic the classical on some characteristic "classical" time scale [1,2], after which the graininess of quantum mechanics becomes manifest. The quantum departures from classical behavior can take the form of tunneling [3-5], dynamical localization [6,7], or quantum revivals [4,8]. In this paper we compare the quantum and classical dynamics of an atom in an optical standing-wave field. We show that the quantum wave packet exhibits nonclassical revivals. These revivals take the form of a superposition of a finite number of semiclassical states. The interaction of electronic and center-of-mass motion of an atom with the electromagnetic field has recently been exploited in the burgeoning field of atomic optics [9]. It is now possible to cool and trap atoms in regions of less than a single optical wavelength, where the number of atomic vibrational energy states is relatively small and the center-of-mass motion is manifestly quantum mechanical. Recent experiments [10,11] have measured the effect of this quantized motion on the fluorescence spectra of trapped atoms. In this paper we focus directly on the center-of-mass motion of the atom as it traverses an optical field. A related problem was recently discussed by Graham, Schlautmann, and Zoller [12], where an external modulation of the cavity mirror position caused chaotic behavior in the equivalent classical model. If the Rabi frequency $\Omega$ is small compared with the atom-field detuning $\Delta$, then the Hamiltonian for a two-level atom interacting with a classical standing-wave field may be written [13]

$$H = \frac{p_x^2}{2m} + \hbar \omega_0 \sigma_z + \frac{\hbar \Omega^2}{4\Delta} \sigma_z \cos 2kx,$$

where $x$ and $p_x$ are the center-of-mass position and momentum operators, $\sigma_z$ is the atomic inversion operator, $\omega_0$ is the atomic transition frequency, and $k$ is the wave number of the electromagnetic field. We have assumed that the field mode function is a plane standing wave with propagation vector along the $x$ axis. Center-of-mass momentum in the plane perpendicular to the propagation vector is a constant of motion, and the corresponding kinetic energy term has been dropped from the Hamiltonian. Previously this system has been studied in the Raman-Nath regime [13], where the spread of the atomic wave function during the time taken for the atom to traverse the cavity is negligible. In this paper we consider the dynamics when this approximation is not possible. The atom is assumed to be in the ground electronic state. As the atomic inversion is a constant of motion we can then make the replacement

$$\sigma_z \to -\frac{i}{2}. \quad (2)$$

It is useful to define the following dimensionless quantities $\tau = \omega_t$, $p = \frac{2k p_x}{m \omega}$, $q = 2k x$, $F = \hbar \Omega^2 k^2 / 2m \Delta \omega^2$, $K = 4 \hbar k^2 / m \omega$, where $\omega$ is a characteristic frequency used to scale $t$. In terms of the new variables the Hamiltonian is

$$H = \frac{p^2}{2} - F \cos q,$$

with

$$[q,p] = iK. \quad (4)$$

Thus $K$ plays the role of a dimensionless Planck constant. This Hamiltonian is the same form as that for a nonlinear pendulum, and thus exhibits bounded motion inside one of the cosine potential wells and unbounded motion over the top of the cosine potential. We define a classical state to be a probability measure on phase space of the form $Q(q,p) dq dp$, where $Q(q,p)$ is the joint probability density. The density then obeys the Liouville equation

$$\frac{\partial Q}{\partial t} = -\{H,Q\} = -p \frac{\partial Q}{\partial q} + F \sin q \frac{\partial Q}{\partial p}, \quad (5)$$

where $\{,\}$ is the usual Poisson bracket. This equation can be solved by the method of characteristics. Let us choose the initial state $Q_0(q,p)$ to be a bivariate Gaussian function centered on $(q_0,p_0)$ with position variance $\sigma_q$ and momentum variance $\sigma_p$. The solution is

$$Q(q,p,\tau) = Q_0(q(q,p,\tau), p(q,p,\tau))$$

where $(q(q,p,\tau), p(q,p,\tau))$ is the trajectory generated by Hamilton's equations
with \( \vec{q}(q,p,0)=q \) and \( \vec{p}(q,p,0)=p \). If the initial density is localized in a region of bounded motion, it will not remain localized but undergo a rotational sheering. This sheering is due to the fact that the points localized at larger energies (further from the origin in phase space) oscillate with different frequencies. For the nonlinear pendulum the nonlinear frequency is a decreasing function of the distance in action from the stable point at the bottom of the cosine potential. This is generic behavior for nonlinear systems, where the frequency of oscillation varies with the classical action [14]. The resulting pattern has been referred to as a "whorl." In Fig. 1 we have plotted the mean and variance of momentum as a function of time for \( F=1.2 \). Since we are not interested in the variation over one classical period, but rather the long-time evolution, we have plotted the variances at times \( \tau=2\pi n \), with \( n \) integer. For obvious reasons \( n \) is referred to as the strobe number. We see that the mean is rapidly damped and the variance rises to a constant value as the initial state is smeared over the classical trajectories. To investigate the quantum dynamics we use the Husimi or \( Q \) function [15] as the appropriate quantum analog of the classical phase-space density. For a quantum state \( |\psi(\tau)\rangle \), it is defined by

\[
Q(q,p,\tau)=\frac{1}{2\pi K}|\langle q,p|\psi(\tau)\rangle|^2.
\]  

The states \( |q,p\rangle \) are coherent states for a simple harmonic oscillator with frequency chosen as \( \omega_0=\sqrt{F} \), i.e., the frequency of linear motion about the stable fixed point of the nonlinear oscillator. As our initial state we choose the wave packet

\[
|\psi(0)\rangle=(2\pi \sigma)^{-1/4}\exp\left\{ \frac{(p-p_0)^2}{4\sigma} - i q_0 \frac{p}{K} \right\}.
\]  

This is a minimum uncertainty state with means \( \langle p \rangle=p_0 \), \( \langle q \rangle=q_0 \). The initial \( Q \) function is then a bivariate Gaussian function centered on \( (q_0,p_0) \) with position variance \( \sigma_q=(K/2\omega_0)^2+(K^2/4\sigma) \), and

\[
\sigma_p=(K\omega_0/2)+\sigma.
\]

In Fig. 2 we have plotted the mean and variance for the momentum as a function of the strobe number for \( F=1.2 \) and \( K=0.24 \). Note that the quantum mean follows the classical mean initially, but at regular intervals exhibits partial revivals of the initial values. Similar behavior was demonstrated for another nonlinear oscillator in Ref. [15]. Further insight into this behavior may be gained by directly computing the \( Q \) function. Averbukh and Perelman [16] have shown that when a classical system performs regular periodic motion with a nonlinear frequency, localized wave packets in the semiclassical regime exhibit what they refer to as "fractional revivals" on a characteristic time scale \( T_{rev} \). At times \( t=nT_{rev} \) and \( t=mT_{rev} \) the quantum state approximates the initial wave packet evolved to time \( t \) according to the linearized dynamics. However, at times \( t=m/nT_{rev} \) with \( m,n \) coprime, the quantum state approximates a superposition of \( l \) copies of the initial state, where

\[
l=\begin{cases} n, & n \text{ odd} \\ n/2, & n \text{ even}. \end{cases}
\]

To illustrate this behavior we show in Fig. 3 the contours of the \( Q \) function at times commensurate with the revivals in the quantum mean and variance. Clearly the state is a superposition of a discrete number of wave packets. Averbukh and Perelman give the following semiclassical estimate of the revival time:

\[
T_{rev}=2T_{cl}\left[ K \frac{\partial \omega_{cl}}{\partial E} \right]^{-1},
\]

where \( T_{cl}, \omega_{cl}, \) and \( E \) are the classical period, frequency, and energy of a point on which the wave packet is localized. For a \( Q \) function centered on \( q_0=-1.5, p_0=0 \) and \( F=1.2, K=0.24 \), we find \( T_{rev}=329 \). This corresponds to a strobe number \( n=52 \). So Fig. 3 corresponds to fractional revivals at times \( \tau=\frac{1}{2}T_{rev}, \frac{3}{2}T_{rev}, \frac{5}{2}T_{rev}, \) and \( T_{rev} \). The revivals of the nonlinear pendulum demonstrated above can be realized experimentally by observing the scattering of atoms from an optical standing wave. As an example we propose using ytterbium atoms [12] with mass \( m=2.9\times10^{-25} \) kg prepared in a two-state system with frequency \( \omega_d/2\pi=5.4\times10^{14} \) Hz, and an atom-
field system with a Rabi frequency $\Omega/2\pi = 1.0 \times 10^8$ Hz, detuning $\Delta/2\pi = 2.9 \times 10^9$ Hz, wavelength $\lambda = 5.56 \times 10^{-7}$ m, and a wave number $k = 1.13 \times 10^7$ m$^{-1}$. For a characteristic frequency of $\omega/2\pi = 1.2 \times 10^5$ Hz we have the dimensionless quantities $F = 1.2$ and $K = 0.24$. A wave packet initially localized at $x = -0.1\lambda$, $\Delta x = 0.02\lambda$, $p_x = 0.0\hbar k$, and $\Delta p_x = 3.6\hbar k$ will have a corresponding revival time of $T_{\text{rev}} = 4.36 \times 10^{-4}$ s. Collimation of the transverse motion of the atom to the degree stated above will be difficult. We require the width of the atomic beam to be 0.01 $\mu$m, whereas current experiments [18] have collimated beams to only 10 $\mu$m. To observe the revival at $t = T_{\text{rev}}$, when the standing wave has a dimension of 10 mm in the direction of atomic beam propagation, the longitudinal velocity of the ytterbium atoms must be 92 ms$^{-1}$. While this is slow it is within reach of current experimental techniques of atomic optics and laser cooling. The standing wave can be produced by retroreflecting a laser beam using a mirror located within the vacuum chamber. The position of the atomic beam relative to the standing-wave antinodes can then be set by adjusting the position of the mirror with the aid of a piezoelectric crystal. The Rabi frequency quoted above corresponds to a laser intensity of 84 W cm$^{-2}$. Intensities of this order are already used in atomic cooling experiments [19]. In Fig. 4 we show the momentum distribution at the four fractional revival times of Fig. 3. These peaks would appear as scattering angle data in an experiment. Our analysis has assumed that dissipation due to spontaneous emission of the atom as it transits the cavity can be neglected. Ytterbium has a spontaneous emission rate of $\gamma/2\pi = 183 \times 10^3$ Hz, and for the experimental parameters described above the interaction time is $t_{\text{int}} \approx 1.1 \times 10^{-4}$ s. Thus the number of photons emitted spontaneously in one transit is on average 0.02 [12], which should be small enough to be negligible, although this would merit further study.

In this paper we have shown that the scattering of atoms in a standing-wave field should show nonlinear quantum effects such as fractional revivals. Previously, quantum revivals have been observed in the electronic motion [17] of an atom. In this paper we have shown that techniques of quantum atomic optics will enable an observation of nonlinear quantum revivals in the center-of-mass motion of an atom, although this will require an atomic beam of smaller width than has currently been achieved.

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