Performance of MIMO Beamforming Transmission Scheme in the Presence of Mutual Coupling

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Abstract

This paper reports investigations into mutual coupling effects on performance of a beamforming transmission scheme of a multiple-input multiple-output (MIMO) system operating under Rician channel conditions. It is shown that the presence of mutual coupling in a transmitting array antenna degrades the radiation pattern. However, it does not adversely affect the system capacity. If the correlated Rayleigh component (NLOS) dominates the channel, the mutual coupling leads to lower capacity. The presence of mutual coupling results in a higher capacity when the LOS component prevails. For some specific ranges of array inter-element spacing, mutual coupling can be seen as beneficial in terms of increasing the capacity when a signal beamforming strategy is applied.

1. Introduction

Driven by the demand for a higher and robust data transmission, multiple-input multiple-output (MIMO) communication systems have received a significant attention of the wireless communication research community in the last decade. By employing multiple element antennas (MEAs) at the transmitter and receiver sides, MIMO systems have shown a significant capacity gain over their traditional single-input single-output (SISO) counterparts [1] [2]. When full or partial Channel State Information (CSI) is available at the transmitter, Beamforming (BF) is a preferable technique for radio signal transmission [3] [4] in a mixed (Line of Sight) LOS and Non-line of Sight (NLOS) signal propagation environment. By directing the main lobe of the array antenna to the desired user it can improve a Signal-to-Interference/Noise Ratio (SINR). Also it can suppress interference in a multiuser environment by filtering undesired signals in the spatial domain.

Most of the works on MIMO Beamforming assume “ideal array antennas” in which the effect of mutual coupling due to finite element spacing is neglected. In real array antennas, Mutual Coupling (MC) is always present and its effects are considerable especially for tightly spaced arrays. The outcome of mutual coupling is that the signals at different antenna elements are correlated. Also the radiation pattern is distorted.

In [5], an electromagnetic (EM) model was proposed and the performance of MIMO system was investigated from an electromagnetic perspective. In [6], an assessment of MC effect on the interference rejection capabilities of a BF array was presented. This paper is the follow up of the work described in [5] and [6]. It investigates the impact of the mutual coupling on the information rate when the Beamforming signal transmission is applied over a Rician MIMO channel.

2. Signal and channel model

2.1. Signal Model

In the undertaken investigations it is assumed that the MIMO system includes $M$ receiving and $N$ transmitting antennas. For this $(M, N)$ MIMO system, the baseband received signal can be presented as

$$ r = Hx + n $$

(1)

where $r \in \mathbb{C}^{Mx1}$ is the received signal vector, $H \in \mathbb{C}^{MxN}$ is the complex channel matrix, $x \in \mathbb{C}^{Nx1}$ is the transmitted signal vector, and $n \in \mathbb{C}^{Mx1}$ is a zero-mean circularly symmetric complex Gaussian noise vector with power spectral density $\sigma_n^2/2$ per element. In addition,

$$ x = ws $$

(2)

where $s \in \mathbb{C}^{Px1}$ is the transmitted symbol vector and $w \in \mathbb{C}^{NxP}$ are the Beamforming weight vectors.
2.2. Channel Model

For the channel, a Rician channel model is assumed to represent a signal propagation scenario. As a result, the channel matrix can be divided into (Line of Sight) LOS and (Non-line of Sight) NLOS components, as given by

\[ H = \sqrt{\frac{K}{K+1}} H_{LOS} + \sqrt{\frac{1}{K+1}} H_{NLOS} \]  

(3)

where \( K \) is the Rician factor defined as the power ratio of LOS and NLOS components. The LOS component is given by [5]

\[ h_{mn}^{LOS} = \exp(-j\beta d_{mn}) \]  

(4)

where \( d_{mn} \) is the distance between the \( m^{th} \) receiving element and the \( n^{th} \) transmitting element and \( \beta = 2\pi/\lambda \) is the wave number.

For the NLOS component, the Jakes fading model is applied. In this model, Base Station (BS) antennas are assumed to be located at a large height above the ground where the influence of scatterers close to the receiver is negligible. In turn, the Mobile Station (MS) is postulated to be surrounded by many scatterers, which can be represented by “circle of influence”.

Using this model, the signal correlation coefficient between any two antennas at the BS is given by [9]

\[ \rho_{BS} = J_0 \left( \frac{2\pi}{\lambda} d_{\text{max}} \cos(\theta) \right) \exp \left( -\frac{2\pi}{\lambda} d \sin(\theta) \right) \]  

(5)

Correspondingly, the signal correlation coefficients between any two antennas at the MS is given by

\[ \rho_{MS} = J_0 \left( \frac{2\pi}{\lambda} d \right) \]  

(6)

In (5) and (6), \( d \) is the antenna spacing distance, \( \lambda \) is the wavelength of the carrier, \( \Delta_{\text{max}} \) is the maximum Angular Spread (AS), \( \theta \) is the Angle of Arrival (AoA) and \( J_0 \) is the 0th order Bessel function.

Assuming the downlink case between a base station (BS) and a mobile station (MS), the correlated channel matrix is given by the following expression [7]

\[ H_{NLOS} = R_{MS}^{1/2} G R_{RS}^{1/2} \]  

(7)

where \( G \in \mathbb{C}^{M \times N} \) is a stochastic spatially white matrix with independent identically distributed (i.i.d.) complex zero-mean, unit variance Gaussian entries. This representation is called the Kronecker model of MIMO channel in which correlation at BS and MS are assumed to be separable.

3. Downlink transmit Beamforming with mutual coupling

3.1. Transmit Beamforming

Since the BS antenna array is assumed to be located at a large height above the ground, the communication between BS and the desired MS is limited for some specific directions. By applying Beamforming, the BS antenna array can exploit the spatial dimension of the mobile radio channel. The signals at the elements of the BS antenna array are weighted to achieve the best signal reception from or optimal transmission to the desired MS. For a single user scenario, the received signal at the BS antenna array can be expressed as

\[ X(t) = \sum_{l=1}^{L} \alpha_l a(\theta) s(t) + n(t) \]  

\[ = \Omega A(\theta) s(t) + n(t) \]  

(8)

where \( X(t) \) is an \( N \times 1 \) vector of measured voltages, \( s(t) \) is the transmitted signal to the target MS, \( n(t) \) is an \( N \times 1 \) noise vector. \( \Omega \) is diagonal matrixes with its diagonal elements \( \alpha_l \) are the gains for the \( L \) wireless paths. \( A(\theta) = [a(\theta_1), a(\theta_2), \cdots, a(\theta_L)] \) is \( N \times L \) matrix whose columns are steering vectors, which can be written as

\[ a(\theta) = \left[ 1, e^{j(2\pi/\lambda) d \sin(\theta_1)}, \cdots, e^{j(2\pi/\lambda) (N-1)d \sin(\theta_L)} \right] \]  

(9)

The correlation matrix for the BS array can be obtained from

\[ R_{XX}(t) = E[X(t)X^H(t)] \]  

(10)

where \( E(\cdot) \) and \( (\cdot)^H \) denote expectation and Hermitian transpose respectively. The array correlation matrix contains the information about how signals between antenna elements are correlated [6]. By performing an
eigenvalue decomposition of $R_{XX}(t)$ into $P$ eigenvectors, the weights can be made by the $P$ largest eigenvalues. As a result, the signals at the array output can be given by

$$y(t) = \frac{1}{M} u^H(t) \tilde{x}(t) = w^H(t) \tilde{x}(t)$$  \hspace{1cm} (11)$$

3.2. Mutual coupling

For a given array antenna, the mutual coupling matrix can be obtained using an electromagnetic and circuit theory described in [6]

$$C = (Z_A + Z_T)(Z + Z_T J_M)^{-1}$$ \hspace{1cm} (12)$$

where $Z_A$ is the element impedance in isolation and $Z_T$ is impedance of the receiver at each element chosen as the complex conjugate of $Z_A$ to obtain the impedance match. $Z$ is the mutual impedance matrix with all the diagonal elements equal to $Z_A + Z_T$, its non-diagonal elements $Z_{nm}$ are the function of physical parameters of antenna elements and their spacing. This representation is valid for single mode antennas such as wire dipoles. Here we assume that the transmitting and receiving array antennas are formed by wire dipoles. For a side-by-side array configuration and dipole length equal to 0.5$\lambda$, the expressions for $\{Z_{nm}\}$ were already given in [5] [6], and thus are not repeated here. It has to be noted that the expressions shown in [5], [6] are obtained by taking into account only EM interactions between two dipoles, one being driven by a source and the other one being open-circuited. This approach neglects the presence of other open-circuited dipoles and hence is valid for the dipole spacing not less than 0.17$\lambda$.

3.3. Transmit Beamforming with mutual coupling

Having determined the mutual coupling matrix $C_R$ at BS, $C_T$ at MS and the Beamforming vectors $w$, the baseband signal (1) is modified to

$$r = C_R H C_T w s + n$$ \hspace{1cm} (13)$$

The capacity of the MIMO system employing beamforming scheme can be calculated from the following formula

$$C = \log_2 \det(I + \rho C_R H C_T w w^H C_R^H H^H C_T^H)$$ \hspace{1cm} (14)$$

where $\rho = P_T/P_N$ with $P_T$ the total transmit power and $P_N$ the noise power at each receiving element. Here, the mean capacity is determined by averaging $C$ over a number of random realizations of the channel matrix $H$.

4. Numerical Results

Using the presented MIMO channel model, computer simulations are performed to investigate the performance of beamforming over Rician MIMO channel without and with the mutual coupling. In the model, dipole antennas are assumed to be 0.5$\lambda$ in length. In the investigations, the array radiation patterns, antennas numbers vs. beamforming capacity, the transmitting array interelement spacing vs. beamforming capacity and Rician factor vs. beamforming capacity are considered. Figure 2 shows the effect of mutual coupling on the beamforming pattern.

The simulations assume a 10-element BS array antenna with spacing of 0.5$\lambda$ and 0.25$\lambda$. Other assumptions are as follows. SNR is 10dB, DOA for LOS is -30$^\circ$, DOA for NLOS is 50$^\circ$, $K=0$dB. One can see from Figure 1 that the beamforming network directs the main lobes to the desired directions. However, with the mutual coupling included, the radiation pattern has higher level of side lobes. This means that the mutual coupling leads to degradation of the radiation pattern due to the mismatched steering vector [6].

![Figure 2. Effect of mutual coupling on the Array Pattern (10 elements, SNR = 10dB, K =0dB)](image)

Figure 3 and Figure 4 shows the plot of MIMO channel capacity versus the number of antenna elements when beamforming is applied for the cases without and with mutual coupling. When the number of receiving antenna elements is fixed the capacity increases with increasing the number of transmit antenna elements. With the presence of mutual coupling, the system has a slightly higher capacity, as seen in Figure 3. If the number of antenna element at BS and MS increase simultaneously, the system
without mutual coupling shows a higher capacity over the system under the effect of mutual coupling, as shown in Figure 4. Also, the difference between the capacity for with and without mutual coupling increases when the number of antennas forming the arrays is increased.

Figure 3. Beamforming Capacity Vs SNR (BS interelement spacing equals 0.5, MS interelement spacing equals 0.5, K =0dB, DOA for LOS is -30°, DOA for NLOS is 50°)

Figure 4. Beamforming Capacity Vs SNR (BS interelement spacing equals 0.5, MS interelement spacing equals 0.5, K =0dB, DOA for LOS is -30°, DOA for NLOS is 50°)

Figure 5 shows the MIMO system performance under different channel conditions when the presence of mutual coupling is taken into account. The plots for specific MIMO configurations with and without mutual coupling have a crossing point at some particular value of K. When K is small, meaning that the correlated Rayleigh component dominates the channel, the existence of mutual coupling leads to lower capacity. When the LOS component prevails, the presence of mutual coupling leads to higher capacity.

Figure 5. Beamforming Capacity Vs Rician Factors (BS interelement spacing equals 1.0, MS interelement spacing equals 0.5, DOA for LOS is -30°, DOA for NLOS is 50°)

Figure 6 shows the plot of capacity versus BS interelement spacing for different configurations, assuming the receive antenna number at MS is fixed to 4. The plots for a specific configuration merge at dt = 1.0 (interelement spacing at BS is equal to 1.0), where the mutual coupling can be ignored. The system with mutual coupling reaches the peak point for 0.3 < dt < 0.4 and it takes minimum at dt = 0.7. In the spacing range of 0.3 < dt < 0.6, mutual coupling can be seen as beneficial with respect to increasing capacity when the beamforming scheme is used.

Figure 6. Beamforming Capacity Vs BS interelement spacing ( MS interelement spacing equals 0.5, K =0dB, DOA for LOS is -30°, DOA for NLOS is 50°)

5. Conclusions

This paper has reported the investigations into the effect of mutual coupling on the capacity of MIMO system operating over a Rician channel when beamforming is applied for signal transmission. It has been shown that the mutual coupling leads to degradation of the beam pattern. However it does not have an adverse effect on capacity. If a correlated
Rayleigh component (NLOS) dominates the channel, the existence of mutual coupling leads to lower capacity. In turn, the mutual coupling increases capacity when the LOS component dominates. In the specific range of small inter-element spacing at BS, mutual coupling can be seen as beneficial in terms of increasing the MIMO capacity when beamforming is applied.

6. References


