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Polarimetric measurement of optical torque

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Abstract

We demonstrate accurate, purely optical measurement of optical torques acting on particles composed of either isotropic or birefringent materials. Since the optical torque primarily depends on changes in the spin angular momentum of the light during scattering, the change in the angular momentum, and hence the optical torque, can be determined by measurement of the polarisation of the scattered field.

1 Introduction

Optical forces and torque are a necessary result of the conservation of momentum and angular momentum when a particle scatters light, changing the momentum or angular momentum. Therefore, if the scattered field can be measured or calculated, the optical force and torque can be found [1]. While such optical forces and torques are small, they are sufficient for useful manipulation of microscopic particles; this is the basis of the widely-used single-beam gradient trap, or *optical tweezers* [2]. Optical tweezers have been used for a wide range of applications, including quantitative measurements of forces on the order of piconewtons [2, 3].

Recently, there has been strong interest in the rotation of microscopic objects, three-dimensionally trapped or otherwise. This introduces the possibility of true three-dimensional manipulation within laser traps—the ability to controllably rotate or orient optically trapped microscopic particles is a major advance in the manipulation possible within a laser trap. This is of interest not only for simple manipulation, but also for the use of rotation as a tool to probe microscopic properties of fluids or biological specimens [4] (requiring quantitative measurement of the optical torque), and the possibility of developing optically powered and controlled micromachines [5] (where measurement of the optical torque is highly desirable for monitoring and feedback).

2 Angular momentum of light

An electromagnetic radiation field—for example, a laser beam—carries angular momentum in two distinct forms: spin angular momentum, associated with the polarisation of the field, and orbital angular momentum, associated with the spatial structure of the beam. Either, or both, can be zero.

The spin angular momentum S varies from $-\hbar$ to $\hbar$ per photon, with $S = -\hbar$ for right-handed circular polarisation, $S = \hbar$ for left circular, $S = 0$ for plane polarisation, and intermediate values for elliptical polarisation.

Since the optical torque resulting from scattering is due to the change in the spin or orbital angular momentum of the beam, such torques can, in principle, be measured. If the angular momentum carried by the incident light is known (or measured), measurement of the angular momentum of the scattered light gives the optical torque directly in absolute terms without the need for any calibration [4]. Since the spin angular momentum depends on the polarisation state of the light, which can be readily and accurately measured, the spin component of optical torque can be found simply. While it is possible in principle to measure the orbital angular momentum of light, the current difficulty of doing so renders it infeasible.

Birefringent particles will alter the polarisation state of light passing through them. Not only does this result in a torque, but the torque can also be optically measured. Significantly, we note that particles composed of optically isotropic material can possess *form birefringence* as a result of asymmetric shape. Thus, such particles generate torque in the same way that particles of birefringent material do.

3 Measurement of optical torque

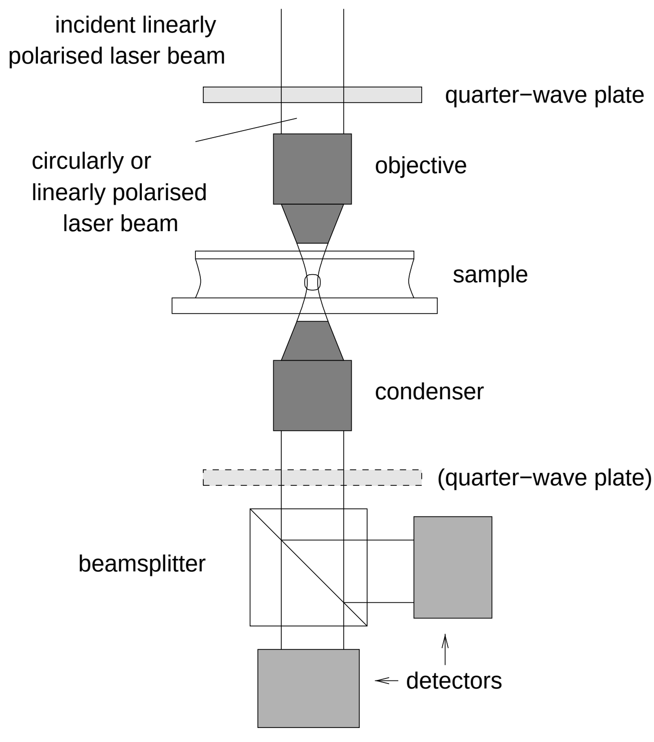

Figure 1: Apparatus for measurement of optical torque

A practical apparatus for the measurement of optical torque is shown in figure 1. Since optical torque is generally applied to microscopic particles using optical tweezers, this apparatus is a modified optical tweezers setup. Firstly, a $\lambda/4$ waveplate is used to control the polarisation of the trapping beam. Secondly, a condenser is used to collect the forward-scattered light. Lastly, a polarising beamsplitter is used to separate the polarisation components of the collected light. A $\lambda/4$ waveplate can be used to obtain the polarisation components in a circular basis.

For the case of a plane polarized incident beam, the incident angular momentum flux is zero. We use a quarter-wave plate and a polarizing beamsplitter to separate the two circularly polarized components, and measure the powers P_{left} and P_{right} of the left- and right-circularly polarized components respectively. If there is no particle present, $P_{\text{left}} = P_{\text{right}}$. Since the outgoing spin angular momentum flux is $(P_{\text{left}} - P_{\text{right}})/\omega$, where ω is the optical frequency, the spin torque acting on a particle is

$$\tau_{\text{LP}} = (P_{\text{right}} - P_{\text{left}})/\omega. \tag{1}$$

In practice, we measure the difference between the signals from the two photodetectors, so we have

$$\tau_{\text{LP}} = \Delta P_{RL}/\omega. \tag{2}$$

When we use a left-circularly polarized incident beam, we could most accurately measure the polarization in a linear basis. If the particle is rotating, this gives a sinusoidally varying signal from the photodetectors [4]. For a stationary particle, a rotatable half-wave plate can be used to obtain the maximum and minimum signals in the photodetectors—equal to the maxima and minima of the sinusoidal signal, which we will denote as P_{high} and P_{low}. Since the particle only changes the polarization by a small amount, the handedness will not reverse. Therefore, the torque is given by

$$\tau_{\text{CP}} = \left[P_{\text{high}} + P_{\text{low}} - 2(P_{\text{high}}P_{\text{low}})^{1/2}\right]/\omega \tag{3}$$

or

$$\tau_{\text{CP}} = \left[P - (P^2 - \Delta P_{HL}^2)^{1/2}\right]/\omega \tag{4}$$

where $P = P_{\text{high}} + P_{\text{low}}$ is the total power and $\Delta P_{HL} = P_{\text{high}} - P_{\text{low}}$ is the difference between the signals.

Since the laser-trapped particle typically only has a small refractive index contrast relative to the surrounding medium, almost all of the light is scattered in the forward direction and can be collected.

3.1 Calculation of optical torque

The optical torque can be readily calculated using the T-matrix method; apart from being computationally well-suited for optical tweezers calculations since the T-matrix for a given particle needs to be calculated only once [1], the mathematical formulation of the T-matrix method is physically enlightening since the vector spherical wavefunctions are simultaneous eigenfunctions of the total angular momentum operator, with eigenvalues $[n(n+1)]^{1/2}$, and the z-component of angular momentum operator, with eigenvalues m.

The normalised torque, or torque efficieny (that is, the torque per photon in units of the photon spin $\hbar$) about the z-axis acting on the trapped particle is

$$\tau_z = \sum_{n=1}^{\infty}\sum_{m=-n}^{n} m(|a_{nm}|^2 + |b_{nm}|^2 - |p_{nm}|^2 - |q_{nm}|^2)/P \tag{5}$$

where a_{nm}, b_{nm}, p_{nm} and q_{nm} are the multipole expansion coefficients of the incident and scattered fields $P = \sum_{n=1}^{\infty}\sum_{m=-n}^{n} |a_{nm}|^2 + |b_{nm}|^2$ is proportional to the incident power (omitting a unit conversion factor which will depend on whether SI, Gaussian, or other units are used). This torque includes contributions from both spin and orbital components; the spin torque about the z-axis is given by [6]

$$\begin{aligned}\sigma_z \;=\; & \frac{1}{P}\sum_{n=1}^{\infty}\sum_{m=-n}^{n}\frac{m}{n(n+1)}(|a_{nm}|^2 + |b_{nm}|^2 - |p_{nm}|^2 - |q_{nm}|^2) \\ & -\frac{2}{n+1}\left[\frac{n(n+2)(n-m+1)(n+m+1)}{(2n+1)(2n+3)}\right]^{\frac{1}{2}} \\ & \times \text{Im}(a_{nm}b^{\star}_{n,m+1} + b_{nm}a^{\star}_{n,m+1} - p_{nm}q^{\star}_{n+1,m} - q_{nm}p^{\star}_{n+1,m}).\end{aligned} \tag{6}$$

The remainder of the torque is the orbital contribution. The axial trapping efficiency Q (that is, the z component of the force per photon, in units of the photon momentum $\hbar k$) is [6]

$$Q \;=\; \frac{2}{P}\sum_{n=1}^{\infty}\sum_{m=-n}^{n}\frac{m}{n(n+1)}\text{Re}(a^{\star}_{nm}b_{nm} - p^{\star}_{nm}q_{nm})$$

$$
-\frac{1}{n+1}\left[\frac{n(n+2)(n-m+1)(n+m+1)}{(2n+1)(2n+3)}\right]^{\frac{1}{2}}
\times \mathrm{Re}(a_{nm}a^\star_{n,m+1} + b_{nm}b^\star_{n,m+1} - p_{nm}p^\star_{n+1,m} - q_{nm}q^\star_{n+1,m}). \quad (7)
$$

This allows us to determine the maximum contribution to the torque by (the essentially unmeasurable) orbital angular momentum. Depending on the size and shape of the particle, the orbital component typically varies from $\approx$ 0–10%. However, as the light is collected by the condensor (which is an axisymmetric non-absorbing scatterer), some of the orbital angular momentum is converted to (measurable) spin angular momentum, with the total torque remaining constant.

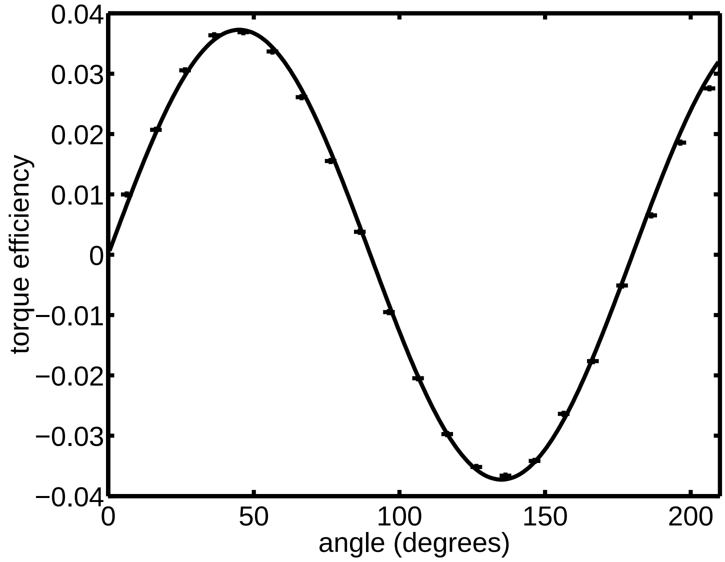

Figure 2: The optical torque per photon, in units of $\hbar$, acting on glass cylinder of radius $0.67 \pm 0.03\,\mu$m, is shown versus the angle between the axis of the cylinder and the plane of polarisation. Experimentally measured values are shown, compared to the theoretical prediction. The angle is measured from the cylinder to the plane of polarisation, so a positive torque acts to reduce the angle—the torque always acts to align the cylinder with the plane of polarisation.

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