Spatial matter-wave interference via adiabatic recombination of atomic wave packets

L. I. Plimak,1,2 Y. V. Rozhdestvensky,1 M. K. Olsen,2 and M. J. Collett2
1Department of Chemical Physics, The Weizmann Institute of Science, 76100 Rehovot, Israel
2Department of Physics, The University of Auckland, Private Bag 92019, Auckland, New Zealand

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We propose and analyze an all-optical atomic beam splitter based on a multistep adiabatic passage in a “tripod” atomic level configuration. The spatial interference patterns obtained may exhibit extremely fine structure due to splittings of many times the photon momentum. The visibility of the obtained interference pattern is also shown to exhibit collapses and revivals as the distance from the interaction region is varied.

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In recent years several versions of atom beam splitters and atom interferometers have been proposed [1–8] and a variety of precision measurements utilizing atom-interferometry techniques have been performed [9,10]. The first atomic beam splitters [2] were based on Rabi oscillations in two-level systems (see also [3]). In a search for an efficient robust wide-angle atomic beam splitter, a number of schemes based on the effect of coherent population transfer [11] [adiabatic passage, or stimulated Raman adiabatic passage (STIRAP)] have been implemented [6,8], following the proposal by Marte et al. [5]. In STIRAP, the populations of optically excited states remain negligible. STIRAP is also insensitive to changes in parameters such as laser powers and pulse shapes. A robust atomic beam splitter based on STIRAP in a “tripod” configuration (cf Fig. 1) was recently demonstrated by Bergmann and co-workers [8]. Experiments by Chu and co-authors [7] showed that, employing STIRAP, momentum splitting can be scaled up by orders of magnitude.

Atom interferometry may have important technological applications in atom lithography [9], for example the deposition of an atomic interference pattern on a substrate to create a periodic structure. To this end, one optimally needs to prepare a wave packet consisting of two coherent momentum components in the same internal atomic state. To our knowledge, the problem of creating such a wave packet has so far not had a satisfactory solution. Methods such as scattering cold atoms from a periodic microfabricated structure [9] convert only a small fraction of atoms into the superposition state sought, and hence require additional state selection. Atomic beam splitters (cf., e.g., Ref. [8]) typically produce atoms in a superposition of different internal states that does not yield a spatial interference pattern.

We ask whether a coherent superposition of momentum components of different states can be converted into that within a single atomic state and explore an idea that consists of a combination of Bergmann et al. [8] and Chu et al.’s [7] techniques. This results in atoms ending up in a superposition of different internal states, with two momentum components in each. The interference patterns due to different states are π out-of-phase and cancel each other. However, the interference patterns due to different internal states also move relative to each other, so that, by simply allowing the atomic wave packet to propagate freely, a revival of interference is achieved. As a result, a spatial interference pattern may be obtained without state selection, merely by locating the substrate in the appropriate position. The scheme we consider here produces a momentum splitting of four times the photon momentum, resulting in an interference pattern with a period of a quarter of the optical wavelength. This splitting may be increased by applying Chu et al.’s technique [7] during intermediate stages of the atomic evolution, resulting in much finer spatial structures.

We analyze an experimental situation that follows closely the conditions of Ref. [8]. In Ref. [8], a beam of metastable Ne atoms in the (2p3s3p, 3P0) state propagating along the x axis, was incident on a combination of three laser beams: a pair of π- and π+ -polarized beams counterpropagating along the z axis, and a π-polarized beam propagating along the y axis. The π-polarized beam was shifted along the x axis so that an atom saw a synchronous pair of π-polarized pulses and an overlapping delayed π-polarized pulse. The states coupled via this pulse sequence are |1,0⟩, |2,0⟩, |3,−1⟩, and |3+,+1⟩ (cf., Fig. 1), where (assuming kω=kπ=k)

|1,n⟩ = |(2p3s3p, 3P0, M = 0,px,pz,−ℏk,pz,+nhk)⟩, (1)
|2,n⟩ = |(2p3s3p, 3P1, M = 0,px,pz,−ℏk,pz,+nhk)⟩, (2)
|3−,n⟩ = |(2p3s3p, 3P2, M = −1,pz,−ℏk,pz,+nhk)⟩, (3)
|3+,n⟩ = |(2p3s3p, 3P2, M = +1,pz,−ℏk,pz,+nhk)⟩. (4)

Assuming that the π pulses have equal Rabi frequencies, Ωπ = Ωσ = Ω′, and that the states (1–4) are on resonance, the tripod configuration as in Fig. 1 exhibits two dark states [8]:

|ΦS⟩ = sin φ/√2(|3+, +1⟩+|3−, −1⟩)−cos φ|1,0⟩, (5)
|ΦA⟩ = 1/√2(|3+, +1⟩−|3−, −1⟩). (6)

Here, S and A stand for symmetric and antisymmetric (in respect to the side M = ±1 states, cf., Fig. 1), and tan φ = Ωπ/(Ω′,√2), where Ωπ is the Rabi frequency of the π-polarized pulse. At the start of the pulse sequence, Ωπ ≫ Ωπ and the symmetric dark state coincides with the “cen-
$u \neq 0$ state, which is exactly the initial atomic state. At the end of the sequence, $\Omega_0 = \Omega_{\pi}$, and the symmetric dark state is a superposition of the side states. Thus this pulse sequence coherently transfers the initial atomic state, $|\Phi_0\rangle = |1,0\rangle$, into the state $|\Phi_1\rangle = -\frac{1}{\sqrt{2}}(|3^+, +1\rangle + |3^-, -1\rangle)$.

The state $|\Phi_1\rangle$ may be transferred back into the state $|\Phi_0\rangle$ by swapping the $\sigma$ and $\pi$-polarized beams in time. However, consider what happens if one also swaps directions of the $\sigma$-polarized beams. In Fig. 2, we show the momentum substates of levels $1, 2, 3^+$, and $3^-$, coupled via light fields if this “inverse” pulse sequence is applied to the state $|\Phi_1\rangle$. We see that the set of coupled substates factorizes into two independent tripod configurations. In each, the incident atomic population is concentrated in a side state of the tripod. Expanding the state $|\Phi_1\rangle$ over the symmetric and antisymmetric dark states of the two tripods in Fig. 2, we obtain

$$|\Phi_1\rangle = -\frac{1}{2}(|\Phi_S^+\rangle + |\Phi_S^−\rangle + |\Phi_A^+\rangle + |\Phi_A^−\rangle),$$

where $|\Phi_{S,A}^\pm\rangle$ are expressed in terms of the central and side states of the corresponding tripods by relations identical to Eqs. (5) and (6). The inverse pulse sequence will convert the symmetric dark states into momentum components of state $1: |\Phi_S^\pm\rangle \rightarrow |1, \pm 2\rangle$. The antisymmetric dark states, which are fully decoupled from the optical fields, remain intact. Hence the resulting atomic state is

$$|\Phi'_1\rangle = \frac{1}{2}(|1, +2\rangle + |1, -2\rangle) - \frac{1}{2\sqrt{2}}(|3^+, +1\rangle + |3^-, -1\rangle + |3^−, +3\rangle + |3^+, -3\rangle).$$

Half of the atomic population thus ends in the metastable state $1$. The remaining half is split 50/50 between the states $3^+$ and $3^-$. Each state contains two momentum components separated by $4hk$. The components of level 1 are in phase, while those of levels $3^\pm$ are in counterphase, so no overall interference pattern is produced. However, consider the time evolution of state (7). Momentum wave functions of the states $1, 3^+$, and $3^−$ in momentum representation have the general form, $\langle p | \psi \rangle = \langle p - p_1 | \psi_0 \rangle + e^{i\varphi} \langle p - p_2 | \psi_0 \rangle$, where $|\psi_0\rangle$ is the initial motional state of the atomic beam (which is no longer assumed to be a pure momentum state). After free evolution for time $t$, the spatial density produced by the two momentum components is

$$|\langle z | \psi, t \rangle|^2 \propto \exp ip_1z/h - ip_t^2t/(2m)z - p_1tt/m |\psi_0, t\rangle$$

$$+ \exp ip_2z/h - ip_t^2t/(2m)z - p_2tt/m |\psi_0, t\rangle|^2$$

$$= 2|\langle z | \psi_0, t \rangle|^2 \{1 + \cos(4k(z - vt + \varphi))\}.$$

Here, $|\psi_0, t \rangle$ is $|\psi_0\rangle$ evolved freely for time $t$, and $v = (p_2 + p_1)/(2m)$ is the average velocity. For state 1, $v = 0$ and $\varphi = 0$, while for states $3^\pm$, respectively, $v = \pm v_R$ and $\varphi = \pi$, where $v_R = hkm/R$ is the recoil velocity. We have also used the momentum splitting $p_1 - p_2 = 4hk$ for all three states. Equation (8) is exact, while Eq. (9) follows if the spatial separation of the components is neglected, $\langle z - p_1tt/m |\psi_0, t\rangle = (z - p_2tt/m |\psi_0, t\rangle = (z |\psi_0, t\rangle$.

Each of the states $1, 3^+$ and $3^−$ thus produces an interference pattern with a period of $\lambda/4$. For $t = 0$, the patterns due to state 1 and states $3^\pm$ are $\pi$ out of phase and cancel each other; the overall spatial atomic distribution simply repeats the initial one. This result is quite natural, since state (7) was found in a Raman-Nath approximation. For $t > 0$, the patterns due to states $3^\pm$ slide (relative to that due to state 1) with velocities $\pm v_R$, resulting in periodic collapses andrevivals of spatial interference. The first revival is at $t = t_{\text{rev}} = \lambda/(8v_R)$, with its completeness depending on the initial atomic collimation. For a perfect revival, the spatial separation of the momentum components of each state, $4v_Rt_{\text{rev}}$, should stay small compared to the transverse coherence length of the wave packet, $\hbar/\Delta p$, where $\Delta p$ is the transverse momentum spread. This yields

$$\Delta p < \frac{\hbar k}{\pi}.$$ 

In Ref. [8], $\Delta p \sim 0.4h\pi$, which is close to the collimation needed. Note that considerations which led to Eq. (7) implied that the momentum substates $|3^\pm, \pm 3\rangle$ (cf. Fig. 2) remained empty after the adiabatic passage $|\Phi_0\rangle \rightarrow |\Phi_1\rangle$. In turn, this
sets a limit on the initial momentum spread along the \( z \) axis, which should be narrow on the scale of \( 2\hbar k \). This condition is weaker than (10).

Since spatial positioning of the revival is velocity dependent, the longitudinal velocity spread will also reduce visibility of the interference pattern. This reduction is, however, limited by 50\%. This will happen in the worst possible case of a very large longitudinal momentum spread, when the \( \text{“moving”} \) patterns coming from states \( 3^+ \) and \( 3^- \) get fully smeared out. A detailed investigation of the robustness of the scheme will be performed elsewhere.

Some remarks are in order here. The first-split-then-combine pulse sequence may be regarded as a generalization of Chu’s techniques [7] to the tripod configuration. In Ref. [7], population was coherently transferred back and forth across a \( \Lambda \) configuration. Using differently directed light beams for the \( \text{“back”} \) and the \( \text{“forth”} \) transitions resulted in an accumulation of photon momenta passed to atoms. Changing directions of the \( \sigma \) beams works in a similar manner. Moreover, Chu’s techniques may be applied to the intermediate state \( |\Phi_i\rangle \) in order to scale up the momentum splitting. For example, without the \( \sigma^- \)-polarized beams, the first-split-then-combine sequence adds \( 2\hbar k \) to the momentum of state \( 3^+ \), leaving the state \( 3^- \) untouched. Repeatedly applying this sequence will move the \( 3^+ \) component of state \( |\Phi_i\rangle \): \( |3^+,+1\rangle \rightarrow |3^+,+3\rangle \rightarrow |3^++2n_+,+1\rangle \). Similarly, applying the first-split-then-combine sequence without the \( \sigma^- \)-polarized beams will move the \( 3^- \) component: \( |3^-,1\rangle \rightarrow |3^-,3\rangle \rightarrow |3^-2n_-,1\rangle \). Now note that positioning of the two independent triplets in Fig. 2 in momentum is determined solely by the encircled states. Hence after the \( \text{“inverse”} \) pulse sequence, \( 2n_+\hbar k \) and \( -2n_-\hbar k \), respectively, will be added to momenta of the positive and negative momentum components of state \( 7 \), resulting in momentum splitting of \( 2(n_++n_+2)\hbar k \), and an interference pattern with a period of \( \sqrt{2} \). In practice, this may require switching from time-of-flight to optical pulse techniques, because conditions like Eq. (10) restrict the time available for scaling up the splitting. Positioning the substrate may also become a problem, since the revival time will be reduced by a factor of \( \sqrt{2} \). Magnetic state selection may then become an easier option.

To verify these considerations beyond the Raman-Nath approximation, we simulated the system evolution numerically. The atom-optical interaction was treated in the dipole approximation neglecting spontaneous processes. The laser beams were regarded as running plane waves on resonance for atoms with \( p_x=p_y=0 \). Atomic motion in the \( z \) and \( y \) directions was treated quantum mechanically. The motion in the \( x \) direction was parametrized by time-dependent Rabi frequencies,

\[
\Omega_{\sigma^+}^{(1)}(t) = \Omega_0 e^{-[(t-t_d)/\Delta \Gamma]^2},
\]

\[
\Omega_{\sigma^-}^{(1)}(t) = \Omega_0 e^{-[(t-t_d)/\Delta \Gamma]^2},
\]

\[
\Omega_{\pi^+}^{(2)}(t) = \Omega_0 e^{-[(t-t_d)/2 \Delta \Gamma]^2},
\]

\[
\Omega_{\pi^-}^{(2)}(t) = \Omega_0 e^{-[(t-t_d)/2 \Delta \Gamma]^2},
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apart from $\Delta p$, the results presented in Figs. 3–5 use the conditions of Ref. [8], so they should be experimentally achievable.

Simulations were also run for a nonzero initial $y$ momentum component. Corresponding changes to the numerical results were negligible.

In conclusion, we have shown that adding the “inverse” laser beam combination to Bergmann’s atomic beam splitter turns it into a 100% efficient all-optical beam splitter, which has potential applications for atom-wave lithography.

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