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# Comment on “Geometric absorption of electromagnetic angular momentum”, C. Konz, G. Benford

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## Abstract

The core theorem on which the above paper is centred—that a perfectly conducting body of revolution absorbs no angular momentum from an axisymmetric electromagnetic wave field—is in fact a special case of a more general result in electromagnetic scattering theory. In addition, the scaling of the efficiency of transfer of angular momentum to an object with the wavelength and object size merits further discussion. Finally, some comments are made on the choice of terminology and the erroneous statement that a circularly polarized plane wave does not carry angular momentum.

*Key words:* angular momentum, electromagnetic scattering

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## 1 Symmetry and the transfer of angular momentum

The authors of the above paper [1] state a theorem:

... a perfectly conducting body of revolution with a piecewise smooth surface around the axis of symmetry (for instance a cone, a disk, a cylinder, or

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a sphere) absorbs *no* angular momentum  $\vec{L}$  from an axisymmetric electromagnetic wave field.

This is in fact a special case of a more general result in electromagnetic scattering theory, namely that a scatterer that is rotationally symmetric about the  $z$  axis does not couple different azimuthal modes [2], that is, it does not couple modes of differing angular momentum about the  $z$  axis. This being the case, no angular momentum about the  $z$  axis will be transferred to a nonabsorbing axisymmetric scatterer, *regardless of the structure of the incident field*.

This result is weaker than the theorem stated by the Konz and Benford [1], since only the  $z$  component of angular momentum is considered, but is more general, applying to any nonabsorbing scatterer. To generalize Konz's result to dielectric scatterers, it is sufficient to consider symmetries of the incident field and the scatterer which lead to zero torque on the scatterer about other axes. For example,

- mirror symmetry of the beam and scatterer about the  $xy$  plane
- mirror symmetry of the beam about any plane containing the  $z$  axis
- rotational symmetry of the beam about the  $z$  axis
- rotational point group symmetry of the beam about the  $z$  axis

all result in zero torque about the origin. Clearly, rotational symmetry of the incident field is not required. It is difficult (and beyond the scope of this comment) to exactly and completely state the conditions under which the total torque about the origin will be zero. However, it should be noted that only very special types of fields—radially and azimuthally polarized beams—are in fact axisymmetric, if one considers the symmetry of the electric and magnetic fields. Broader definitions of symmetry, considering the symmetry of the Poynting vector, or the energy density, can be used, in which case, a wider class of fields can be considered to be axisymmetric. Fields that fail to satisfy any of these criteria can still produce zero torque—for example, higher order Gaussian beams.

## 2 Scaling

As Konz and Benford state [1], the angular momentum of an electromagnetic field is typically on the order of  $\hbar$  per photon. In the case of a circularly polarized beam,  $L = \pm\hbar$  per photon, so  $L = P/\omega$  where  $P$  is the power, and  $\omega$  is the angular frequency. This clearly shows why, *cetera paribus*, lower frequencies are more efficient for exerting torque on scatterers.

However, another important consideration is the fraction of the beam inter-

cepted by the scatterer. The minimum width of a beam of given frequency is on the order of the wavelength, so we can consider the minimum cross-sectional area of the beam to be on the order of  $\lambda^2$ . Thus, the irradiance  $I$  is

$$I \approx P/\lambda^2 \tag{1}$$

and the power actually incident on the scatterer smaller than the beam is

$$P_{\text{inc}} \approx Pd^2/\lambda^2 \tag{2}$$

where  $d$  is the scatterer size. The angular momentum incident on the scatterer will be on the order of

$$L_{\text{inc}} \approx Pd^2/2\pi c\lambda \tag{3}$$

and it can be seen that greater efficiency results from the use of *shorter* wavelengths.

If the scatterer is larger than the beam, then the scatterer can interact with the entire incident beam, and the angular momentum incident on the scatterer will scale as

$$L_{\text{inc}} \approx P\lambda/2\pi c, \tag{4}$$

with greater efficiency at *longer* wavelengths. Accordingly, one would expect maximally efficient angular momentum transfer when the beam is focussed to the maximum possible extent, with the wavelength chosen so that the beam width is on the order of the particle size. This simple argument is also supported by rigorous electromagnetic calculations [3].

So, while electromagnetic angular momentum generally scales with  $1/\omega$ , the transfer of this angular momentum to a scatterer does not follow such simple rules. Rather than lasers being unable to usefully spin objects, as claimed by Konz and Benford [1] optical frequencies are in fact optimal for particles of sizes comparable to optical wavelengths; particles typically rotated within laser traps are of this size [3]. It can also be noted that since Konz and Benford [1] used a beam several wavelengths wide, they could have obtained greater efficiency by using a longer wavelength combined with a more strongly focussed beam, unless the increase in wavelength is accompanied by a sufficient change in the electromagnetic properties of the material. Of course, the added complication of a strongly focussed beam might well make this impractical.

### 3 Angular momentum of a circularly polarized plane wave

Konz and Bedford state that a “circularly polarized incoming wave field which is infinitely extended and homogeneous does not carry angular momentum, since the Poynting flux is parallel to the wave vector.” While a naive calculation of the angular momentum density or flux starting from  $\vec{r} \times \vec{S}$ , where  $\vec{S}$  is the Poynting vector gives a result of zero [4], consideration of the interaction between the field and an absorbing or anisotropic medium shows that torque is exerted on the medium [5–9]. This clearly demonstrates that the wave has non-zero angular momentum.

While this issue is peripheral to the paper being commented on, recognition of the fact that a circularly polarized plane wave does carry angular momentum make the authors’ conclusion that the wedge “absorbs the negative angular momentum  $L_x$  of the reflected wave field” [1] unnecessary.

### 4 A comment on terminology

The terminology “geometric absorption” of angular momentum is perhaps an unfortunate choice, as fundamentally similar processes will act to transfer angular momentum to a scatterer regardless of the angular momentum content of the incident field. The authors introduce the awkward concept of absorption of negative angular momentum from the *scattered* field to account for the angular momentum transfer to the scatterer in the case of an incident field with zero angular momentum; this, however, introduces the surprising concept of absorbing some property of an outward-propagating field. It would seem to be better to use a term such as “geometric transfer” of angular momentum, so as to eliminate any conceptual difficulty associated with the “absorption” of a quantity equal to zero.

In addition, to describe negative “absorption” of angular momentum as “radiation” of angular momentum invites confusion with work involving pure radiation of angular momentum (with no incident field present) [10].

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