Heatline and ‘energy flux vector’ visualization of natural convection in a porous cavity occupied by a fluid with temperature-dependent viscosity

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ABSTRACT
Temperature-dependent viscosity effects in buoyancy driven flow in a porous-saturated enclosure is studied numerically, based on the general model of momentum transfer in a porous medium based on the Arrhenius model, which proposes an exponential form of viscosity-temperature relation. Effects of fluid viscosity variation on isotherms, streamlines, and the Nusselt number are studied. Both cases of viscosity increase and decrease versus temperature is considered. Application of the effective Rayleigh number concept and the reference temperature approach are investigated. Moreover, heatlines and the energy flux vectors are illustrated for a more comprehensive analysis of the problem.

Keywords: Natural convection, Heatline visualization, Temperature-dependent viscosity, Lateral heating
Nomenclature

- \( b \) viscosity variation number
- \( C \) coefficient for Nu-Ra correlation
- \( C_F \) inertia coefficient
- \( Da \) the Darcy number, \( Da = K/L^2 \)
- \( g \) gravitational acceleration
- \( H^* \) heatfunction
- \( H \) dimensionless heatfunction
- \( i,j \) unit vectors in \( x,y \) direction
- \( k \) porous medium thermal conductivity
- \( K \) permeability
- \( L \) cavity height
- \( M \) Brinkman viscosity ratio \( \mu_e/\mu \)
- \( Nu \) the Nusselt number
- \( P^* \) pressure
- \( Pr_c \) modified Prandtl number, \( Pr_c = \nu_c/\alpha \)
- \( q'' \) heat flux
- \( R \) \( R = \psi_{max}/\psi_{max,cp} \)
- \( Ra_f \) the Rayleigh number, \( Ra_f = g\beta(T_H-T_C) L^4/(\nu,\alpha) \)
- \( Ra_q \) the isoflux Rayleigh-Darcy number, \( Ra_q = g\beta q''/(s^2\nu_c,\alpha K) \)
- \( Ra \) Rayleigh-Darcy number, \( Ra = Ra_f/s^2 \).
- \( s \) porous media shape parameter, \( s = Da^{-1/2} \)
- \( S_\phi \) source term for \( \phi \) equation
- \( S_\omega \) source term for vorticity transport equation
- \( T^* \) temperature
- \( u^*,v^* \) \( x^*,y^* \)-velocity
- \( u,v \) \( u^*L/\alpha, v^*L/\alpha \)
- \( |U^*| \) mean velocity \( (u^{*2}+v^{*2})^{1/2} \)
- \( |U| \) dimensionless mean velocity \( (u^2+v^2)^{1/2} \)
- \( (x^*,y^*) \) horizontal and vertical coordinates
- \( (x,y) \) dimensionless coordinates \( (x^*,y^*)/L \)
**Greek symbols**

- $\alpha$: thermal diffusivity of the porous medium
- $\beta$: thermal expansion coefficient
- $\Gamma$: diffusion parameter
- $\Lambda$: inertial parameter $\Lambda = C_f L e^2 / (Pr K_1/2)$
- $\eta$: kinematic viscosity ratio
- $\theta$: dimensionless temperature $(T - T_c) / \Delta T$
- $\mu$: fluid viscosity
- $\mu_e$: effective viscosity
- $\nu$: kinematic viscosity
- $\rho$: fluid density
- $\psi$: stream function
- $\phi$: generic variable
- $\varepsilon$: porosity
- $\omega$: vorticity

**Subscript**

- am: arithmetic mean
- ave: average
- C: of cold wall
- cp: constant property
- eff: effective
- H: of hot wall
1. Introduction

Natural convection in a differentially heated cavity occupied by a fluid-saturated porous medium has attracted the attention of engineers due to its relevance to some applications including groundwater hydrology, petroleum reservoirs, human respiration, sensible heat storage beds, coal combustors, cooling of electronic systems, nuclear waste repositories and thermal performance of solar collectors. As a result, heat and fluid flow in such cavities have been studied extensively for the past several decades. For a recent survey of literature one may consult Nield and Bejan [1], Ingham and Pop [2], Vafai [3], and Bejan et al. [4].

Natural convection is usually associated with a change in the fluid density as a result of a change in the fluid temperature. This raises the question that if the temperature difference is high enough to cause a change in the density how accurate it is to assume a constant value for the fluid viscosity. This issue has been the subject of many studies in the past three decades and one may consult Nield and Bejan [1] or Hooman and Gurgenci [5] for a list of the papers on the topic.

A quick review shows that there are two possible answers. Some authors concluded that considering the effect of temperature-dependent viscosity variation will lead to significant changes in the velocity and temperature distributions, and consequently the Nusselt number, see for example recent work by Lin et al. [6] or Guo and Zhao [7].

On the other hand, others recommended the use of constant-property solutions with a suitable mean value for the fluid viscosity. Chu and Hickox [8] reported that even extreme viscosity variations, in case of localized heating, will not have significant effects on the overall heat transfer coefficient provided the properties be evaluated at the arithmetic mean temperature and a viscosity ratio be applied. This conclusion is in line with what reported for free convection of air in a square enclosure by [9]. Siebers et al. [10] have come up with the same conclusion for laminar natural convection of air along a vertical plate. However, when it came to turbulent natural convection they applied a correction factor for their Nusselt number in terms of a wall/free stream temperature ratio. The drawback of the above suggestion has been outlined by Guo and Zhao [7] where the fluid properties were evaluated at the arithmetic mean temperature (being mean value of hot and cold wall temperature) and still the results showed significant deviations from the constant property counterparts. For example, with a special value of $Da=10^{-4}$ and $Ra=10$, the Nusselt number changed about 75% compared to the constant property case.

Nield [11, 12] introduced the concept of the effective Rayleigh number based on the mean values of the physical properties. According to Nield [12], if the fluid viscosity is evaluated at the harmonic mean value, the critical Rayleigh number would be unaltered so that viscosity variation would not destabilize the problem. Nield [12] then also showed that when viscosity variation lies within one order of magnitude, the concept of the effective Rayleigh number will work out. However, due to the possibility of
localized flow in a part of flow region for the cases of severe property variations, the doubt was it if the same argument could work out for such cases.

It seems that more investigation on natural convection of a fluid with variable viscosity is called for. Applying the general model of [13] and [14], this paper aims at numerical analysis of this problem. The present work also presents a possibility to see the energy flow in the cavity. For this purpose, we have applied the concept of heatlines as introduced by Bejan [15]. One can consult Bejan [16] for over 20 papers cited by the author that followed the same methodology for convection visualization purpose. Similar attempt has been made by authors involved in convection through a porous medium; see Nield and Bejan [1]. Application of the energy flux vectors has been found to be a viable option to see the heat flow. These vectors are similar to the velocity vectors in being tangent to heatlines while the velocity vectors are tangent to the streamlines. A similar attempt was made by Mukhopadhyay et al. [17] where the authors have shown that enthalpy flux vectors are tangential to enthalpy lines. This paper presents a discussion on energy flux vectors and heatlines.

Previous work on the effects of property variation on convection heat transfer, in the case of fluids clear of solid material, has been surveyed in [18].

2. Model equations

Free convection of a fluid with temperature-dependent viscosity in a square enclosure filled with homogeneous, saturated, isotropic porous medium with the Oberbeck–Boussinesq approximation for the density variation in the buoyancy term is considered, as shown in Fig. 1-a. It is assumed that the solid matrix and the fluid are in local thermal equilibrium. The equations that govern the conservation of mass, momentum and energy can be written as follows

\[
\frac{\partial (u^* \phi)}{\partial x^*} + \frac{\partial (v^* \phi)}{\partial y^*} = \frac{\partial}{\partial x^*}(\Gamma_\phi \frac{\partial \phi}{\partial x^*}) + \frac{\partial}{\partial y^*}(\Gamma_\phi \frac{\partial \phi}{\partial y^*}) + S_\phi
\]

where \( \phi \) stands for dependent variables \( u^*, v^*, T^* \) and \( \Gamma_\phi, S_\phi \) are the corresponding diffusion and source terms respectively for the general variable \( \phi \), as summarized in Table 1. Other parameters are defined in the nomenclature.

Further analysis of the problem is impossible unless one assumes a viscosity-temperature relation. Several models have been used in the literature to account for the temperature-dependent viscosity variation. The Arrhenius model proposes an exponential form of viscosity-temperature behavior to provide a good representation for most common fluids as reported by [19, 20]. It is applied here for flow of an incompressible gas or liquid. Both cases of increase or decrease in fluid viscosity with temperature are assumed. The following exponential variation in kinematic viscosity ratio (with temperature) is assumed
\[ \eta = \frac{\nu}{\nu_c} = \exp(b\theta), \]  

(2)

where the viscosity variation number, \( b \), is positive/negative in case of a gas/liquid whose viscosity increases/decreases with an increase in temperature. The cold wall condition is assumed as our reference state so that \( \nu \) is the kinematic viscosity measured at \( T_c \). In line with this choice, our dimensionless temperature is defined as \( \theta = (T - T_c)/(T_H - T_c) \) based on Bejan [16]’s recommendation on selecting the lowest temperature of the system as our reference temperature for heatline visualization. One also notes that the Taylor series expansion for very small values of \( b \) leads to linear or inverse linear relation for viscosity with temperature as

\[ \nu = \nu_c (1 + b\theta), \]

\[ \frac{1}{\nu} = \frac{1}{\nu_c} (1 - b\theta), \]  

(3a,b)

similar to what applied by [12, 21-23].

The dimensionless stream-function is defined as

\[ u = \frac{\partial \psi}{\partial y}, \]

\[ v = -\frac{\partial \psi}{\partial x}. \]  

(4a,b)

With this definition the continuity equation will be satisfied identically while taking the curl of \( x^* \) - and \( y^* \)-momentum equations and eliminating the pressure terms, one finds the dimensionless vorticity transport equation as

\[ u \nabla \omega = Pr_c \left( (\nabla^2 \omega - \omega^2) e^{\omega} - \Lambda \left| U \right| \omega + S_u \right) \]  

(5)

where

\[ S_u = s^2 \left( \frac{\partial \eta}{\partial x} \frac{\partial \psi}{\partial x} + \frac{\partial \eta}{\partial y} \frac{\partial \psi}{\partial y} \right) + \Lambda \left( \frac{\partial |U|}{\partial x} \frac{\partial \psi}{\partial x} + \frac{\partial |U|}{\partial y} \frac{\partial \psi}{\partial y} + Ra \frac{\partial \theta}{\partial x} \left( \frac{\partial \eta}{\partial x} \frac{\partial \psi}{\partial x} + \frac{\partial \eta}{\partial y} \frac{\partial \psi}{\partial y} \right) + \frac{\partial}{\partial x} \left( \frac{\partial \eta}{\partial x} \frac{\partial \psi}{\partial x} + \frac{\partial \eta}{\partial y} \frac{\partial \psi}{\partial y} \right) \right) \]  

(6)

Moreover, the reference modified Prandtl and the clear-fluid Rayleigh numbers, which will not change with temperature-dependent viscosity variation, are \( Pr_c = \nu \nu_c / \alpha \) and \( Ra_f = g \beta \Delta T L^3 / (\nu \alpha) \), respectively. It is also important to note that we preferred to work in terms of the porous medium shape parameter, \( s \), which is related to the Darcy number as \( s = Da^{1/2} \). Using this definition one defines the Rayleigh-Darcy number, or simply \( Ra \) in our work (hereafter), as \( Ra = Ra_f / s^2 \).

The vorticity directed in \( z \) direction is defined as

\[ \omega = -\nabla^2 \psi. \]  

(7)
The thermal energy equation now takes the following form
\[ u \nabla \theta = \nabla^2 \theta. \]  
\[ (8) \]

The average Nusselt number (the ratio of the actual heat transfer to that of pure conduction) is defined as
\[ Nu = \int_0^1 \frac{\partial \theta(0, y)}{\partial x} dy. \]  
\[ (9) \]

Following Bejan [15], the heatfunction concept is applied here. Heatfunction, \( H^*(x^*, y^*) \), is similar to streamfunction in such a way that the former intrinsically satisfies the thermal energy equation while the latter plays the same role for mass continuity equation. Bejan [15] argues that the conventional use of \( T^* = \text{constant} \) lines is not a proper way to visualize heat transfer in the field of convection where the fluid moves and transfers energy. He also notes that applying isotherms to visualize convection heat transfer is similar to an attempt to apply \( p^* = \text{constant} \) lines to visualize fluid flow. Moreover, it is instructive to note that patterns of \( H^* = \text{constant} \) heatlines are lines across which the net flow of energy is zero. Heatlines are expected to be parallel to adiabatic walls and normal to the isothermal wall. For this problem, and in non-dimensional form, the heatfunction is defined as
\[ \frac{\partial H}{\partial y} = u\theta - \frac{\partial \theta}{\partial x}, \]
\[ -\frac{\partial H}{\partial x} = v\theta - \frac{\partial \theta}{\partial y}. \]  
\[ (10a,b) \]

Equations (10a,b) can be combined to form a Poison equation as
\[ \nabla^2 H = \frac{\partial (u\theta)}{\partial y} + \frac{\partial (v\theta)}{\partial x}. \]  
\[ (10c) \]

It is worth noting that in dimensional form, \( H^*(x^*, y^*) \), is given by Eqns. (1-68) and (1-69) of Bejan [16] so that we will just present the dimensionless form of the two equations. The problem is now to solve Eqns. (5-10) subject to the boundary conditions shown in Fig. 1-b.

### 3. Numerical details

Numerical solution for the governing equations for vorticity, streamfunction, dimensionless temperature, and heatfunction are obtained by finite difference method, using the Gauss-Seidel technique with SOR. The governing equations are discretized by applying the second-order accurate central difference schemes. For the numerical integration, algorithms based on the trapezoidal rule are employed. Details of the vorticity-streamfunction method, heatfunction, and applied boundary conditions may be found in [15, 24-26] and are not repeated here.

All runs were performed on a 90 x 90 grid. Our \( Ra \) is limited to 10\(^3\) similar to Guo and Zhao [7] while \( s \) was changed from 10 to 10\(^3\) and the reference Prandtl number, inertia coefficient, and the Brinkman
viscosity ratio were fixed at unity along with what reported in [27]. Grid independence was verified by running different combinations of $s$, $Ra_f$, and $b$ on a 120 x 120 grid to observe that, given a set of parameters, the change in our results is less than 1%.

The convergence criterion (maximum relative error in the values of the dependent variables between two successive iterations) in all test runs was set at $10^{-5}$. Accuracy of the numerical procedure was verified by comparison of the results given in the literature, as shown in Table 2.

### 4. Results and discussion

Figs. 2-3 indicate the effects of the key parameters (being $b$, $s$, $Ra$, and $Ra_f$) on isotherms and streamlines with two $s$ values, 10 and 100, and for both positive and negative values of $b$. The results for three different $Ra$ values (10, 100, and 1000) are shown in separate charts.

To keep $s$ constant, the value of $Ra_f$ has been altered. It is clear that with a fixed value of $s$, an increase in either $Ra$ or $Ra_f$ leads to stronger convective flows. One can easily verify that with $b<0$, which implies a decrease in viscosity with an increase in temperature, the flow patterns are stronger. On the other hand, the converse can be deduced with positive values of $b$. The constant property solution is found to be somewhere between the two cases, as expected.

Fig. 4 shows line diagrams of dimensionless vertical mid-plane velocity and temperature for $s=10$ and $s=100$, again with various $b$ and $Ra$ values, with even more severe change in the fluid viscosity ($\eta_{\text{max}}\approx 7.4$, $\eta_{\text{min}}\approx 0.14$). While in Fig. 4-a $x$ goes from 0 to 0.5, for a better resolution, $x$ changes from 0 to 1 in Fig. 4-b. Based on these figures, with an increase in $Ra$, there is an increase in maximum vertical velocity and a decrease in the dimensionless temperature profile levels. One notes that with $b=0$, the velocity picks are more or less mirrored. However, with negative values of $b$, for example, the maximum velocity occurring near the heated wall is higher than the absolute value of minimum vertical velocity that happens near the other vertical (cold) wall. With positive values of $b$, the situation is reversed and the velocity is higher near the cold wall. For $Ra=10$, similar to the smaller $s$ case with $b=2$ and $s=100$ the dimensionless mid-plane temperature varies almost linearly with $x$ that implies a conduction-dominated heat transfer mechanism.

Fig. 5 is illustrating the effect of changing viscosity on $Nu$ and $\psi_{\text{max}}$. The deviation from constant property solutions is found to be less than 25% for $Nu$ and 60% for $\psi_{\text{max}}$. As seen, with an alteration in $b$, the change trend in the two ratios is not identical. It is clear that, with fixed $s$, for smaller values of $Ra$ the change in $\psi_{\text{max}}$ ratio is more pronounced while for higher values of $Ra$ these changes become smaller. However, the converse is true for $Nu/Nu_{cp}$. Interestingly, when it comes to examine the effects of $s$ on the two ratios, one observes similar trends in such a way that changes in either function become more pronounced with an increase in $s$. 

Fig. 6 shows the effects of $b$ and $Ra$ on heatline distributions for two values of $s$ being $s=10$ and 100. Horizontal heatlines imply conduction-dominated heat transfer and this dominance becomes clearer with positive values of $b$. It could be deduced, for example by Fig. 6-a (top), that a heatline distribution of $H(x,y)=y$ through the flow region shows pure conduction heat transfer and the amount of upward deflection implies convective heat flow strength (Bejan [15] interprets this as ‘heat rises’). One also notes that as $b$ decreases the heatline distribution become denser near the top wall and the range of iso-$H$ values becomes wider implying higher heat transfer rate.

Figs. 7a-c show the energy flux vector plots that are tangent to the heatlines. These vectors are very useful as they show the most important energy ports. One way to see the energy flow is to see the tangential vector paths. It is easy to show that tangents (to heatlines) are normal to the gradient of the heatlines and may be tracked as

$$
\vec{Q} = Q_i \vec{i} + Q_j \vec{j}
$$

where

$$
Q_x = -\frac{\partial H}{\partial y},
$$

$$
Q_y = \frac{\partial H}{\partial x}.
$$

One notes that in this way one does not need to solve a Poison equation, like Eq. (10c), in most cases by iterative methods, to see the energy flow path. At least in view of computational effort, these energy flux vectors are more efficient than the heatfunctions. Figure 7-a shows the energy flux vectors along with the heatlines through the flow region while Figs. 7-b and 7-c show the inflow and outflow of energy by these vectors, respectively. Based on Fig. 7-b, the heat transfer rate to the cavity is higher in the bottom half of the cavity height while the outflow of energy is higher in the lower part which could be explained by simply recalling the fact that heat rises.

The details can be explained as follows. At the bottom of the cavity the temperature is lower than the top and this will enhance the conduction wall heat flux (which is the dominant heat transfer mechanism in the near-wall region for small fluid velocity in the vicinity of the walls) at the heated wall. A similar analysis may be applied to explain the reason for higher heat transfer rate in the upper half of the cavity height at the cooled wall (top left corner of the enclosure).

It is also worth commenting that, similar to the heatlines, the energy flux vectors are normal/parallel to vertical/horizontal walls, as expected. This gives us the impression that energy is transferred along these vectors as they point out the heat transfer direction.

Fig. 8 shows the energy flux vectors for free convection of a fluid with constant viscosity in a porous cavity, with a constant heat flux at the left vertical wall while the other vertical wall is cooled isothermally.
and the horizontal walls are adiabatic, like the problem studied by Hooman and Gurgenci [26]. Interestingly, according to Fig. 8-b, the energy flux vectors are of the same size along the left wall which is kept at a uniform heat flux and this in turn emphasizes that the heat transfer rate is constant. On the other hand, based on Fig. 8-c, the size of the vectors grow near the top left-corner of the cavity and this can be interpreted similar to Fig. 7-c.

Calculating $Ra$ at the cold wall temperature, the apparent destabilizing effect of decreasing $b$ was observed in all figures. It is instructive to see what happens when an average Rayleigh number is used. Examining the effective Rayleigh number theory of Nielsd [11], that recommends using harmonic average for the fluid viscosity, our effective Rayleigh number reads

$$Ra_{\text{eff}} = (Ra_c + Ra_H)/2.$$ \hfill (12)

The subscripts ‘C’ and ‘H’ are applied to show that cold (left) and hot (right) wall temperatures are applied to evaluate the viscosity. One notes that $Ra_c=Ra$ and that applying the wall temperature for the hot (left vertical) wall in Eq. (2) leads to the following relation

$$Ra_H = Ra \exp(-b).$$ \hfill (13)

Equations (12-13) will lead to the following effective Rayleigh number

$$Ra_{\text{eff}} = Ra(1+\exp(-b))/2.$$ \hfill (14)

Table 3 shows a sample of the above effective Rayleigh number for $Ra=100$ with $b=\pm 1$ where $s$ is allowed to change from $s=10$ to $s=100$. Also available in this table is $Nu/Nu_{cp}$ calculated in two ways. The first method is the application of the effective Rayleigh number with $b=0$. The Nusselt number then is divided by $Nu_{cp}$ with a different Rayleigh number (being $Ra=Ra_c$). The second approach (which was taken so far) is dividing $Nu$ (for a case with non-zero value of $b$) by $Nu_{cp}$; this time at the same $Ra$. One verifies that the highest error entering the $Nu$ calculation, by applying $Ra_{\text{eff}}$, is less than 14%.

Table 4 is presented to show the effects of severe viscosity variation on $Nu$ with two $Ra$ values when $s=100$. The results based on $Ra_{\text{eff}}$ are dramatically different from the numerical results hinting that this method will not work very well in this case when $b=\pm 2$.

To examine the arithmetic mean viscosity, one can apply Eq. (2), which gives the viscosity-temperature relation, to see that the ratio of $Ra_{\text{eff}}/Ra_{am}$ is

$$\frac{Ra_{\text{eff}}}{Ra_{am}} = \left( \frac{1+\exp(-b)}{2\exp(-b/2)} \right),$$ \hfill (15)

wherein $Ra_{am}$ is the Rayleigh number with the viscosity being evaluated at the arithmetic mean temperature, i.e. $\theta=0.5$. For small values of $b$, the Rayleigh number ratio is near unity and this can be easily verified by a Taylor series expansion. However, with higher $b$ values the ratio differs substantially from unity. For example, with $b=2$, the Rayleigh ratio becomes 1.543 which leads to nearly 30% change
in the associate Nusselt number when the Darcy model correlation proposed by Lauriat and Prasad [28] is applied. Noting that applying $Ra_{\text{eff}}$ can lead to as much as 50% error in some cases, one realizes that neither the arithmetic nor the harmonic average can lead to accurate results when the viscosity varies with temperature severely. It seems that care should be taken when it comes to apply an average value for fluid properties needed for $Nu-Ra$ correlation for engineering applications.

Mainly for this reason Table 5 presents a new definition $Ra_{\text{ave}}$ which assumes evaluating the fluid viscosity at a reference temperature so that one can still use the constant property results. Based on our numerical results, it is reasonable to expect this reference temperature to change with $s$. This reference temperature is found as

$$T_{\text{ref}} = T_c + 0.45(T_H - T_c) \quad \text{for} \quad s = 10^1,$$

$$T_{\text{ref}} = T_c + 0.4(T_H - T_c) \quad \text{for} \quad s = 10^2,$$

$$T_{\text{ref}} = T_c + 0.32(T_H - T_c) \quad \text{for} \quad s = 10.$$ (16)

leading to

$$Ra_{\text{ave}} = Ra_c \exp(-0.45b) \quad \text{for} \quad s = 10^1,$$

$$Ra_{\text{ave}} = Ra_c \exp(-0.4b) \quad \text{for} \quad s = 10^2,$$

$$Ra_{\text{ave}} = Ra_c \exp(-0.32b) \quad \text{for} \quad s = 10.$$ (17)

A sample of the results based on the above correlation is presented in Table 5 and is found to be more accurate compared to the effective Rayleigh number approach. It may be concluded that one can still apply the constant property solutions available in the literature with the only modification that the fluid property being evaluated at the reference temperature recommended here. Observing the numerical results, we simply propose a rough and ready estimation for the dependence of the reference temperature on the Darcy number as follows

$$T_{\text{ref}} = T_c + 0.5\left(1 - 0.848s^{-0.3}\right)(T_H - T_c)$$ (18)

that leads to

$$Ra_{\text{ave}} = Ra_c \exp\left(-0.5b\left(1 - 0.848s^{-0.3}\right)\right)$$ (19)

Keep in mind that these last two equations are valid for $10 < s < 1000$ while as $s \rightarrow \infty$, i.e. for the Darcy flow model, based on the above correlation, the average Rayleigh number tends to the effective Rayleigh number introduced by Nield.

Another point worthy of comment is that our results are limited within a range of the porous media shape parameter being those relevant to clear fluid ($s \rightarrow 0$) and Darcy flow model ($s \rightarrow \infty$). For these two cases the reference temperatures are $T_{\text{ref}} = T_c + 0.5(T_H - T_c)$ and $T_{\text{ref}} = T_c + 0.25(T_H - T_c)$ with the former being recommended indirectly by Nield [11] (for small values of $b$) for the Darcy-Bénard problem and the latter
proposed by Zhong et al. [9] for the clear fluid natural convection. The dependence of the reference temperature on $s$ is expected as each $s$ value is associated with a unique convection pattern.

5. Conclusion

Numerical simulation of natural convection in a laterally heated porous-saturated square enclosure is presented based on the general momentum equation. The Arrhenius model for the variation of viscosity with the temperature is applied. A reference temperature approach is undertaken to account for viscosity variation. It is found that the reference temperature, at which the fluid properties should be evaluated, is a decreasing function of the porous media shape parameter and is approximately independent of the other parameters considered here. Applying this reference temperature, one can still use the constant property results and this, in turn, will reduce the computational time and expense required for solving a variable property problem. In addition to the application of the heatlines, the energy flux vectors were introduced to improve the salver’s ability to see the energy flow especially at the walls. Besides, application of these vectors reduces the time and the computer resources required to solve the Poison equation to see the heatlines. Such software as Tecplot can be used to do the ‘Streamtrace Placement’ that is, in this case, heatlines, on the energy flux vectors. In view of the above, it seems that application of the energy flux vectors gives us heatline distribution without the need to go through an excessive numerical calculation.

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References

### Table 1. Summary of the solved governing equations

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<th>Sφ</th>
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<td>μ*/ρ</td>
<td>( \frac{1}{\rho} \frac{\partial \check{p}}{\partial x} - \frac{\nu u^<em>}{K} - \frac{C_\nu u^</em></td>
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<tr>
<td>y*-momentum</td>
<td>v*/ε^2</td>
<td>μ*/ρ</td>
<td>( -\frac{1}{\rho} \frac{\partial \check{p}}{\partial y} - \frac{\nu v^<em>}{K} - \frac{C_\nu v^</em></td>
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### Table 2. Present Nu values versus those in the literature.

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<th>Ra</th>
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### Table 3. Calculation of the effective Rayleigh number and the Nusselt number with Ra=100 and b=±1

<table>
<thead>
<tr>
<th>s</th>
<th>b</th>
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<th>Nu/Nu_{cp}</th>
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Table 4 calculation of the effective Rayleigh number and the Nusselt number with \( s=100 \) and \( b=\pm 2 \)

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<th>Ra</th>
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Table 5 Application of the reference temperature approach adopted here for some values of Da, Ra, and b.

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<th>Nu*</th>
<th>Nu</th>
<th>e_{Nu, %}</th>
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Figure 1. Dimensional (a) and dimensionless (b) description of the problem under consideration.
Fig. 2 Isotherms (top) and streamlines (bottom) for s=10 and a) Ra=10 b) Ra=100 c) Ra=1000
Hooman et al. (2007) ICPM2 Heat line and ...

(a)

(b)
Fig. 3 Isotherms (top) and streamlines (bottom) for s=100 and a) Ra=10 b) Ra=100 c) Ra=1000
Hooman et al. (2007) ICPM2 Heat line and …
Fig. 4 Mid-plane velocity (top) and temperature (bottom) for a) $s=10$ and b) $s=100$
Fig. 5 Effects of different parameters on Nu (left) and $\psi$ (right) divided by those of constant property.
Hooman et al. (2007) ICPM2 Heat line and …

(a)

(b)
Hooman et al. (2007) ICPM2 Heat line and ...

(c)

(a)
Fig. 6 Heatlines for $s=10$ (top) and $s=100$ (bottom) for a) $Ra=10$ b) $Ra=100$ c) $Ra=1000$
Hooman et al. (2007) ICPM2 Heat line and …

(a)

(b)
Fig. 7 Heatlines and energy flux vectors a) for the whole cavity and near the b) heated c) cooled wall (s=Ra=10)
Fig. 8 Heatlines and energy flux vectors a) for the whole cavity and near the b) heated c) cooled wall (s=10, $Ra_q=100$)