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An elastic mixed-modes I and II fracture criterion using an artificial neural network database

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Abstract: Many thin structural elements such as various parts of fuselage, tubular supports in oil/gas off-shore platforms, etc may fail under the mixed-modes I and II fracture. Although there are a number of criteria for the mixed-modes I and II fracture, none of them are universally accepted and applied to practical problems. This is mainly due to lack of knowledge about the fracture toughness under various mode-mixity and rather complexity of the pertinent relationships. This paper proposes a new and relatively simple criterion for the mixed-modes I and II fracture that uses an artificial neural network database to obtain the pertinent fracture toughness. The application of the criterion is demonstrated by applying it to an example.

Keywords: artificial neural network, mixed-modes I-II fracture.

1 Introduction

Normally industrial components contain crack-like defects. The cracks may be created during manufacturing, assembling, and/or operational stages. It is uneconomic to detect and repair all of the cracks existing in an industrial component. Thus there exists a need to assess the integrity of defected components and identify and take remedial actions about those cracks that if left unrepaired would cause failure of the component.

It is well established that fracture of components containing crack-like defects can occur under mode I (tensile) loading alone, mode II (in-plane shear) loading alone, mode III (out-of-plane shear) loading alone or a combination of these modes. For cracks in complex industrial components, the loads may not be applied in one of these modes alone. Instead a defected component may be subjected to a combination of the tension and in-plane shear, i.e., mixed-modes I and II. This is usually the case for thin components such as various sections of a fuselage, thin tubes, etc. This paper concentrates on linear elastic fracture mechanics (LEFM) when the crack-tip plasticity is relatively small and negligible. To date, the majority of studies have been focused on mode I alone. However, there exist several fracture criteria for mixed-modes I and II, see for example [1] to [15], which employ: stress intensity factor (K), or strain energy density release rate ($G = \partial \overline{U} / \partial a$ where $\overline{U}$ is strain energy per unit thickness and a is the crack length), or crack-tip opening displacement (d), or Intensity of strain energy density ($S = U / r$ where U is the strain energy density and r is the radial distance from the crack-tip) as the crack-tip driving force parameter but none of these criteria have universally been accepted as yet. This appears to be mainly due to lack of knowledge of the pertinent measure of the fracture toughness under mixed-modes I and II. Note that fracture toughness is assumed to be a material property. Strictly, however, fracture toughness depends on the thickness of the component (which is a geometrical parameter) as well as mode-mixity (which is a loading ratio parameter, see (6) below), i.e., the portion of mode I loading relative to mode II loading that is applied to the component.

One may argue that among the various mixed-modes I-II fracture criteria, the one based on S and originally proposed by Sih [3] is more pragmatic. This is because S being independent from r represents the intensity of the strain energy density near the crack-tip, which is a scalar quantity whereas K represents stresses near the crack-tip, which are tensor quantities and d represents displacements near the crack-tip, which are vector quantities. Although $G = \partial \overline{U} / \partial a$ also represents scalar quantities but its computation, in general, is more difficult than S as it involves crack (virtual or actual) extension and/or numerical differentiation of $\overline{U}$. Note also that the above-mentioned criteria require knowing and/or predicting the direction of fracture, which make them somewhat convoluted. Therefore, this paper has two objectives. Firstly, it modifies Sih's criterion that is based on S to make it more pragmatic. To this end, the modified criterion concentrates on predicting the on-set of fracture and ignores the direction of crack growth during the fracture process. This is consistent with an integrity analysis where the primary objective of the analyst is to predict the fracture load and predicting the direction of fracture is not as important as predicting the fracture load. Secondly, to address the lack of current knowledge about the fracture toughness for various materials under mixed-modes I-II loading, the authors are setting up a database of mixed-mode fracture toughness based on S_C (i.e., the critical value of S at the on-set of fracture) that uses artificial neural network (ANN) concept. As mentioned above fracture toughness depends on mode-mixity (see (6) below). ANN database is particularly suited to interpret between the data values in the database to obtain the sought fracture toughness. The main merit of ANN is that it does not assume a pre-defined and explicit relationship between inputs and outputs to the network. The developed ANN database uses a spline wavelet transformation to alleviate local minimization problem [16]. In the following, first the proposed mixed-modes I-II fracture that is based on modifying Sih's criterion is defined (note that for the sake of brevity the Sih's criterion is not described here in full and the reader is referred to [3]). Then the ANN database for determining the fracture toughness under mixed-modes I-II loading is described. Finally, the application of proposed criterion and database is demonstrated by applying it to an example.

2 Proposed fracture criterion

Figure 1 shows a crack that is loaded in a mixed-modes I-II under LEFM, i.e., the crack-tip plasticity is ignored. Consider a small damaged-zone of arbitrary shape at the crack-tip represented by its characteristic dimension d where $d/a << 1$. As mentioned before, Sih [3] argues that the crack-tip driving force is S.

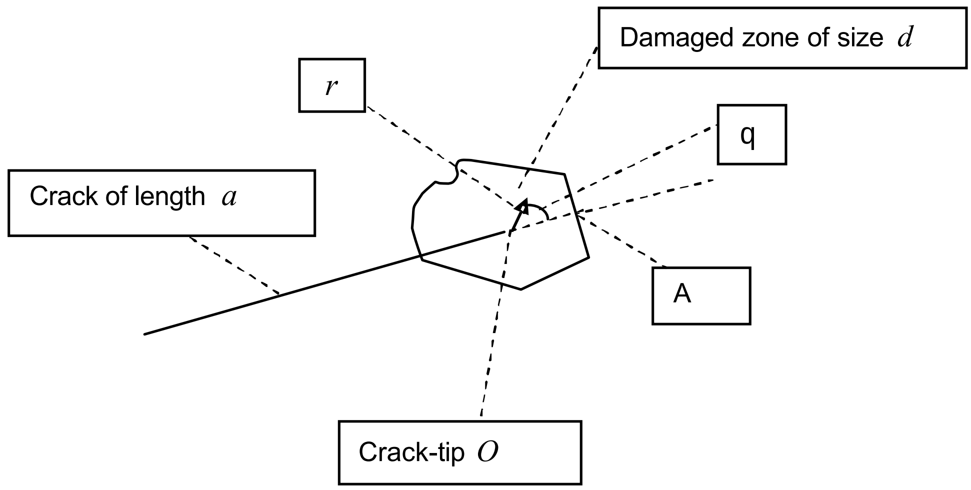

Figure 1 – A mixed-modes I-II crack and its damaged zone

Near crack-tip, we know that [17]:

$$s_{ij} \propto \frac{1}{\sqrt{r}} \quad (1)$$

$$e_{ij} \propto \frac{1}{\sqrt{r}} \quad (2)$$

Where s_{ij} are stress components near the crack-tip and e_{ij} are strain components near the crack-tip. Therefore:

$$U = \frac{1}{2}\sigma_{ij}\varepsilon_{ij} \propto \frac{1}{r} \quad (3)$$

Or

$$U = \frac{1}{r}S \quad (4)$$

where S is a function of the angular coordinate θ (see Figure 1) but it is independent from the radial coordinate r and damaged-zone size d. Note that having computed σ_{ij} and ε_{ij} (analytically or by a numerical method such as FEM) then (3) and (4) may be used to determine U and S at any point of interest within the damaged-zone. Sih [3] argues that the fracture initiates at an angle that makes S minimum and it uses this condition to determine this particular minimum value, $\theta_{\min}$, and therefore the corresponding minimum of S, i.e., $S_{\min}$. According to Sih's model fracture occurs when $S_{\min} = S_c$ where as mentioned above, S_c is the critical intensity of strain energy density at fracture representing the fracture toughness. But this procedure makes the involved equations somewhat complex and less useful for practical integrity assessment. To simplify the fracture assessment, this paper postulates that every point within the damaged-zone contributes to fracture and each point within the damaged-zone should reach its respective critical loading (damaged) condition when fracture occurs. Because all the points within the damaged-zone should reach their respective critical damaged condition for fracture occurring, then it does not matter which point within the damaged-zone one takes as the reference point for determining S and S_C. Therefore, in determining S, the present model considers a point along the crack-plane within the damaged-zone for which $\theta = 0$. Note that because S is independent from r, the exact radial location of this reference point is immaterial as far as it lies along the crack-plane and within the damaged-zone, i.e., within the proximity of the crack-tip where d / a << 1. Therefore, to determine S one uses $\theta = 0$, and (2). To determine S_c, one uses:

$\theta = 0$, (3) and (4) for each applied mode-mixity β where:

$$\beta = \arctan\frac{U_I}{U_{II}} = \arctan\frac{S_I}{S_{II}} \quad (5)$$

where U_I and S_I are the mode I strain energy density and mode I intensity of strain energy density at a point along the crack-plane within the damaged-zone respectively and U_{II} and S_{II} are the mode II strain energy density and mode II intensity of strain energy density at the same point respectively. The model is then postulates that fracture occurs if:

$$S \geq S_c \quad (6)$$

It should be noted that β varies from 0 (pure mode II) to 90° (pure mode I) and fracture toughness (i.e., S_C) depends on β.

3 Database of toughness using artificial neural network

In applying (6) for assessing integrity of industrial components, one needs to determine S and S_C as described in Section 2. S (and U) can easily be computed using FEM. To obtain the material property S_C, a database for various materials is being set up at UNSW. The database uses the experimental data available in the literature to obtain S_C for any applied mode-mixity (β) and the component thickness (h). The database uses ANN concept similar to that developed by Zarrabi [16]. One of the strengths of ANN is the ability to model scattered data without assuming a mathematical distribution. When data are assumed to fit a mathematical distribution, one is adding information that helps the analyst to model the available data. This, however, may be misleading if one has chosen the wrong distribution. ANN consists of a training stage and a simulation stage. ANN models the data that are presented to it during the training stage without assuming a particular distribution. After the network is trained it is used to simulate or predict S_C values using the inputs to ANN. The inputs to ANN are: b and h. The output is S_C. All current data in the database are at room-temperature. ANN has a parallel processing architecture that is composed of many non-linear computational elements (neurons). It is naturally suited to tackle complex and non-linear problems. The elements or neurons in ANN are arranged in patterns reminiscent of biological brain cells. The present investigation uses a back propagation, feed forward ANN with input, hidden, and output layers [16]. In operation, ANN learns a predefined set of input-output example pairs by using a two-phase propagate-adapt cycle. As mentioned before, the development of ANN consists of two stages, viz., a learning and a prediction stages. During the learning stage, first, the inputs are supplied to the input layer where they are acted upon by input transfer functions, weights and biases at each neuron; then they propagate through hidden and output layers. At hidden and output layers the variables are acted upon by the corresponding transform functions, weights, and biases. Biases are normally set to unity for all three layers. At the output layer, the variables are combined to produce an output. This output is then compared with the desired output and an error signal (ER) is computed. ER is then minimized with respects to weights and the process is iterated until ER is less than a desired value. The final weights are used in the predication stage to compute the desired output variable. Before the training of the network both input and output variables (V) are normalized within the range 0–1 using:

$$V_n = \frac{2V - V_{\max} - V_{\min}}{V_{\max} - V_{\min}} \qquad (7)$$

where V_n is the normalized value of variable V, $V_{\max}$ is the maximum value of the variable and $V_{\min}$ is the minimum value of the variable. So far, the ANN database is set for aluminum alloys and PMMA as the bulk of the available pertinent experimental data in the literature are related to these materials ([18] – [25]). Currently the database contains 120 data sets for aluminum alloys and 50 for PMMA. The database is being extended as more data becomes available from literature and this should improve the accuracy of prediction of S_C by ANN in time.

4 Application

Consider a long and thin tube of mean radius $R = 508\ mm$ and thickness $h = 50.8\ mm$ containing a through-thickness crack of length $2a = 508\ mm$ oriented at an angle $j = 45^\circ$ to the longitudinal axis of the tube and subjected to uniform internal pressure $p = 0.689\ MPa$, see Figure 2.

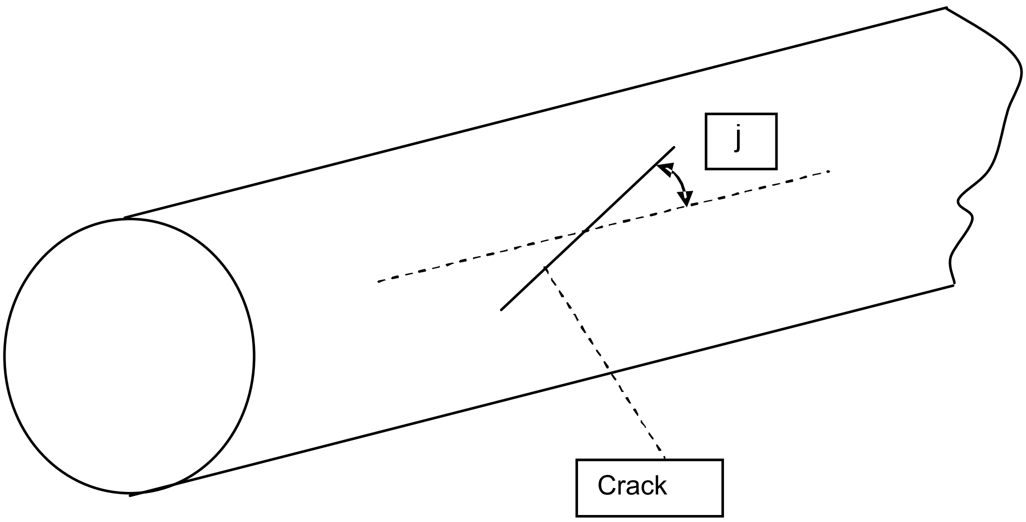

Figure 2 – A thin tube containing a through-thickness crack

The tube is made of aluminium alloy 7075-T6 with modulus of elasticity $E = 73100\ MPa$ and Poisson's ratio $\nu = 0.3$. NASTRAN [26] is used to perform the finite element analysis of the tube. The tube is considered as a thin shell ($\frac{R}{h} = 10$) and therefore 19,556 shell elements (QUAD4) with element-size at and near the crack-tip being $b = 0.5\,mm$. This resulted in $\frac{b}{2a} = 0.000984$ that though is sufficient to generate fine mesh around the crack-tip for obtaining accurate results; see Figure 3. Note that the crack-tips are sufficiently away from the tube ends and therefore the boundary conditions will have negligible effects on the strain energy at the crack-tips.

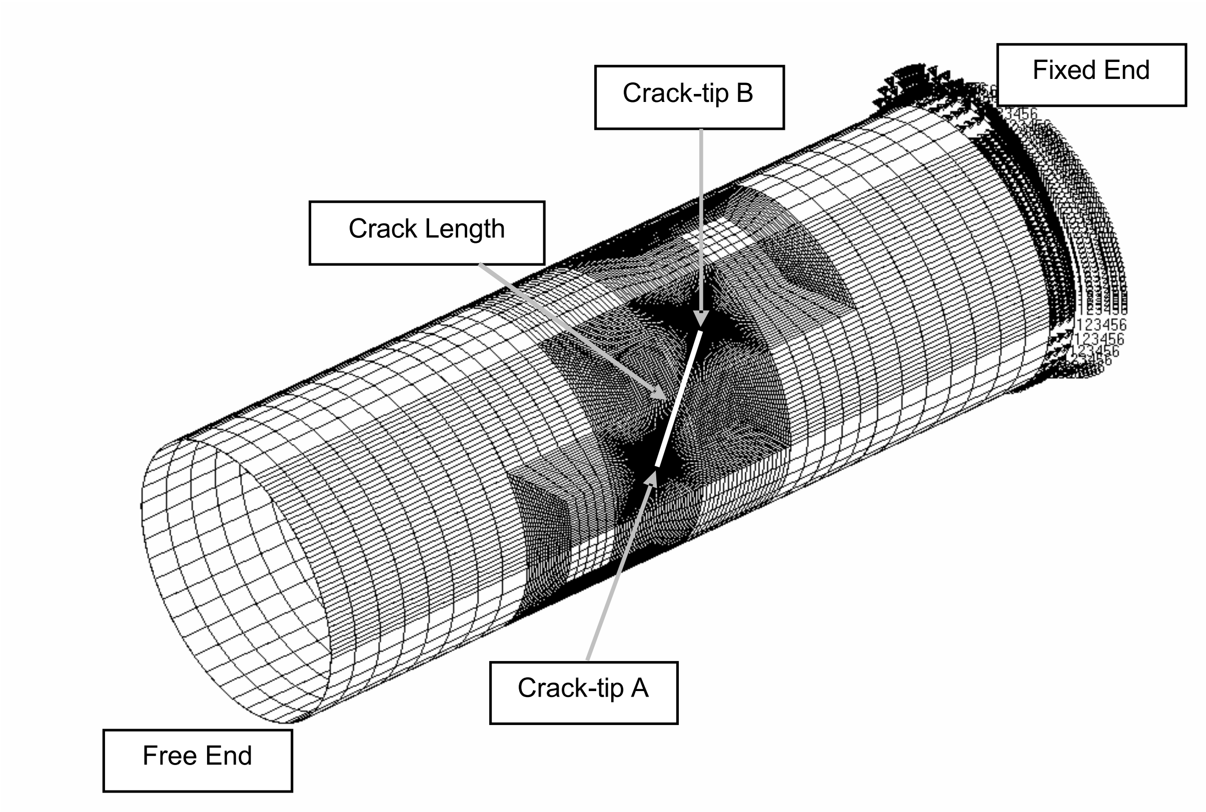

Figure 3 – Finite element model of the cracked thin tube

The computed intensities of the strain energy density are plotted in Figure 4, which give the average intensities as $S_A = 0.0782\,mJ/mm^2$ and $S_B = 0.0502\,mJ/mm^2$ for the crack-tips A and B respectively. This means that the fracture commence from the crack-tip A first. Using the ANN, it is found that: $S_C = 0.0419\,mJ/mm^2$. Because $S_C < S_A \,\&\, S_B$ then the pressure loading of $p = 0.689\ MPa$ will cause fracture. Noting that LEFM prevails, then one can calculate the critical value of pressure that will cause fracture from $p_C = \frac{S_C}{S_A} p = 0.369\ MPa$.

5 Conclusions

This paper proposes a pragmatic model for predicting the fracture load under mixed-modes I and II when the linear elastic fracture mechanics prevail. To obtain the pertinent fracture toughness that depends on mode-mixity, the paper suggests an ANN database approach. Finally the application of proposed models has been demonstrated by applying them to a cracked tube.

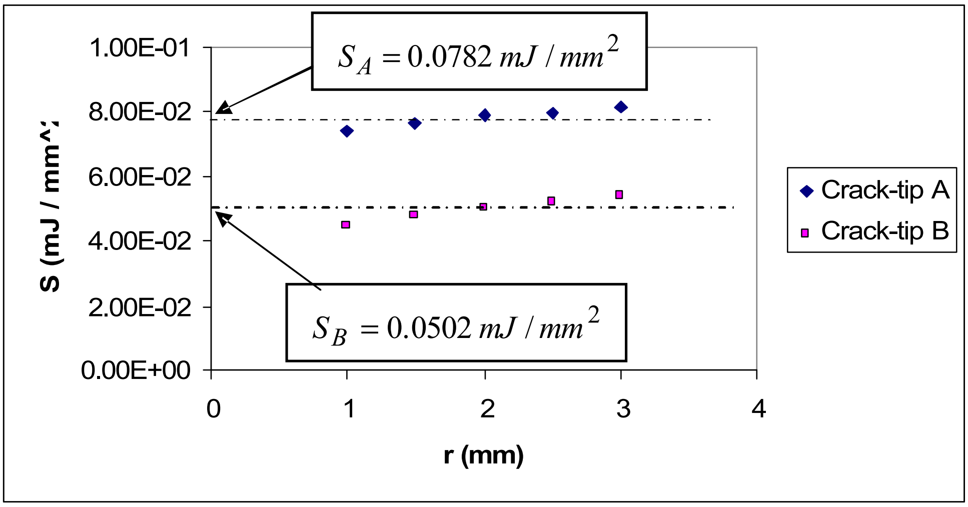

Figure 4 – Intensity of strain energy density (S) versus radial distance

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