Title: Perspectives on technology mediated learning in secondary school mathematics classrooms

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Abstract

The introduction of technology resources into mathematics classrooms promises to create opportunities for enhancing students’ learning through active engagement with mathematical ideas; however, little consideration has been given to the pedagogical implications of technology as a mediator of mathematics learning. This paper draws on data from a three year longitudinal study of senior secondary school classrooms to examine pedagogical issues in using technology in mathematics teaching – where “technology” includes not only computers and graphics calculators but also projection devices that allow screen output to be viewed by the whole class. We theorise and illustrate four roles for technology in relation to such teaching and learning interactions – master, servant, partner, and extension of self. Our research shows how technology can facilitate collaborative inquiry, during both small group interactions and whole class discussions where students use the computer or calculator and screen projection to share and test their mathematical understanding.
1. Introduction

This paper reports on aspects of a three year longitudinal study that investigated the role of electronic technologies (graphics calculators and computers) in supporting students’ exploration of mathematical ideas and in mediating their social interactions with teachers and peers. Numerous research studies have examined the effects of technology usage on students’ mathematical achievements and attitudes, and their understanding of mathematical concepts (e.g. Adams, 1997; Lesmeister, 1996; Quesada & Maxwell, 1994; Weber, 1998). However, the quasi-experimental design of many of these studies is based on the assumption that the same instructional objectives and methods are valid for both pen and paper and technology enhanced tasks. Less is known about how the availability of technology, especially graphics calculators and their peripheral devices, has affected teaching approaches (Penglase & Arnold, 1996). Some studies have found changes in classroom dynamics leading to a less teacher centred and more exploratory environment (e.g. Simonsen & Dick, 1997). However, it appears that negotiation of such a pedagogical shift is mediated not only by teachers’ mastery of the technology itself, but also by their personal philosophies of mathematics and mathematics education (Tharp, Fitzsimmons & Ayres, 1997; Thomas, Tyrrell & Bullock, 1996).

Unlike much previous research in this area, our study explicitly addresses technology as a tool that is integral to the mathematical practice of students and teachers in particular learning environments. We theorise four roles for technology in relation to teaching and learning interactions – “master”, “servant”, “partner”, and “extension of self” – to show how technology re-organises interactions between human and technological agencies, and changes
the ways that knowledge is produced, shared, and tested. In contrast with other similar studies (see Doerr & Zangor, 2000), our findings suggest that technology can facilitate collaborative inquiry through both small group conversations and whole class discussions where students use screen projection devices to present their work publicly for critical scrutiny.

2. Theoretical perspective

Mathematics curriculum and policy documents now place increased emphasis on the processes of problem solving, reasoning, and communication, and endorse student discussion of mathematical ideas as a means of developing and reflecting on their understanding (Australian Education Council, 1991; National Council of Teachers of Mathematics, 2000). These moves for curriculum reform are supported by current research in mathematics education that draws on sociocultural theories of learning (Vygotsky, 1978; Wertsch, 1985; Wertsch & Rupert, 1993). From this theoretical perspective, all human development involves learning from others and the culture that precedes us, and thinking and reasoning are mediated by cultural tools – material artefacts or sign systems such as language, symbol systems, diagrams, and so on (Lerman, 2000). Thus mathematics teaching and learning requires the formation of a classroom community of learners where the epistemological values and discourse conventions of the wider mathematical community are progressively appropriated and enacted (Brown, Stein & Forman, 1995; Goos, Galbraith & Renshaw, 1999; Forman, 1996; Schoenfeld, 1989). In such classrooms, discussion and collaboration are valued in building a climate of intellectual challenge. Rather than relying on the teacher as an unquestioned authority, students are expected to propose and defend mathematical ideas and conjectures, and to respond thoughtfully to the mathematical arguments of their peers.

The increasing availability and power of electronic technologies such as computers and graphics calculators offers new opportunities for students to communicate and analyse their mathematical thinking, since the objects generated on the screen can act as a common referent
for discussion (National Council of Teachers of Mathematics, 2000). Most importantly, technology can foster conjecturing, justification, and generalisation by enabling fast, accurate computation, collection and analysis of data, and exploration of multiple representational forms (e.g. numerical, symbolic, graphical). Consistent with our sociocultural perspective, we regard technology as one of several types of cultural tools that not only amplify, but also re-organise, cognitive processes through their integration into the social and discursive practices of a knowledge community (Resnick, Pontecorvo & Säljö, 1997). The amplification effect may be observed when technology simply supplements the range of tools already available in the mathematics classroom, for example, by speeding tedious calculations or verifying results obtained by hand. By contrast, cognitive re-organisation occurs when learners’ interaction with technology as a new semiotic system qualitatively transforms their thinking; for example, use of spreadsheets and graphing software can alter the traditional privileging of algebraic over graphical or numerical reasoning. Accordingly, learning becomes a process of appropriating tools that change the ways in which individuals formulate and solve problems. How such appropriation might occur in technology enriched mathematics classrooms is the subject of the remainder of this paper.

3. Research methodology

3.1. Background to the study

Data collection over three years from 1998-2000 involved five senior secondary mathematics classrooms from two government schools and one independent school in a large Australian city. Students participating in the study were in either Year 11 or Year 12, the final two years of secondary schooling. The study gathered data from three Mathematics B classrooms (two classes in a government school and one class in the independent school) and two Mathematics C classrooms (one class in a government school and one in the independent school). Mathematics B is a calculus and statistics subject required for entrance to tertiary
courses in science, business, and engineering, while Mathematics C is an advanced subject, usually chosen by students wishing to specialise in mathematics at university, that must be taken in conjunction with Mathematics B.

While at the time of the study the syllabuses for both mathematics subjects did not yet mandate the use of graphics calculators and computers, teachers were strongly encouraged to make use of these technologies wherever appropriate. All classes had ready access to either desktop or laptop computers equipped with generic (e.g. spreadsheet) and mathematical (e.g. graphing) software. The independent school and one of the government schools provided students with graphics calculators for use both at school and at home. The other school owned several class sets of calculators that were made available to students only during mathematics lessons when their use was planned in advance.

3.2. Data collection methods

Since the aim of the study was to investigate students’ and teachers’ use of technology in specific classroom environments, we employed research methods that drew on ethnographic techniques such as participant observation, interviews, survey instruments, and collection of video and audio taped records (Burns, 1997). In addition, individual cases – bounded systems such as a single classroom, or a group of students who worked on a specific task – were selected to gain a deeper understanding of the meanings participants ascribed to their own and each other’s actions (Stake, 1988).

At least one lesson every week was videotaped and observed for each participating classroom, and selected segments of the tapes were transcribed for later analysis. Field notes of each lesson were also kept to record details of classroom tasks, teacher actions, and student actions involving technology usage. More frequent classroom visits were scheduled if the teacher planned a technology intensive approach to the topic. For example, every lesson was observed and videotaped in a two week unit of work that introduced some of the mathematics
of chaos theory, because in every lesson students were using spreadsheets to investigate numeric iteration processes in a variety of contexts (compound interest, population growth, radioactive decay, approximate methods for solving equations.) Sample episodes from some of these lessons are analysed in a later section of the paper.

Students completed a questionnaire on their attitudes towards technology and its role in learning mathematics at key times throughout the study: (a) the beginning of Year 11; (b) the end of Year 11, after one year of exposure to technology enhanced mathematics lessons; and (c) the end of Year 12 when students were about to leave school. (See Geiger, 1998, for details of the questionnaire pilot study; and Galbraith, Goos, Renshaw & Geiger, 2001, for a discussion of questionnaire results from the present study). In addition, audiotaped interviews were conducted with individuals and groups of students to examine the extent to which they thought technology contributed to their understanding of mathematics, and their perceptions of how technology changed the teacher’s role in the classroom. The scheduling of these interviews was not pre-determined, but was contingent upon observed classroom events. For example, individuals or groups were interviewed after lessons in which they were the focus of videotaping, to discover their interpretation of specific episodes of interest to the research team. Interviews were transcribed in full so that students’ responses could be integrated with lesson observation notes and video transcripts.

This paper makes use of questionnaire responses, lesson observation data (field notes, videotape records, and transcribed video segments) and interviews with students (audiotape records and full transcripts), to illustrate and compare different ways in which technology enters into teaching-learning interactions.

3.3. Emerging analytical framework

Few studies have investigated how and why students use technology to learn mathematics in specific classroom contexts, and how the roles of students and teachers might
change when technology is integrated into the mathematics curriculum. Amongst these, Doerr and Zangor (2000) in an observational case study of two pre-calculus classrooms identified five * modes of graphics calculator use: * computational tool, transformational tool, data collection and analysis tool, visualising tool, and checking tool. Taking a somewhat different approach, Guin and Trouche (1999) categorised their observations of students using graphic and symbolic calculators into *profiles of behaviour*, in order to understand how students transformed the material tool into an instrument of mathematical thought that re-organised their activity. The nature of this transformation varied according to whether the student displayed a random, mechanical, rational, resourceful, or theoretical behaviour profile in terms of their ability to interpret and coordinate calculator results. With respect to *classroom interactions*, Farrell (1996) observed a shift in both teachers’ and students’ roles towards that of consultant and fellow investigator, accompanied by a similar movement away from teacher exposition towards planned or informal group work.

Our own conceptualisation of technology usage in mathematics classrooms differs from analytical frameworks developed in previous research in that it encompasses interactions between teachers and students, amongst students themselves, and between people and technology, in order to investigate how different participation patterns offered opportunities for students to engage constructively and critically with mathematical ideas. Our analysis of technology focused classroom interactions is framed by four metaphors we have developed to theorise the varying degrees of sophistication with which teachers and students work with technology: technology as “master”, “servant”, “partner”, and “extension of self”. These metaphors are suggestive of different ways in which teachers and students might appropriate technology into classroom mathematical practices.

Since our aim was to inductively derive theory from data, our data collection and analysis was consistent with principles of theoretical sampling shared with the grounded
theory approach to qualitative research (see Strauss & Corbin, 1990) and approximated Cobb and Whitenack’s (1996) methodological approach to longitudinal analysis of classroom video recordings and transcripts. This involves:

1. continually testing and refining inferences and conjectures from initial categories in the light of subsequent data gathering and analysis;

2. long term engagement of the researchers with the participants of the study in order to gain insights into teachers’ and students’ actions;

3. subjecting the developing analysis to critique by peers, for example via publication and conference presentations (e.g. see Goos, Galbraith, Renshaw & Geiger, 2000a; Goos, Galbraith, Renshaw & Geiger, 2000b; Goos, Galbraith, Renshaw & Geiger, 2001).

Thus, observation was initially exploratory in nature, but became increasingly focused and selective as patterns emerged in the data. A consequence of this process is the gradual refinement of the categories used to interpret the data. Throughout the first year of the study, the research team regularly reviewed lesson field notes and videotapes to create initial categories for teacher-student-technology interactions. Categories were progressively tested and refined against further observations and students’ questionnaire responses.

The questionnaire investigated students’ attitudes towards and preferred ways of working with technology, and included sections containing structured Likert items and questions designed to elicit open-ended responses. Questions in the latter section were modified throughout the life of the study to gain more detailed information in response to our increasingly focused observations of classrooms. For example, the open-ended section of the questionnaire administered at the start of the study (beginning of Year 11) simply asked
students to write what they thought about using technology to learn mathematics. At the end of the first year (end of Year 11) this section instead posed the following questions:

*Are there any advantages/disadvantages in using technology instead of pen and paper? Use examples to illustrate how it helps/gets in the way of learning.*

*Are there ways in which you believe technology helps you to think differently?*

*Does using technology change the teacher’s role in the classroom? In what way(s)?*

At the end of the second year (end of Year 12), we also included in this section of the questionnaire brief descriptions of the metaphors for working with technology we labelled “master”, “servant”, “partner” and “extension of self”, and students were asked to identify, with reasons, which best fitted the way they used technology in the classroom. Students’ responses to these open ended questions in each version of the questionnaire were categorised as illustrating one or more of our four emerging metaphors.

4. **Four Metaphors for Technology-Mediated Learning**

We now offer our descriptions of these metaphors, illustrated with data from classroom observations and student questionnaires.

4.1. **Technology as Master.** Teachers and students may be subservient to the technology if their knowledge and usage are limited to a narrow range of operations over which they have technical competence. In the case of students, subservience may become dependence if lack of mathematical understanding prevents them from evaluating the accuracy of the output generated by the calculator or computer.

The way in which technology could prove the master for teachers became clear to us from our observations of one of the project classrooms. This teacher admitted very little expertise with using a graphics calculator, to the extent that he regularly called on a student “expert” to demonstrate calculator procedures via the overhead projection panel. While the
teacher lacked personal autonomy in the use of technology he nevertheless retained tight control of the lesson agenda through the medium of the student presenter – often providing the mathematical commentary and explanations accompanying the student’s silent display. Because of syllabus and research project expectations, this teacher felt obliged to include technology-based learning activities in his lessons; however, his own lack of knowledge and experience in this area made him reluctant to allow students to use technology to explore unsanctioned mathematical territory.

Through their questionnaire responses students acknowledged that there could be disadvantages in using technology if they lacked specific technology skills or if its use led to mathematical dependence:

*I’m hopeless with computers. I find if we involve graphing calculators it makes work harder than what it already is.* (Lack of skills; Beginning Year 11)

*Somedtimes I don’t know how to use the technology which means I can’t get anything done.* (Lack of skills, End Year 12)

*People may become too dependent on it. Instead of wanting to know how and why they just want to do it.* (Mathematical dependence, End Year 11).

*Somedtimes you learn a technique using technology that you don’t really understand, and then you don’t grasp the concept.* (Mathematical dependence, End Year 12)

These comments suggest a degree of subservience corresponding to technology in the role of master.

4.2. Technology as Servant. Here technology is used as a fast, reliable replacement for mental or pen and paper calculations, but the tasks of the classroom remain unchanged. That is,
technology is a supplementary tool that amplifies cognitive processes but is not used in creative ways to change the nature of activities.

This mode of working is reflected in students’ questionnaire responses identifying advantages of using technology compared with pen and paper. They commented that technology helped with large and repetitive calculations, allowed them to calculate more quickly and efficiently, reduced calculation errors, and was useful in checking answers:

*Technology can help us to calculations and graphs easier [...] and help us arrive at the answer faster.* (Large calculations, Speed and efficiency; Beginning Year 11)

*No chance of simple mathematical errors.* (Error reduction; End Year 11)

From the teacher’s perspective, technology is a *servant* if it simply supports preferred teaching methods; for example, if the overhead projection panel is used as an electronic chalkboard, providing a medium for the teacher to demonstrate calculator operations to the class. Nevertheless, we have noted interesting variations in the way teachers operate with technology in this mode. One emergent property of the graphics calculator involves its use in conjunction with other material resources in ways that further enhance the calculator’s capacity for linking multiple representations of a concept. For example, one teacher used transparent grid paper, plastic cut out polygons, and the overhead projector to physically demonstrate the results of matrix transformations such as

\[
\begin{pmatrix}
1 & 0 \\
0 & -1
\end{pmatrix}
\quad \text{and} \quad
\begin{pmatrix}
1 & -1 \\
\sqrt{2} & \sqrt{2} \\
\sqrt{2} & \sqrt{2}
\end{pmatrix}
\]

on the polygons’ vertex vectors. Students then investigated further with their own polygons and grid paper by recording the coordinates of the vertices before and after transformation, with the graphics calculator taking care of the matrix calculations so that conjectures on the
geometric meaning of the transformations could be formulated and tested. While the technology is subsumed into the teacher’s preferred approach involving manipulable materials, it becomes an intelligent servant that complements the effective features of more conventional instruction.

4.3. Technology as Partner. Here technology is used creatively to increase the power students exercise over their learning (Templer, Klug & Gould, 1998); for example, by providing access to new kinds of tasks or new ways of approaching existing tasks. This cognitive re-organisation effect may involve using technology to facilitate understanding or to explore different perspectives, as the following student questionnaire responses illustrate:

By displaying things in different ways [technology] can help you to understand things more easily. (Facilitate understanding; End Year 11)

[Without technology] the study of chaos theory would have been virtually impossible as the graphs enable us to visualise the functions more clearly. [Facilitate understanding; End Year 12)

[Technology] may help you approach problems differently in the sense that you can visualise functions. (Different perspectives; End Year 12)

It helps us to explore concepts in greater depth. (Facilitate understanding; End Year 12).

Technology may also act as a partner by mediating mathematical discussion in the classroom. For example, instead of functioning as a transmitter of teacher input, the overhead projection panel can become a medium for students to present and examine alternative mathematical conjectures (cf the master and servant examples in sections 4.1 and 4.2). This is illustrated by the practice in one of the project classrooms of inviting students to compare and evaluate programs they have written to simplify routine calculations, such as finding the angle between two three-dimensional vectors. In this classroom we observed how the public display
of student work facilitated whole class discussion with the student-presenters themselves leading the dialogue and trying out different command lines in response to suggestions from peers in the audience.

In small group interactions, calculator or computer output also promoted peer discussion as students clustered together to compare their screens, hold up graphics calculators side by side or pass them back and forth to neighbours to emphasise a point or compare their working. Some students seemed to develop a distinct rapport with technology, often appearing to interact directly with their graphics calculator as it responded to their commands – for example, with error messages that demanded investigation. Here the calculator acted as a stimulus for students to verbalise their thinking and seek help from peers in the process of locating and correcting such errors. (See Goos, Galbraith, Renshaw & Geiger, 2000a, for a detailed analysis of one such classroom episode.)

4.4. Technology as Extension of Self. The most sophisticated mode of functioning, this involves users incorporating technological expertise as a natural part of their mathematical and/or pedagogical repertoire. From the teacher’s perspective, writing courseware to support an integrated teaching program would be an example of operating at this level. Similarly, students may integrate a variety of technological resources into the construction of a mathematical argument so that powerful use of computers and calculators forms an extension of the individual’s mathematical prowess. In describing how technology helped them to think differently, students in their questionnaire responses referred to this aspect of technology use as mind expanding and according them freedom to explore at will:

[Technology] allows me to expand my mind because I know I have the power to do complex techniques. (Mind expanding; End Year 12)
Technology allows you to expand ideas and to do the work your own way. (Mind expanding, Freedom; End Year 12)

This sense of autonomy and the blurring of boundaries between mind and technology was also mentioned by students who identified extension of self as the metaphor that best described their way of working with technology:

It allows you to explore and go off in your own direction. (Freedom; End Year 12)

My calculator is practically a part of myself. It’s like my third brain. (Mind expanding; End Year 12)

The development of these metaphors was largely concerned with understanding how teachers and students interacted with technology; however they are also bound up with the ways in which human participants interact with each other in a technology-enriched classroom. We take up this idea in the following section, which examines episodes from one of the project classrooms in which students worked on a challenging spreadsheet task.

5. Classroom Case Study

This analysis applies the metaphors of technology as master, servant, partner, and extension of self to demonstrate different ways in which technology as a cultural tool can mediate teachers’ and students’ engagement with mathematical tasks and with each other. In particular, it highlights the vital role of the teacher in moving students towards more thoughtful and powerful ways of working with technology.

5.1. Classroom learning environment

As has been noted elsewhere (e.g. Doerr & Zanger, 2000; Tharp, Fizsimons & Ayres, 1997; Thomas, Tyrell & Bullock, 1996), teachers’ beliefs about mathematics and mathematics education influence their pedagogical strategies in making use of technology. In
an earlier study (Goos, Galbraith & Renshaw, 1999), we highlighted the significance of this particular teacher’s espoused beliefs, and the ways in which these beliefs were enacted as he established a classroom community of mathematical practice with his Year 11 and Year 12 students. From our observations of classroom interaction patterns and interviews with the teacher, we identified a number of pedagogical assumptions that appeared to be crucial to the formation of the classroom culture:

1. Mathematical thinking is an act of sense making, and rests on the processes of specialising and generalising, conjecturing and justifying;
2. The processes of mathematical inquiry are accompanied by habits of individual reflection and self-monitoring;
3. Mathematical thinking develops through teacher scaffolding of the processes of inquiry;
4. Mathematical thinking can be generated and tested by students themselves through participation in equal-status peer partnerships;
5. Interweaving of familiar and formal knowledge helps students to adopt the conventions of mathematical communication.

In the technology focused episodes that follow, the teacher’s and students’ actions should be interpreted in the light of the belief structures and social and communicative norms, outlined above, that operated within this classroom.

5.2. Chaos task

This episode spans two consecutive lessons in a Year 11 Mathematics C classroom in the independent school referred to earlier. The teacher (the fourth author of this paper) was an expert and innovative user of technology with considerable experience in curriculum design. As the option existed within the Mathematics C syllabus for schools to design and teach a
topic of their choice, the teacher had chosen to introduce students to iteration as one of the central ideas of chaos theory. This topic was presented as a teacher-prepared booklet containing a series of spreadsheet examples and tasks for students to work through at their own pace. One particularly challenging task involved using iterative methods to find approximate roots of equations such as $x^3 - 8x - 8 = 0$. The equation may be expressed in the form $x = F(x)$, and a first approximation to the solution is obtained by estimating the point of intersection of the curves $y = x$ and $y = F(x)$. This approximate solution is used as the initial value in a two column spreadsheet, where the first column provides input $x$-values for $F(x)$ in the second column, and the output of $F(x)$ becomes the input of subsequent iterations.

Figure 1 shows the calculation when $F(x) = \frac{x^3}{8} - 1$. Cell B5 contains the formula $= (1/8)*((A3)^3)-1$ and cell A6 contains =$B5$, both these formulae then being copied down into the other cells in these columns.

Figure 1. Spreadsheet method for solving $x^3 - 8x - 8 = 0$, rearranged as $F(x) = \frac{x^3}{8} - 1$, with initial value $x = 1$
Depending on the way in which the original equation is rearranged and the initial value chosen, the iteration may converge on a solution (as in Figure 1), or generate increasingly divergent outputs and hence no solution (for example, see Figure 2).

Figure 2. Spreadsheet method for solving \(x^3 - 8x - 8 = 0\), rearranged as \(F(x) = \frac{x^3}{8} - 1\), with initial value \(x = 4\)

Rearranging \(x^3 - 8x - 8 = 0\) as \(x = \frac{x^3}{8} - 1\) yields only one of the three roots \((-1.236)\).

To find the other roots of this cubic equation \((-2\) and \(3.236\)), students must investigate other rearrangements and a range of initial values. Thus the task afforded the use of technology as a \textit{partner} in the sense that the spreadsheet approach provided a new way for students to tackle the task of solving cubic equations.

In an earlier study conducted in the same teacher’s classroom, it was found that students attempting this task embraced technology as a \textit{partner} to an even greater extent when they quickly discovered that they could create an alternative, graphical, representation of the
19 problem with the graphing software installed on the school’s computers (Goos, 1998). Plotting the graphs of $y = x$ and $y = F(x)$ enabled students to make a realistic first approximation to the roots of the equation (see Figure 3). In addition to spreadsheets and function graphing software, students participating in the present study chose to use their TI-83 graphing calculators to tackle this task.

![Figure 3](image)

**Figure 3.** Graphical representation of iterative solution to $x^3 - 8x - 8 = 0$, rearranged as

$$F(x) = \frac{x^3}{8} - 1$$

Episodes involving one group of students have been reconstructed with the aid of the videotape record, video transcript and lesson observation notes, and the transcript from a group interview conducted soon after the lessons in question. Interview questions and student responses are integrated into the following account, and distinguished by *italics*. (The first author observed the lessons and interviewed students.)
5.3. Lesson 1

Four students (Hayley, Nerida, Sally, David) clustered around a laptop computer, sharing the responsibilities of pencil-and-paper and keyboard work. (Other similar groups were working on the same task in the classroom.) They ignored the written instructions, on how to use the spreadsheet method, that accompanied the task, and instead launched the graphing software installed on the laptop computer:

Hayley: Should we be using the spreadsheet?

Nerida: I don’t think so ... the spreadsheet’s just a way of checking.

The students rearranged $x^3 - 8x - 8 = 0$ as $x = \frac{x^3}{8} - 1$ and plotted it on the same axes as $y = x$. Three intersection points were clearly visible (see Figure 3), much to their dismay:

Sally: Oh no no! It’s gone through it too many times!

They zoomed in on only one intersection point to find the $x$-coordinate, and obtained an approximate value of 3.24. Ignoring the other solutions, they used the TI-83’s Equation Solver with this value entered as an initial guess. The group accepted this as “the” solution – there was no attempt to explore other two intersections. They then moved on to the next problem.

After a few minutes Nerida reminded the others that they zoomed in on only one intersection point for the cubic equation.

Nerida: We ignored the other two. Why did the Solver only pick up one?

The students seemed unaware of the limitations of the calculator’s Equation Solver, which yields one solution that is closest to an initial guess within specified bounds. The lesson ended before this anomaly could be explored further.
In this segment, the students deferred to the graphing calculator (technology as master) and blindly accepted the output produced by the Equation Solver without monitoring its reasonableness in the light of the graphical evidence before them.

5.4. Lesson 2

At the start of the lesson the observer mentioned to the teacher that this group of students had not used spreadsheets at all. The teacher repeated the task instructions to the whole class, emphasising the importance of the spreadsheet approach.

Interviewer: You accepted this (i.e. $x = 3.24$) as the only solution ... Did it occur to you to explore the possibility of other solutions at all?

Sally: We didn’t realise! We only did when [the teacher] told us to.

Here the teacher simply wanted the students to follow the task instructions and begin to apply the spreadsheet as a tool to carry out the repetitive calculations involved in the iteration process. His intervention at this point moved the students away from their uncritical acceptance of the Equation Solver answer from the previous lesson, towards using technology as a servant in order to demonstrate the utility of a spreadsheet in performing time consuming calculations.

The students started on the cubic problem again, this time using a spreadsheet. They entered a formula equivalent to their original rearrangement of the equation ($x = \frac{x^3}{8} - 1$) and “filled down” the columns until the values converged. However, their answer, –1.23 (see Figure 1), did not match the graphical result obtained earlier:

Sally: But we got 3.24!

Hayley reminded the group that there were three intersection points visible on the graph, and suggested they might find the other two solutions if they continued scrolling down their spreadsheet. When this was not successful they called the teacher over and requested
clarification as to how the spreadsheet worked. He re-focused the group on the important elements of the task, and issued a challenge:

Teacher: Is it possible to use the spreadsheet to get all three solutions?

By juxtaposing the spreadsheet, showing only one solution, with the graph, which displayed all three, the teacher attempted to have the students use technology as a partner to re-organise their thinking and engage with the task in the way he had originally intended.

The students found that trying different initial values made no difference to their position: the spreadsheet values either converged on –1.23 or became increasingly large. David reproduced the graph previously plotted on the computer with the aid of the TI-83, thus enabling the graph and spreadsheet to be viewed simultaneously.

Interviewer: I noticed you used the TI-83 to draw graphs.

David: It’s quicker than multi-tasking!

Nerida: Otherwise we’d have to swap around (i.e. between spreadsheet and graphing program) using the computer and it takes ages.

David’s words seem to imply he viewed the TI-83 as a technological servant that provided a more efficient way of viewing both representations at the same time. However, the very act of coordinating different types of technology in this way also resonates with the metaphor of technology as a partner that transforms the nature of mathematical tasks and hence the reasoning processes students need to employ in solving them.

The students continued trying different initial values, to no avail. After conferring once more, they called on the teacher again:

David: Are you going to tell us what to do now?

Teacher: No ... I’m going to tell you to take a walk around the class and see how other people have done it.

Nerida: Have they done it?

Teacher: Other people are trying it. It might interest you to see how.
Through his intervention at this point the teacher reinforced the role of technology as a partner in mediating mathematical discussion between students. He was aware that other groups of students had rearranged the cubic equation in different ways and thus obtained different solutions, and, realising that the focus group of students had exhausted their own intellectual resources, he wished to prompt further discussion focused on other groups’ computer screens.

The four students dispersed to consult with other groups, and discovered two other ways of rearranging the equation: $x = \sqrt{8x + 8}$ and $x = \frac{8x + 8}{x^2}$. These gave the “missing” spreadsheet solutions of 3.24 and –2 respectively.

Interviewer: Would you have thought have doing that (i.e. visiting other groups) on your own?

David & Sally: [in unison] No – We’re too self-centred!

On reconvening the group, the students pieced together the information they had obtained, set up the relevant spreadsheets and confirmed they had found all three solutions. This resulted in some excitement as no other group had managed to do so.

Making a spur of the moment decision, the teacher asked the group to connect their laptop computer to the data projector and present their findings to the class. The students quickly decided who would operate the computer keyboard, data projector remote control (which permits scrolling and zooming independently of the computer), and laser pen. Although they had no time to prepare explanations, a communally constructed argument emerged through questioning by the teacher and other members of the class. The teacher’s comments and queries had the effect of drawing attention to salient aspects of the task and ensuring that other students saw how different technologies created different representations of the task:
David: (showing spreadsheet) Basically the very first equation we that we used we reorganised from the basic equation was eight minus $x$ cubed over negative eight, and that was just using all terms and stuff. We rearranged it –

Teacher: OK slow down. So what we are establishing here are that there are different ways of arranging the equation, which is a very important thing. Most people don’t recognise that for a start.

Nerida: We found that there are three different ways ...

Teacher: There are at least three different ways?

David: Yes. To start with our group actually used the graph to find the three intersections.

Teacher: Have you got the graphs there?

David: (shows graph) And that shows the three intersection points.

Mathematical and communications technologies were thus seamlessly integrated to share and support argumentation on behalf of the group of students, suggesting that technology became an extension of self for the members of this group.

Interviewer: What made this task exciting compared with other things you’d been doing?

Students: [overlapping] It was new! Like a prac, very hands on. You didn’t have to sit there and listen. And we got involved because we were working with friends. We were doing it ourselves, not just listening to the teacher. And seeing something visual helped our understanding.

Hayley: You feel you’ve achieved something when you did it all by yourself!

Interviewer: So you created something that was yours, very uniquely yours.

David: We’ll call it Sally’s conjecture! (referring to the teacher’s practice of naming conjectures after the students who propose them)

The students’ recollections of this experience hint at the sense of autonomy and power associated with appropriating technology into one’s personal repertoire of mathematical practice, that is, as an extension of self.

5.5. Implications for Learning and Teaching
The analysis presented above is consistent with a sociocultural perspective on learning as the product of tool-mediated social activity, in that students’ task performance was shaped by the tools available to them (graphing software, spreadsheet, graphics calculator) and by the sociocultural context of the classroom. In particular, the teacher’s actions in orchestrating students’ interaction with the task, the technology, and their peers proved to be crucial to their success in finding a solution to the cubic equation. The impact of four instances of teacher intervention could be summarised as follows.

1. The teacher directed the students to explore the problem with a spreadsheet, in addition to their first choice of a graphing program, so they would come to terms with the mechanics of the iteration process and recognise the limitations of the graphics calculator’s equation solving algorithm. This initiated the students’ transition from working with technology as master – a black box that produced an incomplete answer – to technology as servant – an efficient and time saving calculation tool.

2. He insisted that students try to find all three roots with spreadsheet methods, to highlight potential connections between numerical and graphical representations of the task and challenge students’ understanding of what counts as a “solution”. This altered students’ mode of working with technology as servant to technology as a partner in re-organising cognitive processes.

3. The teacher strategically withheld assistance and encouraged the students to consult with other groups, thus reminding students of his commitment to collaborative inquiry and reinforcing the role of technology as a partner in mediating mathematical discussion.

4. He invited the group to present their findings to the rest of the class for public and critical scrutiny. This represented a transition to embracing technology as an extension
of self, where the spreadsheet, graphing software, and data projector were integral to the production of the mathematical argument.

The teacher’s interventions listed above were wholly consistent with his previously articulated and demonstrated beliefs, concerning students making sense of mathematics (e.g. resolving the apparent contradiction between solutions returned by the Equation Editor, graphical, and spreadsheet methods), teacher encouragement of conjecturing and justification (e.g. the final presentation of the group’s findings), and the role of peer interaction and discussion in developing deep understanding (e.g. his scaffolding of intra-group and inter-group interaction).

6. Discussion

The NCTM’s *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000) discusses the role of technology as one of six overarching principles describing features of high quality mathematics education. This Principle states that “technology is essential in teaching and learning mathematics”; it enhances mathematics learning, supports effective mathematics teaching, and influences what mathematics is taught (pp. 24-26). Our research contributes to this discussion by identifying various modes of technology use by teachers and students within specific classroom learning environments.

The relationship between technology usage and teaching/learning environments is not one of simple cause and effect. The four metaphors of *master, servant, partner, and extension of self* are intended to capture some of the different ways in which technology enters into the mathematical practices of secondary school classrooms. Note that these modes of working are not necessarily tied to the level of mathematics taught, or the sophistication of the technology available. In addition, we have observed that teachers and students do not necessarily remain attached to a single mode of working with technology – as the classroom case study in the previous section demonstrates.
Whereas Doerr and Zangor (2000) in a similar study found that use of the graphics calculator as a private device led to the breakdown of small group interactions, our own observations show that graphics calculators as well as computers could facilitate communication and sharing of knowledge in both private and public settings, especially when the technology was treated as a partner or extension of self. In these cases students interacted both with and around the technology; for example, the calculator became a stimulus for, and partner in, face to face discussions when students worked together in groups. Similarly, when teachers invited students to share their work publicly via the overhead projection panel or data projector the technology was transformed from a presentation device to a discourse tool that mediated whole class discussion. Clearly, the teacher’s own pedagogical beliefs and values play an important part in shaping technology-mediated learning opportunities, whether this results in technology being used as a servant to reinforce existing teaching approaches or as a partner or extension of self to change the way teachers and students interact with other and with tasks.

These findings have theoretical and practical implications for mathematics teaching and learning. Theoretically, we have elaborated different ways in which technology may be appropriated as a cultural tool by teachers and students. From a practical perspective, our study demonstrates that graphics calculators, computers, and projection units are not passive or neutral objects, as they can re-shape interactions between teachers, students, and the technology itself. This highlights a number of challenges for teachers in integrating new technologies into their practice in addition to the obvious requirement to gain technical expertise. More attention needs to be directed to the inherent mathematical and pedagogical challenges in technology-enhanced classrooms if the goal of an investigative and collaborative learning environment is to be realised. Perhaps the most significant challenge for teachers lies in orchestrating collaborative inquiry so that control of the technology, and
the mathematical argumentation it supports, is shared with students. Our analysis highlights
important issues concerning the negotiation of power and authority with respect to the
production and validation of knowledge in classrooms where technology mediates
collaborative inquiry.

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